# CSIR-UGC-NET/JRF | GATE

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# Assignment

# Classical Mechanics

## PHYSICAL SCIENCES

CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED

**South Delhi Centre:**
28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## CSIR-UGC-NET/JRF | GATE PHYSICS

## Assignment-1: Classical Mechanics (General)

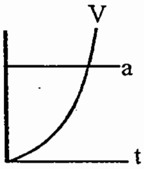
1. A ray of light is reflected from a mirror. If the mirror is slowly rotated with an angular velocity w, then the angle between the incident and the reflected rays changes at the rate
(a) –w (b) w (c) 2w (d) w/2

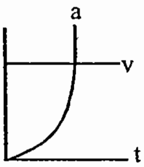
2. Two similar cars of mass m each, moving with a uniform speed v, make a head-on collision and come to rest. The damage suffered by the cars is directly proportional to
(a) v (b) $v^2$ (c) $m^2v^2$ (d) mv

3. A person sitting on a chair cannot get up without bending forward because,
(a) Human body is not rigid. (b) Newtons third law is operative.
(c) Newtons second law is operative (d) Gravitational attraction is strong.

4. The work done in moving an object along a vector $2\hat{i}-3\hat{j}+4\hat{k}$, when a force $\vec{F}=\hat{i}-2\hat{j}-2\hat{k}$ is applied is:
(a) Zero (b) 29 (c) $\sqrt{38}$ (d) 3

5. An object is dropped from certain height. Among the following which graph correctly explain change in magnitude of acceleration and velocity before hitting the ground

(a)


(b)


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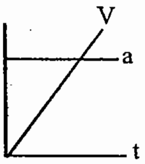

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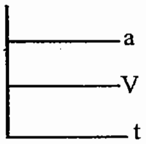
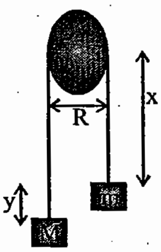


6. A plane flying at certain altitude completes a complete turn around the earth at equator in 24 hours. What would be time taken to complete revolution around 60° N of earth flying at same speed and altitude
(a) 24 hrs (b) 18 hrs (c) 12 hrs (d) 6 hrs.

7. A plane is flying at height 10 km with speed of 1000 km/hr. What would be angular velocity of plane when it is observed by person standing on earth
(a) 1 rad/sec (b) 36 rad/sec (c) $\pi$ rad/sec (d) $\pi/2$ rad/sec

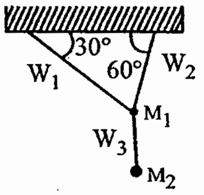
8. Two particles approach each other with different velocities. After collision, one of them is found to have momentum $\vec{p}$ in their centre of mass frame. In the same reference frame, the other particle must have momentum
(a) Zero (b) $\frac{-\vec{p}}{2}$ (c) $-\vec{p}$ (d) $-2\vec{p}$

9. A heavy uniform rope of length L and mass per unit length $\mu$ goes over a frictionless pulley of diameters R, and has two masses M and m attached to its two ends as shown. In terms of the distance x, the equilibrium position is given by



(a) $\frac{1}{2}\left(L-\frac{\mu L}{m+M}+\frac{\pi R}{2}\right)$ (b) $\frac{1}{2}\left(L+\frac{m+M}{\mu}\right)$ (c) $L-\frac{\pi R}{2}$ (d) $\frac{1}{2}\left(L-\frac{m-M}{\mu}-\frac{\pi R}{2}\right)$

10. Two masses $M_1$ and $M_2$ $(M_1 < M_2)$ are suspended from a perfectly rigid horizontal support by a system of three taut massless wires $W_1, W_2$ and $W_3$, as shown in the figure. All the three wires have identical cross-section and elastic properties and are know to be very strong.



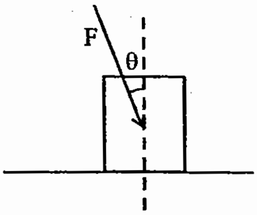
If the mass $M_2$ is increased gradually, but without limit, we should expect the wires to break in the following order :

(a) First $W_2$, then $W_1$ (b) First $W_1$, then $W_2$ (c) First $W_2$, then $W_3$ (d) First $W_3$

11. A high-velocity missile, travelling in a horizonal line with a kinetic energy of 3.0 Giga-Joules (GJ), explodes in flight and breaks into two pieces A and B of equal mass. One of these Pieces (A) flies of in a straight line perpendicular to the original direction in which the missile was moving and its kinetic energy is found to be 2.0 GJ. If gravity can be neglected for such high-velocity projectiles, it follows that the other piece (B) flew off in a direction at an angle with the original direction of

(a) 30° (b) 33°, 24' (c) 45° (d) 60°

12. A block of negligible mass rests on a horizontal surface with coefficient of friction $\mu$. A force F is applied downwards, at an angle $\theta$ to one vertical (see figure).



The maximum value of the angle at which the force cannot make the block slip is:

(a) $\theta = \tan^{-1}\mu$ (b) $\theta = \cot^{-1}\mu$ (c) $\theta = \sin^{-1}\mu$ (d) $\theta = \cos^{-1}\mu$

13. In an experiment it is found that the combination $\frac{F}{\rho v^2}$ increases with a spatial dimension R as $R^{\alpha}$. Here F is a force, $\rho$ is a mass density and v is a velocity. The value of $\alpha$ is

(a) 4 (b) 1 (c) 3 (d) 2

14. A rope of negligible mass goes around a frictionless pulley. A monkey is on one end of the rope, while at the other end hangs a mass that is of the same weight as the monkey. When the monkey starts to climb up the rope, then

(a) Both monkey and the mass move up, but the mass moves up at a rate exacty half that of the monkey
(b) The monkey remains at the same height, the mass moves up
(c) The monkey moves down, the mass moves up
(d) The monkey and the mass move up by the same distance.

15. A girl measures the period of simple pendulum inside a stationary lift and finds it to be T seconds. If the lift accelerates upward with an acceleration g/4, then the period of the pendulum will be

(a) T (b) T/4 (c) $2T/\sqrt{5}$ (d) $2T\sqrt{5}$

16. A particle of mass m is thrown upward with velocity v and there is retarding air resistance proportional to the square of the velocity with proportionality constant k. If the particle attains a maximum height after time t, and *g* is the gravitational acceleration, what is the velocity v?

(a) $\sqrt{\frac{k}{g}}\tan(\sqrt{\frac{g}{k}}t)$ (b) $\sqrt{gk}\tan(\sqrt{\frac{g}{k}}t)$ (c) $\sqrt{\frac{g}{k}}\tan(\sqrt{gkt})$ (d) $\sqrt{gk}\tan(\sqrt{gkt})$

17. The free fall time of a test mass on an object of mass *M* from a height 2*R* to *R* is

(a) $(\pi/2+1)\sqrt{\frac{R^3}{GM}}$ (b) $\sqrt{\frac{R^3}{GM}}$ (c) $(\pi/2)\sqrt{\frac{R^3}{GM}}$ (d) $\pi\sqrt{\frac{2R^3}{GM}}$

18. A small mass *M* hangs from a thin string and can swing like a pendulum. It is attached above the window of a car. When the car is at rest, the string hangs vertically. The angle made by the string with the vertical when the car has a constant acceleration $a = 1.2\ \text{m/s}^2$ is approximately.

(a) 1° (b) 7° (c) 15° (d) 90°

19. The period of a simple pendulum inside a stationary lift is *T*. If the lift accelerates downwards with an acceleration *g*/4, the period of the pendulum will be

(a) *T* (b) *T*/4 (c) $2T/\sqrt{3}$ (d) $2T/\sqrt{5}$

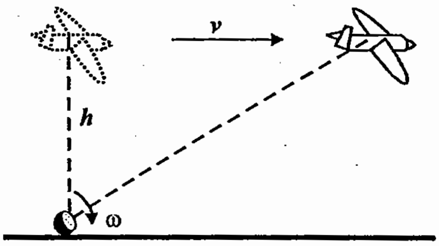
20. Spherical particles of a given material of density $p$ are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, i.e, $F_d = 6\pi\eta R v$, where $\eta$ is the viscosity of the medium, R the radius of a particle and v its instantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass *m* to reach half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is

(a) 8 (b) 1/8 (c) 4 (d) 1/4

21. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If {,} represents the Poisson bracket, the value of $\{A,\{B,C\}\} - \{\{A,B\},C\}$ is given by

(a) 0 (b) $\{B,\{C,A\}\}$ (c) $\{A,\{C,B\}\}$ (d) $\{\{C,A\},B\}$

22. The directed beam from a small but powerful searchlight placed on the ground tracks a small plane flying horizontally at a fixed height 'h' above the ground with a uniform velocity v, as shown in the figure below.



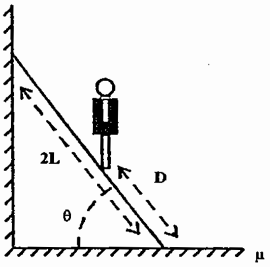
If the searchlight starts rotating with an instantaneous angular velocity $\omega_0$ at time $t = 0$ when the plane was directly overhead, then at a later time 't' its instantaneous angular velocity $\omega(t)$ is given by

(a) $\omega_0 \exp(-\omega_0 t)$ (b) $\frac{\omega_0}{1+\tan\omega_0 t}$ (c) $\frac{\omega_0}{1+\omega_0^2 t^2}$ (d) $\frac{\omega_0}{1-\omega_0 t+\frac{1}{2}\omega_0^2 t^2}$

23. The conservation principles for energy, linear momentum and angular momentum arise from the necessity that

(a) The laws of physics should not involve infinite quantities.
(b) Internal forces on a body should cancel out, by Newton's (third) law of action and reaction
(c) Physical measurements should be independent of the origin and orientation of the coordinate system.
(d) The laws of physics should be independent of the state of rest or motion of the observer.

24. A uniform ladder of length 2L and mass 'm' leans against a wall in a vertical plane at an angle $\theta$ to the horizontal. The floor is rough, having a coefficient of static friction μ. A person of mass M stands on the ladder at a distance D from its base (see figure). If the wall is frictionless, the maximum distance ($D_{max}$) up the ladder that the person can reach before the ladder slips is



(a) $2\mu L\left(1+\frac{m}{M}\right)\tan\theta$ (b) $\left\{2\mu\left(1+\frac{m}{M}\right)\tan\theta-\frac{m}{M}\right\}L$

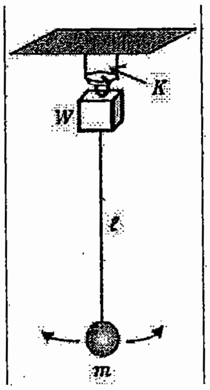
(c) $\mu L\tan\theta$ (d) $2\mu L\frac{m}{M}\tan\theta$

25. A body of mass m falls from rest at a height 'h' under gravity (acceleration due to gravity g) through a dense medium which provides a resistive force $F = -kv^2$, where k is a constant and v is the speed. It will hit the ground with a kinetic energy

(a) $\frac{m^2 g}{2k}\exp\left(-\frac{2kh}{m}\right)$ (b) $\frac{m^2 g}{2k}\tanh\frac{2kh}{m}$

(c) $\frac{m^2 g}{2k}\left\{1+\exp\left(-\frac{2kh}{m}\right)\right\}$ (d) $\frac{m^2 g}{2k}\left\{1-\exp\left(-\frac{2kh}{m}\right)\right\}$

26. A weight W is suspended from a rigid support by a hard string with stiffness constant K. The spring is enclosed in a hard plastic sleeve, which prevents horizontal motion, but allows vertical oscillations (see figure). A simple pendulum of length '$l$' with a bob of of mass m(mg <<W) is suspended from the weight W and is set oscillating in a horizontal line with a small amplitude. After some time has passed, the weight W is observed to be oscillating up and down with a large amplitude (but not hitting the sleeve).
It follows that the stiffness constant K must be



(a) $K = \frac{4W}{\ell}$ (b) $K = \frac{2W}{\ell}$ (c) $K = \frac{W}{\ell}$ (d) $K = \frac{W}{2\ell}$

27. Assume that the Earth is a uniform sphere of radius R, rotating about its axis with a uniform angular velocity $\omega$. A rocket is launched from the equator in a direction due North. If it keeps on flying at a uniform speed 'v' (neglecting air resistance), the highest latitude that can be achieved as

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} - (\pi - 2)\frac{\omega R}{v}$ (c) $\frac{\pi}{2} - (\pi + 2)\frac{\omega R}{v}$ (d) $\frac{\pi}{2}\left(1 - \frac{2\omega R}{v}\right)$

28. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be

(a) 0° to the vertical and $\sqrt{3}T$ (b) 30° to the vertical and T/2
(c) 60° to the vertical and $T/\sqrt{3}$ (d) 0° to the vertical and $T/\sqrt{3}$

29. The time evolution of a one-dimensional dynamical system is described by $\frac{dx}{dt} = -(x+1)(x^2 - b^2)$

If this has one stable and two unstable fixed points, then the parameter 'b' satisfies
(a) 0 < b < 1 (b) b > 1 (c) b < −1 (d) b = 2

30. A particle of mass 'm' is in a potential given by $V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$

where 'a' and '$r_0$' are positive constants. When distrubed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

## Answer Key: Classical Mechanics (General)

1. (c) 2. (d) 3. (a, b) 4. (a) 5. (c) 6. (c) 7. (a)
8. (c) 9. (d) 10. (d) 11. (a) 12. (a) 13. (d) 14. (d)
15. (d) 16. (c) 17. (a) 18. (b) 19. (c) 20. (c) 21. (d)
22. (c) 23. (c) 24. (b) 25. (d) 26. (a) 27. (b) 28. (c)
29. (c) 30. (a)

## CSIR-UGC-NET/JRF | GATE PHYSICS

## Assignment-2: Classical Mechanics (Lagrangian Dynamics)

1. The Lagrangian of a particle of mass m moving in a central potential V(r) is

$$L=\frac{1}{2}m\left(\dot{r}^2+r\dot{\theta}^2+r^2\sin^2\theta\dot{\varphi}^2\right)-V(r)$$

(a) $\theta$ is a cyclic coordinate (b) $\theta$ and $\varphi$ are cyclic coordinates
(c) $\varphi$ is a cyclic coordinate (d) r is a cyclic coordinate

2. The Lagrangian of a particle of mass m moving in two dimension is $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\frac{k}{2}\left(x^2+y^2\right)$

If the particle has a finite angular momentum *l* about the origin, then we may conclude that it executes
(a) Oscillatory motion about the origin $r=0$ (b) Periodic motion with a constant value of r
(c) Oscillatory motion along the x-axis. (d) Oscillatory motion along the y-axis.

3. A particle is moving on a sphere of fixed radius. The number of degrees of freedom for the particle will be
(a) 3 (b) 4 (c) 2 (d) 1

4. If a generalised co-ordinate q is ignorable or cycle in a Lagrangian L, then the following is conserved.
(a) $\frac{\partial L}{\partial q}$ (b) $\frac{\partial L}{\partial t}$ (c) $\frac{\partial L}{\partial \dot{q}}$ (d) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$

5. The Lagrangian of a particle moving in a central field is given by $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2\right)-U(r)$
Which of the following is/are constant?
(a) Radial momentum (b) Potential energy
(c) Azimuthal momentum (d) Radial and azimuthal momentum

6. A system exhibits invariance under translation in space and time. The following quantities will remain conserved.
(a) Linear momentum and angular momentum.
(b) Angular momentum and energy.
(c) Linear momentum and energy.
(d) Linear momentum, angular momentum and energy.

7. One of the generalized co-ordinates $Q_j$ of a system is cyclic. Then its
(a) Energy is not conserved. (b) Conjugate momentum $P_j$ is also cyclic.
(c) Conjugate momentum $P_j$ is conserved. (d) Conjugate momentum $P_j$ is indeterminate.

8. A cylinder of mass M and radius r rolls in the x direction from the point $x=0$ on a horizontal plane without friction. Its Lagrangian is:

(a) $\frac{1}{2}M\dot{x}^2$ (b) $\frac{1}{2}M\dot{x}^2+\frac{1}{2}Mr^2\dot{\theta}^2$

(c) $\frac{1}{2}M\dot{x}^2+\frac{1}{2}M\dot{r}^2\dot{\theta}^2$ (d) $\frac{1}{2}Mr\dot{\theta}^2$

9. Consider a six particle system with five particles arranged as regular pentagon with 6th one at the centre. If all the five particles are connected to the central one by rigid rods, then the number of degrees of freedom for the system are given by

(a) 13 (b) 6 (c) 5 (d) 18

10. The Lagrangian of a free particle having a rest $m_0$ and relativistic speed v is given by

(a) $\frac{-m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ (b) $m_0c^2\sqrt{c-v}$ (c) $m_0c\sqrt{c^2-v^2}$ (d) $-m_0c\sqrt{c^2-v^2}$

11. Ignorable coordinates of the system corresponding to the Lagrangian

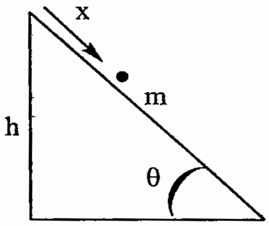
$L=\frac{1}{2}\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{1}{2}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2-k\cos\theta$, where $\theta, \phi$ and $\psi$ are generalized coordinates and k is a constant parameter, are

(a) $\theta$ and $\phi$ (b) $\phi$ and $\psi$ (c) $\psi$ and $\theta$ (d) $\theta, \phi$ and $\psi$

12. Conservation of angular momentum is related to the invariance of Lagrangian under

(a) Rotation in space (b) Translation in space (c) Inversion in space (d) Translation in time

13. A particle of mass m is sliding on a frictionless inclined plane of angle of inclination θ with the horizontal and height h. The distance x traveled by the particle is measured from the top of the inclined plane and along the inclination. If the potential energy is zero at the bottom of the inclined plane, the Lagrangian of the particle is given by



(a) $L=\frac{1}{2}m\dot{x}^2-mg\left(x\sin\theta-h\right)$ (b) $L=\frac{1}{2}m\dot{x}^2-mg\left(h-x\sin\theta\right)$

(c) $L=\frac{1}{2}m\dot{x}^2-mg\left(h-x\cos\theta\right)$ (d) $L=\frac{1}{2}m\dot{x}^2-mg\left(x\cos\theta-h\right)$

14. The Lagrangian of a particle with respect to a stationary frame S is L. If the motion of the particle is studied from another inertial frame $S'$ moving with a constant relative velocity with respect to S under the same potential, then

(a) both the Lagrangian and the equations of motion remain unchanged.
(b) the Lagrangian is modified by addition of a total time derivative of a scalar function and the equations of motion change.
(c) the Lagrangian is modified by addition of a total time derivative of a scalar function and the equations of motion remain unchanged
(d) the Lagrangian is modified by a constant factor and the equations of motion change.

15. The dynamics of a system with generalized coordinates (x, y, z) is described by a Lagrangian $L=L(\dot{x},\dot{y},\dot{z},x,y,t)$ where t is time. Which of the following statements is correct?

(a) The total energy and x-component of the linear momentum are conserved.
(b) The total energy is not conserved but y-component of the linear momentum is conserved.
(c) The total energy is conserved but z-component of linear momentum is not conserved.
(d) The total energy is not conserved but z-component of linear momentum is conserved.

16. The Lagrangian of the particle of mass m is:

$$L=\frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right]-\frac{V}{2}\left(x^2+y^2\right)+W\sin\omega t,$$

where V, W and $\omega$ are constants. The conserved quantities are:

(a) Energy and z-component of linear momentum only.
(b) Energy and z-component of angular momentum only.
(c) z-components of both linear and angular momenta only.
(d) Enregy and z-components of both linear and angular momenta.

17. Three particles of mass m each situated at $x_1(t)$, $x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-streteched length $\ell$. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is:

(a) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-\ell)^2+\frac{k}{2}(x_3-x_2-\ell)^2$

(b) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_3-\ell)^2+\frac{k}{2}(x_3-x_2-\ell)^2$

(c) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2+\ell)^2-\frac{k}{2}(x_3-x_2+\ell)^2$

(d) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-\ell)^2-\frac{k}{2}(x_3-x_2-\ell)^2$

**Statement for Linked Q. 18 and Q.19.**

A particle of mass m is constrained to move in a vertical plane along a trajectory given by $x=A\cos\theta$, $y=A\sin\theta$, where A is a constant:

18. The Lagrangian of the particle is:

(a) $\frac{1}{2}mA^2\dot\theta^2-mgA\cos\theta$
(b) $\frac{1}{2}mA^2\dot\theta^2-mgA\sin\theta$
(c) $\frac{1}{2}mA^2\dot\theta^2$
(d) $\frac{1}{2}mA^2\dot\theta^2+mgA\cos\theta$

19. The equation of motion of the particle is:

(a) $\ddot\theta+\frac{g}{A}\cos\theta=0$ (b) $\ddot\theta+\frac{g}{A}\sin\theta=0$ (c) $\ddot\theta=0$ (d) $\ddot\theta-\frac{g}{A}\sin\theta=0$

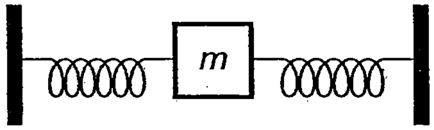
20. A particle of mass m moves in a potential $V(x)=\frac{1}{2}m\omega^2x^2+\frac{1}{2}m\mu v^2$ where x is the position coordinate, v is the speed, and $\omega$ and $\mu$ are constants. The canonical (conjugate) momentum of the particle is:

(a) $p=m(1+\mu)v$ (b) $p=mv$ (c) $p=m\mu v$ (d) $p=m(1-\mu)v$

21. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L=\frac{1}{2}m\left(\dot r^2+r^2\dot\theta^2\right)-V(r)$. The generalized momenta corresponding to r and $\theta$ are given by:

(a) $m\dot r$ and $mr^2\dot\theta$ (b) $m\dot r$ *and* $mr\,\dot\theta$ (c) $m\dot r^2$ and $mr^2\dot\theta$ (d) $m\dot r^2$ and $mr^2\dot\theta^2$

22. A mass m is connected on either side with a spring each of spring constants $k_1$ and $k_2$. The free ends of springs are tied to rigid supports. The displacement of the mass is x from equilibrium position. Which one of the following is TRUE?



(a) The force acting on the mass is $-(k_1, k_2)^{1/2} x$.

(b) The angular momentum of the mass is zero about the equilibrium point and its Lagrangian is
$$\frac{1}{2}m\dot{x}^2 - \frac{1}{2}(k_1 + k_2)x^2.$$

(c) The total energy of the system is $\frac{1}{2}m\dot{x}^2$

(d) The angular momentum of the mass is $mx\dot{x}$ and the Lagrangian of the system is $\frac{m}{2}\dot{x}^2 + \frac{1}{2}(k_1 + k_2)x^2$.

23. The Lagrangian of a particle of mass m moving in a plane is given by $L = (½)\, m\, [v_x^2 + v_y^2] + a(x\, v_y - y\, v_x)$, where $v_x$ and $v_y$ are velocity components and a is a constant. The canonical momenta of the particle are given by:

(a) $p_x = m\, v_x$ and $p_y = m\, v_y$
(b) $p_x = m\, v_x + ay$ and $p_y = m\, v_y + a\, x$
(c) $p_x = m\, v_x - a\, y$ and $p_y = m\, v_y + a\, x$
(d) $p_x = m\, v_x - a\, y$ and $p_y = m\, v_y + a\, x$

24. Two particles of equal mass are connected by an inextensible string of length L. One of the masses is constrained to move on the surface of a horizontal table. The string passes through a small hole in the table and the other mass is hanging below the table. The only constraint is that the first mass moves on the surface of the table. The number of degrees of freedom of the mass-string system is:

(a) five
(b) four
(c) two
(d) one

25. The homogeneity of time leads to the law of conservation of:

(a) Linear momentum
(b) Angular momentum
(c) Energy
(d) Parity

26. The Lagrangian for the Kepler problem is given by:
$$L = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu}{r}, (\mu > 0)$$
where (r, θ) denote the polar coordinates and the mass of the particle is unity. Then:

(a) $p_\theta = 2r^2\dot{\theta}$
(b) $p_r = 2\dot{r}$
(c) The angular momentum of the particle about the centre of attraction is a constant
(d) The total energy of the particle is time dependent.

27. with reference to a cylindrical polar coordinate system, a particle of charge q and momentum $\bar{p}$ is moving in an electric potential $\phi(r)$. Which of the following are conserved?

(a) $p_r$
(b) $p_\theta$
(c) $p_z$
(d) $rp_\theta$

28. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and $q_n$ is a generalized coordinate, then the generalized momentum is:

(a) a cycle cooridnat
(b) a constant of motion
(c) equal to $\frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$
(d) undefined

29. The potential energy of a classical particle moving in one dimension is $kx^4$, where k is a constant. If the particle moves from a point $x_1$ at time $t_1$ to a point $x_2$ at time $t_2$, the actual path followed by the particle is that which makes the following integral extremum.

(a) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2+kx^4\right)dt$ (b) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2-kx^4\right)dt$

(c) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2+4kx^3\right)dt$ (d) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2-4kx^3\right)dt$

30. Lagrangian of the Sun-Earth System is:

(a) $\frac{1}{2}m\dot{r}+\frac{1}{2}mr^2\dot{\theta}^2-\frac{GMm}{r}$ (b) $\frac{1}{2}m\dot{r}^2+\frac{1}{2}mr^2\dot{\theta}^2+\frac{GMm}{r}$

(c) $\frac{1}{2}m\dot{r}^2-\frac{GMm}{r}$ (d) $\frac{1}{2}mr^2\dot{\theta}^2+\frac{GMm}{r}$

Here r is the Sun-Earth distance, M and m are the masses of the Sun and Earth respectiely. $\dot{\theta}$ is the angular speed, and G the gravitational constant.

31. A pair is constrained to move along the inner surface of a hemisphere number of degrees of freedom of the particle is:

(a) one (b) two (c) three (d) four

32. The Lagrangian of a system is given by $L=\frac{1}{2}\dot{q}^2+q\dot{q}-\frac{1}{2}q^2$. It describes the motion of

(a) A harmonic oscillator. (b) A damped harmonic oscillator.
(c) An anharmonic oscillator (d) A system with unbounded motion.

33. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ. The number of generalize coordinates required to describe the motion of this system.

(a) 1 (b) 2 (c) 4 (d) 6

34. The lagrangian of a particle of mass m moving in two dimension is

$$L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\frac{1}{2}k\left(x^2+y^2\right)$$

If the particle has a finite angular momentum *l* about the origin, then we may conclude that it executes

(a) Oscillatory motion about the origin $r=0$ (b) Periodic motion with a constant value of r.
(c) Oscillatory motion along the x-axis. (d) Oscillatory motion along the y-axis.

35. A particle is moving under the action of a generalized potential, $V(q,q)=\frac{(1+\dot{q})}{q^2}$. The magnitude of the generalized force is:

(a) $\frac{2(1+\dot{q})}{q^3}$ (b) $\frac{2(1-\dot{q})}{q^3}$ (c) $\frac{2}{q^3}$ (d) $\frac{\dot{q}}{q^3}$

36. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic fields is given by $L=\frac{1}{2}m\vec{v}^2+e\vec{A}.\vec{v}-e\phi$, where $\vec{A}$ and $\phi$ are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct?

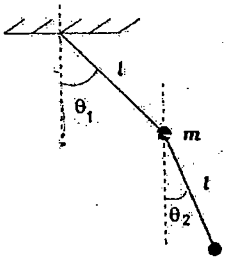
(a) The canonically conjugate momentum of the particle is given by $\vec{p}=m\vec{v}$

(b) The Hamiltonian of the particle is given by $H = \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{A}.\vec{p} + e\phi$

(c) L remains unchanged under a gauge transformation of the potentials.

(d) Under a gauge transformation of the potentials, L changes by the total time derivative of a function of $\vec{r}$ and t.

37. A double pendulum consists of two point masses m attached by massless strings of length $l$ as shown in the figure:



The kinetic energy of the pendulum is:

(a) $\frac{1}{2}m\ell^2\left[\dot{\theta}_1^2 + \dot{\theta}_2^2\right]$
(b) $\frac{1}{2}m\ell^2\left[2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos\left(\dot{\theta}_1 - \dot{\theta}_2\right)\right]$

(c) $\frac{1}{2}m\ell^2\left[\dot{\theta}_1^2 + 2\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos\left(\dot{\theta}_1 - \dot{\theta}_2\right)\right]$
(d) $\frac{1}{2}m\ell^2\left[2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos\left(\dot{\theta}_1 + \dot{\theta}_2\right)\right]$

38. A particle of mass 'm' moves inside a bowl. If the surface of the bowl is given by the equation $z = \frac{1}{2}a\left(x^2 + y^2\right)$, where $a$ is a constant, the Lagrangian of the particle is:

(a) $\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2 - gar^2\right)$
(b) $\frac{1}{2}m\left[\left(1 + a^2r^2\right)\dot{r}^2 + r^2\dot{\phi}^2\right]$

(c) $\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2 - gar^2\right)$
(d) $\frac{1}{2}m\left[\left(1 + a^2r^2\right)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2\right]$

39. The Lagrangian for a symmetric top is given by $L = \frac{1}{2}I_1\left(\dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta\right) + \frac{1}{2}I_3\left(\dot{\varphi} + \dot{\varphi}\cos\theta\right)^2 - Mgl\cos\theta$

where $\theta, \varphi, \psi$ are the Euler angles. Then the following are conserved quantities:

(a) $P_\varphi$ and $P_\psi$
(b) $P_\theta$ and $P_\varphi$
(c) $P_\theta$ and $P_\psi$
(d) $P_\theta, P_\varphi$ and $P_\psi$

40. A system with d degrees of freedom can have at most k independent constants of motion. The value of k is equal to

(a) $d-1$
(b) $2d$
(c) $2d-1$
(d) d

41. The invariance of the Lagrangian $L = \frac{1}{2}m\left(\frac{dz}{dt}\right)\left(\frac{dz^*}{dt}\right) - V\left(zz^*\right)$ (where $z = re^{i\theta} = x + iy$ is a complex coordinate under the transformation $Z \rightarrow e^{i\alpha}z, z^* \rightarrow e^{-i\alpha}z^*$ (for a real number $\alpha$), leads to the conservation of

(a) Angular momentum
(b) linear momentum along x.
(c) linear momentum along y
(d) energy

42. The Lagrangian of a 2-dimensional system is given by $L = \mu\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \gamma\left(x_1 + x_2\right)$ where $\mu$ and $\gamma$ are constants

(a) The total energy E as well as $\left(\dot{x}_1 + \dot{x}_2\right)$ are conserved

(b) The total energy E as well as $\left(\dot{x}_1 - \dot{x}_2\right)$ are conserved

(c) Only the total energy E is conserved

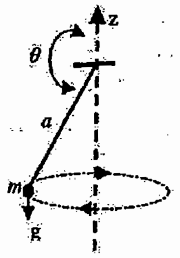
(d) Only the sum $\left(\dot{x}_1 + \dot{x}_2\right)$ is conserved.

43. A particle of mass m is attached to a fixed point 'O' by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure.

The Lagrangian of the particle is

$$L(\theta, \phi) = \frac{1}{2} m a^2 \left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) - mga\cos\theta$$

where $\theta$ and $\phi$ are the polar angles.



-- The Hamiltonian of the particle is:

(a) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) - mga\cos\theta$

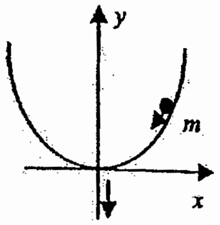
(b) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

(c) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) - mga\cos\theta$

(d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) + mga\cos\theta$

**Statement for Linked Answer Q.44 and Q. 45**

A particle of mass m slides under the gravity without friction along parabolic path $y = ax^2$ as shown in the figure. Here a is a constant.



44. The Lagrangian for this particle is given by,

(a) $L = \frac{1}{2} m\dot{x}^2 - mgax^2$

(b) $L = \frac{1}{2} m\left(1 + 4a^2x^2\right)\dot{x}^2 - mgax^2$

(c) $L = \frac{1}{2} m\dot{x}^2 + mgax^2$

(d) $L = \frac{1}{2} m\left(1 + 4a^2x^2\right)\dot{x}^2 + mgax^2$

45. The Lagrange's equation of motion of the particle is

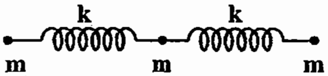
(a) $\ddot{x} = 2gax$

(b) $m\left(1 + 4a^2x^2\right)\ddot{x} = -2mgax - 4ma^2x\,\dot{x}^2$

(c) $m\left(1 + 4a^2x^2\right)\ddot{x} = 2mgax + 4ma^2x\,\dot{x}^2$

(d) $\ddot{x} = -2gax$

46. Three particles of equal mass 'm' are connected by two identical massless springs of stiffness constant 'k' as shown in the figure.



If $x_1$, $x_2$ and $x_3$ denote the displacements of the masses from their respective equilibrium positions, the potential energy of the system is:

(a) $\frac{1}{2}k\left(x_1^2+x_2^2+x_3^2\right)$ (b) $\frac{1}{2}k\left[x_1^2+x_2^2+x_3^2-x_2\left(x_1+x_3\right)\right]$

(c) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2+2x_2\left(x_1+x_3\right)\right]$ (d) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2-2x_2\left(x_1+x_3\right)\right]$

47. The Lagrangian of a particle of mass m moving in one dimensions is given by

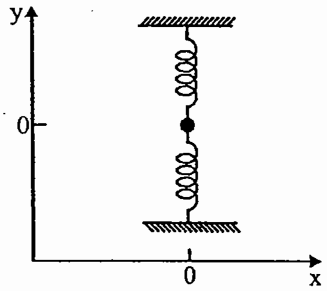
$$L=\frac{1}{2}m\dot{x}^2-bx$$

where b is a positive constant. The coordinate of the particle x(t) at time t is given by: (in the following $c_1$ and $c_2$ are constants)

(a) $-\frac{b}{2m}t^2+c_1t+c_2$ (b) $c_1t+c_2$

(c) $c_1\cos\left(\frac{bt}{m}\right)+c_2\sin\left(\frac{bt}{m}\right)$ (d) $c_1\cosh\left(\frac{bt}{m}\right)+c_2\sinh\left(\frac{bt}{m}\right)$

48. Consider a particle of mass *m* attached to two identical springs each of length *l* and spring constant k (see the figure below). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations?



(a) $m\ddot{x}+\frac{kx^3}{l^2}=0$ (b) $m\ddot{x}+kx=0$

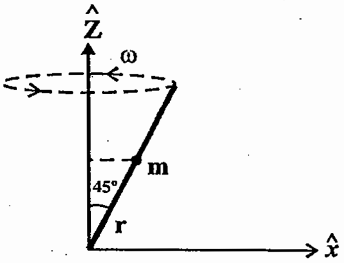
(c) $m\ddot{x}+2kx=0$ (d) $m\ddot{x}+\frac{kx^2}{l}=0$

49. The Lagrangian of a system with one degree of freedom $q$ is given by $L=\alpha\dot{q}^2+\beta q^2$, where $\alpha$ and $\beta$ are non-zero constants. If $p_q$ denotes the canonical momentum conjugate to $q$ then which one of the following statements is CORRECT?

(a) $p_q=2\beta q$ and it is a conserved quantity. (b) $p_q=2\beta q$ and it is not a conserved quantity.

(c) $p_q=2\alpha\dot{q}$ and it is a conserved quantity. (d) $p_q=2\alpha\dot{q}$ and it is not a conserved quantity.

50. A bead of mass ‘m’ can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed $\omega$. At any instant, $r$ is the distance of the bead from the origin. The momentum conjugate to ‘r’ is



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

## Answer Key: Classical Mechanics (Lagrangian Dynamics)

1. (c) 2. (b) 3. (c) 4. (c) 5. (c) 6. (c) 7. (c)
8. (b) 9. (a) 10. (d) 11. (b) 12. (a) 13. (b) 14. (a)
15. (d) 16. (c) 17. (d) 18. (b) 19. (a) 20. (d) 21. (a)
22. (b) 23. (c) 24. (c) 25. (c) 26. (c) 27. (c) 28. (b)
29. (b) 30. (b) 31. (b) 32. (a) 33. (b) 34. (b) 35. (a)
36. (d) 37. (b) 38. (d) 39. (a) 40. (d) 41. (a) 42. (c)
43. (b) 44. (b) 45. (b) 46. (d) 47. (a) 48. (d) 49. ()
50. (a)

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## Assignment-3: Classical Mechanics (Hamiltonian Dynamics)

1. A canonical transformation in classical Hamiltonian dynamics
(a) Cannot be made if there is more than one degree of freedom.
(b) Leaves the canonical Poisson brakcet relations unchanged.
(c) Can only be made for the cartesian components of the coordinates and momenta.
(d) Cannot be time-dependent

2. If $P_x$ and $L_z$ are constants of motion, then their Poisson bracket $[P_x, L_z]$ is :
(a) Also a constant of motion (b) Not a constant of motion.
(c) Zero (d) Undefined.

3. For a Hamiltonian $H = \frac{p^2}{2m} - \frac{e^2}{r} + k\left(x^2 + y^2\right)$, where $r^2 = x^2 + y^2 + z^2$ and k is a constant, the following quantities are constants of motion.
(a) $E, L^2, L_z$ (b) $E, L_z$ (c) $E, L^2$ (d) E only

4. The Hamiltonian of a charge particle of charge q, mass m, moving in an electromagnetic field $\left(\phi, \vec{A}\right)$ is given by
(a) $\frac{1}{2m}\left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 + q\phi$ (b) $\frac{1}{2m}\left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 - q\phi$
(c) $\frac{1}{2m}\left(\vec{P} + \frac{q}{c}\vec{A}\right)^2 - q\phi$ (d) $\frac{\vec{P}^2}{2m} + q\phi$

5. The Hamiltonian of a particle moving in a plane is given by

$$H = \frac{\vec{P}^2}{2m} + \frac{1}{2}m\omega^2 r^2 \sin^2\theta$$

where m is the mass, $\vec{p}$ the momentum, $\omega$ a constant, and $(r\ \theta)$ the radial and polar coordinates. The following quantity/quantities will be conserved.
(a) Energy (b) Angular momentum
(c) Energy and angular momentum (d) Energy and linear momentum.

6. The value of Poisson brakcet $\left[x^2, p_x^2\right]$ is :
(a) $4x^2$ (b) $4p_x^2$ (c) $4xp_x$ (d) $xp_x$

7. If the Lagrangian of a system has only one term which has a linear dependence on velocity, then its Hamiltonian
(a) Also has only one term depending on velocity.
(b) Is zero.
(c) Is not defined.
(d) Is independent of velocity and is constant.

8. A particle of mass $m$ and coordinate $q$ has the Lagrangian

$$L=\frac{1}{2}m\dot{q}^2-\frac{\lambda}{2}q\dot{q}^2$$

where $\lambda$ is a constant. The Hamiltonian for the system is given by

(a) $\frac{p^2}{2m}+\frac{\lambda q p^2}{2m^2}$ (b) $\frac{p^2}{2(m-\lambda q)}$ (c) $\frac{p^2}{2m}+\frac{\lambda q p^2}{2(m-\lambda q)^2}$ (d) $\frac{pq}{2}$

9. The action variable J for a simple harmonic oscillator having hamiltonian $H=\frac{p^2}{2m}+\frac{1}{2}kq^2$, is related to its energy E by

(a) $J=\frac{1}{2\pi}\sqrt{\frac{k}{m}}E$ (b) $J=\sqrt{\frac{m}{k}}E$ (c) $J=\sqrt{\frac{k}{m}}E$ (d) $J=2\pi\sqrt{\frac{m}{k}}E$

10. A particle is being acted upon by the inverse square force $\vec{F}=-\frac{k}{r^2}\hat{r}$. The Hamiltonian of the system is:

(a) $H=\frac{\vec{p}.\vec{p}}{2m}-\frac{k}{r}$ (b) $H=\frac{\vec{p}.\vec{p}}{2m}+\frac{k}{r}$ (c) $H=\frac{\vec{p}.\vec{p}}{2m}-\frac{k}{r^2}$ (d) $H=\frac{\vec{p}.\vec{p}}{2m}+\frac{k}{r^2}$

11. The Hamiltonian of a system as a function of the canonical conjugate cordinate q and momentum p is given by H(q,p). The Hamilton's canonical equations are given by

(a) $p=\frac{\partial H}{\partial q}, q=\frac{\partial H}{\partial p}$ (b) $p=-\frac{\partial H}{\partial q}, q=\frac{\partial H}{\partial p}$

(c) $\dot{p}=-\frac{\partial H}{\partial q}, \dot{q}=\frac{\partial H}{\partial p}$ (d) $\dot{p}=\frac{\partial H}{\partial q}, \dot{q}=-\frac{\partial H}{\partial q}$

12. Two sets of transformation of conjugate position and momentum are given as (i) Q = p, P = – q and (ii) Q = p + iq, P = p–iq. Which of the following statements is correct?

(a) Both (i) and (ii) represent canonical transformations.
(b) (i) represent a canonical transformation but (ii) does not represents a canonical transformation.
(c) (i) does not represent a canonical transformation but (ii) does not representsa canonical transformation.
(d) Neither (i) nor (ii) represents any canonical transformation.

13. The Hamiltonian of a particle is $H=\frac{p^2}{2m}+pq$, where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is:

(a) $\frac{m}{2}\left(\frac{dq}{dt}+q\right)^2$ (b) $-\frac{m}{2}\left(\frac{dq}{dt}-q\right)^2$ (c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2+q\frac{dq}{dt}-q^2\right]$ (d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2-q\frac{dq}{dt}-q^2\right]$

14. A particle of charge q, mass m and linear momentum $\vec{p}$ enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential $\phi$. The Hamiltonian of the particle is:

(a) $\frac{p^2}{2m}+q\phi+\frac{A^2}{2m}$ (b) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+q\phi$ (c) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+\vec{p}\cdot\vec{A}$ (d) $\frac{p^2}{2m}q\phi-\vec{p}\cdot\vec{A}$

15. For the given transformations (i) Q = p, P = –q and (ii) Q = p, P = q, where p and q are canonically conjugate variables, which one of the following statements is true?

(a) Both (i) and (ii) are canonical (b) Only (i) is canonical
(c) Only (ii) is canonical (d) Neither (i) nor (ii) is conical

16. The Hamiltonian corresponding to the Langrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is:

(a) $\frac{p_x^{\;2}}{2a} + \frac{p^2{}_y}{2b} + kxy$ (b) $\frac{p^2{}_x}{4a} + \frac{p^2{}_y}{4b} - kxy$ (c) $\frac{p^2{}_x}{4a} + \frac{p^2{}_y}{4b} + kxy$ (d) $\frac{p^2{}_x + p^2{}_y}{4ab} + kxy$

17. The value of the Poisson bracket $\left[\vec{a}.\vec{r}, \vec{b}.\vec{p}\right]$, where $\vec{a}$ and $\vec{b}$ are constant vectors, is:

(a) $\vec{a}\,\vec{b}$ (b) $\vec{a} - \vec{b}$ (c) $\vec{a} + \vec{b}$ (d) $\vec{a}.\vec{b}$

18. Hamilton canonical equations of motion for a conservative system are:

(a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (b) $\frac{dp_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

where $q_i$, $p_i$ and H are generalized cooridnate, generalized momentum and Hamiltonian respectively.

19. A linear transformation of a generalized coordinate q and the corresponding momentum p to Q and P given by

$Q = q + p;\; P = q + \alpha p$

is canonical if the value of the constant $\alpha$ is:

(a) $-1$ (b) 0 (c) $+1$ (d) $+2$

20. $q_1$ and $q_2$ are generalised coordinate and $p_1$, $p_2$ are the corresponding generalised moments. The Poisson braket $\{X, Y\}$ of $X = q_1^2 + q_2^2$ and $Y = 2p_1 + p_2$ is:

(a) $(q_1^2 + q_2^2)p_1$ (b) $3(q_1^2 + q_2^2)$ (c) $4q_1 + 2q_2$ (d) 0

21. For a simple harmonic oscillator the Lagrangian is given by $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$. If $A(p,d) = \frac{p + iq}{\sqrt{2}}$ and H(p,d) is the Hamiltonian of the system, the Poisson bracket $\{A(p,q), H(p,q)\}$ is given by

(a) iA(p,q) (b) $A^*(p,q)$ (c) $-iA^*(p,q)$ (d) $-iA(p,q)$

**Statement for Linked Q.22 and Q.23.**

The Lagrangian for a simple pendulum is given by: $L = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell(1 - \cos\theta)$

22. Hamilton's equations are then given by

(a) $\dot{p}_\theta = -mg\ell\sin\theta;\quad \dot{\theta} = \frac{p_\theta}{m\ell^2}$ (b) $\dot{p}_\theta = mg\ell\sin\theta;\quad \dot{\theta} = \frac{p_\theta}{m\ell^2}$

(c) $\dot{p}_\theta = -mg\ddot{\theta};\quad \dot{\theta} = \frac{p_\theta}{m}$ (d) $\dot{p}_\theta = -\left(\frac{g}{\ell}\right)\theta;\quad \dot{\theta} = \frac{p_\theta}{m\ell}$

23. The Poisson bracket between $\theta$ and $\dot{\theta}$ is:

(a) $\{\theta, \dot{\theta}\} = 1$ (b) $\{\theta, \dot{\theta}\} = \frac{1}{m\ell^2}$ (c) $\{\theta, \dot{\theta}\} = \frac{1}{m}$ (d) $\{\theta, \dot{\theta}\} = \frac{g}{\ell}$

24. Given the Lagrangian $L = -\sqrt{1-\left(\frac{dx}{dt}\right)^2}$

where 'x' is a co-ordinate and t is the time, the Hamiltonian is

(a) $\frac{\left(\frac{dx}{dt}\right)^2}{\sqrt{1-\left(\frac{dx}{dt}\right)^2}}$ (b) $\sqrt{1-\left(\frac{dx}{dt}\right)^2}$ (c) $\frac{1}{\sqrt{1-\left(\frac{dx}{dt}\right)^2}}$ (d) $\left(\frac{dx}{dt}\right)^2$

25. Let $(p,q)$ and $(P,Q)$ be two pairs of canonical variables. The transformation

$$Q = q^{\alpha}\cos(\beta p);\ P = q^{\alpha}\sin(\beta p)$$

is canonical for

(a) $\alpha = 2, \beta = 1/2$ (b) $\alpha = 2, \beta = 2$ (c) $\alpha = 1, \beta = 1$ (d) $\alpha = 1/2, \beta = 2$

26. The Hamiltonian of a particle of unit mass moving in the xy-plane is given to be:

$H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$ in suitable units. The initial values are given to be $(x(0), y(0)) = (1, 1)$ and

$(p_x(0), p_y(0)) = \left(\frac{1}{2}, -\frac{1}{2}\right)$. During the motion, the curves traced out by the particles in the xy-plane and the

$p_xp_y$-plane are

(a) Both straight lines (b) A straight line and a hyperbola respectively
(c) A hyperbola an ellipse, respectively (d) Both hyperbolas.

27. A Dynamical system with two degrees of freedom, has generalised coordinates $q_1$ and $q_2$, and kinetic energy

$$T = \lambda\dot{q}_1\dot{q}_2$$

If the potential energy is $V(q_1, q_2) = 0$, the correct form of the Hamiltonian for this system is:

(a) $p_1p_2/\lambda$ (b) $\lambda\dot{q}_1\dot{q}_2$ (c) $(p_2\dot{q}_1 + p_2\dot{q}_2)/2$ (d) $(p_1q_2 + p_2q_1)/2$

28. For what value of $\alpha$ will the transformation $q \to Q = q^{\alpha}\cos 2p$ and $p \to P = q^{\alpha}\sin 2p$ be cononical

(a) $\alpha = 2$ (b) $\alpha = 1/2$ (c) $\alpha = -1$ (d) $\alpha = 1$

29. The Lagrangian of a relativistic particle of mass m and velocity v is $L = -mc^2\sqrt{1-\frac{v^2}{c^2}}$. Its Hamiltonian H is

(a) $H = mc^2\sqrt{1-\frac{v^2}{c^2}}$ (b) $H = \frac{mc^2}{\sqrt{1-v^4/c^2}}$ (c) $H = \sqrt{m^2c^2 + p^2c^2}$ (d) $H = \frac{m^2c^4}{\sqrt{m^2c^4 + p^2c^2}}$

30. A spring made of a given material has the restoring force $F = -kx^3$. The total energy of a point of mass m at the end
of this spring is :

(a) $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}kx^4$ (b) $E = \frac{1}{2}m\dot{x}^2 - \frac{1}{4}kx^4$ (c) $E = \frac{1}{2}m\dot{x}^2 + kx^3$ (d) $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

31. The phase space orbit of a body sliding down a frictionless inclined plane in uniform gravity is

(a) a straight line (b) an ellipse (c) a parabola (d) a hyperbola

32. A second order differential equation in one variable of the form $s(t)\ddot{q}+p(t)\dot{q}+r(t)q=0$ where s, p and r are smooth functions of time t with $s(t)\neq 0$ can be written in the from of Hamilton's equations

(a) always (b) only if $s(t)\equiv 1$ (c) only if $p(t)\equiv 0$ (d) only if $r(t)=1$

33. Consider a canonical transformation $q\rightarrow Q=-q$ and $p\rightarrow P=-p$. The generating function for this transformation is given by

(a) $F_3=-pQ$ (b) $F_1=-qQ$ (c) $F_2=-qP$ (d) $F_4=-pP$

34. What is the Poisson bracket between the position x and x-component of the angular momentum $L_x$

(a) x (b) y (c) z (d) 0

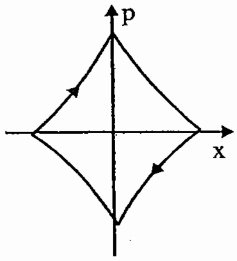
35. The Hamiltonian of a system is independent of the coordinate q and is given by $H=ap^4$ where a is a constant. The corresponding Lagrangian is then given by

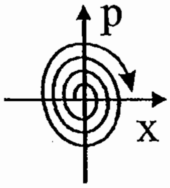
(a) $L=m\dot{q}^2-ap^4$ (b) $L=\frac{1}{2}m\dot{q}^2-ap^4$ (c) $L=a\dot{q}^4$ (d) $L=\frac{3\dot{q}}{4}\left(\frac{\dot{q}}{4a}\right)^{1/3}$

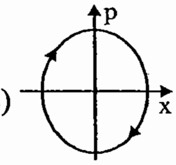
36. If the coordinate q and the momentum p form a canonical pair (q,p). which one of the sets given below also forms a canonical pair ?

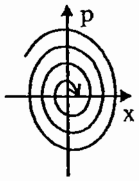
(a) $(q,-p)$ (b) $(q^2,p^2)$ (c) $(p,-q)$ (d) $(q^2,-p^2)$

37. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum?

(a) 

(b) 

(c) 

(d) 

(a) FIGURE (b) FIGURE (c) FIGURE (d) FIGURE

38. If the Lagrangian of a particle moving in one dimensions is given by $L=\frac{\dot{x}^2}{2x}-V(x)$, the Hamiltonian is:

(a) $\frac{1}{2}xp^2+V(x)$ (b) $\frac{\dot{x}^2}{2x}+V(x)$ (c) $\frac{1}{2}\dot{x}^2+V(x)$ (d) $\frac{p^2}{2x}+V(x)$

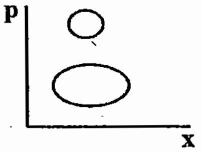
39. The Poisson bracket $\{|r|,|p|\}$ has the value

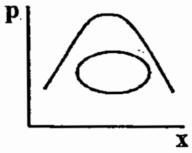
(a) $|r||p|$ (b) $\vec{r}.\vec{p}$ (c) 3 (d) 1

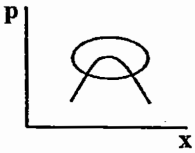
40. The Hamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length $l$ is $H=\frac{p_\theta^2}{2m\ell^2}+mg\ell(1-\cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is :

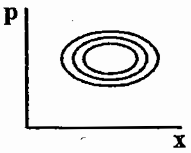
(a) $-\frac{2g}{\ell}p_\theta\sin\theta$ (b) $-\frac{g}{\ell}p_\theta\sin2\theta$ (c) $\frac{g}{\ell}p_\theta\cos\theta$ (d) $\ell p_\theta^2\cos\theta$

41. Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)?

(a)


(b)


(c)


(d)


42. In an observer's rest frame, a particle is moving towards the observer with an energy $E$ and momentum $p$. If $c$ denotes the velocity of light in vacuum, the energy of the particle in another frame moving in the same direction as the particle with a constant velocity $v$ is

(a) $\frac{(E+vp)}{\sqrt{1-(v/c)^2}}$ (b) $\frac{(E-vp)}{\sqrt{1-(v/c)^2}}$ (c) $\frac{(E+vp)}{[1-(v/c)^2]^2}$ (d) $\frac{(E-vp)}{[1-(v/c)^2]^2}$

43. The Hamiltonian of a relativistic particle of rest mass $m$ and momentum $p$ is given by $H=\sqrt{p^2+m^2}+V(x)$, in units in which the speed of light $c = 1$. The corresponding Lagrangian is

(a) $L=m\sqrt{1+\dot{x}^2}-V(x)$ (b) $L=-m\sqrt{1-\dot{x}^2}-V(x)$

(c) $L=\sqrt{1+m\dot{x}^2}-V(x)$ (d) $L=\frac{1}{2}m\dot{x}^2-V(x)$

44. A system is governed by the Hamiltonian $H=\frac{1}{2}(p_x-ay)^2+\frac{1}{2}(p_y-bx)^2$

where a and b are constants and $p_x$, $p_y$ are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved?

(a) a = –3, b = 2 (b) a = 3, b = –2 (c) a = 2, b = –3 (d) a = –2, b = 3

45. The coordinates and momenta $x_i, p_i$ $(i=1,2,3)$ of a particle satisfy the canonical Poissoa bracket relations $\{x_i,p_j\}=\delta_{ij}$. If $C_1=x_2p_3+x_3p_2$ and $C_2=x_1p_2-x_2p_1$ are constants of motion, and if $C_3=\{C_1,C_2\}=x_1p_3+x_3p_1$, then

(a) $\{C_2,C_3\}=C_1$ and $\{C_3,C_1\}=C_2$ (b) $\{C_2,C_3\}=-C_1$ and $\{C_3,C_1\}=-C_2$

(c) $\{C_2,C_3\}=-C_1$ and $\{C_3,C_1\}=C_2$ (d) $\{C_2,C_3\}=C_1$ and $\{C_3,C_1\}=-C_2$

46. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q = q^2$ and $P = p/2q$. The corresonding time-independent generating function is

(a) $\frac{P}{q^2}$ (b) $q^2P$ (c) $q^2/P$ (d) $qP^2$

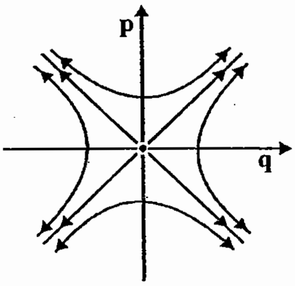
47. The Hamilton's canonical equations of motion in terms of Poisson Brackets are

(a) $\dot{q}=\{q,H\};\dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\};\dot{p}=\{H,q\}$

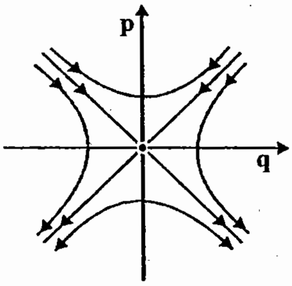
(c) $\dot{q}=\{H,p\};\dot{p}=\{H,q\}$ (d) $\dot{q}=\{p,H\};\dot{p}=\{q,H\}$

48. The Hamiltonian of a particle of mass 'm' is given by $H=\frac{p^2}{2m}-\frac{\alpha q^2}{2}$. Which of the folloiwng figures describes the motion of the particle in phase space?

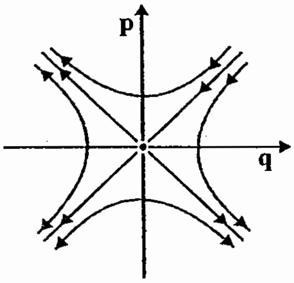
(a)



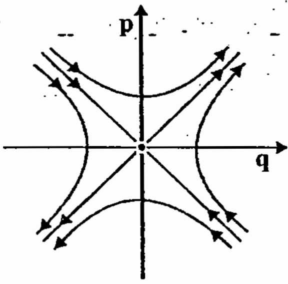
(b)



(c)



(d)



49. Given that the linear transformation of a generalized coordinate 'q' and the corresponding momentum p,

$$Q = q + 4ap$$
$$p = q + 2p$$

is canonical, the value of the constant 'a' is ___________

## Answer Key: Classical Mechanics (Hamiltonian Dynamics)

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (b)** | **2. (b)** | **3. (b)** | **4. (a)** | **5. (c)** | **6. (c)** | **7. (b)** |
| **8. (b)** | **9. (d)** | **10. (a)** | **11. (c)** | **12. (b)** | **13. (b)** | **14. (b)** |
| **15. (b)** | **16. (c)** | **17. (d)** | **18. (d)** | **19. (d)** | **20. (c)** | **21. (a)** |
| **22. (a)** | **23. (b)** | **24. (c)** | **25. (d)** | **26. (d)** | **27. (a)** | **28. (b)** |
| **29. (c)** | **30. (a)** | **31. (c)** | **32. (b)** | **33. (c)** | **34. (d)** | **35. (d)** |
| **36. (c)** | **37. (b)** | **38. (a)** | **39. (b)** | **40. (a)** | **41. (c)** | **42. (b)** |
| **43. (b)** | **44. (d)** | **45. (d)** | **46. (b)** | **47. (a)** | **48. (d)** | |
| **49. (0.24 to 0.26)** | | | | | | |

## CSIR-UGC-NET/JRF | GATE PHYSICS

## Assignment-4: Classical Mechanics (Special Theory of Relativity)

1. A relativistic particle of rest mass $m_0$ is moving with a speed v. The value of v at which its kinetic energy is equal to its rest energy is:

(a) $v=\frac{c}{2}$ (b) $v=\frac{c}{4}$ (c) $v=\frac{\sqrt{3}}{2}c$ (d) $v=2\left(\sqrt{2}-1\right)c$

2. A meter rod is moving along the positive x axis with velocity 0.6C. Its length as measured by a stationary observer in meters, is:

(a) 0.6 (b) 0.8 (c) 1.0 (d) 1.2

3. An event is space-like if the two world points are separated such that

(a) $|\vec{r}_1-\vec{r}_2|^2 \le c^2(t_1-t_2)^2$ (b) $|\vec{r}_1-\vec{r}_2|^2 \le c^2(t_1+t_2)^2$

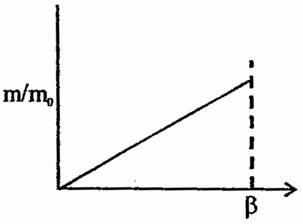
(c) $|\vec{r}_1-\vec{r}_2|^2 > c^2(t_1-t_2)^2$ (d) $|\vec{r}_1-\vec{r}_2|^2 > c^2(t_1-t_2)^2$

4. A particle moves with a relativistic speed such that its energy is twice the rest mass energy. The speed of the particle is:

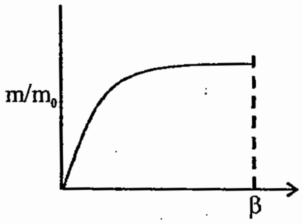
(a) $v=\frac{c}{2}$ (b) $v=\frac{3}{4}c$ (c) $v=\frac{\sqrt{3}}{2}c$ (d) $v=c$

5. For a particle moving with relativistic speed, the ratio of its relativistic mass to its rest mass $(m/m_0)$ varies with the parameter $\beta = v/c$ as

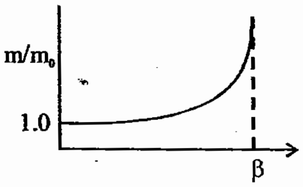
(a)



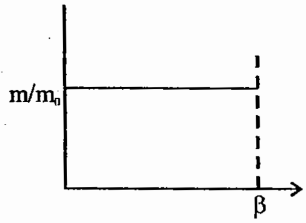
(b)



(c)



(d)



6. **Length contraction:** space ship length $l$. What is the new form if it varies like cube

(a) $\ell=\frac{1}{2}\ell_0\sqrt{1-\frac{v^2}{c^2}}$ (b) $\ell^3={\ell_0}^3\sqrt{1-\frac{v^2}{c^2}}$ (c) $\ell^3={\ell_0}^3\left(1-\frac{v^2}{c^2}\right)^{3/2}$ (d) $\ell=2\ell_0\sqrt{1-\frac{v^2}{c^2}}$

7. A particle of mass M at rest decays into two identical particles of mass m each such that 2m is less than M. The speed of each particle after decay

(a) $v=0$ (b) $v=c\sqrt{1+(2m/M)^2}$ (c) $v=c\sqrt{M/m}$ (d) $v=c\sqrt{1-(2m/M)^2}$

**Statement for Linked Q.8 and Q.9.**

In the laboratory fram, a particle P of rest mass m0 is moving in the positive x direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q moving in the negative x direction with a speed of $\frac{2c}{5}$.

8. The speed of the particle P in the rest frame of the particle Q is:

(a) $\frac{7c}{95}$ (b) $\frac{13c}{85}$ (c) $\frac{3c}{5}$ (d) $\frac{63c}{95}$

9. The energy of the particle P in the rest frame of the particle Q is:

(a) $\frac{1}{2}m_0c^2$ (b) $\frac{5}{4}m_0c^2$ (c) $\frac{19}{13}m_0c^2$ (d) $\frac{11}{9}m_0c^2$

10. The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where $m_0$ is its rest mass. The linear momentum of the particle is:

(a) $2m_0c$ (b) $\frac{2m_0c}{\sqrt{3}}$ (c) $\frac{m_0c}{\sqrt{3}}$ (d) $m_0c$

11. A rod of length $\ell_0$ makes an angle $\theta_0$ with the y-axis in its rest frame, while the rest frame moves to the right along the x-axis with relativistic speed v with respect to the lab frame. If $\gamma = (1 - v^2/c^2)^{-1/2}$, the angle q in the lab frame is:

(a) $\theta=\tan^{-1}(\gamma\tan\theta_0)$ (b) $\theta=\tan^{-1}(\gamma\cot\theta_0)$ (c) $\theta=\tan^{-1}\left(\frac{1}{\gamma}\tan\theta_0\right)$ (d) $\theta=\tan^{-1}\left(\frac{1}{\gamma}\cot\theta_0\right)$

12. An electron gains energy so that it mass becomes $2m_0$. Its speed is:

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{3}{4}c$ (c) $\frac{3}{2}c$ (d) $\sqrt{\frac{3}{2}}c$

13. The speed of a particle whose kinetic energy is equal to its rest mass energy is given by (c is the speed of light in vacuum)

(a) c/3 (b) $\sqrt{2}c/3$ (c) c/2 (d) $\frac{\sqrt{3}}{2}c$

14. Which of the following equations is relatively invariant? (α, β, δ and d are constants of suitable dimensions)

(a) $\frac{\partial\phi(x,t)}{\partial t}=\alpha\frac{\partial^2\phi(x,t)}{\partial x^2}$ (b) $\frac{\partial^2\phi(x,t)}{\partial t^2}=\beta\frac{\partial^2\phi(x,t)}{\partial x^2}$

(c) $\frac{\partial^2\phi(x,t)}{\partial t^2}=\gamma\frac{\partial^2\phi(x,t)}{\partial x}$ (d) $\frac{\partial\phi(x,t)}{\partial t}=\delta\frac{\partial^3\phi(x,t)}{\partial x^3}$

15. A relativeistic particle of mass m and velocity c/2 $\hat{z}$ is moving towards a wall. The wall is moving with a velocity c/3 $\hat{z}$. The velocity of the particle after it suffers an elastic collision is $\upsilon\hat{z}$ with $\upsilon$ equal to:
(a) c/2 (b) c/5 (c) c/7 (d) c/15

(All the velocities refer to the laboratory frame of reference)

16. The momentum of an electron (mass m) which has the same kinetic energy as its rest mass energy is (c is velocity of light)
(a) $\sqrt{3}$ mc (b) $\sqrt{2}$ mc (c) mc (d) $mc/\sqrt{2}$

17. In a system of units in which the velocity of light c = 1, which of the following is a Lorentz transformation?
(a) $x' = 4x, y = y', z' = z, t' = 0.25t$
(b) $x' = x - 0.5t, y' = y, z' = z, t' = t + x$
(c) $x' = 1.25x - 0.75t, y' = y$
$z' = z, t' = 0.75t - 1.25x$
(d) $x' = 1.25x - 0.75t, y' = y$
$z' = z, t' = 1.25t - 0.75x$

18. Kinetic energy of a relativistic particle of rest mass m moving with speed v is:
(a) $\frac{1}{2}mv^2$ (b) $\frac{mc^2}{\sqrt{1-v^2/c^2}}$ (c) $\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$ (d) $\frac{1}{2}m(v^2 - c^2)$

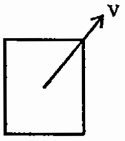
19. A particle has rest mass $m_0$, and momentum $m_0c$, where c is the velocity of light. The total energy and the velocity of the particle are respectively
(a) $\sqrt{2}m_0c^2$ and c/2 (b) $2m_0c^2$ and $c/\sqrt{2}$ (c) $\sqrt{2}m_0c^2$ and $c/\sqrt{2}$ (d) $2m_0c^2$ and c/2

20. An observer O is at the origin of an inertial frame of reference I and rotating with angular velocity $\omega\hat{z}$ relative to it. A satellite of mass M is in a circular orbit of radius R about the origin with angular velocity $-\omega\hat{z}$ as seen from I. The total force $M\bar{\tau}$ on the satellite in O's rest frame is:
(a) $M\omega^2R\hat{r}$ (b) $-M\omega^2R\hat{r}$ (c) $4M\omega^2R\hat{r}$ (d) $-4M\omega^2R\hat{r}$

21. Two particles, each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of final entity is:
(a) 5m/4 (b) 2m (c) 5m/2 (d) 25m/8

22. A constant force F is applied to a relativistic particle of rest mass m. If the particle starts from rest at t = 0, its speed after a time t is
(a) $Ft/m$ (b) $c\tanh\left(\frac{Ft}{mc}\right)$ (c) $c\left(1-e^{-Ft/mc}\right)$ (d) $\frac{Fct}{\sqrt{F^2t^2+m^2c^2}}$

23. A uniform cube of side $l$ (in its rest frame) is moving at a constant velocity v along the direction shown in the figure with respect to a stationary observer.



The volume of the cube as seen by her is
(a) $\ell^3\sqrt{1-\frac{v^2}{c^2}}$ (b) $\ell^3\left(1-\frac{v^2}{c^2}\right)$ (c) $\ell^3\left(1-\frac{v^2}{c^2}\right)^{3/2}$ (d) $\ell^3$

24. A pair of identical relativistic particles with mass m each and momenta $p\hat{x}$ and $-p\hat{x}$ respectively collide. After collision, they are transformed to a pair of new particles with mass M each. If the magnitudes of the final state momenta are $P_1$ & $P_2$ respectively, then

(a) $P_1 = P_2 = p + (m - M)c$
(b) $P_1^2 = P_2^2 = p^2 + (M^2 - m^2)c^2$
(c) $P_1^2 = P_2^2 = p^2 + (m^2 - M^2)c^2$
(d) $P_1^2 = p^2 + (m^2 - M^2)c^2$ and $P_2^2 = p^2 + (M^2 - m^2)c^2$

25. A rod of proper length $\ell_0$ oriented parallel to the x-axis moves with speed 2c/3 along the x-axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

(a) $0.35\,\ell_0$ (b) $0.48\,\ell_0$ (c) $0.87\,\ell_0$ (d) $0.97\,\ell_0$

26. Two events, separated by a (spatial) distance $9\times10^9$ m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0.8c (where the speed of light $c = 3\times10^8$ m/s) is:

(a) 60 s (b) 40 s (c) 20 s (d) 0 s

27. Let v, p and E denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass 'm'. Then

(a) $\frac{dE}{dp} = \text{constant}$ (b) $p = mv$ (c) $v = \frac{cp}{\sqrt{p^2 + m^2c^2}}$ (d) $E = mc^2$

28. Although mass-energy equivalence of special relativity allows conversion of photon to an electron-positron pair, such a process cannot occur in free space because:

(a) The mass is not conserved
(b) The energy is not conserved
(c) The momentum is not conserved
(d) The charge is not conserved

29. An electron is accelerated from rest by 10.2 million volts. The percent increase in its mass is:

(a) 20,000 (b) 2,000 (c) 200 (d) 20

30. A circle of radius 5m lies at rest in $x - y$ plane in the laboratory. For an observer moving with a uniform velocity $\upsilon$ along the y direciton, the circle appears to be an ellipse with an equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

The speed of the observer in terms of the velocity of light c is:

(a) 9c/25 (b) 3c/5 (c) 4c/5 (d) 16c/25

31. What is the proper time interval between the occurence of two events if in one inertial frame the events are separated by $7.5\times10^8$ m and occur 6.5 s apart?

(a) 6.50 s (b) 6.00 s (c) 5.75 s (d) 5.00 s

32. The relativistic form of Newton's second law of motion is

(a) $F = \frac{mc}{\sqrt{c^2 - v^2}}\frac{dv}{dt}$ (b) $F = \frac{m\sqrt{c^2 - v^2}}{c}\frac{dv}{dt}$ (c) $F = \frac{mc^2}{c^2 - v^2}\frac{dv}{dt}$ (d) $F = m\frac{c^2 - v^2}{c^2}\frac{dv}{dt}$

33. A light beam is propagating through a block of glass with index of refraction $n$. If the glass is moving at constant velocity $v$ in the same direction as the beam, the velocity of the light in the glass block as measured by an observer in the laboratory is approximately

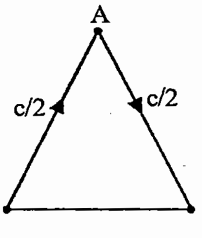
(a) $u = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)$ (b) $u = \frac{c}{n} - v\left(1 - \frac{1}{n^2}\right)$

(c) $u = \frac{c}{n} + v\left(1 + \frac{1}{n^2}\right)$ (d) $u = \frac{c}{n}$

34. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of $c$, the speed of light in vacuum)

(a) $\sqrt{3}c/2$ (b) $3c/4$ (c) $\sqrt{3/5}c$ (d) $c/\sqrt{2}$

35. In the laboratory frame, two observers A and B are moving along the sides of an equilateral triangle with equal speeds c/2, as shown in the figure



The speed of B as measured by A will be

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{4}{3\sqrt{3}}c$ (c) $\frac{\sqrt{13}}{7}c$ (d) $\frac{\sqrt{5}}{3}c$

36. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is

(a) $c$ (b) $-c$ (c) $0.15c$ (d) $-0.15c$

37. If a Higgs boson of mass $m_H$ with a speed $\beta = \frac{v}{c}$ decays into a pair photons, then the invariant mass of the photon pair is

[**Note:** The invariant mass of a system of two particles, with four-momenta $p_1$ and $p_2$ is $(p_1 + p_2)^2$]

(a) $\beta m_H$ (b) $m_H$ (c) $m_H/\sqrt{1-\beta^2}$ (d) $\beta m_H/\sqrt{1-\beta^2}$

38. A light source is switched on and of at a constant frequency $f$. An observer moving with a velocity $u$ with respect to the light source will observe the freuqency of the switching to be

(a) $f\left(1 - \frac{u^2}{c^2}\right)^{-1}$ (b) $f\left(1 - \frac{u^2}{c^2}\right)^{-1/2}$ (c) $f\left(1 - \frac{u^2}{c^2}\right)$ (d) $f\left(1 - \frac{u^2}{c^2}\right)^{1/2}$

39. Which one of the following quantities is invariant under Lorentz transformation?

(a) Charge density (b) Charge (c) Current (d) Electric field

40. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is $5\times10^{-8}$s, then the proper half-life is _____$\times10^{-8}$ s. ($c = 3\times10^{8}$ m/s)

# Answer Key: Classical Mechanics

## [Special Theory of Relativity]

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (c) | 4. (c) | 5. (c) | 6. (b) | 7. (d) |
| 8. (c) | 9. (b) | 10. (c) | 11. (a) | 12. (a) | 13. (d) | 14. (b) |
| 15. (b) | 16. (a) | 17. (d) | 18. (c) | 19. (c) | 20. (d) | 21. (c) |
| 22. (d) | 23. (b) | 24. (c) | 25. (d) | 26. (a) | 27. (c) | 28. (c) |
| 29. (d) | 30. (c) | 31. (b) | 32. (a) | 33. (a) | 34. (a) | 35. (d) |
| 36. (b) | 37. (b) | 38. (d) | 39. (b) | 40. (2.1 to 2.3) | | |

## CSIR-UGC-NET/JRF | GATE PHYSICS

## Assignment-5: Classical Mechanics (Central Forces)

1. In Kepler's problem of planetary motion, the orbits depend on the various values of the eccentricity (e) and energy (E). What values of e and E belong to a hyperbolic orbit?
(a) $E>0, e=1$ (b) $E<0, e=1$ (c) $E>0, e=0$ (d) $E>0, e>1$

2. The vector force corresponding to a potential energy $V=kr^3$ is given by
(a) $-3k\,r\,\vec{r}$ (b) $-3k\,r^2\,\vec{r}$ (c) $3k\,r^2\,\vec{r}$ (d) $3k\,r\,\vec{r}$

3. A particle of mass m moves in a central potential $V(r)=Kr^3$ in a circle of radius 'a' its kinetic energy is:
[✓ (a) $\frac{3Km\,a^3}{2}$] (b) $\frac{Km\,a^3}{2}$ (c) $\frac{3Km\,a^2}{2}$ (d) $\frac{Km\,a^2}{2}$

4. Motion of a particle in spherically symmetric attractive potential is:
(a) Always bounded though not in circular orbits
(b) Always in circular orbits and bounded
(c) May be bounded or unbounded depending upon the energy and angular momentum of the particle.
(d) Not bounded at all.

5. The central forces, which support closed orbits for bound particles, are given by the following, where k>0.
(a) $f(r)=\frac{k}{r^{(3-n)}}, n=1 \text{ and } 4$ (b) $f(r)=-\frac{k}{r^{3-n}}, n=1 \text{ to } 4$
(c) $f(r)=-\frac{k}{r^{3-n}}, n=1 \text{ and } 4$ (d) $f(r)=\frac{k}{r^{1-n}}, n=1 \text{ and } 4$

6. Two heavenly bodies of masses 6 and 4 units move under the influence of that mutual central force separating out the centre of mass motion. The resultant one-body motion corresponds to a particle of mass
(a) 10 units (b) 2.4 units (c) 6 units (d) 4 units.

7. Potential energy $V=kr^n$ then the relation between K.E. & P.E. is
(a) $\langle T\rangle=\langle V\rangle$ (b) $\langle T\rangle=\frac{n}{2}\langle V\rangle$ (c) $\langle T\rangle=\frac{3}{2}\langle V\rangle$ (d) $\langle T\rangle=2\langle V\rangle$

8. If a planet sweeps in equal area in equal time then the following orbit is represented.
(a) Circular orbit (b) Elliptical (c) Hyperbola (d) Parabola.

9. A particle moves under a central force field with the total energy E and the angular momentum $\vec{L}$. For this particle,
(a) E is conserved but $\vec{L}$ is not conserved. (b) E is not conserved but $\vec{L}$ is conserved.
(c) both E and $\vec{L}$ are conserved. (d) neither E nor $\vec{L}$ is conserved.

10. A space station moving in a circular orbit around the Earth goes into a new bound orbit by firing its engine radially outwards. The orbit is:
(a) A large circle (b) A smaller circle (c) An ellipse (d) A parabola

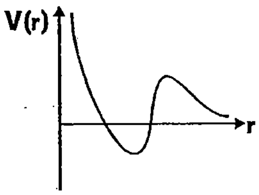
11. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is:
(a) Circular (b) Elliptical (c) Parabolic (d) Hyperbolic

12. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where $\alpha$ is a positive constant. In a stationary state, the expectation value of the kinetic energy $\langle T\rangle$ of the particle is:
(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = 2\langle V\rangle$ (c) $\langle T\rangle = 3\langle V\rangle$ (d) $\langle T\rangle = 4\langle V\rangle$

13. Consider two particles with position vectors $\bar{r}_1$ and $\bar{r}_2$. The force exerted by particle 2 on particle 1 is, $\bar{f}(\bar{r}_1, \bar{r}_2) = (\dot{r}_2 - \dot{r}_1)(\bar{r}_2 - \bar{r}_1)$. The force is:
(a) Central and conservative (b) Non-central and conservative
(c) Central and non-conservative (d) Non-central and non-conservative.

14. A particle moves in a central force field $\bar{f} = -kr^n\,\hat{r}$, where k is a constant, r, the distance of the particle from the origin and $\hat{r}$ is the unit vector in the direction of position vector $\bar{r}$. Closed stable orbits are possible for:
(a) $n = 1$ and $n = 2$ (b) $n = 1$ and $n = -1$ (c) $n = 2$ and $n = -2$ (d) $n = 1$ and $n = -2$

15. A particle moves in the central potential V(r) shown in the figure below
Which of the following statements is true?
(a) Both a stable circular orbit and an unstable circular orbit are possible.
(b) Only a stable circular orbit is possible.
(c) Only an unstable circular orbit is possible.
(d) No circular orbit is possible.

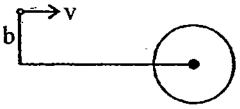


16. A particle of mass m is located at a distance z along the axis of a uniform disk of mass M and radius R. The gravitational force felt by the mass m is given by

(a) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2+R^2)^{1/2}} - 1\right)$ (b) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2-R^2)^{1/2}} - 1\right)$

(c) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2+R^2)^{1/2}} + 1\right)$ (d) $\dfrac{2GMm}{R^2}\left(\dfrac{z}{(z^2+R^2)^{1/2}}\right)$

17. A small asteroid is approaching a massive star with a speed v, from a very large distance, at an impact parameter b as shown below.



If the mass of the star is M and its radius is R, then what is the minimum value of b such that the asteroid will miss the star?
(a) $R \times \sqrt{1 + 2GM/v^2R}$ (b) $R / \sqrt{1 - 2GM/v^2R}$
(c) R (d) R/2

18. Two gravitating bodies A and B with masses $m_A$ and $m_B$, respectively, are moving in circular orbit. Assume that $m_B >> m_A$ and let the radius of the orbit of body A be $R_A$. If the body A is losing mass adiabatically, its orbital radius $R_A$ is proportional to

(a) $1/m_A$ (b) $1/m_A^2$ (c) $m_A$ (d) $m_A^2$

19. Consider a spherical planet, rotating about an axis passing through its centre. The velocity of a point on its equator is $v_{eq}$. If the acceleration due to gravity 'g' measured at the equator is half value of 'g' measured at one of the poles, then the escape velocity for a particle shot upwards from that pole will be

(a) $v_{eq}/2$ (b) $v_{eq}/\sqrt{2}$ (c) $\sqrt{2}\,v_{eq}$ (d) $2v_{eq}$

20. Two particles of identical mass move in circular orbits under a central potential $V(r)=\frac{1}{2}kr^2$. Let $\ell_1$ and $\ell_2$ be the angular momenta and $r_1, r_2$ be the radii of the orbits respectively. If $\ell_1/\ell_2 = 2$, the value of $r_1/r_2$ is:

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$ (c) $2$ (d) $1/2$

21. Consider a particle with a finite angular momentum $l$ moving in a central field defined by a linear restoring force. The minimum allowed energy $E_{min}$ scales as a function of $l$ as

(a) $E_{min} \propto \frac{1}{\ell}$ (b) $E_{min} \propto \sqrt{\ell}$ (c) $E_{min} \propto \ell$ (d) $E_{min} \propto \frac{1}{\sqrt{\ell}}$

22. With what angular velocity $\omega$ should a spherical planet of mass M and radius R rotate so that the effective gravitational acceleration on its surface at the equator is zero?

(a) $\omega = \sqrt{\frac{GM}{R^3}}$ (b) $\omega = \frac{GM}{R^3}$ (c) $\omega = \frac{\sqrt{GM}}{R}$ (d) $\omega = \frac{\sqrt{GM}}{R^2}$

23. In a central force potential $V(r) = -k/r$, the equation for the orbit of a particle of mass m and angular momentum $l$ is given by $r(\theta) = \frac{\ell^2/mk}{1+\sqrt{1+\frac{2E\ell^2}{mk^2}}(\cos\theta)}$. If R is the radius of a circular orbit, the distance d at minimum approach for parabolic orbit is

(a) $d = 2R$ (b) $d = \frac{R}{2}$ (c) $d = \frac{R}{\sqrt{2}}$ (d) $d = \frac{R}{1+\sqrt{2}}$

24. An object moving in an unknown potential filed $V(\vec{r})$ executes a closed orbit confined to a plane. It sweeps out the equal areas in equal time intervals. From this, one can conclude that

(a) $V(\vec{r})$ is a central potential (b) The orbit must be elliptical

(c) $V(\vec{r}) \alpha \frac{1}{|\vec{r}|}$ (d) $V(\vec{r}) = V(\theta, \varphi)$

25. For a signal particle under a central force which varies as $r^2$, the average kinetic energy T and average potential energy V obey the relation

(a) T=V (b) $T = \frac{3}{2}V$ (c) $T = \frac{1}{2}V$ (d) T=2V

26. In a central force field the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by $\frac{1}{r}=\frac{m}{L^2}(1+\varepsilon\cos\theta)$

Where, $\varepsilon$ is the eccentricity of the particle's motion. Which one of the following choices for $\varepsilon$ gives rise to a parabolic trajectory?

(a) $\varepsilon=0$ (b) $\varepsilon=1$ (c) $0<\varepsilon<1$ (d) $\varepsilon>1$

27.✓ A Planet orbits a massive star in a highly elliptical orbit. i.e., the total orbital energy E is close to zero. The initial distance of closest approach is $R_0$. energy is dissipated through tidal motions until the orbit is circularized with a final radius of $R_f$. Assume that orbital angular momentum is conserved during the circularization process. Then

(a) $R_f=R_0/2$ (b) $R_f=R_0$ (c) $R_f=\sqrt{2}R_0$ (d) $R_f=2R_0$

28. A binary system consists of two stars of equal mass m orbiting each in a circular orbit under the influence of gravitational forces. The period of the orbit is $\tau$. At $t=0$, the motion is stopped and the stars are allowed to fall towards each other. After what time t, expressed in terms of $\tau$ do they collide?

The following integral may be useful $\left(x=r^{1/2}\right)$, $\int\frac{x^2dx}{\sqrt{a-x^2}}=-\frac{x}{2}\sqrt{a-x^2}+\frac{a}{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

(a) $\sqrt{2}\tau$ (b) $\frac{\tau}{\sqrt{2}}$ (c) $\frac{\tau}{2\sqrt{2}}$ (d) $\frac{\tau}{4\sqrt{2}}$

29. A binary star system consists of two stars $S_1$ and $S_2$, with masses m and 2m respectively separated by a distamnce 'r'. If both $S_1$ and $S_2$ individually follow circular orbits around the centre of the mass with intantaneous speeds $v_1$ and $v_2$ respectively, the ratio of speeds $v_1/v_2$ is:

(a) $\sqrt{2}$ (b) 1 (c) 1/2 (d) 2

30. A planet of mass 'm' moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axes of the orbit are 'a' and 'b' respectively, the angular momentum of the planet is:

(a) $\sqrt{2GMm^2(a+b)}$ (b) $\sqrt{2GMm^2(a-b)}$ (c) $\sqrt{\frac{2GMm^2ab}{(a-b)}}$ (d) $\sqrt{\frac{2GMm^2ab}{(a+b)}}$

31. A planet of mass m moves in the inverse square central force field of the Sun of mass M. If the semi-major and semi-minor axes of the orbit are a and b, respectively, the total energy of the planet is:

(a) $-\frac{GMm}{2a}$ (b) $-GMm\left(\frac{1}{a}+\frac{1}{b}\right)$ (c) $-\frac{GMm}{a}\left(\frac{1}{b}-\frac{1}{a}\right)$ (d) $-GMm\frac{(a-b)}{(a+b)^2}$

32. A planet of mass m and an angular momentum L moves in a circular orbit in a potential, $V(r)=-k/r$, where k is a constant. If it is slightly perturbed radially, the angular frequency of radial oscillations is

(a) $mk^2/\sqrt{2}L^3$ (b) $mk^2/L^3$ (c) $\sqrt{2}mk^2/L^3$ (d) $\sqrt{3}mk^2/L^3$

33. If, in a Kepler potential, the pericentre distance of a particle in a parabolic orbit is $r_p$ while the radius of the circular orbit with the same angular momentum is $r_c$, then

(a) $r_c=2r_p$ (b) $r_c=r_p$ (c) $2r_c=r_p$ (d) $r_c=\sqrt{2}r_p$

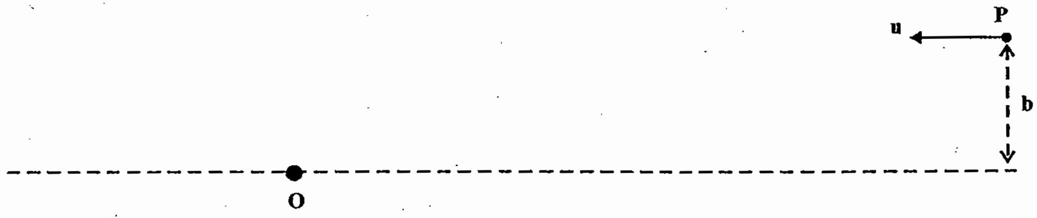
34. A planet of mass 'm' moves in a circular orbit of radius $r_0$ in the gravitational potential $V(r) = -\frac{k}{r}$ where 'k' is a positive constant. The orbital angular momentum of the planet is

(a) $2r_0km$ (b) $\sqrt{2r_0km}$ (c) $r_0km$ (d) $\sqrt{r_0km}$

35. A particle P of mass m moves under the influence of a central potential, centred at the origin O, of the form

$$V(r) = -\frac{k}{3r^3}$$

where 'k' is a positive constant.



If the particle P comes in from infinity with initial velocity 'u' and impact parameter b(see figure), then the largest value of b for which the particle gets captured by the potential is

(a) $\left(\frac{3k^2}{m^2u^4}\right)^{1/6}$ (b) $\left(\frac{k}{3mu}\right)^{1/3}$ (c) $\left(\frac{2k^2}{m^2u^4}\right)^{1/6}$ (d) $\left(\frac{2k}{3mu}\right)^{1/3}$

## Answer Key: Classical Mechanics (Central Forces)

1. (d) 2. (a) 3. (a) 4. (a) 5. (c) 6. (b) 7. (b)
8. (b) 9. (a) 10. (c) 11. (d) 12. (a) 13. (d) 14. (d)
15. (a) 16. (a) 17. (b) 18. (b) 19. (c) 20. (a) 21. (c)
22. (a) 23. (b) 24. (b) 25. (b) 26. (b) 27. (d) 28. (d)
29. (d) 30. (d) 31. (a) 32. (b) 33. (a) 34. (d)
35. (a)

## CSIR-UGC-NET/JRF | GATE PHYSICS

## Assignment-6: Classical Mechanics (Small Oscillations)

1. The total energy E of a particle of mass m executing small oscillations about the origin along on the x-direction is given by, $E = \frac{1}{2}mv^2 + V_0 \cosh\left(\frac{x}{L}\right)$, where $V_0$ and L are positive constants. The time period T of oscillation is

(a) $T = \frac{1}{2\pi}\sqrt{\frac{m}{V_0}}$ (b) $T = 2\pi\sqrt{\frac{L}{m}}$ (c) $T = \pi L\sqrt{\frac{m}{E}}$ (d) $T = 2\pi\sqrt{\frac{mL^2}{V_0}}$

2. A particle moves in one dimension in a potential $V(x) = x^2/2 + x^3/3$, in suitable units. If E is the total energy of the particle, then the motion is:

(a) Always bounded if $0 \le E \le \frac{1}{6}$ (b) Always unbounded if $0 \le E \le \frac{1}{6}$

(c) Always bounded if $E < 0$

(d) Bounded if $0 \le E \le \frac{1}{6}$ and the initial position satisfies $-1 < x(0) < \frac{1}{2}$

3. Two simple pendula each of length L and mass m are suspended from a horizontal support. If they are coupled by a massless spring of spring constnat k, then the higher frequency of oscillation of the system is:

(a) $\omega = \sqrt{\frac{g}{2L} + \frac{2k}{m}}$ (b) $\omega = \sqrt{\frac{2g}{L} + \frac{2k}{m}}$ (c) $\omega = \sqrt{\frac{g}{L} + \frac{k}{m}}$ (d) $\omega = \sqrt{\frac{g}{L} + \frac{2k}{m}}$

4. In the free vibration of a linear triatomic molecule, the central atom takes part in the oscillatory motion for
(a) All three modes (b) Two modes
(c) One mode only (d) Does not take part at all

5. In a normal mode of vibration of a molecule, all atoms move
(a) With the same frequency (b) With the same amplitude
(c) In the same direction (d) Opposite to each other.

6. A column of mercury of total length L partially fills a U tube. If the mercury is set into small oscillation, its frequency is given by

(a) $2\pi\sqrt{\frac{2g}{L}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2g}{L}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{g}{L}}$ (d) $2\pi\sqrt{\frac{g}{L}}$

7. The equation of motion of a simple pendulum is
(a) Linear for small and large amplitude oscillations.
(b) Nonlinear for small and large amplitude oscillations.
(c) Linear for small amplitude and nonlinear for large amplitude oscillations.
(d) Non linear for small amplitude and linear for large amplitude oscillations.

8. A linear symmetric triatomic molecule consisting of masses 1, 2 and 1 units connected by springs of unit force constant admits normal modes of following nonzero normal frequencies:

(a) 1, 1 (b) 1, 2 (c) $1, \sqrt{2}$ (d) $1, \sqrt{3}$

9. For a system of 12 degrees of freedom, the number of nonzero frequency normal modes are

(a) 12 (b) 10 (c) 9 (d) 6

10. Consider two simple pendulums each consisting of a point mass m attached to a string of length $\ell$. The masses are coupled by a spring of spring constant k. The kinetic energy T and the potential energy V of the system are

given by $T = m\ell^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $V = m\ell^2\begin{pmatrix} \frac{g}{\ell}+\frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{\ell}+\frac{k}{m} \end{pmatrix}$

The frequencies of small oscillation of this system are

(a) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{g/\ell + 2k/m}$ (b) $\omega_1 = \sqrt{2k/m}, \omega_2 = \sqrt{g/\ell + 2k/m}$

(c) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{k/m}$ (d) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{g/\ell + k/m}$

**Statement for Linked Answer Q.11 and Q.12:**

The Lagrangian of two coupled oscillators of mass m each is:

$$L = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right) + m\omega_0^2\mu x_1 x_2$$

11. The equations of motion are:

(a) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_2$ (b) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_2, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_1$

(c) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_2$ (d) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_1$

12. The normal modes of the system are:

(a) $\omega_0\sqrt{\mu^2 - 1}, \omega_0\sqrt{\mu^2 + 1}$ (b) $\omega_0\sqrt{1 - \mu^2}, \omega_0\sqrt{1 + \mu^2}$

(c) $\omega_0\sqrt{\mu - 1}, \omega_0\sqrt{\mu + 1}$ (d) $\omega_0\sqrt{1 - \mu}, \omega_0\sqrt{1 + \mu}$

13. A particle of mass m is moving in a potential of the form $V(x, y, z) = (1/2)\, m\omega^2\,(3x^2 + 3y^2 + 2z^2 + 2xy)$. The oscillation frequencies of the three normal modes of the particle are given by:

(a) $\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$ (b) $\sqrt{2}\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$ (c) $\sqrt{2}\omega, \sqrt{2}\omega$ and $2\omega$ (d) $\sqrt{2}\omega, 2\omega$ and $2\omega$

14. The Lagrangian for a three particle system is given by $L = \frac{1}{2}\left(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2\right) - a^2\left(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3\right)$,

where a is real. Then one of the normal cooridnates has a frequency $\omega$ given by:

(a) $\omega^2 = a^2$ (b) $\omega^2 = \frac{a^2}{2}$ (c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}a^2$

15. The motion of a simple pendulum undergoing large oscillations can be described as:

(a) harmonic, non-conservative (b) harmonic, consrevative

(c) anharmonic, non-conservative (d) anharmonic, conservative

16. A loop of radius R is pivoted at a point on the circumference. The period of small oscillations in the plane of the hoop is

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{4g}}$ (c) $2\pi\sqrt{\frac{R}{g}}$ (d) $2\pi\sqrt{\frac{9R}{7g}}$

17. A simple pendulum is in the form of a unit mass at the end of a massless rigid rod of unit length. If $\theta$ is the angle with the downward vertical, the kinetic energy T of the pendulum is given by

(a) $T = 2g\sin^2 \theta/2$ (b) $T = 2g\cos^2 \theta/2$ (c) $T = g\sin^2 \theta$ (d) $T = g\cos^2 \theta$

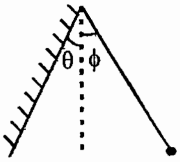
18. A particle moves in one dimension in a potential $V(x) = x^2/2 + x^3/3$, in suitable units. If E is the total energy of the particle, then the motion is

(a) Always bounded if $0 \le E \le \frac{1}{6}$ (b) Always unbounded if $0 \le E \le \frac{1}{6}$

(c) Always bounded if $E < 0$

(d) Bounded if $0 \le E \le \frac{1}{6}$ and the initial position satisfies $-1 < x(0) < \frac{1}{2}$

19. A simple pendulum of length $l$ is suspended from a hook mounted on a slanted wall. The wall makes a small angle $\theta$ with the vertical. The pendulum is displaced from the vertical by a small angle $\phi\ (\phi > \theta)$ and released. Assuming that the collision of the bob is elastic, the time period of oscillation is:



(a) $2\pi\sqrt{\frac{\ell}{g}}$ (b) $2\sqrt{\frac{\ell}{g}}\left[\pi/2+\sin^{-1}\frac{\theta}{\phi}\right]$ (c) $2\pi\sqrt{\frac{\ell}{g}\frac{\theta}{\phi}}$ (d) $2\pi\sqrt{\frac{\ell}{g}}\left[\pi/2+\cos^{-1}\frac{\theta}{\phi}\right]$

20. A point particle of mass m carrying an electric charge $q$ is attached to a spring of stiffness constant $k$. A constant electric field $E$ along the direction of the spring is switched on for a time interval T (where $T \ll \sqrt{m/k}$). Neglecting radiation loss, the amplitude of oscillation after the field is switched off is:

(a) $qE/k$ (b) $qET^2/m$ (c) $qE\sqrt{m}/Tk^{3/2}$ (d) $qET/\sqrt{mk}$

21. Consider a simple pendulum executing small oscillations (in vacuum) with a time period $T_0$. If a charge +q is pasted onto the pendulum bob, then the resultant (modified) pendulum

(a) continues to execute non- sinusoidal small oscillations with the same time period $T_0$.

(b) executes damped oscillation (c) executes sinusoidal oscillatoions with $T > T_0$.

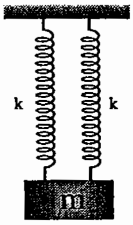
(d) executes sinusoidal oscillations with $T < T_0$.

22 Two object each of mass m are connected by a spring of spring constant k and are capable of moving in a line. The normal modes have the frequencies.

(a) $\sqrt{\frac{2k}{m}}$ and 0 (b) $\sqrt{\frac{k}{m}}$ and 0 (c) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}$ and 0

23. Consider a mass m attached to a spring . When the spring is extended from its equilibrium position, the restoring force is given by $F = -k_1x - k_2x^3$. Where $k_1$ and $k_2$ are positive constants. The spring is now extended slightly and let go. Which of the following is true?
(a) The mass executes sinusoidal oscillations
(b) The mass executes periodic, but non sinusoidal oscillations
(c) The mass executes a damped oscillatory motion
(d) the total energy of the system decreases with time.

24. Consider a bar of mass m attached to two identical massless springs (of spring constant k) in parallel, as shown in the figure, The bar would oscillate with a frequency.



(a) $\sqrt{k/m}$ (b) $\sqrt{\frac{2k}{m}}$ (c) $\sqrt{\frac{k}{2m}}$ (d) $2\sqrt{k/m}$

25. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{3\pi}{2}$ (d) $2\pi$

26. For small angular displacements (i.e, $\sin\theta \approx \theta$) a simple pendulum oscillates harmonically. For larger displacements. the motion
(a) becomes aperiodic
(b) remains periodic with the same period
(c) remains periodic with a higher period
(d) remains period with a lower period.

27. Consider the motion of a classical particle in a one dimensional double-well potential $V(x) = \frac{1}{4}(x^2 - 2)^2$.
If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then
(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$
(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$
(c) the particle will switch between the right and left wells
(d) the particle will approach the bottom of the right well and settle there

28. A solid cylinder of height H, radius R and density $\rho$, floats vertically on the surface of a liquid of density $\rho_0$. The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

(a) $\frac{\rho g}{\rho_0 H}$ (b) $\frac{\rho}{\rho_0}\sqrt{\frac{g}{H}}$ (c) $\sqrt{\frac{\rho g}{\rho_0 H}}$ (d) $\sqrt{\frac{\rho_0 g}{\rho H}}$

29. Consider small oscillations of a triatomic molecule. Which of the following is true?
(a) There cannot be any mode of zero frequency.
(b) There may not be any mode of zero frequency.
(c) There must be mode of zero frequency
(d) There must be a mode of imaginary frequency.

# Answer Key: Classical Mechanics (Small Oscillation)

1. (d) 2. (b) 3. (d) 4. (c) 5. (b) 6. (b) 7. (a)
8. (c) 9. (d) 10. (a) 11. (b) 12. (d) 13. (b) 14. (d)
15. (d) 16. (a) 17. (a) 18. (d) 19. (b) 20. (d) 21. (a)
22. (a) 23. (b) 24. (b) 25. (b) 26. (b) 27. (b) 28. (d)
29. (c)

## CSIR-UGC-NET/JRF | GATE PHYSICS

### Assignment-7: Classical Mechanics (Rigid Body)

1. The motion of a particle of mass m is described in a non-inertial frame of reference that is rotating with a uniform angular velocity $\omega$. If $r$ denotes the position of the particle in the non-inertial frame

(a) The centrifugal force on the particle is $-m\omega^2\dot{r}$

(b) The centrifugal force on the particle is $-m\omega\times(\omega\times r)$

(c) The Coriolis force on the particle is $-m\left(\omega\times\frac{dr}{dt}\right)$

(d) The Coriolis force on the particle $-2m\omega^2 r$

2. An asymmetric rigid body has three distinct principal moments of inertia, with $I_x < I_y < I_z$. If we consider rotation with uniform angular velocity about the x, y and z axes, respectively, then the motion is:

(a) Stable about the y-axis, but not about the x and z axes.
(b) Stable about the x and y axes, but not about the z-axis.
(c) Stable about the y and z axes, but not about the x-axis.
(d) Stable about the x and z axes, but not about the y-axis.

3. A particle having angular momentum L is placed in a rotating frame having angular velocity $\vec{\omega}$. It will have lowest energy when

(a) L is parallel to $\vec{\omega}$
(b) L is anti-parallel to $\vec{\omega}$
(c) L and $\vec{\omega}$ are perpendicular to each other.
(d) L oscillates from parallel to anti parallel.

4. An N-particle system is being acted upon by external forces. The applied torque is zero but the total external force is not. Let $\bar{P}$ be the total linear momentum and $\bar{L}$ be the total angular momentum. Then which of the following statement is true?

(a) There is no conserved quantity
(b) $\bar{P}$ is conserved.
(c) $\bar{L}$ is conserved.
(d) Both $\bar{P}$ and $\bar{L}$ are conserved.

5. A particle of mass m is free to slide along a thin rod. The rod rotates with a constant angular velocity $\omega$ about a fixed axis passing through one end of the rod. The angle between the rod and the axis of rotation remains fixed. The number of generalized coordinates required to describe the motion is

(a) 1
(b) 2
(c) 3
(d) 4

6. For torque free motion of a symmetric body of moment of inertia tensor $\vec{I}$ and angular velocity $\vec{\omega}$ can be described by the Euler's equations given in

(a)
$$I_{12}\dot{\omega}_1 = (I_{11} - I_{33})\omega_1\omega_3$$
$$I_{13}\dot{\omega}_2 = (I_{33} - I_{22})\omega_3\omega_2$$
$$I_{23}\dot{\omega}_3 = (I_{22} - I_{11})\omega_1\omega_2$$

(b)
$$I_{11}\dot{\omega}_1 = (I_{12} - I_{13})\omega_2\omega_3$$
$$I_{22}\dot{\omega}_2 = (I_{32} - I_{21})\omega_3\omega_1$$
$$I_{33}\dot{\omega}_3 = (I_{31} - I_{23})\omega_1\omega_3$$

(c) $I_{11}\dot{\omega}_1 = (I_{22} - I_{33})\omega_2\omega_3$

(d) $I_{11}\dot{\omega}_1 = (I_{12} - I_{33})\omega_2\omega_3$

$$I_{22}\dot{\omega}_2 = (I_{33} - I_{11})\omega_3\omega_1 \qquad\qquad I_{22}\dot{\omega}_2 = (I_{23} - I_{11})\omega_3\omega_1$$

$$I_{33}\dot{\omega}_3 = (I_{11} - I_{22})\omega_1\omega_2 \qquad\qquad I_{33}\dot{\omega}_3 = (I_{13} - I_{22})\omega_1\omega_2$$

7. A system of four particles is in x-y plane. Of these, two particles each of mass m are located at (1, 1) and (1, −1). The remaining two particles each of mass 2m are located at (1, 1) and (−1, −1). The xy-component of the moment of inertia tensor of this system of particles is:

(a) 10 m (b) −10 m (c) 2m (d) −2m

8. A particle of mass M moving in a straight line with speed υ collides with a stationary particle of the same mass. In the center of mass coordinate system, the first particle is deflected by 90°. The speed of the second particle, after collision, in the laboratory system will be:

(a) $v/\sqrt{2}$ (b) $\sqrt{2}v$ (c) v (d) v/2

9. A rod of twisting couple C per unit twist connects two identical spheres of moment of inertia I each (about the length of the rod). This dumbell shapped arrangement is placed on a horizontal frictionless table and is given a twist causing the spheres to execute simple harmonic motion. The time period of oscillation is:

(a) $2\pi\sqrt{(I/2C)}$ (b) $2\pi\sqrt{(2I/C)}$ (c) $2\pi\sqrt{(4I/C)}$ (d) $2\pi\sqrt{(I/4C)}$

-Gate

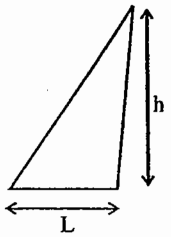
10. The moment of inertia tensor of a rigid body is given by $I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$. The magnitude of the moment of inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is:

(a) 6 (b) 5 (c) 2 (d) 8/3

11. A rigid body is rotating about its centre of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is $\vec{L}$, the rate of change of its kinetic energy is

(a) $\frac{1}{2}\vec{\tau}.\vec{\omega}$ (b) $\frac{1}{2}\vec{L}.\vec{\omega}$ (c) $\frac{1}{2}\left(\vec{\tau}.\vec{\omega} + \vec{L}.\vec{\alpha}\right)$ (d) $\frac{1}{2}\vec{L}.\vec{\alpha}$

12. A triangle of uniform mass density of base L and height h is shown below. The centre of mass of the triangle lies at this distance above the base:



(a) $\frac{h}{2}$ (b) $\frac{2h}{2}$ (c) $\frac{h}{3}$ (d) $\frac{h}{6}$

13. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given by $I_1, I_2$ and $I_3$ is rotating freely about a fixed point inside the body. The Euler equations for the components of its angular velocity $(\omega_1, \omega_2, \omega_3)$ are $\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\,\omega_3,\ \dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\,\omega_3,\ \dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_1\,\omega_2$

**A.** The equilibrium points in $(\omega_1, \omega_2, \omega_3)$ space are

(a) $(1,-1,0), (-1,0,1)$ and $(0,-1,1)$ (b) $(1,1,0), (1,0,1)$ and $(0,1,1)$

(c) $(1,0,0), (0,1,0)$ and $(0,0,1)$ (d) $(1,1,1), (-1,-1,-1)$ and $(0,0,0)$

**B.** The constants of motion are

(a) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1\omega_1 + I_2\omega_2 + I_3\omega_3$ (b) $I_1\omega_1^2 + I_2 w_2^2 + I_3\omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

(c) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $\omega_1 + \omega_2 + \omega_3$ (d) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

14. An annulus of mass M made of a material of uniform density has inner and outer radii 'a' and 'b' respectively. Its principal momnet of inertia along the axis of symmetry perpendicular to the plane of the annulus is :

(a) $\frac{1}{2}M\frac{(b^4 + a^4)}{(b^2 - a^2)}$ (b) $\frac{1}{2}M\pi(b^2 - a^2)$ (c) $\frac{1}{2}M(b^2 - a^2)$ (d) $\frac{1}{2}M(b^2 + a^2)$

15. The principal moments of inertia of a rigid body of an arbitrary shape are given by, $I_1, I_2$ and $I_3$. which of the following represents a valid set of values for $I_i$ (in some arbitrary units)?

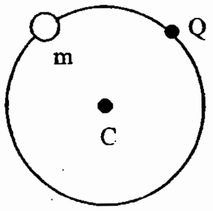
(a) $I_1 = -3, I_2 = 4, I_3 = 1$ (b) $I_1 = 3, I_2 = 3, I_3 = 8$

(c) $I_1 = 3, I_2 = 7, I_3 = 4$ (d) $I_1 = 0, I_2 = 0, I_3 = 1$

16. A circus motorcyclist rides on the inner surface of a large globe of radius R along a vertical longitude. In order that he stays on the innner surface, his minimum speed should be

(a) $2\sqrt{Rg}$ (b) $\sqrt{5Rg}$ (c) $\pi\sqrt{5Rg}$ (d) $2\pi\sqrt{Rg}$

17. A point mass m is moving along a circle of radius r and centred at a point C. Q represents a fixed point on the circle (see figure). If $\omega_C = 2\omega_Q$ be the instantaneous angular velocity of the mass around the points C and Q respectively, then



(a) $\omega_Q = 2\omega_C$ (b) $\omega_Q = \omega_C$ (c) $\omega_Q = \frac{\omega_C}{2}$ (d) $\omega_Q$ and $\omega_C$ are independent

18. Two uniform thin rods of equal length L, and masses $M_1$ and $M_2$ are joined together along the length. The moment of inertia of the combined rod of length 2L about an axis passing through the mid-point perpendicular to the length of the rod is,

(a) $(M_1 + M_2)\frac{L^2}{12}$ (b) $(M_1 + M_2)\frac{L^2}{6}$ (c) $(M_1 + M_2)\frac{L^2}{3}$ (d) $(M_1 + M_2)\frac{L^2}{2}$

19. A horizontal circular platform rotates with a constant angular velocity $\Omega$ directed vertically upwards. A person seated at the centre shoots a bullet of mass 'm' horizontally with speed 'v'. The acceleration of the bullet, in the reference frame of the shooter, is

(a) $2v\Omega$ to his right (b) $2v\Omega$ to his left

(c) $v\Omega$ to his right (d) $v\Omega$ to his left

20. Two bodies of equal mass 'm' are connected by a massless rigid rod of length '$\ell$' lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency $\omega$, its angular momentum is

(a) $m\ell^2\omega/4$ (b) $m\ell^2\omega/2$ (c) $m\ell^2\omega$ (d) $2m\ell^2\omega$

21. A jet of gas consists of molecules of mass m, speed v and number density n all moving co-linearly. This jet hits a wall at an angle $\theta$ to the normal. The pressure exerted on the wall by the jet assuming elastic collision will be

(a) $p = 2mnv^2\cos^2\theta$ (b) $p = 2mnv^2\cos\theta$

(c) $p = \sqrt{(3/2)}mnv\cos^2\theta$ (d) $p = mnv^2$

22. The number of degrees of freedom of a rigid body in d space-dimensions is

(a) 2d (b) 6 (c) d(d + 1)/2 (d) d!

23. A particle of mass m is at the stable equilibrium position of its potential energy $V(x) = ax - bx^3$ where a, b are positive constants. The minimum velocity that has to be imparted to the particle to render its motion unstable is

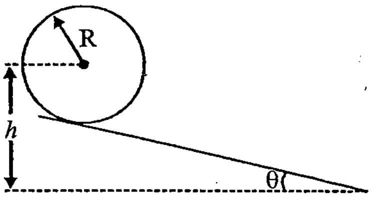
(a) $(64a^3 / 9m^2b)^{1/4}$ (b) $(64a^3 / 27m^2b)^{1/4}$

(c) $(16a^3 / 27m^2b)^{1/4}$ (d) $(3a^3 / 64m^2b)^{1/4}$

24. A uniform circular disk of radius $R$ and mass $M$ is rotating with angular speed $\omega$ about an axis, passing through its center and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is,

(a) $\frac{\sqrt{3}}{4}\omega MR^2$ (b) $\frac{\sqrt{3}}{8}\omega MR^2$ (c) $\frac{\sqrt{7}}{8}\omega MR^2$ (d) $\frac{\sqrt{7}}{4}\omega MR^2$

25. A ring of mass $m$ and radius $R$ rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height $h$, the angular velocity when the ring reaches the base is



(a) $\sqrt{g/(h-R)}\tan\theta$ (b) $\sqrt{g/(h-R)}$

(c) $\sqrt{g(h-R)/R^2}$ (d) $\sqrt{2g/(h-R)}$

26. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length $l$ attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

(a) $2\pi\sqrt{\dfrac{l+R}{g}}$

(b) $\dfrac{2\pi}{\sqrt{g}}(l^2+R^2)^{1/4}$

(c) $2\pi\sqrt{\dfrac{2R^2+2Rl+l^2}{g(R+l)}}$

(d) $\dfrac{2\pi}{\sqrt{g}}(2R^2+2Rl+l^2)^{1/4}$

## Answer Key: Classical Mechanics (Rigid Body)

1. (b) 2. (d) 3. (b) 4. (d) 5. (b) 6. (c) 7. (c)
8. (c) 9. (a) 10. (b) 11. (a) 12. (c) 13_A(c) 13_B(d)
14. (d) 15. (c) 16. (a) 17. (d) 18. (c) 19. (b) 20. (b)
21. (b) 22. (c) 23. (b) 24. (c) 25. (c) 26. (c)

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## Classical Mechanics (Course Structure)

### Syllabus:

**Part A:** Newton's laws. Dynamical systems, Phase space dynamics, stability analysis. Central force motions.Two body Collisions - scattering in laboratory and Centre of mass frames. Rigid body dynamics, momentof inertia tensor. Non-inertial frames and pseudo forces

**Part B:** Special theory of relativity: Lorentz transformations, relativistic kinematics and mass–energy equivalence.

**Part C:** Variational principle. Generalized Coordinates. Lagrangian and Hamiltonian formalism and equations of motion. Conservation laws and Cyclic coordinates. Symmetry, invariance and Noether's theorem. Poisson brackets and canonical transformations. Periodic motion: Small oscillations, normal modes.

### Lecture plan:

- We have divided the syllabus of **Classical Mechanics** into three parts which we plan to cover sequentially starting with part A.
- You will be given objective problems sheets on every topic.
- We will also give you topic wise assignment sheets.
- After completion of each part (A,B,C) there will be a TEST.
- Some unannounced (surprise) quizzes will also be taken.

**Physical Science Paper:** Max Marks: 200 (All MCQ): Part A (No. of question 20, to be answered 15, each question carries 2 marks), Part B (No. of question 25, to be answered 20, each question carries 3.5 marks), Part C (No. of question 30, to be answered 20, each question carries 5 marks). **–Ve marking:** 25%.

Questions asked from this topic in previous exams.

| Year | 2012 June | 2012 Dec | 2013 June | 2013 Dec | 2014 June |
|---|---|---|---|---|---|
| Marks | 5x3.5+4x5=37.5 | 4x3.5+4x5=34 | 4x3.5+3x5=29 | 3×3.5+3×5=25.5 | 3×3.5+3×5=25.5 |

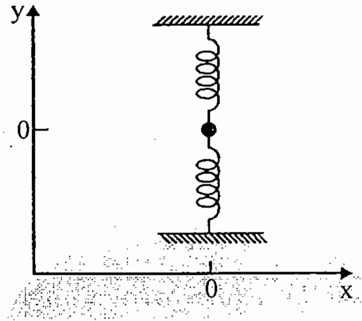
Every year weight-age of this topic has been about 15%. Therefore we plan to cover this topic very well.

**Reference Books:** (in class notes we cover all the topics of syllabus however if required one may refer to following books)

1. Classical Mechanics : David Morin
2. Classical Mechanics: J.C Upadhyaya or Gupta Kumar Sharma
3. Classical Mechanics: Rana & Joag (TMH) or Kiran C. Gupta
4. Concept of Physics Vol.1: H C Verma (for fundamentals) (for parts A and B)
5. Mechanics: Kleppner and Kolenkov: (for parts A & B)

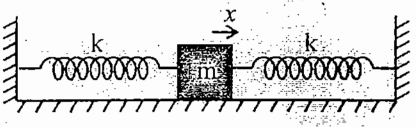
## CSIR-UGC-NET/JRF-GATE-Physical Sciences
## Assignment-1: Classical Mechanics

1. Consider a particle of mass $m$ attached to two identical springs each of length $l$ and spring constant k (see the figure below). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations?



[✓ (a) $m\ddot{x}+\frac{kx^3}{l^2}=0$] (b) $m\ddot{x}+kx=0$ (c) $m\ddot{x}+2kx=0$ (d) $m\ddot{x}+\frac{kx^2}{l}=0$

2. In the figure shown equation of motion of block which is displaced horizontally is

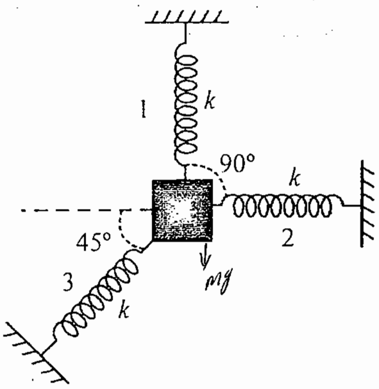


(a) $\ddot{x}+\frac{k}{m}x=0$ (b) $\ddot{x}+\frac{k}{2m}x=0$

(c) $\ddot{x}-\frac{k}{2m}x=0$ [✓ (d) $\ddot{x}+\frac{2k}{m}x=0$]

3. In the figure shown the block is given a small displacement '$x$' along the spring 3. Equation of motion of the block is (Assume that system is lying on a table)

$-kx\sin\theta$



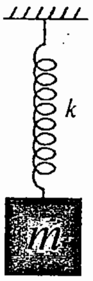
(a) $\ddot{x}+\left(1+\sqrt{2}\right)\frac{k}{m}x=0$

(b) $\ddot{x}+\left(\sqrt{2}-1\right)\frac{k}{m}x=0$

(c) $\ddot{x}+\frac{2k}{m}x=0$

(d) $\ddot{x}+\frac{3k}{\sqrt{2}m}x=0$

4. In the figure shown if the block is displaced from equilibrium position by a distance '$x$' its equation of motion will be



(a) $\ddot{x}-\frac{k}{m}x-g=0$ (b) $\ddot{x}+\frac{k}{m}x+g=0$

[✓ (c) $\ddot{x}+\frac{k}{m}x=0$] (d) $\ddot{x}+\frac{k}{m}x-g=0$

5. In previous question if $y$ be the elongation in spring at any instant of time then in terms of $y$ equation of motion can be written as

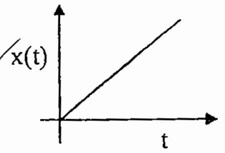
(a) $\ddot{y}-\frac{k}{m}y-g=0$ (b) $\ddot{y}+\frac{k}{m}y+g=0$ (c) $\ddot{y}-\frac{k}{m}y+g=0$ [✓ (d)] $\ddot{y}+\frac{k}{m}y=0$

6. Spherical particles of a given material of density $p$ are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, i.e, $F_d=6\pi\eta Rv$, where $\eta$ is the viscosity of the medium, R the radius of a particle and v its instantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass $m$ to reach half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is

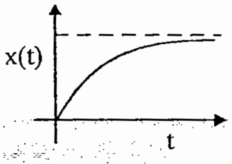
(a) 8 (b) 1/8 [✓ (c)] 4 (d) 1/4

7. A particle is observed after it has been moving for a long time under the influence of a constant force in a medium that applies a drag force proportional to the square of its velocity. Distance versus time graph made by the observer will look like.

[✓ (a)]



(b)



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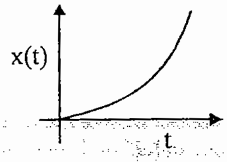
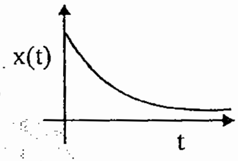


(d)



8. A body of mass 'm' falls from rest at a height 'h' under gravity (acceleration due to gravity g) through a dense medium which provides a resistive force $F=-kv^2$, where 'k' is a constant and 'v' is the speed. It will hit the ground with a kinetic energy

(a) $\frac{m^2g}{2k}\exp\left(-\frac{2kh}{m}\right)$ (b) $\frac{m^2g}{2k}\tanh\left(\frac{2kh}{m}\right)$

(c) $\frac{m^2g}{2k}\left\{1+\exp\left(-\frac{2kh}{m}\right)\right\}$ [✓ (d)] $\frac{m^2g}{2k}\left\{1-\exp\left(-\frac{2kh}{m}\right)\right\}$

9. A particle of mass m is thrown upward with velocity v and there is retarding air resistance proportional to the square of the velocity with proportonality constank k. If the particle attains a maximum hight after time t, and g is the gravitaltional acceleration, acceleration, what is the velocity v?

(a) $\sqrt{\frac{k}{g}}\tan\left(\sqrt{\frac{g}{k}}t\right)$ (b) $\sqrt{gk}\tan\left(\sqrt{\frac{g}{k}}t\right)$ [✓ (c)] $\sqrt{\frac{g}{k}}\tan\left(\sqrt{gk}t\right)$ (d) $\sqrt{gk}\tan\left(\sqrt{gk}t\right)$

10. The free fall time of a test mass on an object of mass M from a hight 2R to R is

[✓ (a)] $(\pi/2+1)\sqrt{\frac{R^3}{GM}}$ (b) $\sqrt{\frac{R^3}{GM}}$ (c) $(\pi/2)\sqrt{\frac{R^3}{GM}}$ (d) $\pi\sqrt{\frac{2R^3}{GM}}$

$$\int_0^{t_f}\sqrt{\frac{GM}{R}}\,dt = -\int_{2R}^{R}\sqrt{\frac{x}{2R-x}}\,dx$$

## PHYSICAL SCIENCES

## Basic Questions_Newton's Laws

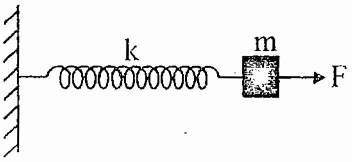
1. A particle of mass 2kg moves along a straight line such that the position coordinate of the particle depends on time as $x = 2t^3 - 3t^2 + 4t$ meter, force on the particle at $t = 1$ sec is
(a) 6N [✓ (b) 12N] (c) 8N (d) 4N

2. Position vector of particle of mass 3kg varies with time as $\vec{r} = t^2\hat{i} - 2t^3\hat{j} + \hat{k}$. Force acting on the particle at the instant when acceleration is perpendicular to its velocity is
(a) 6N (b) 3N (c) 2N (d) 0N

3. For a spring mass system or simple pendulum which of the statement is correct regarding acceleration
(a) It is maximum at mean position (b) It is maximum extreme positions
(c) It same every where (d) It is never zero.

4. A block of mass 5kg is lifted with a rope which can bear maximum tension of 100N. Maximum possible acceleration of block is
(a) 20 m/s$^2$ (b) 10 m/s$^2$ (c) 2 m/s$^2$ (d) 5 m/s$^2$

5. In the figure shown the block is pulled with a constant force F. What is speed of the the block at the instant when acceleration is zero.

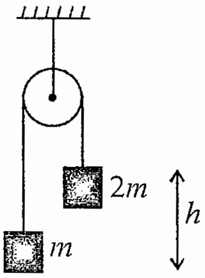


(a) $\frac{F}{\sqrt{mk}}$ (b) $\frac{2F}{\sqrt{mk}}$

(c) $\frac{F}{\sqrt{2mk}}$ (d) $\frac{F}{2\sqrt{mk}}$

6. In the previous question maximum elongation of spring is
(a) $\frac{F}{k}$ (b) $\frac{2F}{k}$ (c) $\frac{F}{2k}$ (d) $\frac{F}{4k}$

7. In the figure shown the two blocks are released from the position shown. After what time the two will cross each other. [Assume pulley and string to be light and smooth]

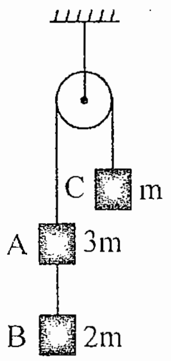


(a) $\sqrt{\frac{3h}{g}}$ (b) $\sqrt{\frac{6h}{g}}$

(c) $\sqrt{\frac{3h}{2g}}$ (d) $\sqrt{\frac{2h}{3g}}$

8. In previous question what is speed of each block at the instant they cross each other.
(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{2h}{3}}$ (c) $\sqrt{\frac{2gh}{3}}$ (d) $\sqrt{3gh}$

9. In the figure shown what is tension in the string between block A and B



(a) $\frac{mg}{3}$

(b) $2mg$

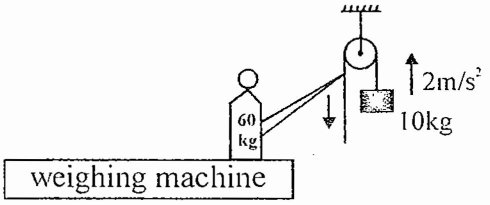
(c) $5mg$

(d) $\frac{2mg}{3}$

10. A projectile is thrown at some angle. What is tangential acceleration of projectile at the instant its velocity vector makes 30° with its acceleration vector.

(a) $\frac{g}{2}$ (b) $\frac{\sqrt{3}g}{2}$ (c) g (d) $-\frac{\sqrt{3}g}{2}$

11. A person standing on a weighing machine as shown in the figure pulls the string attached to a block in vertically downward direction. What is reading shown by the machine.



(a) 40 kg (b) 38 kg (c) 80 kg (d) 100 kg

12. A block slides down an inclined plane of inclination $\theta$. If coefficient of friction between the block and the plane is $\mu$, acceleration of the block is

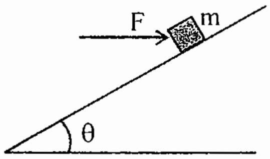
(a) $g\sin\theta$ (b) $g(\sin\theta-\mu\cos\theta)$

(c) $g(\sin\theta+\mu\cos\theta)$ (d) $g(\cos\theta-\mu\sin\theta)$

13. A block of thrown up an inclined plane due to which it slides upto a certain height and then returns to its initial position. If $t_1$ and $t_2$ be time for ascent and descent and $\mu$ and $\theta$ be coefficient of friction and inclination of plane. Then value of $t_1/t_2$ is

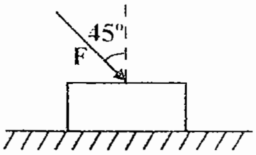
(a) $\frac{\sin\theta+\mu\cos\theta}{\sin\theta-\mu\cos\theta}$ (b) $\sqrt{\frac{\sin\theta+\mu\cos\theta}{\sin\theta-\mu\cos\theta}}$ (c) $\sqrt{\frac{\sin\theta-\mu\cos\theta}{\sin\theta+\mu\cos\theta}}$ (d) $\frac{\sin\theta-\mu\cos\theta}{\sin\theta+\mu\cos\theta}$

14. In the figure shown coefficient of friction between the block and inclined plane is $\mu$. What should be value of F so that block slides up the plane with constant speed.



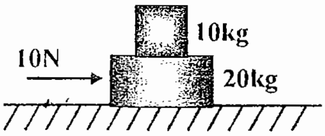
(a) $mg\tan\theta$ (b) $\frac{mg(\sin\theta-\mu\cos\theta)}{(\cos\theta+\mu\sin\theta)}$ (c) $\frac{mg(\sin\theta+\mu\cos\theta)}{(\cos\theta-\mu\sin\theta)}$ (d) $\frac{mg(\cos\theta-\mu\cos\theta)}{(\sin\theta+\mu\cos\theta)}$

15. A 2kg block is lying on a rough surface. If coefficient of static friction between the block and ground is 0.2. What maximum force can be applied on the block without moving it



(a) 5N (b) 7N (c) 9N (d) 11N

16. In the figure shown coefficient of static friction between the blocks is 0.5 while ground is smooth. Which of the following statement is true



(a) Friction force on 10 kg block is 50 N

(b) Friction force on 10 kg block is $\frac{10}{3}N$

(c) The blocks do not move on the floor

(d) 10 kg block slides on 20 kg block

17. In previous question what maximum force can be applied on the lower block so that their is no relative motion between the blocks

(a) 50N (b) 100N (c) 150 N (d) 200 N

18. What would be answer of previous question if force is applied to the upper block

(a) 50 N (b) 75N (c) 125N (d) 250 N

19. A block is given an initial speed 20 m/s on a rough horizontal surface where coefficient of friction is 0.2. For how long the block moves?

(a) 1 sec (b) 2 sec (c) 0.5 sec (d) 0.2 sec

20. A uniform rope of length L is hanging off the edge of a rough table having coefficient of static friction μ. What should be minimum length of hanging part so that the rope starts sliding down.

(a) $\frac{\mu L}{2}$ (b) $\frac{\mu L}{\mu+1}$ (c) $\left(\frac{1}{\mu}-1\right)L$ (d) $\frac{\mu L}{2-\mu}$

## Answer Key (Basic Questions_Newton's Laws

| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | b | a | b | b | a | b | a | b | d | b |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Ans. | b | b | c | c | b | b | c | b | a | b |

## PHYSICAL SCIENCES

### Assignment - 3: Classical Mechanics (Newton's Laws)

**1.** A horizontal force $F = \beta t$ is applied on an object of mass $m$ which is initially at rest on a smooth horizontal surface. Distance moved by object in time $t_0$ is

(a) $\frac{\beta t_0^3}{2m}$ (b) $\frac{\beta t_0^3}{3m}$ (c) $\frac{\beta t_0^3}{6m}$ (d) $\frac{\beta t_0^3}{4m}$

**2.** A spring mass system (spring constant K and mass $m$) lies on a smooth horizontal surface with one end of spring being rigidly fixed. At $t = 0$ the mass is pulled with a constant horizontal force F. Speed of the mass after time $t$ is

(a) $\frac{Ft}{m}$ (b) $\frac{F}{\sqrt{Km}}\sin\left(\sqrt{\frac{K}{m}}t\right)$

(c) $\frac{2F}{\sqrt{Km}}\sin\left(\sqrt{\frac{K}{m}}t\right)$ (d) $\frac{F}{\sqrt{Km}}\cos\left(\sqrt{\frac{K}{m}}t\right)$

**3.** A simple pendulum is deflected aside to a position where string makes 60° with downward vertical and released. Tension in the string when bob passes from lowest position is (mass of bob = $m$)

(a) $mg$ (b) $\frac{mg}{2}$ (c) $2mg$ (d) $\frac{3mg}{2}$

**4.** Tension in the string of simple pendulum is 5mg at the lowest position where $m$ is mass of bob. What is tension in the string when string is horizontal.

(a) $mg$ (b) $2mg$ (c) $3mg$ (d) $4mg$

**5.** Two particles of masses $m_1, m_2$ and charges $q_1, q_2$ are placed $r_0$ distance apart on a smooth horizontal surface. Due to electrostatic repulsion they move away from each other. Ratio of their kinetic energy at a later time is

(a) $\left(\frac{m_1}{m_2}\right)^2$ (b) $\frac{m_2}{m_1}$ (c) $\frac{q_2}{q_1}$ (d) $\left(\frac{q_2}{q_1}\right)^2$

**6.** In previous question speed of $m_1$ when their separation becomes $2r_0$ is

(a) $\sqrt{\frac{2Gm_2^2}{(m_1+m_2)r_0}}$ (b) $\sqrt{\frac{q_1q_2 \cdot m_1}{4\pi \epsilon_0 r_0 m_2(m_1+m_2)}}$

(c) $\sqrt{\frac{q_1q_2m_2}{4\pi \epsilon_0 r_0 m_1(m_1+m_2)}}$ (d) $\sqrt{\frac{q_1q_2}{4\pi \epsilon_0 r_0(m_1+m_2)}}$

7. A small object of mass $m$ falls from a height equal to radius of earth $R_e$. If $M_e$ be mass of earth, time taken by the particle to reach the earth's surface is (take $m << M_e$)

(a) $\left(\frac{\pi}{2}+1\right)\sqrt{\frac{2R_e^3}{GM_e}}$

(b) $\left(\frac{\pi}{2}+1\right)\sqrt{\frac{R_e^3}{GM_e}}$

(c) $\frac{\pi}{2}\sqrt{\frac{R_e^3}{GM}}$

(d) $\sqrt{\frac{2R_e^3}{GM}}$

8. In previous question if $m = M_e$, then time of fall will be

(a) $\left(\frac{\pi}{2}+1\right)\sqrt{\frac{R_e^3}{2GM_e}}$

(b) $\left(\frac{\pi}{2}+1\right)\sqrt{\frac{2R_e^3}{GM_e}}$

(c) $\frac{\pi}{2}\sqrt{\frac{R_e^3}{GM}}$

(d) $\sqrt{\frac{2R_e^3}{GM}}$

9. In question No. 8, what is speed of the object when it strikes the earth's surface.

(a) $\sqrt{\frac{GM_e}{R_e}}$ (b) $\sqrt{\frac{2GM_e}{R_e}}$ (c) $\sqrt{\frac{3GM_e}{2R_e}}$ (d) $\sqrt{\frac{GM_e}{2R_e}}$

0. A point particle of mass $m$ carrying an electric charge $q$ is attached to a spring of stiffness constant $K$. A constant electric field $E$ along the direction of spring is switched on for a time interval $T$ (where $T << \sqrt{\frac{m}{K}}$). Neglecting radiation loss, the amplitude of oscillation after the field is switched off is

(a) $\frac{qE}{K}$ (b) $\frac{qET^2}{m}$ (c) $\frac{qE\sqrt{m}}{TK^{3/2}}$ (d) $\frac{qET}{\sqrt{mK}}$

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## PHYSICAL SCIENCES (NET-JRF/GATE)

## Assignment-4: Classical Mechanics

1. A particle of mass 'm' is moving in a potential

$$V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$$

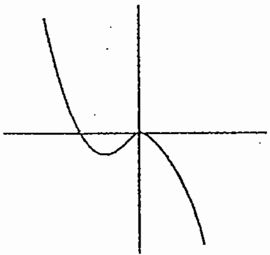
What is minimum potential energy of the particle.

(a) $\omega_0\sqrt{a}$ (b) $2\omega_0\sqrt{a}$ (c) $\frac{\omega_0\sqrt{a}}{2}$ (d) $2\omega_0 a$

2. In the previous question what is frequency of oscillation about stable point?

(a) $\sqrt{2}\,\omega_0$ (b) $2\omega_0$ (c) $4\omega_0$ (d) $4\sqrt{2}\,\omega_0$

3. A particle of mass $\frac{2}{3}kg$ is subjected to a potential $V(x)=3x^2-2x^3$. Which of the following $V(x)$ versus $x$ plot is correct?

(a)


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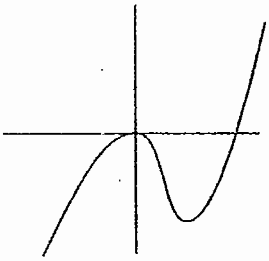

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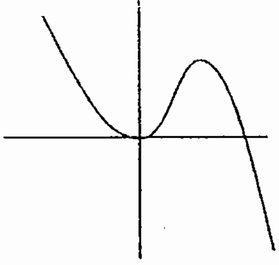

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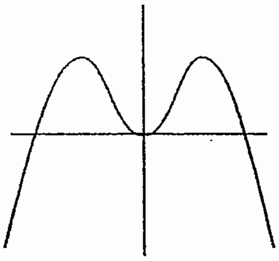

4. In the previous question if particle is released at $x=\frac{4}{3}$, what is its velocity when it reaches point $x=\frac{3}{2}$

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{2}$

5. In **Q.3**, what is maximum value of potential energy.

(a) 0.2 (b) 1 (c) 2 (d) 0

6. A particle of mass 'm=4' moves along the $x$-axis under the influence of the potential $V(x)=2\left(e^{-\frac{3}{2}x}-2e^{-\frac{3}{4}x}\right)$, if the particle oscillates with small amplitude around the minimum potential, what is the period of the oscillation.

(a) 0.12 (b) 1.33 (c) 8.37 (d) 11.17

7. Force acting on a particle along $x$-axis is given to be $F(x)=2bx^3-ax$. If the particle is released from a point $x=-\sqrt{\dfrac{a}{3b}}$ it will move towards which of the following points.

(a) 0 (b) $+\sqrt{\dfrac{a}{2b}}$ (c) $-\sqrt{\dfrac{a}{2b}}$ (d) $-\sqrt{\dfrac{a}{b}}$

8. A particle of mass $m$ is moving in one dimension so that its potential energy is $U(x)=A-Bx+Cx^3$. When particle is at the unstable equilibrium position, it is slightly displaced. What is its speed when it reaches the stable equilibrium position?

(a) $\left[\dfrac{4B^3}{27m^2C}\right]^{1/4}$ (b) $\left[\dfrac{64B^3}{27m^2C}\right]^{1/4}$ (c) $\left[\dfrac{4B^3}{9m^2C}\right]^{1/4}$ (d) $\left[\dfrac{16B^3}{9m^2C}\right]^{1/4}$

9. Consider the motion of a classical particle in a one dimensional double-well potential $V(x)=\dfrac{1}{4}(x^2-2)^2$. If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then

(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega=\sqrt{2}$

(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega=2$

(c) the particle will switch between the right and left wells

(d) the particle will approach the bottom of the right well and settle there

10. The total energy E of a particle of mass m executing small oscillations about the origin along on the x-direction is given by, $E=\dfrac{1}{2}mv^2+V_0\cosh\left(\dfrac{x}{L}\right)$, where $V_0$ and L are positive constants. The time period T of oscillation is

(a) $T=\dfrac{1}{2\pi}\sqrt{\dfrac{m}{V_0}}$ (b) $T=2\pi\sqrt{\dfrac{L}{m}}$ (c) $T=\pi L\sqrt{\dfrac{m}{E}}$ (d) $T=2\pi\sqrt{\dfrac{mL^2}{V_0}}$

11. A particle of mass 'm' is moving under a one dimensional potential $V(x)=ax^3-bx$. Due to the force acting on the particle its kinetic energy changes as the particle moves from one point to the other. What can be maximum change in K.E. of the particle in this case.

(a) $\sqrt{\dfrac{16b^3}{3a}}$ (b) $\sqrt{\dfrac{8b^3}{3a}}$ (c) $\sqrt{\dfrac{4b^3}{3a}}$ (d) $\sqrt{\dfrac{8b^3}{3a}}$

12. A particle of mass 2 kg is released at $x=0m$. The particle is acted upon by a force whose potential is expressed as $V(x)=2x-4x^3$ joule. The force on the particle at the point where it will again come to rest for the first ime is

(a) -4 newton (b) +4 newton (c) zero newton (d) +2 newton

13. A particle is placed in a region with the potential $U(x)=\frac{1}{2}kx^2-\frac{\lambda}{3}x^3$, where $k, \lambda > 0$. Then,

(a) $x=0$ and $x=\frac{k}{\lambda}$ are points of stable equilibrium

(b) $x=0$ is a point of stable equilibrium and $x=\frac{k}{\lambda}$ is a point of unstable equilibrium.

(c) $x=0$ and $x=\frac{k}{\lambda}$ are points of unstable equilibrium.

(d) There are no points of stable or unstable equilibrium.

14. A particle of mass 'm' is moving under potential $V(x)=ax^3-bx$, frequency of oscillation about the stable equilibrium position is

(a) $\left[\frac{12ab}{m^2}\right]^{1/4}$  (b) $\left[\frac{6ab}{m^2}\right]^{1/4}$  (c) $\left[\frac{4ab}{m^2}\right]^{1/4}$  (d) $\left[\frac{3ab}{m^2}\right]^{1/4}$

15. In the previous question force on the particle is maximum at

(a) $x=0$  (b) $x=\sqrt{\frac{b}{a}}$  (c) $x=\sqrt{\frac{b}{3a}}$  (d) $x=-\sqrt{\frac{b}{3a}}$

## Answer Key

### Assignment - 3: Classical Mechanics (Newton's Laws)

**1.** (c)  **2.** (b)  **3.** (c)  **4.** (b)  **5.** (b)
**6.** (c)  **7.** (b)  **8.** (a)  **9.** (d)  **10.** (d)

### Assignment- 4: Classical Mechanics

**1. (a)**  **2. (b)**  **3. (c)**  **4. (b)**  **5. (b)**
**6. (c)**  **7. (a)**  **8. (b)**  **9. (b)**  **10. (d)**
**11. (a)**  **12. (b)**  **13. (b)**  **14. (a)**  **15. (a)**

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## PHYSICAL SCIENCES (NET-JRF/GATE)

## Assignment-4: Classical Mechanics

1. A particle of mass 'm' is moving in a potential

$$V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$$

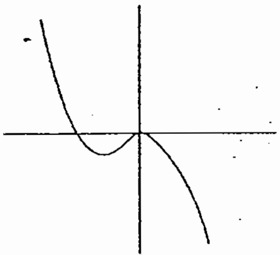
What is minimum potential energy of the particle.

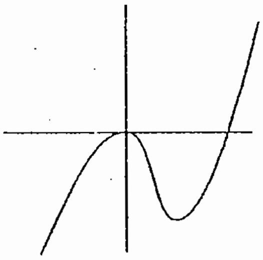
(a) $\omega_0\sqrt{a}$ (b) $2\omega_0\sqrt{a}$ (c) $\frac{\omega_0\sqrt{a}}{2}$ (d) $2\omega_0 a$

2. In the previous question what is frequency of oscillation about stable point?

(a) $\sqrt{2}\,\omega_0$ (b) $2\omega_0$ (c) $4\omega_0$ (d) $4\sqrt{2}\,\omega_0$

3. A particle of mass $\frac{2}{3}kg$ is subjected to a potential $V(x)=3x^2-2x^3$. Which of the following $V(x)$ versus $x$ plot is correct?

(a)


(b)


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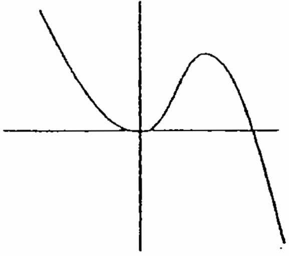

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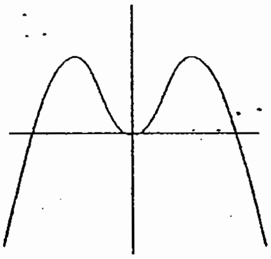


4. In the previous question if particle is released at $x=\frac{4}{3}$, what is its velocity when it reaches point $x=\frac{3}{2}$

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{2}$ (put Conservation of energy)

5. In Q.3, what is maximum value of potential energy.

(a) 0.2 (b) 1 (c) 2 (d) 0

6. A particle of mass 'm=4' moves along the $x$-axis under the influence of the potential $V(x)=2\left(e^{-\frac{3}{2}x}-2e^{-\frac{3}{4}x}\right)$, if the particle oscillates with small amplitude around the minimum potential, what is the period of the oscillation.

(a) 0.12 (b) 1.33 [✓ (c) 8.37] (d) 11.17

7. Force acting on a particle along $x$-axis is given to be $F(x) = 2bx^3 - ax$. If the particle is released from a point $x = -\sqrt{\frac{a}{3b}}$ it will move towards which of the following points.

(a) 0 (b) $+\sqrt{\frac{a}{2b}}$ (c) $-\sqrt{\frac{a}{2b}}$ (d) $-\sqrt{\frac{a}{b}}$

8. A particle of mass $m$ is moving in one dimension so that its potential energy is $U(x) = A - Bx + Cx^3$. When particle is at the unstable equilibrium position, it is slightly displaced. What is its speed when it reaches the stable equilibrium position?

(a) $\left[\frac{4B^3}{27m^2C}\right]^{1/4}$ (b) $\left[\frac{64B^3}{27m^2C}\right]^{1/4}$ (c) $\left[\frac{4B^3}{9m^2C}\right]^{1/4}$ (d) $\left[\frac{16B^3}{9m^2C}\right]^{1/4}$

9. Consider the motion of a classical particle in a one dimensional double-well potential $V(x) = \frac{1}{4}(x^2 - 2)^2$. If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then

(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$.
(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$
(c) the particle will switch between the right and left wells
(d) the particle will approach the bottom of the right well and settle there

10. The total energy E of a particle of mass m executing small oscillations about the origin along on the x-direction is given by, $E = \frac{1}{2}mv^2 + V_0 \cosh\left(\frac{x}{L}\right)$, where $V_0$ and L are positive constants. The time period T of oscillation is

(a) $T = \frac{1}{2\pi}\sqrt{\frac{m}{V_0}}$ (b) $T = 2\pi\sqrt{\frac{L}{m}}$ (c) $T = \pi L\sqrt{\frac{m}{E}}$ (d) $T = 2\pi\sqrt{\frac{mL^2}{V_0}}$

11. A particle of mass 'm' is moving under a one dimensional potential $V(x) = ax^3 - bx$. Due to the force acting on the particle its kinetic energy changes as the particle moves from one point to the other. What can be maximum change in K.E. of the particle in this case.

(a) $\sqrt{\frac{16b^3}{3a}}$ (b) $\sqrt{\frac{8b^3}{3a}}$ (c) $\sqrt{\frac{4b^3}{3a}}$ (d) $\sqrt{\frac{8b^3}{3a}}$

12. A particle of mass 2 kg is released at $x = 0m$. The particle is acted upon by a force whose potential is expressed as $V(x) = 2x - 4x^3$ *joule*. The force on the particle at the point where it will again come to rest for the first time is

(a) -4 newton (b) +4 newton (c) zero newton (d) +2 newton

13. A particle is placed in a region with the potential $U(x)=\frac{1}{2}kx^2-\frac{\lambda}{3}x^3$, where $k, \lambda > 0$. Then,

(a) $x=0$ and $x=\frac{k}{\lambda}$ are points of stable equilibrium

(b) $x=0$ is a point of stable equilibrium and $x=\frac{k}{\lambda}$ is a point of unstable equilibrium.

(c) $x=0$ and $x=\frac{k}{\lambda}$ are points of unstable equilibrium.

(d) There are no points of stable or unstable equilibrium.

14. A particle of mass 'm' is moving under potential $V(x)=ax^3-bx$, frequency of oscillation about the stable equilibrium position is

(a) $\left[\frac{12ab}{m^2}\right]^{1/4}$ (b) $\left[\frac{6ab}{m^2}\right]^{1/4}$ (c) $\left[\frac{4ab}{m^2}\right]^{1/4}$ (d) $\left[\frac{3ab}{m^2}\right]^{1/4}$

15. In the previous question force on the particle is maximum at

(a) $x=0$ (b) $x=\sqrt{\frac{b}{a}}$ (c) $x=\sqrt{\frac{b}{3a}}$ (d) $x=-\sqrt{\frac{b}{3a}}$

## Answer Key

### Assignment - 3: Classical Mechanics (Newton's Laws)

| | | | | |
|---|---|---|---|---|
| 1. (c) | 2. (b) | 3. (c) | 4. (b) | 5. (b) |
| 6. (c) | 7. (b) | 8. (a) | 9. (d) | 10. (d) |

### Assignment- 4: Classical Mechanics

| | | | | |
|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (c) | 4. (b) | 5. (b) |
| 6. (c) | 7. (a) | 8. (b) | 9. (b) | 10. (d) |
| 11. (a) | 12. (b) | 13. (b) | 14. (a) | 15. (a) |

## CSIR-UGC-NET/JRF|GATE- PHYSICAL SCIENCES

### Assignment - 05 Centrifugal Coriolis Forces

1.✓ A person standing on a rotating disc sees that everything around him is rotating which of the following statements given by the person is correct.

(a) Both coriolis and centrifugal forces on all objects are radially outward

(b) Both coriolis and centrifugal forces on all objects are radially inward

(c) On all objects coriolis force is radially inward and centrifugal force is radially outward

(d) Centrifugal force and coriolis force are equal in magnitude but opposite direction.

2. A disc of radius R is rotating clockwise with angular speed $\omega$. An insect of mass $m$ crawls along the circumference in anticlockwise sense with a speed $v_0$ relative to the disc. What is centrifugal

(a) $m\omega^2 R$ radially outward

(b) $m\omega^2 R$ radially inward

(c) $m\omega^2 R$ tangential

(d) $m\omega^2 R$ parallel to the axis of rotation of disc

3. In the previous question coriolis force on the disc is

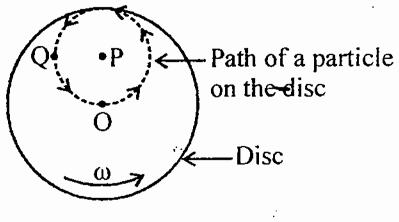
(a) $2m\omega v_0$ radially outward

(b) $2m\omega v_0$ radially inward

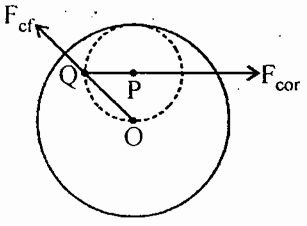
(c) $2m\omega v_0$ tangential

(d) $2m\omega v_0$ parallel to the axis of rotation

4. A circular disc is rotating in anticlockwise sense as shown in the figure. On the disc a particle moves in anticlockwise circle with center at P. At the instant particle at Q, which of the following options correctly represent directions of centrifugal and coriolis forces. ['O' is center of the disc]



(a)



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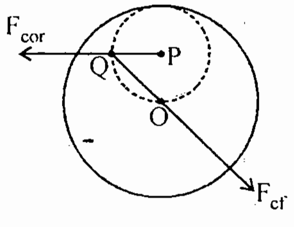

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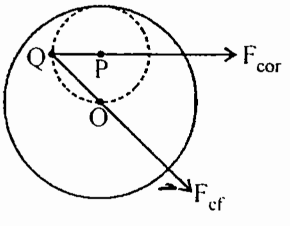
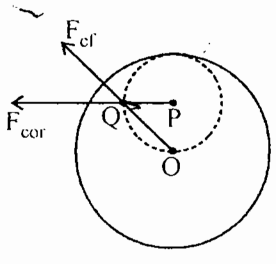


(d)



5. A particle of mass $m$ is lying on earth's surface at a location where latitude is $\lambda$. If $\omega$ be angular velocity of earth's spinning motion and R be the radius of the earth then, centrifugal force on the particle is

(a) $m\omega^2 R\sin\lambda$ (b) $M\omega^2 R\cos\lambda$

(c) $2m\omega^2 R\sin\lambda$ (d) $2m\omega^2 R\cos\lambda$

6. In the previous question if the particle starts moving along the latitude from east to west with constant speed $v_0$, coriolis force on the particle will be (magnitude)

(a) $2m\omega v_0$ (b) $2mv_0\omega\cos\lambda$

(c) $2mv_0\omega\sin\lambda$ (d) $2mv_0\omega\sin\lambda\cos\lambda$

7. A stone is dropped from a height 100m at equator. What is east-ward deviation of the particle (approximately)

(a) 3.15 cm (b) 2.17 cm (c) 1.08 cm (d) 2.73 cm

8. In the previous question if the location is at latitude 60° in northern hemisphere then eastward deviation will be

(a) 3.15 cm (b) 2.17 cm (c) 1.08 cm (d) 2.73 cm

9. If $\omega$ be angular velocity of the earth's spinning motion and R be the radius of the earth, with what speed should a train move along equator so that magnitude of centrifugal and coriolis forces and acting on the train become equal.

(a) $\omega R$ (b) $\frac{\omega R}{2}$ (c) $2\omega R$ (d) $\frac{2\omega R}{3}$

10. A person is running at equator with speed $\frac{\omega R}{2}$ where $\omega$ and $R$ have their usual meaning. In which direction should the person run so that his apparent weight is equal to his true weight.

(a) N to S (b) W to E (c) E to W (d) S to N

11.✓ Particle of mass 2kg is moving in north east direction on earth's surface at a location where latitude is 45° N. Find magnitude of coriolis force experience by the particle if its speed is 200 m/s.

(a) 0.05 N (b) 0.5 N (c) 5 N (d) 0.25 N

12. A stone is thrown in meridian plane at an angle $\phi$ with the horizontal. If $v_0$ is its speed and $m$ is mass of the stone, what is coriolis force on the stone at the moment it is thrown. Latitude of location is $\lambda$.

(a) $2m\omega v_0\sin(\lambda-\phi)$ (b) $2m\omega v_0\sin(\lambda+\phi)$

(c) $2m\omega v_0\cos(\lambda-\phi)$ (d) $2m\omega v_0\cos(\lambda+\phi)$

13. Angular velocity of a frame is $\vec{\omega}=(2\hat{i}+3\hat{j})$ rad/s. A particle of mass 2kg lies at a point whose co-ordinate in lab frame is (3, 4, 0). What is centrifugal force on the mass as seen by a person standing on the rotating frame.

(a) $2\sqrt{13}$ N (b) $2\sqrt{5}$ N (c) $2\sqrt{7}$ N (d) $2\sqrt{3}$ N

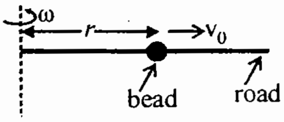
14. A body of mass $m$ starts sliding down from top of a smooth sphere of radius $R$. The sphere rotates with a constant angular velocity $\omega$ about a vertical axis passing through its center. What is the centrifugal force acting on the body at the moment it breaks off the surface as seen from the frame fixed to the sphere.

(a) zero (b) $\frac{2}{3}m\omega^2 R$ (c) $\frac{\sqrt{5}}{3}m\omega^2 R$ (d) $2m\omega^2 R$

**15.** In the previous question what is coriolis force on the body as seen from the frame fixed to the sphere at that moment.

(a) zero (b) $\frac{\sqrt{5}}{3}m\omega^2R$ (c) $\frac{2}{3}m\omega^2R$ (d) $\frac{2}{3}m\omega^2R\sqrt{5+\frac{8g}{3\omega^2R}}$

**16.** A thin rod is rotating with uniform angular velocity $\omega$ about its one end. A bead of mass $m$ slides along the length of rod with constant speed $v_0$ as shown in the figure. If there is no gravity in the space, what is normal reaction between the bead and the rod.

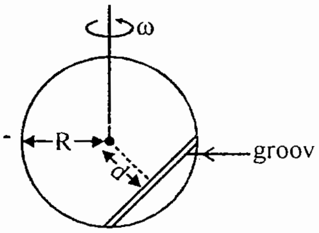


(a) zero (b) $m\omega^2r$ (c) $2m\omega v_0$ (d) $mg$

**17.** In the previous question what is friction force on the bead at the instant its distance from the axis of rotation is $r$.

(a) $2m\omega v_0$ (b) zero (c) $m\omega^2r$ (d) None of these

**18.** A circular disc is rotating about its axis with uniform angular velocity $\omega$. There is a groove along a chord of the disc. An insect starts crawling along the groove. If the insect starts at $t = 0$ from the middle point, and moves with constant speed $v_0$ w.r.t. the disc. What is centrifugal force on the insect at time $t$.



(a) $m\omega^2d$ (b) $m\omega^2v_0t$ (c) $m\omega^2(d+v_0t)$ (d) $m\omega^2\sqrt{d^2+v_0^2t^2}$

**19.** The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be

(a) 0° to the vertical and $\sqrt{3}T$ (b) 30° to the vertical and T/2

(c) 60° to the vertical and $T/\sqrt{2}$ (d) 0° to the vertical and $T/\sqrt{3}$

**20.** Two points N and S are located in the northern and southern hemisphere, respectively, on the same longitude. Projectiles P and Q are fired from N and S, respectively, towards each other. Which of the following options is correct for the projectiles as they approach the equator?

(a) Both P and Q will move towards the east
(b) Both P and Q will move towards the west
(c) P will move towards the east and Q towards the west
(d) P will move towards the west and Q towards the east

21.✓ An observer is sitting on a horizontal platform which is rotating with a constant angular velocity. He puts an object on the smooth frictionless floor of the platform, away from the axis of rotation, with zero initial velocity with respect to him. Let the time at this instant be t = 0. In the frame of the platform, the object would

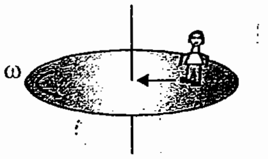
(a) Remain at rest for all t > 0

(b) Accelerate purely in a radial direction outwards for all t > 0.

(c) Accelerate purely in a tangential direction for all t > 0.

✓(d) Accelerate radially in the outward direction at t = 0, however the direction of acceleration changes for t >0.

22. A circular platform is rotating with a uniform angular speed $\omega$ counterclockwise about an axis passing through its centre and perpendicular to its plane as shown in the figure. A person of mass 'm' walks radially inward with a uniform speed 'v' on the platform. The magnitude and the direction of hte Corolis force (with respect to the direction along which the person walks) is:



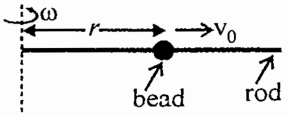
(a) $2m\omega v$ towards his left

(b) $2m\omega v$ towards his front.

(c) $2m\omega v$ towards his right

(d) $2m\omega v$ towards his back.

23. A thin rod is rotating with uniform angular velocity $\omega$ about its one end. A bead of mass $m$ slides along the length of rod with constant speed $v_0$ as shown in the figure. If there is no gravity in the space, what is coefficientof friction between the bead and the rod.



(a) $\frac{\omega r}{v_0}$ (b) $\frac{\omega r}{2v_0}$ (c) $\frac{v_0}{\omega r}$ (d) $\frac{2v_0}{\omega r}$

24. A horizontal circular platform rotates with a constant angular velocity $\Omega$ directed vertically upwards. A person seated at the centre shoots a bullet of mass 'm' horizontally with speed 'v'. The acceleration of the bullet, in the reference frame of the shooter, is

(a) $2v\Omega$ to his right

(b) $2v\Omega$ to his left

(c) $v\Omega$ to his right

(d) $v\Omega$ to his left

25. The motion of a particle of mass m is described in a non-inertial frame of reference that is rotating with a uniform angular velocity $\omega$. If $r$ denotes the position of the particle in the non-inertial frame

(a) The centrifugal force on the particle is $-m\omega^2\dot{r}$

(b) The centrifugal force on the particle is $-m\omega\times(\omega\times r)$

(c) The Coriolis force on the particle is $-m\left(\omega\times\frac{dr}{dt}\right)$

(d) The Coriolis force on the particle $-2m\omega^2 r$

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## CSIR-UGC-NET/JRF|GATE- PHYSICAL SCIENCES

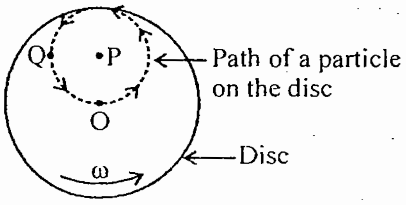
## Assignment - 05 Centrifugal Coriolis Forces

1. A person standing on a rotating disc sees that everything around him is rotating which of the following statements given by the person is correct.
   (a) Both coriolis and centrifugal forces on all objects are radially outward
   (b) Both coriolis and centrifugal forces on all objects are radially inward
   (c) On all objects coriolis force is radially inward and centrifugal force is radially outward
   (d) Centrifugal force and coriolis force are equal in magnitude but opposite direction.

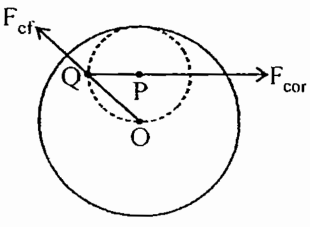
2. A disc of radius R is rotating clockwise with angular speed $\omega$. An insect of mass $m$ crawls along the circumference in anticlockwise sense with a speed $v_0$ relative to the disc. What is centrifugal
   (a) $m\omega^2 R$ radially outward
   (b) $m\omega^2 R$ radially inward
   (c) $m\omega^2 R$ tangential
   (d) $m\omega^2 R$ parallel to the axis of rotation of disc

3. In the previous question coriolis force on the disc is
   (a) $2m\omega v_0$ radially outward
   (b) $2m\omega v_0$ radially inward
   (c) $2m\omega v_0$ tangential
   (d) $2m\omega v_0$ parallel to the axis of rotation

4. A circular disc is rotating in anticlockwise sense as shown in the figure. On the disc a particle moves in anticlockwise circle with center at P. At the instant particle at Q, which of the following options correctly represent directions of centrifugal and coriolis forces. ['O' is center of the disc]



(a)



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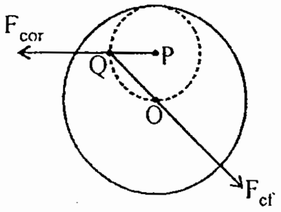

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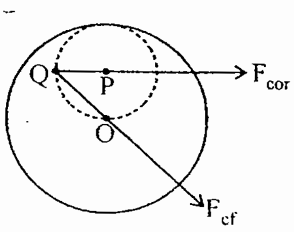
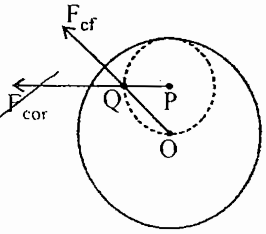


(d)



5. A particle of mass $m$ is lying on earth's surface at a location where latitude is $\lambda$. If $\omega$ be angular velocity of earth's spinning motion and R be the radius of the earth then, centrifugal force on the particle is

(a) $m\omega^2 R\sin\lambda$ [✓ (b) $M\omega^2 R\cos\lambda$]

(c) $2m\omega^2 R\sin\lambda$ (d) $2m\omega^2 R\cos\lambda$

6. In the previous question if the particle starts moving along the latitude from east to west with constant speed $v_0$, coriolis force on the particle will be (magnitude)

[✓ (a) $2m\omega v_0$] (b) $2mv_0\omega\cos\lambda$

(c) $2mv_0\omega\sin\lambda$ (d) $2mv_0\omega\sin\lambda\cos\lambda$

7. A stone is dropped from a height 100m at equator. What is east-ward deviation of the particle (approximately)

(a) 3.15 cm (b) 2.17 cm (c) 1.08 cm [✓ (d) 2.73 cm]

8. In the previous question if the location is at latitude 60° in northern hemisphere then eastward deviation will be

(a) 3.15 cm (b) 2.17 cm [✓ (c) 1.08 cm] (d) 2.73 cm

9. If $\omega$ be angular velocity of the earth's spinning motion and R be the radius of the earth, with what speed should a train move along equator so that magnitude of centrifugal and coriolis forces and acting on the train become equal.

(a) $\omega R$ [✓ (b) $\frac{\omega R}{2}$] (c) $2\omega R$ (d) $\frac{2\omega R}{3}$

10. A person is running at equator with speed $\frac{\omega R}{2}$ where $\omega$ and $R$ have their usual meaning. In which direction should the person run so that his apparent weight is equal to his true weight.

(a) N to S (b) W to E [✓ (c) E to W] (d) S to N

11. Particle of mass 2kg is moving in north east direction on earth's surface at a location where latitude is 45° N. Find magnitude of coriolis force experience by the particle if its speed is 200 m/s.

[✓ (a) 0.05 N] (b) 0.5 N (c) 5 N (d) 0.25 N

12. A stone is thrown in meridian plane at an angle $\phi$ with the horizontal. If $v_0$ is its speed and $m$ is mass of the stone, what is coriolis force on the stone at the moment it is thrown. Latitude of location is $\lambda$.

(a) $2m\omega v_0\sin(\lambda-\phi)$ (b) $2m\omega v_0\sin(\lambda+\phi)$

(c) $2m\omega v_0\cos(\lambda-\phi)$ (d) $2m\omega v_0\cos(\lambda+\phi)$

13. Angular velocity of a frame is $\vec{\omega}=(2\hat{i}+3\hat{j})$ rad/s. A particle of mass 2kg lies at a point whose co-ordinate in lab frame is (3, 4, 0). What is centrifugal force on the mass as seen by a person standing on the rotating frame.

(a) $2\sqrt{13}$ N (b) $2\sqrt{5}$ N (c) $2\sqrt{7}$ N (d) $2\sqrt{3}$ N

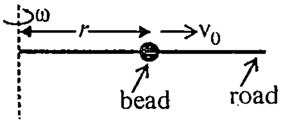
14. A body of mass $m$ starts sliding down from top of a smooth sphere of radius $R$. The sphere rotates with a constant angular velocity $\omega$ about a vertical axis passing through its center. What is the centrifugal force acting on the body at the moment it breaks off the surface as seen from the frame fixed to the sphere.

(a) zero (b) $\frac{2}{3}m\omega^2 R$ (c) $\frac{\sqrt{5}}{3}m\omega^2 R$ (d) $2m\omega^2 R$

15. In the previous question what is coriolis force on the body as seen from the frame fixed to the sphere at that moment.

(a) zero (b) $\frac{\sqrt{5}}{3}m\omega^2 R$ (c) $\frac{2}{3}m\omega^2 R$ (d) $\frac{2}{3}m\omega^2 R\sqrt{5+\frac{8g}{3\omega^2 R}}$

16. A thin rod is rotating with uniform angular velocity $\omega$ about its one end. A bead of mass $m$ slides along the length of rod with constant speed $v_0$ as shown in the figure. If there is no gravity in the space, what is normal reaction between the bead and the rod.

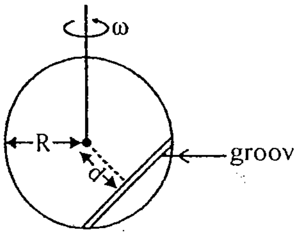


(a) zero (b) $m\omega^2 r$ (c) $2m\omega v_0$ (d) $mg$

17. In the previous question what is friction force on the bead at the instant its distance from the axis of rotation is $r$.

(a) $2m\omega v_0$ (b) zero (c) $m\omega^2 r$ (d) None of these

18. A circular disc is rotating about its axis with uniform angular velocity $\omega$. There is a groove along a chord of the disc. An insect starts crawling along the groove. If the insect starts at $t = 0$ from the middle point, and moves with constant speed $v_0$ w.r.t. the disc. What is centrifugal force on the insect at time $t$.



(a) $m\omega^2 d$ (b) $m\omega^2 v_0 t$ (c) $m\omega^2(d+v_0 t)$ (d) $m\omega^2\sqrt{d^2+v_0^2 t^2}$

19. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be

(a) 0° to the vertical and $\sqrt{3}T$ (b) 30° to the vertical and T/2

(c) 60° to the vertical and $T/\sqrt{2}$ (d) 0° to the vertical and $T/\sqrt{3}$

20. Two points N and S are located in the northern and southern hemisphere, respectively, on the same longitude. Projectiles P and Q are fired from N and S, respectively, towards each other. Which of the following options is correct for the projectiles as they approach the equator?

(a) Both P and Q will move towards the east

(b) Both P and Q will move towards the west

(c) P will move towards the east and Q towards the west

(d) P will move towards the west and Q towards the east

21. An observer is sitting on a horizontal platform which is rotating with a constant angular velocity. He puts an object on the smooth frictionless floor of the platform, away from the axis of rotation, with zero initial velocity with respect to him. Let the time at this instant be $t = 0$. In the frame of the platform, the object would

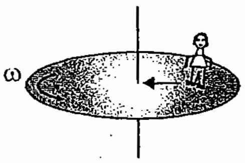
(a) Remain at rest for all $t > 0$

(b) Accelerate purely in a radial direction outwards for all $t > 0$.

(c) Accelerate purely in a tangential direction for all $t > 0$.

[✓ (d) Accelerate radially in the outward direction at $t = 0$, however the direction of acceleration changes for $t > 0$.]

22. A circular platform is rotating with a uniform angular speed $\omega$ counterclockwise about an axis passing through its centre and perpendicular to its plane as shown in the figure. A person of mass 'm' walks radially inward with a uniform speed 'v' on the platform. The magnitude and the direction of hte Corolis force (with respect to the direction along which the person walks) is: the force on the person



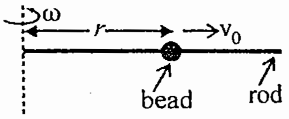
(a) $2m\omega v$ towards his left

(b) $2m\omega v$ towards his front.

[✓ (c) $2m\omega v$ towards his right]

(d) $2m\omega v$ towards his back.

23. A thin rod is rotating with uniform angular velocity $\omega$ about its one end. A bead of mass $m$ slides along the length of rod with constant speed $v_0$ as shown in the figure. If there is no gravity in the space, what is coefficient of friction between the bead and the rod.



(a) $\dfrac{\omega r}{v_0}$  (b) $\dfrac{\omega r}{2v_0}$  (c) $\dfrac{v_0}{\omega r}$  (d) $\dfrac{2v_0}{\omega r}$

Neteal

24. A horizontal circular platform rotates with a constant angular velocity $\Omega$ directed vertically upwards. A person seated at the centre shoots a bullet of mass 'm' horizontally with speed 'v'. The acceleration of the bullet, in the reference frame of the shooter, is

[✓ (a) $2v\Omega$ to his right]

(b) $2v\Omega$ to his left

(c) $v\Omega$ to his right

(d) $v\Omega$ to his left

25. The motion of a particle of mass m is described in a non-inertial frame of reference that is rotating with a uniform angular velocity $\omega$. If $r$ denotes the position of the particle in the non-inertial frame

(a) The centrifugal force on the particle is $-m\omega^2\dot{r}$

(b) The centrifugal force on the particle is $-m\omega\times(\omega\times r)$

(c) The Coriolis force on the particle is $-m\left(\omega\times\dfrac{dr}{dt}\right)$

(d) The Coriolis force on the particle $-2m\omega^2 r$

## CSIR-UGC-NET/JRF|GATE PHYSICS

### Assignment-6: Moment of Inertia & Inertia Tensor

1. Linear mass density of a thin rod of length 2L varies with distance from its centre as $\lambda = \lambda_0 |x|$. Moment of inertia of the rod about a perpendicular axis through its centre is

(a) $\frac{\lambda_0 L^4}{2}$ (b) $\frac{3\lambda_0 L^4}{4}$ (c) $\frac{3\lambda_0 L^4}{2}$ (d) $\frac{\lambda_0 L^4}{4}$

2. A square is made by joining four thin rods of length L and mass M. Moment of inertia of square about one of its diagonal is

(a) $\frac{4}{3}ML^2$ (b) $\frac{2}{3}ML^2$ (c) $\frac{ML^2}{2}$ (d) $\frac{ML^2}{3}$

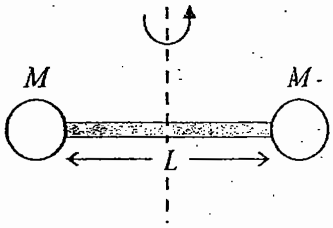
3. A circular dis (of mass M and radius R) rolls on a rough horizontal surface. Moment of inertia of disc about an instaneous axis through point of contact and perpendicular to plane of disc is

(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{4}$ (c) $\frac{5MR^2}{4}$ (d) $\frac{3MR^2}{2}$

4. For given volume of a solid uniform cylinder, what should be ratio of length and radius of cylinder so that moment of inertia of cylinder about an axis perpendicular to its length and passing through its centre is minimum.

(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) 2 : 1

5. A dumbell consists of two solid sphere of equal masses M and radii R connected by a light rod of length L. Moment of inertia of system about a perpendicular axis through the centre of rod is



(a) $\frac{2}{5}MR^2\left[7+\frac{1}{R}+\frac{5}{4}\left(\frac{L}{R}\right)^2\right]$ (b) $\frac{2}{5}MR^2\left[5+\frac{1}{R}+\frac{3}{4}\left(\frac{L}{R}\right)^2\right]$

(c) $\frac{2}{5}MR^2\left[3+\frac{L}{R}+\frac{5}{4}\left(\frac{L}{R}\right)^2\right]$ (d) $\frac{2}{5}MR^2\left[7+\frac{L}{R}+\frac{3}{4}\left(\frac{L}{R}\right)^2\right]$

6. Moment of inertia of a solid cube about an axis perpendicular to its face and passing through its centre is [Mass of cube = M, side length = a]

(a) $\frac{Ma^2}{12}$ (b) $\frac{Ma^2}{6}$ (c) $\frac{Ma^2}{3}$ (d) $\frac{Ma^2}{4}$

7. A circular disc of radius R and thickness R/6 has moment of inertia I about its perpendicular axis through centre. The disc is melted and recasted into a sphere M.I. of sphere about its axis is

(a) I/2 (b) I/5 (c) I/6 (d) I/7

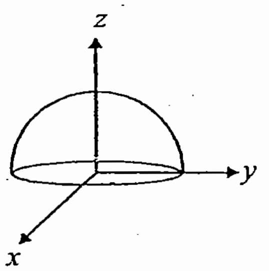
8. Two right circular solid cones have same base radius. Ratio of their heights is 2 : 3. If the two cones have uniform density and their masses are equal, then ratio of moment of inertia about a their axes axis.

(a) 2 : 3 (b) 3 : 2 (c) 9 : 4 (d) 1 : 1

9. A thick hollow sphere of mass 'M' has inner and outer adii $R_1$ and $R_2$. Moment of inertia of sphere about its dimeter is

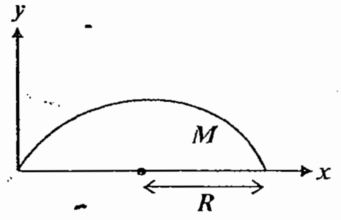
(a) $\frac{1}{5}M\left(R_1^2+R_2^2\right)$ (b) $\frac{2}{3}M\left(R_1^2+R_2^2\right)$ (c) $\frac{1}{3}M\left(R_1^2+R_2^2\right)$ [✓ (d) $\frac{2}{5}M\left(\frac{R_2^5-R_1^5}{R_2^3-R_1^3}\right)$]

10. In the figure a thin hemisphere shell of mass M and radius R is kept on x-y plane with its centre at origin. Moment of inertia of shell about x and z axes are respectively.



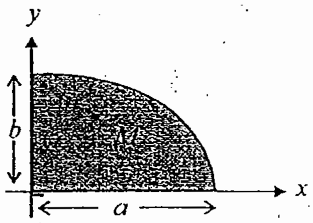
(a) $\frac{5}{12}MR^2, \frac{2}{3}MR^2$ (b) $\frac{5}{12}MR^2, \frac{1}{3}MR^2$ [✓ (c) $\frac{2}{3}MR^2, \frac{2}{3}MR^2$] (d) $\frac{1}{3}MR^2, \frac{1}{3}MR^2$

11. A semicircular disc of radius R and mass M is kept in x-y plane as shown in figure. The product of inertia $I_{xy}$ is:



(a) $-\frac{2Ma^2}{3}$ (b) $-\frac{4Ma^2}{3}$ (c) $-\frac{4}{3}\frac{Ma^2}{\pi}$ (d) $-\frac{2}{3}\frac{Ma^2}{\pi}$

12. A plate shaped as a quadrant of an ellipse is lying in x-y plane as shown in figure. Value of $I_{xy}$ is:



(a) $-\frac{Mab}{2\pi}$ (b) $-\frac{Mab}{\pi}$ (c) $-\frac{Mab}{4\pi}$ (d) $-\frac{Mab}{6\pi}$

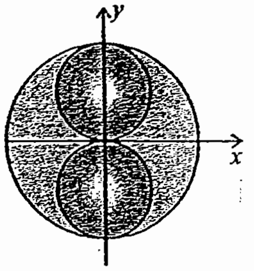
13. A rod has moment of inertia $I_0$ about an axis perpendicular to it and passing through its center. If linear mass density is halved and length is doubled, the moment of inertia now will be

(a) $2I_0$ (b) $4I_0$ (c) $I_0/2$ (d) $I_0/4$

14. M.I. of a disc of mass M and radius R is $MR^2$ about an axis perpendicular to its surface. What is distance of the axis from center of the disc?

(a) $\frac{R}{2}$ (b) $\frac{\sqrt{3}R}{2}$ (c) $\frac{R}{\sqrt{2}}$ (d) $\frac{R}{4}$

15. From a solid sphere of density ρ and radius R two smaller spheres of radius R/2 are cut as shown in the figure. M.I. of resulting body about $y$ axis is

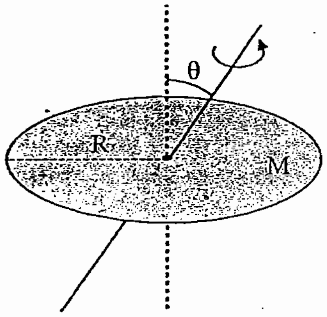


[✓ (a)] $\frac{\pi\rho R^5}{2}$ (b) $\frac{5\pi\rho R^4}{12}$ (c) $\frac{7\pi\rho R^5}{12}$ (d) $\frac{3\pi\rho R^5}{4}$

16. In the previous question M.I. of object about $x$ axis is

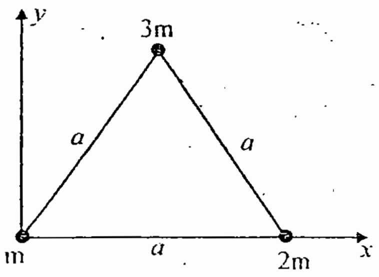
(a) $\frac{\pi\rho R^5}{4}$ (b) $\frac{5\pi\rho R^5}{12}$ [✓ (c)] $\frac{7\pi\rho R^5}{12}$ (d) $\frac{3\pi\rho R^5}{4}$

17. A circular disc is rotating about an axis shown in the figure. M.I. of the disc about that axis is:



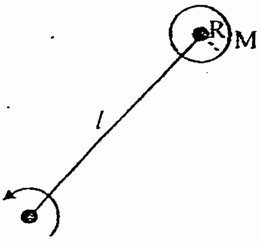
(a) $\frac{MR^2}{2}\cos^2\theta$ (b) $\frac{MR^2}{2}\left(1-\frac{\sin^2\theta}{2}\right)$ (c) $\frac{MR^2}{4}\cos^2\theta$ (d) $\frac{MR^2}{2}\sin^2\theta$

18. In the figure shown value of $I_{xy}$ is:



(a) $\frac{-3\sqrt{3}ma^2}{2}$ [✓ (b)] $\frac{-3\sqrt{3}ma^2}{4}$ (c) 0 (d) $\frac{-\sqrt{3}ma^2}{4}$

19. A disc of radius R and mass M is fixed rigidly at the end of a rod of same mass and length $l = 3R$. The rod is pivoted at the other end and is rotating on a frictionless horizontal table with angular velocity $\omega$ such that the plane of the disc is parallel to the table. Angular momentum of the system about the pivot is

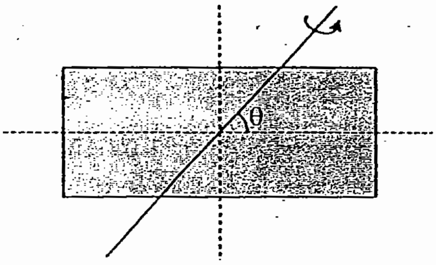


(a) $4MR^2\omega$ (b) $\frac{7}{2}MR^2\omega$ (c) $\frac{25}{2}MR^2\omega$ (d) $\frac{19}{2}MR^2\omega$

20. A solid cylinder of mass M and radius R has volume mass density $\rho = \alpha r$ where $r$ = distance from axis of cylinder. M.I. about its axis is

(a) $\frac{3}{5}MR^2$ (b) $\frac{2MR^2}{5}$ (c) $\frac{3MR^2}{7}$ (d) $\frac{5MR^2}{6}$

21. A rectangular plate of mass M and sides $a$ and $b$ $(a < b)$ is rotating about an axis which passes through its center and lies in its plane as shown in the figure. M.I. of the plate about this axis is;



(a) $\frac{M}{12}(a^2+b^2)$

(b) $\frac{Ma^2}{12}\sin^2\theta$

(c) $\frac{Mb^2}{12}\cos^2\theta$

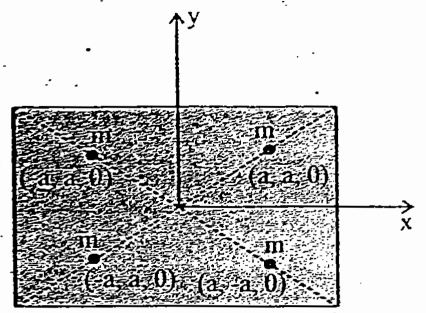
[✓ (d)] $\frac{Ma^2}{12}\cos^2\theta+\frac{Mb^2}{12}\sin^2\theta$

(e) $\frac{Ma^2}{12}\sin^2\theta+\frac{Mb^2}{12}\cos^2\theta$

22. What is moment of inertia of a rectangular plate about its diagonal (take parameters of previous questions)

(a) $\frac{M}{6}(a^2+b^2)$ [✓ (b)] $\frac{Ma^2b^2}{6(a^2+b^2)}$ (c) $\frac{Ma^2b^2}{24(a^2+b^2)}$ (d) $\frac{M}{12}(a^2+b^2)$

23. EFGH is a thin square plate of uniform density $\sigma$ and side $4a$. Four point masses, each of mass $m$, are placed on the plate as shown in the figure. In the moment of inertia matrix I of the composite system,



(a) only $I_{xy}$ is zero
(b) only $I_{xz}$ and $I_{yz}$ are zero
(c) All the product of inertia terms are zero
(d) None

24. Four thin rods each of mass M and length $l$ are joined together to form a square loop. M.I. of the loop about its axis is (axis is $\perp$ to plane through centre)

(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{4}$ (c) $\frac{4ML^2}{3}$ (d) $\frac{3ML^2}{4}$

25. What is M.I. of the loop in previous question about its diagonal.

(a) $\frac{ML^2}{6}$ (b) $\frac{2}{3}ML^2$ (c) $\frac{ML^2}{3}$ (d) $\frac{ML^2}{4}$

26. A thin wire of length L and mass M is bent to form a circular loop. M.I. of the loop about a tangent perpendicular to its plane is

(a) $\frac{ML^2}{4\pi^2}$ (b) $\frac{ML^2}{2\pi^2}$ (c) $\frac{3ML^2}{2\pi^2}$ (d) $\frac{3ML^2}{4\pi^2}$

27. We have two spheres, one of which is hollow and the other is solid. The two have equal masses and equal moment of inertia about their diameters, ratio of their radii is (hollow to solid)

(a) 5 : 7 (b) 3 : 5 (c) $\sqrt{3}:\sqrt{5}$ (d) $\sqrt{3}:\sqrt{7}$

### Assignment-6: Moment of Inertia & Inertia Tensor

### ANSWER KEY

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (c) | 2. (b) | 3. (d) | 4. (c) | 5. ( ) | 6. (b) | 7. (b) |
| 8. (d) | 9. (d) | 10. (a) | 11. (c) | 12. (a) | 13. (b) | 14. (c) |
| 15. (a) | 16. (b) | 17. (b) | 18. (b) | 19. (c) | 20. (a) | 21. (d) |
| 22. (c) | 23. (c) | 24. (c) | 25. (b) | 26. (b) | 27. (c) | |

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## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-7: Rigid Body Dynamics

1. A solid sphere rolls on a horizontal surface with constant speed. Ratio of rotational kinetic energy and total K.E. is

(a) $\frac{2}{7}$ (b) $\frac{5}{7}$ (c) $\frac{10}{7}$ (d) $\frac{1}{2}$

2. In previous question, ratio of angular momentum of the sphere about a point on its path to its rotational angular momentum is

(a) $\frac{7}{5}$ (b) $\frac{2}{5}$ (c) $\frac{7}{2}$ (d) $\frac{5}{7}$

3. A circular disc of mass M and radius R rotates about an axis lying in its plane with constant angular velocity $\omega$. If angular momentum of the disc about this axis is $\frac{3}{4}MR^2\omega$, distance of axis from centre of ~~ring~~ disc is

(a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{3}$ (d) $\frac{R}{\sqrt{3}}$

4. A solid sphere and a hollow sphere roll down an inclined plane. On reaching the bottom of inclined plane ratio of their speed will be

$I_h = \frac{2}{3}MR^2$

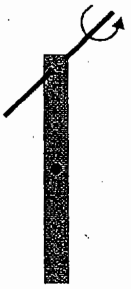
$I_s = \frac{2}{5}MR^2$

(a) $\sqrt{\frac{25}{14}}$ (b) $\sqrt{\frac{25}{7}}$ (c) $\sqrt{\frac{25}{21}}$ (d) $\sqrt{\frac{5}{7}}$

5. A solid cylinder starts rolling down an inclined plane of inclination $\theta$ from a height $h$. Time taken to reach the bottom is

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sqrt{\frac{3h}{g}}$ (c) $\frac{1}{\sin\theta}\sqrt{\frac{3h}{2g}}$ (d) $\frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}$

6. A uniform rod of length L is suspended from one end. The rod can rotate, freely about horizontal axis through point of suspension. What minimum angular velocity should be given to the rod so that it completes a full rotation



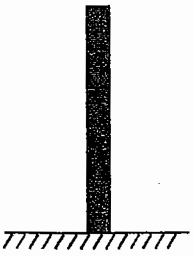
(a) $\sqrt{\frac{3g}{L}}$ (b) $\sqrt{\frac{6g}{L}}$

(c) $\sqrt{\frac{2g}{L}}$ (d) $\sqrt{\frac{4g}{L}}$

7. A rod of length 'L' is translatting with speed 'v' perpendicular to its length if one end of the rod is suddenly fixed what will be angular velocity rod about that end

(a) $\frac{v}{L}$ (b) $\frac{2v}{L}$ (c) $\frac{3v}{2L}$ (d) $\frac{2v}{3L}$

8. A rod of length L is held vertical with its one end fixed one ground. The rod can rotate about fixed end. If the rod is released from this position, what is its speed when it is about to hit the ground



[✓ (a)] $\sqrt{\frac{3g}{L}}$ (b) $\sqrt{\frac{2g}{L}}$

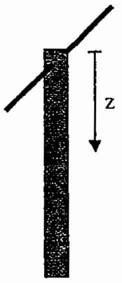
(c) $\sqrt{\frac{g}{L}}$ (d) $\sqrt{\frac{g}{2L}}$

9. A thin rod of length L is suspended from one end. Time period of oscillation of the rod is

(a) $2\pi\sqrt{\frac{L}{g}}$ (b) $2\pi\sqrt{\frac{L}{2g}}$ [✓ (c)] $2\pi\sqrt{\frac{2L}{3g}}$ (d) $2\pi\sqrt{\frac{3L}{2g}}$

$T = 2\pi\sqrt{\frac{I}{mgd}}$

10. A thin rod of length L is suspended from one end as shown in figure. The linear mass density of rod varies with distance from point of suspension as $\lambda = \alpha z$. Time period of oscillation of the rod is



[✓ (a)] $2\pi\sqrt{\frac{3L}{4g}}$ (b) $2\pi\sqrt{\frac{2L}{g}}$

(c) $2\pi\sqrt{\frac{3L}{2g}}$ (d) $2\pi\sqrt{\frac{L}{g}}$

11. A circular disc of radius R is suspended from a point on its surface such that it oscillates about an axis passing through point of suspension and perpendicular to its plane. For what distance of point of suspendion from centre of the disc time period of oscillation is minimum

(a) $\frac{R}{2}$ (b) $\frac{R}{3}$ (c) $\frac{R}{\sqrt{2}}$ (d) $\frac{R}{\sqrt{3}}$

12. A hollow sphere and a solid cylinder are releasd one by one from same point on an inclined plane. Assume rolling without slipping. Ratio of time taken by cylinder to that by sphere to reach the bottom of inclined plane is

(a) $\sqrt{\frac{10}{9}}$ (b) $\sqrt{\frac{5}{6}}$ (c) $\sqrt{\frac{4}{3}}$ (d) $\sqrt{\frac{3}{5}}$

13. A solid sphere of mass 'M' rolls down an inclined plane of inclination $\theta$. Friction force on the sphere is

(a) $\frac{2}{5}g\sin\theta$ (b) $\frac{2}{3}g\sin\theta$ (c) $\frac{5}{7}g\sin\theta$ (d) $\frac{2}{7}g\sin\theta$

14. A small thin ring of radius 'r' rolls down an inclined plane of inclination $\theta$. Angular acceleration of ring on the inclined plane is

(a) $\frac{g\sin\theta}{r}$ (b) $\frac{g\sin\theta}{2r}$ (c) $\frac{g\sin\theta}{3r}$ (d) $\frac{2g\sin\theta}{r}$

15. A circular loop of inner and outer radii $r_1$ and $r_2$ rolls down an inclined plane of inclination $\theta$. Angular acceleration of loop on the inclined plane is

[✓ (a)] $\frac{2r_2}{3r_2^2+r_1^2}g\sin\theta$ (b) $\frac{2r_1}{3r_2^2+r_2^2}g\sin\theta$ (c) $\frac{r_2}{r_2^2+r_1^2}g\sin\theta$ (d) $\frac{r_1}{r_2^2+r_1^2}g\sin\theta$

16. A solid cylinder is rolling on a horizontal surface. What fraction is its rotational kinetic energy of total kinetic energy

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

17. A hollow sphere is rolling on a rough horiozontal surface such that its centre is moving with linear speed 'v'. If M be mass and R be radius of sphere, its angular momentum about any point along its path is

(a) $\frac{3}{2}MvR$ (b) $\frac{7}{5}MvR$ (c) $\frac{5}{3}MvR$ (d) $MvR$

18. A circular disc is rotating in its plane with angular velocity $\omega$, its centre being at rest. If a point on the rim is suddenly fixed, what is new angular velocity of the disc about the point on the rim.

(a) $\omega$ (b) $\frac{\omega}{2}$ (c) $\frac{\omega}{3}$ (d) $\frac{2\omega}{3}$

19. A right circular cone has mass M, radius R, and height h. Moment of inertia of cone about a diameter of its base is

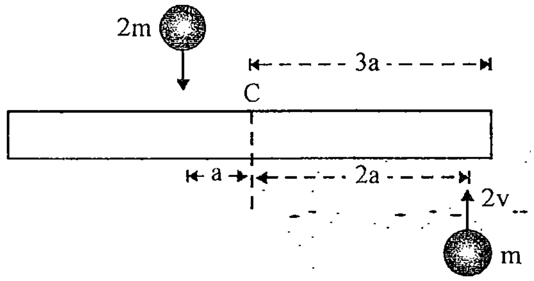
(a) $\frac{M}{20}\left(3R^2+2h^2\right)$ (b) $\frac{M}{20}\left(3h^2+2R^2\right)$ (c) $\frac{M}{10}\left(3R^2+2h^2\right)$ (d) $\frac{M}{10}\left(3h^2+2R^2\right)$

20. A disc of mass M and radius R is pivoted about a horizontal axis through its centre and a small body of the same mass M is attached to the rim of the disc. If the disc is released from rest with the small body at the end of a horizontal radius, the angular speed when the small body is at the bottom is

(a) $\sqrt{\left(\frac{g}{4R}\right)}$ (b) $\sqrt{\left(\frac{g}{2R}\right)}$ (c) $\sqrt{\left(\frac{3g}{4R}\right)}$ (d) $\sqrt{\left(\frac{4g}{3R}\right)}$

21. A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two poin masses 'm' and '2m' moving in the same horioznta1 plane with speed '2v' and 'v' respectively strike the bar (see figure) and stick to the bar after collision.

TFIB



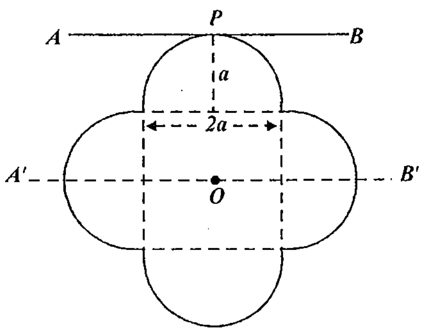
Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by $\omega$, $E$ and $v_c$ respectively, we have after collision

if ask angular moment

if they ask energy

(a) $v_c = 0$ (b) $\omega = \frac{3}{5}\left(\frac{v}{a}\right)$ (c) $\omega = \frac{v}{5a}$ (d) $E = \frac{3}{5}mv^2$

22. A symmetric lamina of mass M consists of a square shape with a semicircular section over each of the edge of the square as shown in figure. The side of the square is 2a. The moment of inertia of the lamina about an axis through its centre of mas and perpendicular to the plane is 1.6 Ma². The moment of inertia of the lamina about the tangent AB in the plane of the lamina is



(a) 4.8 Ma² (b) 3.2 Ma²

(c) 6.4 Ma² (d) 1.6 Ma²

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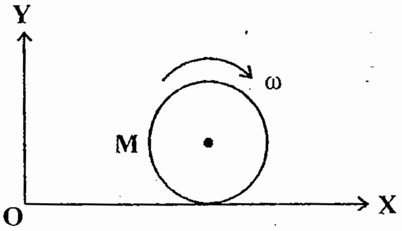
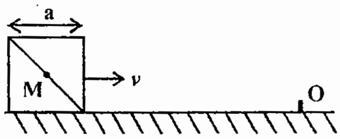
23. A disc of mass M and radius R is rolling with angular speed $\omega$ on a horizontal plane as shown in figure. The magnitude of angular momentum of the disc about the origin O is



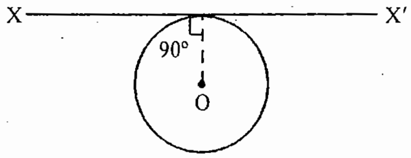
(a) $\frac{1}{2}MR^2\omega$ (b) $MR^2\omega$ (c) $\frac{3}{2}MR^2\omega$ (d) $2MR^2\omega$

24. A cubical block of side 'a' is moving with velocity 'v' on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is



(a) $\frac{3v}{(4a)}$ (b) $\frac{3v}{(2a)}$ (c) $\frac{\sqrt{3}v}{(\sqrt{2}a)}$ (d) zero

25. A thin wire of length L and uniform linear mass density $\rho$ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis $XX'$ is:



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

**ANSWER KEY**

**1. (a) 2. (c) 3. (b) 4. (c) 5. (d) 6. (b) 7. (c)**

**8. (a) 9. (c) 10. (a) 11. (c) 12. (a) 13. (d) 14. (b)**

**15. (a) 16. (b) 17. (c) 18. (c) 19. (b) 20. (d)**

**21. (a, c, d ) 22. (a) 23. (c) 24. (a) 25. (d)**

# CAREER ENDEAVOUR
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## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-8: Classical Mechanics (Special Theory of Relativity)

1. Apparent length of a rod is half of its proper length. If direction of velocity of the rod makes 30° with the length of the rod then velocity of rod is equal to

(a) C (b) C/2 (c) $\frac{\sqrt{3}}{2}C$ (d) $\frac{2C}{3}$

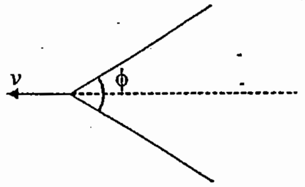
2. An equilateral triangle of side '$a$' is translated parallel to its plane with speed $v=\frac{\sqrt{3}}{2}C$. The velocity vector is parallel to one of its sides. Apparent perimeter of the triangle is

(a) $\frac{3a}{2}$ (b) $\frac{a}{2}\left(1+\frac{\sqrt{13}}{2}\right)$ (c) $\frac{a}{2}\left(1+\sqrt{13}\right)$ (d) $\frac{a}{2}\left(1+\sqrt{7}\right)$

3. In the previous question apparent area of the triangle

(a) $\frac{\sqrt{3}a^2}{4}$ (b) $\frac{\sqrt{3}a^2}{2}$ (c) $\frac{\sqrt{3}a^2}{8}$ (d) $\frac{\sqrt{39}a^2}{8}$

4. A wire is bent as shown in the figure and if is moved along the bisector of the angle with speed $v$. Apparent angle is



(a) $\tan^{-1}\frac{\tan\phi}{\sqrt{1-\frac{v^2}{c^2}}}$

(b) $2\tan^{-1}\frac{\tan\frac{\phi}{2}}{\sqrt{1-\frac{v^2}{c^2}}}$

(c) $\tan^{-1}\left[\frac{2\tan^{-1}\frac{\phi}{2}}{\sqrt{1-\frac{v^2}{c^2}}}\right]$

(d) $\tan^{-1}\left[\frac{\tan\frac{\phi}{2}}{\sqrt{1-\frac{v^2}{c^2}}}\right]$

5. A cube of side '$a$' is moved with speed $\frac{4c}{5}$ along it body diagonal apparent volume of the cube is

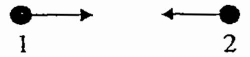
(a) $\frac{3a^3}{10}$ (b) $\frac{3a^3}{5}$ (c) $\sqrt{\frac{33}{75}}a^3$ (d) $\left(\frac{59}{75}\right)^{3/2}a^3$

3.

6. At what speed mass density of an object will increase by 25%.

(a) $\frac{4c}{5}$ (b) $\frac{3c}{5}$ (c) $\frac{2c}{5}$ (d) $\frac{c}{5}$

7. Two photons are moving along $x$ axis as shown in the figure. Velocity of photon 1 with respect to photon 2 is

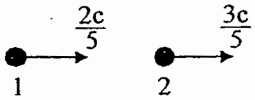


(a) $c$ (b) $-c$ (c) $2c$ (d) $-2c$

8. In the previous question velocity of approach of the two photons is

(a) $c$ (b) $2c$ (c) $\frac{3c}{2}$ (d) zero

9. Two electrons are moving with speeds $\frac{2c}{5}$ and $\frac{3c}{5}$ as shown in the figure. Velocity of 2 with respect to 1 is

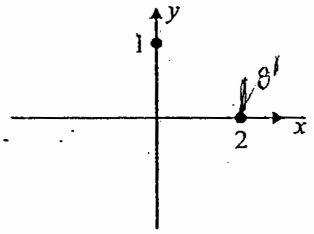


(a) $\frac{5c}{19}$ (b) $\frac{-5c}{19}$ (c) $\frac{c}{5}$ (d) $-\frac{c}{5}$

10. In the previous question what is velocity of approach as seen by an observer on ground.

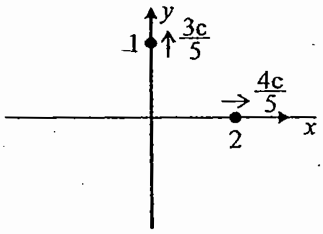
(a) $c$ (b) $\frac{c}{5}$ (c) $\frac{5c}{19}$ (d) $\frac{2c}{3}$

11. Two photons are moving as shown in the figure. Velocity of 1 with respect to 2 is



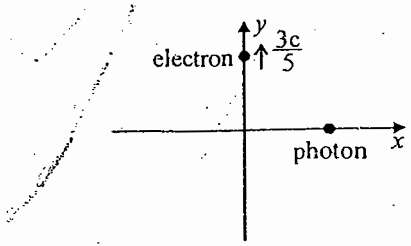
(a) $c(\hat{i}+\hat{j})$ (b) $c(-\hat{i}+\hat{j})$ (c) $-c\hat{i}$ (d) $c\hat{j}$

12. Two electrons are moving as shown in the figure. Velocity of 1 with respect to 2 is



(a) $\frac{12c}{13}$ (b) $\frac{20c}{13}$ (c) $\frac{\sqrt{481}}{25}c$ (d) $c$

13. An electron and a photon are moving as shown in the figure. Velocity of electron with respect to the photon is



(a) $\frac{3c}{5}$ (b) $\frac{2c}{5}$ (c) $\frac{\sqrt{13}}{5}c$ (d) $c$

14. Two rods of rest length $L_0$ are moving towards each other with speeds $\frac{3c}{5}$ and $\frac{4c}{5}$. What is time taken by the two rods to cross each other as seen by a person on the ground

(a) $\frac{2L_0}{3c}$ (b) $\frac{3L_0}{5c}$ (c) $\frac{L_0}{c}$ (d) $\frac{2L_0}{c}$

15. In the previous question what is time taken by the two rods to cross each other as seen by the rod of speed $\frac{3c}{5}$

(a) $\frac{7L_0}{5c}$ (b) $\frac{3L_0}{5c}$ (c) $\frac{5L_0}{7c}$ (d) $\frac{5L_0}{3c}$

16. Two fire crackers burn on ground at points A and B. If co-ordinates of A and B are (2, 0, 0) (64,0,0) meters and time recorded by a clock on the ground be $t_A = 3.5$ sec, $t_B = 3.5 + 1 \times 10^{-10}$ sec then with what speed should a plane be flying and in which direction so that the two events appear simultaneous in the frame of reference of the plane.

(a) $1.5 \times 10^5$ m/s (A to B) (b) $1.5 \times 10^5$ m/s (B to A)
(c) $3 \times 10^5$ m/s (A to B) (d) $3 \times 10^5$ m/s (B to A)

17. Two cars A and B are at rest at a separation of 1000 meters. Clocks kept in two cars are initially synchronized. The two cars start moving towards each other with speeds $\frac{c}{2}$. By how much time the two clocks differ at the moment the cars start moving as measured by an observer on the ground.

(a) 2μs (b) 4.5μs (c) 6.8μs (d) 12.5μs

18. In the previous question what is time difference as measured by observer in one the cars.

(a) 2μs (b) 4.5μs (c) 6.8μs (d) 12.5μs

19. A rod of proper length $l_0$ oriented parallel to the $x$-axis moves with speed $u$ along the $x$-axis in S. What is length measured by an observer in $S'$ which is moving along $+x$ direction w.r.t. S with speed $v$.

(a) $l_0\sqrt{1-\frac{v^2}{c^2}}\sqrt{1+\frac{u^2}{c^2}}$ (b) $\frac{l_0\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{uv}{c^2}}$ (c) $\frac{l_0\sqrt{1+\frac{v^2}{c^2}}\sqrt{1+\frac{u^2}{c^2}}}{1+\frac{uv}{c^2}}$ (d) $l_0\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}$

20. Two electrons are moving with equal speeds $\frac{c}{2}$ at 60°. Their relative velocity is equal to

(a) $\frac{c\sqrt{13}}{7}$ (b) $\frac{c\sqrt{5}}{7}$ (c) $\frac{c\sqrt{3}}{7}$ (d) $\frac{c\sqrt{11}}{7}$

21. Two events are separated by a distance of $6 \times 10^5$ km if the first event occurs 1's before the second event. The interval between the two events

(a) is time like
(b) is light like (null)
(c) is space like
(d) cannot be determined from the information given

22. Relativistic mechanics reduces to non-relativistic mechanics in the limit

(a) $C \to 0$ (b) $C \to \infty$ (c) $v \to 0$ (d) $C \to \infty$

23. Two particles are moving towards each other with equal velocities $\frac{C}{2}$. Their relative velocity is

(a) C (b) $\frac{C}{2}$ (c) $\frac{4C}{5}$ (d) $\frac{C}{4}$

24. In the above question what is the velocity of approach

(a) C (b) $\frac{C}{2}$ (c) $\frac{4C}{5}$ (d) $\frac{C}{4}$

25. A rod of length $L_0$ (when at rest) rotates about a perpendicular axis passing through one of its ends. The length of the rotating rod appears to be

(a) $L_0$ (b) $L_0\sqrt{1-\frac{\omega^2L^2}{C^2}}$ (c) $\frac{L_0}{\sqrt{1-\frac{\omega^2L^2}{C^2}}}$ [✓ (d)] None

26. Life time of a muon in laboratory frame is $2.5 \times 10^{-6}$ sec. Muon flies with speed $\frac{3C}{5}$. What is life time of muon in its rest frame.

(a) $2 \times 10^{-6}$ sec (b) $1.5 \times 10^{-6}$ sec (c) $1 \times 10^{-6}$ sec (d) $2.5 \times 10^{-6}$ sec

27. Kinetic energy of a relativistic particle of rest mass *m* moving with speed *v* is

(a) $\frac{1}{2}mv^2$ (b) $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ (c) $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}-mc^2$ (d) $\frac{1}{2}m(v^2-c^2)$

28. Although mass-energy equivalence of special relativity allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because

(a) the mass is not conserved
(b) the energy is not conserved
(c) the momentum is not conserved
(d) the charge is not conserved

29. Lowest order relativistic correction to the Hamiltonian of a particle is

[✓ (a)] $-\frac{p^4}{8m_0^3c^2}$ (b) $\frac{p^4}{8m_0^3c^2}$ (c) $\frac{p^4}{4m_0^3c^2}$ (d) $-\frac{p^4}{4m_0^2c^2}$

30. Lagrangian of a relativistic particle is $\boxed{L = -m_0 c^2 \sqrt{1-v^2/c^2}}$

(a) $m_0\sqrt{1-\frac{v^2}{c^2}}$ (b) $-m_0 c^2\sqrt{1-\frac{v^2}{c^2}}$ (c) $\frac{m_0v^2}{2}$ (d) $\frac{m_0v^2}{2\sqrt{1-\frac{v^2}{c^2}}}$

31. Charge density of an object is $\rho_0$ if the object moves with speed $v$, its charge density will become.

(a) $\rho_0\sqrt{1-\frac{v^2}{c^2}}$ (b) $\rho_0\sqrt{1+\frac{v^2}{c^2}}$ (c) $\frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}}$ (d) $\frac{\rho_0}{\sqrt{1+\frac{v^2}{c^2}}}$

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32. Lowest order relativistic correction to the kinetic energy of a particle is

(a) $\frac{1}{8}\frac{m_0v^4}{c^2}$ [✓ (b) $\frac{3}{8}\frac{m_0v^4}{c^2}$] (c) $\frac{5}{8}\frac{m_0v^4}{c^2}$ (d) $\frac{7}{8}\frac{m_0v^2}{c^2}$

33. An electron annihilates with a positron. Wavelength of radiation emitted is of the order of

(a) nano meter [✓ (b) pico meter] (c) Fento meter (d) micro meter

34. A particle is subjected to a constant force. The displacement versus time plot for relativistic motion will be

(a) Straight line [✓ (b) Paraboli] (c) Hyperbolic (d) Circ

35. A square loop of area $A_0$(when at rest) is moving with speed $\frac{C}{\sqrt{5}}$ in a direction parallel to one of its diagonal area of the moving loop is

(a) $A_0$ (b) $\sqrt{\frac{3}{5}}A_0$ (c) $\sqrt{\frac{4}{5}}A_0$ (d) $\sqrt{\frac{9}{10}}A_0$

36. Speed of an object cannot be greater than speed of light in

(a) Vacuum (b) a medium

(c) both vacuum and medium (d) a medium denser than air

37. Relativistic mechanics rules out the concept of

(a) Rigidity (b) Conservation of linear momentum

(c) Conservation of charge (d) Conservation of energy

38. If we define $\frac{v}{c}=\tan h\theta$, for relative motion along $x$-axis the Lorentz transformation matrix can be written as (taking $c = 1$)

[✓ (a)] $\begin{pmatrix} \cos h\theta & -\sin h\theta & 0 & 0 \\ -\sin h\theta & \cos h\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} \cos h\theta & -\sin h\theta & 0 \\ \sin h\theta & \cos h\theta & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} \sin h\theta & -\cos h\theta & 0 & 0 \\ -\cos h\theta & \sin h\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} \sin h\theta & -\cos h\theta & 0 \\ \cos h\theta & \sin h\theta & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

39. Two particles are moving with constant speeds equal to $\frac{C}{2}$ along +X and +Y direction respectively. Magnitude of their relative velocity is

(a) $\frac{C}{2}$ (b) $\frac{\sqrt{7}}{5}C$ (c) $\frac{\sqrt{7}}{4}C$ (d) $\frac{\sqrt{7}}{8}C$

0. In the above question instead of two particles if two photons are moving along +X and +Y direction. What will be magnitude of their relative velocity.

(a) C (b) $\frac{C}{\sqrt{2}}$ (c) $\frac{C}{2}$ (d) $\frac{C}{4}$

41. The ordering of two events in absolute (does not change) if the invariant interval is

(a) space-like (b) time-like

(c) light-like (d) either space-like or time-like

42. For a particle of zero rest mass momentum and energy are related as

(a) $p=\frac{E}{2c}$ (b) $p=\frac{E}{c}$ (c) $p=\frac{2E}{c}$ (d) $p=\frac{E}{c^2}$

43. An object of rest mass $m_0$ collides perfectly inelastically with another object kept at rest and having same rest mass. The rest mass of the combined system will be

(a) equal to $2m_0$ (b) greater than $2m_0$ (c) smaller than $2m_0$ (d) $m_0$

44. An electron is projected at relativistic speed into a uniform magnetic field such that the direction of velocity is perpendicular to the field. The electron will move

(a) along a straight line

(b) along a circle

(c) along a curved path with increasing radius of curvature

(d) along a curved path with decreasing radius of curvature

45. Length of a meter stick appears to be 50cm. How fast is it moving?

(a) $\frac{\sqrt{3}}{2}c$ (b) $\sqrt{3}c$ (c) $\frac{\sqrt{3}}{4}c$ (d) $\frac{c}{2}$

46. Which of the following equations is invariant under Lorentz transformation.

(a) $\nabla^2\psi(\vec{r},t)+\frac{1}{c^2}\frac{\partial^2\psi(\vec{r},t)}{\partial t^2}=0$ (b) $\nabla^2\psi(\vec{r},t)-\frac{1}{c^2}\frac{\partial^2\psi(\vec{r},t)}{\partial t^2}=0$

(c) $\nabla^2\psi(\vec{r},t)-i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=0$ (d) $\nabla^2\psi(\vec{r},t)+i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=0$

47. Rest mass of a relativistic particle can be expressed in terms of its momentum and K.E as

(a) $\frac{(pc)^2-T^2}{2Tc^2}$ (b) $\frac{(pc)^2+T^2}{2Tc^2}$ (c) $\frac{(pc)^2+T^2}{Tc^2}$ (d) $\frac{(pc)^2-T^2}{Tc^2}$

48. Energy equivalent of an electron is

(a) ~~5110 Kev~~ (b) 511 Kev (c) 51100 Kev (d) 511000 Kev

49. Density of an object appears to be 25% greater. Its velocity is

(a) $\frac{3}{5}c$ (b) $\frac{2}{5}c$ (c) $\frac{4}{5}c$ (d) $\frac{1}{5}c$

50. A Particle A is moving in x-y plane with speed $c/\sqrt{2}$ such that its direction makes 45° with +x direction in the first quadrant. Another particle B is moving with speed c/2 along +x direction. Velocity of particle B with respect to A is,

(a) $c/2\hat{j}$ (b) $c/\sqrt{3}\hat{j}$ (c) $-c/2\hat{j}$ (d) $-c/\sqrt{3}\hat{j}$

2/1. In an inertial frame S, a stationary rod makes an angle $\theta$ with the x-axis. Another inertial frame $S'$ moves with a velocity 'v' with respect ot S along the common $x-x'$ axis. As observed from $S'$ the angle made by the rod with the $x'$–axis is $\theta'$. Which of the following statements is correct?

(a) $\theta' < \theta$ (b) $\theta' > \theta$

(c) $\theta' < \theta$ if v is negative and $\theta' > \theta$ if v is positive

(c) $\theta' > \theta$ if v is negative and $\theta' < \theta$ if v is positive.

52. A beam of light moves in a slab of glass of refractive index n in the positive x-direction. The slab itself is also moving in the positive with a speed v in laboratory frame. What is the speed of the beam of light as measured in the laboratory frame ?

(a) c (b) $\left(c^2 n + cv\right)/\left(c + nv\right)$

(c) $c\left(1 - \frac{1}{n}\right)$ (d) $\left(c^2 + vcn\right)/\left(cn + v\right)$

53. In the special theory of relativity, consider a Lorentz boost by a velocity 'v' along the x-direction. If $u = ct + x$, then the boosted value $u' = ct' + x'$ is:

(a) $u' = \sqrt{\frac{1+v/c}{1-v/c}}u$ (b) $u' = \sqrt{\frac{1-v/c}{1+v/c}}u$ (c) $u' = \frac{1+v/c}{1-v/c}u$ (d) $u' = \frac{1-v/c}{1+v/c}u$

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54. A galaxy is receding relative to us at a speed of 3000 km/s. It emits hydrogen redline of wavelength 6560 Å. When seen by us, the wavelength of this radiation will appear to be

(a) higher by approximately 65Å (b) lower by approximately 65Å

(c) lower by approximately 6Å (d) higher by approximately 6Å

55. Planck's constant (h), the speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants of nature. Which of the following combinations has the dimensions of length?

(a) $\sqrt{hG}/c^{3/2}$ (b) $\sqrt{hG}/c^{5/2}$ (c) $\sqrt{hc/G}$ (d) $\sqrt{Gc}/h^{3/2}$

56. Vector form of Lorentz Transformation can be expressed as, (here $\left[\vec{v} = v\hat{n}\right]$)

(a) $\underset{\text{vector}}{\vec{r}'} = \gamma\left[\underset{\text{Scalar}}{\left(\hat{n}.\vec{r}\right)}\underset{\text{scalar}}{\hat{n}.\vec{v}t}\right]$, $t' = \gamma\left[t - \vec{v}.\vec{r}/c^2\right]$

(b) $\vec{r}' = \gamma\left[\left(\hat{n}.\vec{r}\right)\hat{n} - \vec{v}t\right] + \left[\vec{r} - \left(\hat{n}.\vec{r}\right)\hat{n}\right]$, $t' = \gamma\left[t - \vec{v}.\vec{r}/c^2\right]$

(c) $\vec{r}' = \gamma\left[\vec{r} - \left(\hat{n}.\vec{r}\right)\hat{n}\right]$, $t' = \gamma\left[t - \left(\hat{n}.\vec{v}\right)/c^2\right]$

(d) $\vec{r}' = \gamma\left[\vec{r} - \left(\hat{n}.\vec{v}\right)\hat{n}\right]$, $t' = \gamma\left[t - \left(\hat{n}.\vec{r}\right)v/c^2\right]$

57. Dynamic mass of a particle is measured to be $\sqrt{3}$ times its rest mass. Ratio of kinetic energy and total energy is

(a) $\frac{\sqrt{5}-1}{\sqrt{5}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{3}}$ (c $\frac{\sqrt{2}-1}{\sqrt{2}}$ (d) $\frac{\sqrt{7}-1}{\sqrt{7}}$

8. A pion of mass $m_\pi$ at rest decays into a muon of mass $m_\mu$ and a neutrino of zero mass. Speed of muon is

[✓ (a) $\frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}c$] (b) $\frac{m_\pi - m_\mu}{m_\pi + m_\mu}c$ (c) $\frac{m_\pi}{m_\pi + m_\mu}c$ (d) $\frac{m_\mu}{m_\pi + m_\mu}c$

9. A constant force $F$ is applied to a relativistics particle of rest mass $m_0$. If the particle starts from rest at $t = 0$, its dynamic mass after a time 't' is

(a) $\frac{m_0}{\sqrt{1+\frac{F^2t^2}{m_0^2c^2}}}$ (b) $\frac{1}{c}\sqrt{F^2t^2 + m_0^2c^2}$ (c) $\frac{m_0}{\sqrt{1-\frac{F^2t^2}{m_0^2c^2}}}$ (d) $m_0 \tan h \frac{Ft}{m_0c}$

NEST

0. An atom of rest mass 'M' goes to excited state of rest mass $M + \Delta$, the transition from ground state to excited state takes place by absorption of a photon. Kinetic energy of atom in excited state is

(a) $\Delta c^2$ (b) $\frac{\Delta^2c^2}{M}$ [✓ (c) $\frac{\Delta^2c^2}{2M}$] (d) $\Delta c^2\left(1+\frac{\Delta}{2M}\right)$

NEST

1. Consider the decay process $\tau^- \to \pi^- + \nu_\tau$. The rest masses of $\tau^-$, $\pi^-$ and $\nu_\tau$ are, $M_\tau, M_\pi$ and zero respectively. The energy of $\pi^-$ is:

(a) $\frac{(M_\tau^2 - M_\pi^2)c^2}{2M_\tau}$ (b) $\frac{(M_\tau^2 + M_\pi^2)c^2}{2M_\tau}$ (c) $(M_\tau + M_\pi)c^2$ (d) $\sqrt{M_\tau M_\pi}c^2$

12. Velocity of a particle is $\vec{v} = \frac{c}{\sqrt{2}}(\hat{i} + \hat{j})$ in S frame and $\vec{v}' = \frac{c}{\sqrt{2}}(-\hat{i} + \hat{j})$ in $S'$ frame which is moving along x-direction w.r.t S. Velocity of $S'$ w.r.t $S$ frame is

(a) $\frac{\sqrt{2}\,c}{3}$ [✓ (b) $\frac{2\sqrt{2}\,c}{3}$] (c) $\frac{c}{\sqrt{2}}$ (d) $\frac{c}{2}$

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## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-9: Classical Mechanics (Special Theory of Relativity)

1. A thin rod is translating with speed $v = \frac{\sqrt{3}}{2}c$ such that directions of velocity makes an angle of 60° with length of the rod as measured in lab frame. Length of rod in lab frame is $l$. Proper length of the rod must be

(a) $\frac{\sqrt{5}}{2}\ell$ (b) $\sqrt{3}\ell$ (c) $\frac{\sqrt{7}}{2}\ell$ (d) $\frac{4\ell}{\sqrt{13}}$

2. Velocity of a particle as seen from frame S and $S'$ are $\frac{c}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ and $\frac{c}{\sqrt{2}}\left(-\hat{i}+\hat{j}\right)$. Then which of the following is correct.

(a) $x' = 3x - 2\sqrt{2}ct$ (b) $x' = 2\sqrt{2} - 3ct$ (c) $x' = \sqrt{2}x - ct$ (d) $x' = x - \frac{c}{2}t$

3. What work has to be done to accelerate a particle from rest to a speed $\frac{3c}{5}$. Rest mass of the particle is $m_0$.

(a) $\frac{5m_0c^2}{4}$ (b) $\frac{3m_0c^2}{4}$ (c) $\frac{m_0c^2}{4}$ (d) $\frac{2m_0c^2}{3}$

4. A source of light is moving towards an observer with speed $v << c$. Value of $\frac{\Delta\lambda}{\lambda_0}$ is

(a) $\frac{2v}{c}$ (b) $\frac{v}{c}$ (c) $\frac{v}{2c}$ (d) $\frac{v^2}{c^2}$

5. To a physicist a red light signal $\left(\lambda = 6700\text{Å}\right)$ appears green $\left(\lambda = 5250\text{Å}\right)$. What is speed of the physicist.

(a) 0.63c (b) 0.24c (c) 0.02c (d) 0.85c

6. A person is sitting at the centre of a circle. And a light source emitting a light of wavelength 589 nm is moving along the circle with speed 3c/5. What is wavelength of the light observed by the person

(a) 736 nm (b) 393 nm (c) 1178 nm (d) 295 nm

7. A pion moving with speed 3c/5 emits a muon. As seen in the lab frame muon moves perpendicular to pion with speed 4c/5. With what speed relative to pion the muon is emitted.

(a) (b) (c) (d)

8. Proper life time of a muon is $2\times10^{-6}$ sec. It is travelling with speed $\frac{4c}{5}$. What is the distance moved past the muon as seen by the muon.

(a) 480 meter (b) 288 meter (c) 384 meter (d) 256 meter

9. At what speed should an object travel so that it will appear twice as dense as it was at rest

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{c}{2}$ (c) $\frac{c}{\sqrt{2}}$ (d) $\frac{\sqrt{5}}{3}c$

10. A particle of charges Q and rest mass $M_0$ is moving at relativistic speed. If numerical value of mass density and charge density in lab frame are same while in motion. What is speed of the particle

(a) $c\sqrt{1-\frac{Q^2}{M_0^2}}$ (b) $c\sqrt{1-\frac{M_0^2}{Q^2}}$ (c) $\frac{c}{\sqrt{1-\frac{Q^2}{M_0^2}}}$ (d) $\frac{c}{\sqrt{1-\frac{M_0^2}{Q^2}}}$

11. Dynamic mass (effective mass) of a photon of wavelength 600 nm is

(a) $4.51\frac{eV}{c^2}$ (b) $2.07\frac{eV}{c^2}$ (c) $6.32\frac{eV}{c^2}$ (d) $12.53\frac{eV}{c^2}$

12. Which of the following is invariant under Lorentz transformation

(a) Amplitude of a wave (b) Phase of wave

(c) Frequency of a wave (d) wavelength of a wave

13. Which of the following is invariant under Lorentz transformation

(a) $\frac{p}{E}$ (b) $\frac{p^2}{E}$ (c) $\frac{d^3\vec{p}}{E}$ (d) $Ed^3\vec{p}$

14. A galaxy is moving away from the earth at such a speed that blue light $(\lambda=400nm)$ appears to be $\lambda=600\ nm$. The speed of galaxy is

(a) $\frac{3c}{5}$ (b) $\frac{5c}{13}$ (c) $\frac{5c}{9}$ (d) $\frac{4c}{9}$

15. A Radio active particle has proper mean life of 1μsec moves through the laboratory at a speed $2.7\times10^8$ meter/sec. What will be its lifetime observed by an observer in laboratory.

(a) 10 μs (b) 0.1μs (c) 2.30μs (d) 0.43μs

16. A shooter fires a bullet with velocity 'u' in the $\hat{x}$ direction at a target. The target is moving with a velocity 'v' in the $\hat{x}$ direction relative to the shooter and at a distance 'L' from the shooter at the instant when bullet is fired. How long from the instant at which the bullet is fired, will it take to hit the target in the target's frame of reference?

(a) $\frac{\gamma Lc}{uv}$ (b) $\frac{\gamma L\left(1-\frac{uv}{c^2}\right)}{(u-v)}$ (c) $\frac{L\left(1+\frac{c^2}{uv}\right)}{\gamma(u-v)}$ (d) $\frac{L}{c}$

17. An electron of rest mass $9.1\times10^{-31}kg$, is moving with a speed of 0.99c. The ratio of Newtonian kinetic energy to relativistic kinetic energy, will be

(a) 0.08 (b) 0.13 (c) 0.19 (d) 0.26

18. A star of radius ‘r’ contracts to a radius ‘r′’, while its temperature remains unchanged. Assuming uniform density for the star, the energy radiated by the star will be

(a) $mc^2\left(\frac{r'}{r}\right)^3$ (b) $mc^2\left[1-\left(\frac{r'}{r}\right)^3\right]$ (c) $mc^2\left[1-\left(\frac{r}{r'}\right)^3\right]$ (d) $mc^2\left(\frac{r}{r'}\right)^3$

19. Two photons approaches each other. The relative velocity of them will be
(a) c/2 (b) c/3 (c) c/4 (d) c

20. A pion of mass $m_\pi$ at rest, decays into a muon of mass $m_\mu$ and a neutrino of mass $m_\nu$.
The momentum of the muon will be

(a) $\frac{c\left(m_\pi^2-m_\mu^2\right)}{2m_\pi}$ (b) $\frac{c\left(m_\pi^2-m_\mu^2\right)}{m_\pi}$ (c) $\frac{c\left(m_\pi^2+m_\mu^2\right)}{m_\pi}$ (d) $\frac{c\left(m_\pi^2+m_\mu^2\right)}{2m_\pi}$

21. A particle of mass ‘m’ which is at rest in the laboratory decays into two particles, one of which has mass ‘m/2’ while other is massless. The magnitude of the momentum of each of the particles in the laboratory frame will be
(a) 3mc/8 (b) mc/8 (c) 3mc/4 (d) mc/4

### Assignment-9: Classical Mechanics

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (c)** | **2. (a)** | **3. (c)** | **4. (b)** | **5. (b)** | **6. (a)** | **7. ( )** |
| **8. (a)** | **9. (a)** | **10. (b)** | **11. (b)** | **12. (b)** | **13. ( )** | **14. ( )** |
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## Assignment-9: Classical Mechanics

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (c)** | **2. (a)** | **3. (c)** | **4. (b)** | **5. (b)** | **6. (a)** | **7. ()** |
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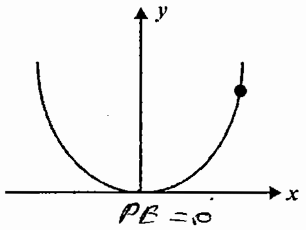
## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-10: Lagrangian Formation

1. A bead of mass 'm' slides along a wire kept in vertical plane as shown in figure. The equation of the wire is $y = \alpha x^2$. Lagrangian of the bead is



(a) $\frac{1}{2}m\dot{x}^2\left(1+4\alpha^2x^2\right)-mg\alpha x^2$ ✓

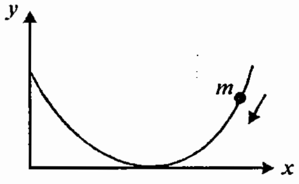
(b) $\frac{1}{2}m\dot{x}^2\left(1+4\alpha^2x^2\right)+mg\alpha x^2$

(c) $\frac{1}{2}m\dot{y}^2\left(1-\frac{1}{4y\alpha}\right)-mgy$ (d) $\frac{1}{2}m\dot{y}^2\left(1+\frac{1}{4y\alpha}\right)+mgy$

2. A projectile is thrown with intial speed 'u' at angle $\theta$ with horizontal. The projectile moves in x-y plane where 'x' axis is taken in horizontal and y-axis in vertical upward direction with origin being at point of projection. Lagrangian of projectile is:

(a) $\frac{1}{2}mu^2-mgy$ (b) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy$ (c) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)+mgy$ (d) $\frac{1}{2}mu^2-mgy\cos\theta$

3. A bead of mass 'm' slides in a cycloid shape wire kept in vertical plane as shown in figure. The parametric equation of the wire is $x = a(\theta-\sin\theta)$, $y = a(1+\cos\theta)$
Lagrangian of bead is



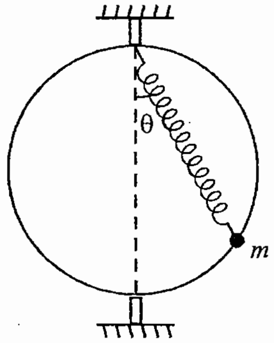
(a) $ma^2\dot{\theta}^2(1-\cos\theta)+mga$

(b) $ma^2\dot{\theta}^2(1+\cos\theta)-mga(1+\cos\theta)$

(c) $ma^2\dot{\theta}^2(1-\cos\theta)-mga(1+\cos\theta)$

(d) $ma^2\dot{\theta}^2-mga(1+\cos\theta)$

4. A bead of mass moves along a fixed vertical circle of radius 'R'. The bead is attached to highest point of the circle with a light spring of spring constant 'k'. If $\theta$ shown in figure is taken as generalised co-ordinate. what is lagrangian of the system [Natural length of spring is 2R and gravitation potential energy is taken to be zero at the centre of circle].



(a) $2mR^2\dot{\theta}^2-2kR^2(1-\cos\theta)^2+mgR\cos2\theta$

(b) $\frac{1}{2}mR^2\dot{\theta}^2-2kR^2(1-\cos\theta)^2+mgR\cos2\theta$

(c) $2mR^2\dot{\theta}^2-2kR^2(1-\cos\theta)^2+mgR\cos2\theta$

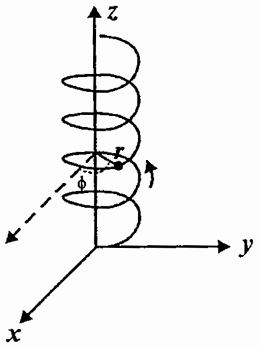
(d) $\frac{1}{2}mR^2\dot{\theta}^2-2kR^2(1-\cos\theta)^2+mgR\cos\theta$

5. In previous question, equation of motion of bead for smal value of $\theta$ is

[✓ (a)] $\ddot{\theta}+\frac{g}{R}\theta=0$ (b) $\ddot{\theta}+\frac{2g}{R}\theta$

(c) $\ddot{\theta}+\frac{g}{2R}\theta=0$ (d) $\ddot{\theta}+\frac{4g}{R}\theta=0$

6. A particle of mass 'm' moves under gravity along a spiral curve shown in figure. The equation of the spiral is $z=a\phi$ and $r=b=$ constant. Lagrangian of the system is



(a) $\frac{1}{2}m\left(a^2+b^2\right)\dot{\phi}^2+mga\phi$

[✓ (b)] $\frac{1}{2}m\left(a^2+b^2\right)\dot{\phi}^2-mga\phi$

(c) $\frac{1}{2}m\left(\frac{b^2}{a^2}+1\right)\dot{z}^2+mgz$ (d) $\frac{1}{2}m\left(\frac{b^2}{a^2}+1\right)\dot{z}^2-mgz$

7. A thin circular disc of mass 'm' and radius 'R' rolls on a fixed sphere without sliding. Equation of motion of the disc is

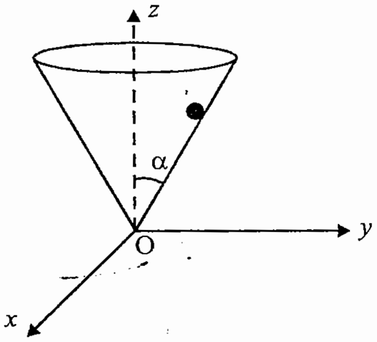
[✓ (a)] $\ddot{\theta}+\frac{4}{3}\frac{g}{(R+r)}\sin\theta=0$ (b) $\ddot{\theta}+\frac{3}{5}\frac{g}{(R+r)}\sin\theta=0$

(c) $\ddot{\theta}+\frac{1}{2}\frac{g}{(R+r)}\sin\theta=0$ (d) $\ddot{\theta}+\frac{2}{3}\frac{g}{(R+r)}\sin\theta=0$

8. Lagrangian of a system is $L=e^{\gamma t}\left(\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2\right)$ where $\gamma, m$ and $k$ are positive constants. Equation of motion of system is,

(a) $\ddot{x}-\gamma\dot{x}+\frac{k}{m}x=0$ (b) $\ddot{x}+\gamma\dot{x}-\frac{k}{m}x=0$ [✓ (c)] $\ddot{x}+\gamma\dot{x}+\frac{k}{m}x=0$ (d) $\ddot{x}-\gamma\dot{x}-\frac{k}{m}x=0$

9. A particle of mass 'm' moves on the inner surface of inverted cone of half vertex angle $\alpha$, if spherical polar co-ordinates are taken as generalised co-ordinates, Lagrangian of the system particle is



[✓ (a)] $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2\sin^2\alpha\right)-mgr\cos\alpha$

(b) $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2\right)-mgr\cos\alpha$

(c) $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2\sin^2\alpha\right)+mgr\cos\alpha$

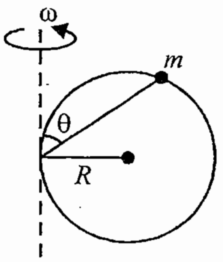
(d) $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2\cos^2\alpha\right)-mgr\cos\alpha$

10. In cylinderical coordinate system lagrangian of bead in previous question is

(a) $\frac{1}{2}m\left[\dot{z}^2\tan^2\alpha+z^2\dot{\phi}^2\sec^2\alpha\right]+mgz$ [✓ (b)] $\frac{1}{2}m\left[\dot{z}^2\sec^2\alpha+z^2\dot{\phi}^2\tan^2\alpha\right]-mgz$

(c) $\frac{1}{2}m\left[\dot{z}^2\sec^2\alpha+z^2\dot{\phi}^2\tan^2\alpha\right]+mgz$ (d) $\frac{1}{2}m\left[\dot{z}^2\sec^2\alpha+z^2\dot{\phi}^2\tan^2\alpha\right]-mgz$

11. A bead of mass 'm' is sliding on a vertical circular loop of radius R. The loop is rotated with constant angular velocity $\omega$ about a tangential axis shown in figure. Lagrangian of the bead is:



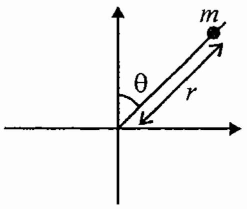
[✓ (a)] $2mR^2\left(\dot{\theta}^2+\sin^4\theta\omega^2\right)-mgR\sin 2\theta$

(b) $2mR^2\left(\dot{\theta}^2+\sin^4\theta.\cos^2\theta\omega^2\right)-mgR\sin 2\theta$

(c) $mR^2\left(\dot{\theta}^2+\sin^4\theta\,\omega^2\right)-mgR\sin 2\theta$

(d) $2mR^2\left(\dot{\theta}^2+\cos^4\theta\,\omega^2\right)-mgR\sin 2\theta$

12. A thin wire is free to rotate in a vertical plane as shown in figure. A bead of mass 'm' slides on the wire. Lagrangian of the bead is



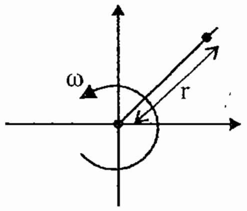
(a) $\frac{1}{2}m\left[\dot{r}^2+r^2\dot{\theta}^2\right]+mgr\cos\theta$

[✓ (b)] $\frac{1}{2}m\left[\dot{r}^2+r^2\dot{\theta}^2\right]-mgr\cos\theta$

(c) $\frac{1}{2}m\left[\dot{r}^2+r^2\sin^2\theta\dot{\theta}^2\right]+mgr\cos\theta$

(d) $\frac{1}{2}m\left[\dot{r}^2+r^2\sin^2\theta\,\dot{\theta}^2\right]-mgr\cos\theta$

13. A thin wire is rotated with uniform angular velocity $\omega$ in horizontal plane. A bead of mass 'm' slides on the wire. Distance of the bead from the fixed end of the wire as a function of time can be written as



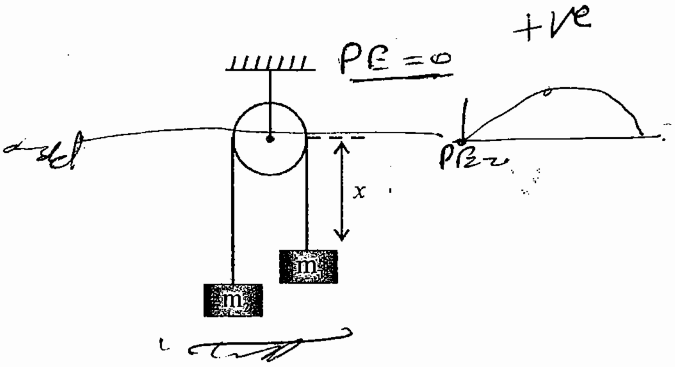
(a) $r=a\cos\omega t+B\sin\omega t$

(b) $r=A\,e^{\omega t}$

[✓ (c)] $r=A\,e^{-\omega t}+B\,e^{\omega t}$

(d) $r=A\tanh(\omega t)$

14. In the figure shown, two masses are suspended from a light pully of radius R, length of string is $l$. Lagrangian of the system in terms of '$x$' is



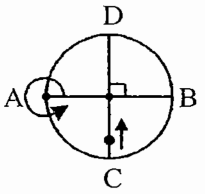
(a) $\frac{1}{2}(m_1+m_2)\dot{x}^2+m_1gx+m_2g(\ell-x-\pi R)$

(b) $\frac{1}{2}(m_1+m_2)\dot{x}^2-m_1gx-m_2g(\ell-x-\pi R)$

(c) $\frac{1}{2}(m_1-m_2)\dot{x}^2-m_1gx-m_2g(-x-\pi R)$

(d) $\frac{1}{2}(m_1-m_2)\dot{x}^2+m_1gx+m_2g(\ell-x-\pi R)$

15. A ring of mass 'M' and radius 'R' lying on a smooth table is free to rotate about a point A on its rim as shown in figure. CD is a light wire along the diameter of ring on which an insect of mass 'm' crawls starting from point C. If angle rotated by the ring $\phi$ and distance travelled by the insect '$x$' are taken as generalised coordinates, which of the following option is correct.



(a) Both $\phi$ and x are cyclic coordinates

(b) Only x is cyclic

(c) $\frac{d\phi}{dx}=-\left[\frac{2Rm}{2MR^2+m\left[(R-x)^2+R^2\right]}\right]$

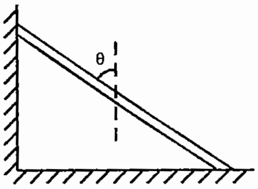
(d) $\frac{d\phi}{dx} = -\left[\frac{2RM}{2MR^2 + M\left[(R-x)^2 + R^2\right]}\right]$

16. If $F_r$ be radial component of resultant forcce acting on a particle of mass 'm' then in spherical polar coordinate system Newton's law in radial direction is written as,

(a) $F_r = m\ddot{r}$  $G_K = F \cdot \frac{\partial r}{\partial q_K}$  (b) $F_r = m\left(\ddot{r} - r\,\dot{\theta}^2\right)$  $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = Q_j$

(c) $F_r = m\left(\ddot{r} - r\,\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2\right)$  (d) $F_r = m\left(\ddot{r} - r\,\dot{\theta}^2 - r\sin\theta\,\dot{\phi}^2\right)$  $Q_j$ – Generalized force

17. A uniform ladder of length '2a' and mass 'M' slides against a smooth vertical wall as shown in the figure. If angle made with vertical is taken as generalised coordinate, Lagrangian of the rod is



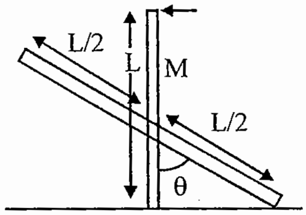
(a) $\frac{2}{3}Ma^2\dot{\theta}^2 - Mga\cos\theta$

(b) $\frac{Ma^2}{3}\dot{\theta}^2 - Mga\cos\theta$

(c) $\frac{4}{3}Ma^2\dot{\theta}^2 - Mga\cos\theta$  (d) $\frac{Ma^2\dot{\theta}^2}{6} - Mga\cos\theta$

18. A thin rod of mass 'M' and length 'L' is hold vertical on a smooth table. The rod is slightly displaced from its position due to which its centre falls vertically downward. If angle $\theta$ made with vertical is taken as generalised coordinate Lagrngian of the rod is

(Special condition)



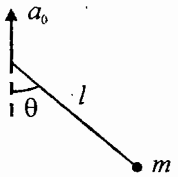
(a) $\frac{mL^2}{4}\left(\frac{1}{3} + \frac{1}{2}\sin^2\theta\right) - \frac{MgL\sin\theta}{2}$

(b) $\frac{ML^2}{4}\left(\frac{1}{3} + \frac{1}{2}\sin^2\theta\right) - \frac{MgL\cos\theta}{2}$

(c) $\frac{ML^2}{4}\left(\frac{1}{3} + \frac{1}{2}\cos^2\theta\right) - \frac{MgL\cos\theta}{2}$

(d) $\frac{ML^2}{4}\left(\frac{1}{3} + \frac{1}{2}\cos^2\theta\right) - \frac{MgL\sin\theta}{2}$

19. The point of suspension of a simple pendulum is moved vertically upward with constant acceleration $a_0$. Lagrangian of the pendulum is



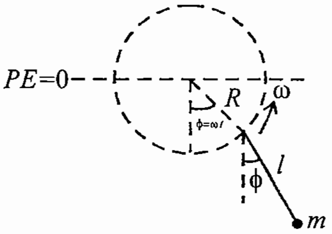
(a) $\frac{1}{2}m\ell^2\dot{\theta}^2 + m(g + a_0)\ell\cos\theta$

(b) $\frac{1}{2}m\left[\ell^2\dot{\theta}^2 + a_0^2t^2 + 2\ell\dot{\theta}a_0t\sin\theta\right] + mg\ell\cos\theta$

(c) $\frac{1}{2}m\left[\ell^2\dot{\theta}^2 + a_0^2t^2 + 2\ell\dot{\theta}a_0t\sin\theta\right] + m(g + a_0)\ell\cos\theta$

(d) $\frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell\cos\theta$

20. Point of suspension of a simple pendulum is moved along a vertical circle of radius R with constant angular velocity $\omega$. The pendulum oscillates in the plane of the circle considering the figure shown Lagrangian of pendulum can be written as,



(a) $\frac{1}{2}m\left[\omega^2R^2+\ell^2\dot{\phi}^2+2\omega R\ell\dot{\phi}\cos(\omega t-\phi)\right]+mg(R\cos\omega t+\ell\cos\phi)$

(b) $\frac{1}{2}m\ell^2\dot{\phi}^2+mg(R\cos\omega t+\cos\phi)$

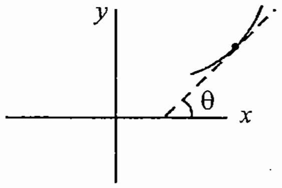
(c) $\frac{1}{2}m\left[\omega^2R^2+\ell^2\dot{\phi}+2\omega R\ell\dot{\phi}\cos\phi\right]+mg\left[R\cos\omega t+\ell\sin\phi\right]$

(d) $\frac{1}{2}m\ell^2\dot{\phi}^2+mg\ell\cos\phi$

21. Lagrangian of a system is $L=\dot{q}_k\,q_k-\sqrt{1-\dot{q}_k^2}$, equation of motion of the system is

(a) $\ddot{q}_k+q_k=0$ (b) $\ddot{q}_k=0$ (c) $\ddot{q}_k-q_k=0$ (d) $\ddot{q}_k+q_k\dot{q}_k=0$

22. A particle of mass 'm' moves in horizontal plane along the curve $s=\alpha\theta$ as shown in figure, where $\theta$ is angle made by tangent to the curve with x-axis and $s$ is the distance coverved along the curve. Lagrangian of particle is



$\frac{1}{2}mv^2$

$v=\frac{ds}{dt}=\alpha\dot{\theta}$

(a) $\frac{1}{2}m\alpha^2\dot{\theta}^2$ (b) $\frac{1}{2}m\alpha^2\sin^2\theta\,\dot{\theta}^2$

(c) $\frac{1}{2}m\alpha^2\cos^2\theta\,\dot{\theta}^2$ (d) $m\alpha^2\sin^2\theta\,\dot{\theta}^2$

23. In previous question if equation of path of the particle be $s=4\alpha\sin\theta$ then its lagrangian is

(a) $4m\alpha^2\dot{\theta}^2(1+\cos 2\theta)$ (b) $4m\alpha^2\dot{\theta}^2(1-\cos 2\theta)$

(c) $2m\alpha^2\dot{\theta}(1+\cos 2\theta)$ (d) $2m\alpha^2\dot{\theta}(1-\sin 2\theta)$

24. A planet of mass 'm' revolves around the sun of mass 'M', in an elliptical orbit. If motion of planet is confined in one plane, lagrangian of system is

(a) $\frac{1}{2}\left(\dot{r}^2+r^2\dot{\theta}^2\right)+\frac{GMm}{r}$ (b) $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-\frac{GMm}{r}$

(c) $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)+\frac{GMm}{r^2}$ (d) $\frac{1}{2}m\dot{r}^2+\frac{GMm}{r}$

25. Lagrangian of a system is $L=a\dot{x}^2-bx^2$ then

(a) $x=c_1t+c_2t^2+c_3$ (b) $x=c_1e^{-c_2t}+c_3$

(c) $x=c_1\sin(c_2t+c_3)$ (d) $x=c_1\,e^{-c_2t}\sin(c_2t+c_3)$

## CSIR-UGC-NET/JRF-PHYSICAL SCIENCES

## Assignment-10 : Lagrangian Formation

### Answer Key

1. (a)
2. (b)
3. (c)
4. (a)
5. (a)
6. (b)
7. (d)
8. (c)
9. (a)
10. (b)
11. (a)
12. (b)
13. (c)
14. (a)
15. (c)
16. (c)
17. (c)
18. (b)
19. (b)
20. (a)
21. (b)
22. (a)
23. (a)
24. (a)
25. (c)

## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-11: Lagrangian and Hamiltonian Formulation

1. Lagrangian of a system is $e^{\gamma t}\left[\dot{x}^2 - x^2\right]$, hamiltonian of the system is:

(a) $p_x^2\, e^{-\gamma t} + x^2 e^{\gamma t}$ (b) $\frac{p_x^2}{2} e^{\gamma t} - \frac{x^2}{2} e^{-\gamma t}$ [✓ (c) $\frac{p_x^2}{4} e^{-\gamma t} + x^2 e^{\gamma t}$] (d) $\frac{p_x^2}{2} e^{\gamma t} + \frac{x^2}{2} e^{-\gamma t}$

2. Lagrangian of a system is $L = \frac{1}{2} m v^2 - \alpha \vec{r}.\vec{v}$ where $\vec{v}$ is velocity and $\alpha$ is a constant. Hamiltonian of the system is:

(a) $\frac{(\vec{p} - \alpha\vec{r})^2}{2m}$ (b) $\frac{(\vec{p} + \alpha\vec{r})^2}{2m}$ (c) $\frac{p^2}{2m} + \frac{\alpha\vec{p}.\vec{r}}{m}$ (d) $\frac{p^2}{2m} - \frac{\alpha\vec{p}.\vec{r}}{m}$

3. Lagrangian of a system is $L = \frac{1}{2} m(\vec{v} + \vec{a})^2$, hamiltonian of the system is ($\vec{a}$ is constant and $\vec{v}$ is velocity of particle)

(a) $\frac{p^2}{2m} + \vec{a}.\vec{p}$ (b) $\frac{p^2}{2m} - \vec{a}.\vec{p}$ (c) $\frac{(\vec{p} + m\vec{a})^2}{2m}$ (d) $\frac{(\vec{p} - m\vec{a})^2}{2m}$

4. Lagrangian of a system is $L = -\alpha\sqrt{1 - \dot{x}^2} - V(x)$, hamiltonian of the system is

(a) $\sqrt{p_x^2 - \alpha^2} + V(x)$ (b) $\sqrt{p_x^2 + \alpha^2} + V(x)$

(c) $\frac{p_x}{\sqrt{1 - p_x^2/\alpha^2}}$ (d) $\frac{p_x + \alpha}{\sqrt{1 - p_x^2/\alpha^2}}$

5. A particle of mass m is moving under a central potential $V(r)$. In spherical polar coordinate $(r, \theta, \phi)$ which of the following is constant of motion?

(a) $\frac{p_r^2}{m}$ (b) $p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}$ (c) $p_\phi^2 + \frac{p_\theta^2}{\sin^2\theta}$ (d) $\frac{p_r^2}{m} + \frac{p_\theta^2}{mr^2\sin^2\theta}$

6. Hamiltonian for a case is given to be $H(\vec{p}, \vec{r}) = \frac{c\vec{p}}{n(\vec{p}, \vec{r})}$ where $c$ is a constant. Equations of motion for the case are

(a) $\dot{\vec{r}} = \frac{c\vec{p}}{np}$, $\dot{\vec{p}} = \frac{cp}{n^2}\vec{\nabla}_r n$ (b) $\dot{\vec{r}} = \frac{c\vec{p}}{np} - \frac{c\vec{p}}{np^2}\vec{\nabla}_p n$, $\dot{\vec{p}} = \frac{cp}{n^2}\nabla_r n$

(c) $\dot{\vec{r}} = \frac{cp}{n^2}\vec{\nabla}_r n$, $\dot{\vec{p}} = \frac{cp}{n^2}\vec{\nabla}_r n$ (d) $\dot{\vec{r}} = \frac{cp}{n^2}\vec{\nabla}_r n$, $\dot{\vec{p}} = \frac{cp}{n^2}\vec{\nabla}_p^2 n$

7. Lagrangian of a system is $L(x,\dot{x}) = e^{-x^2}\left(e^{-\dot{x}^2} + 2\dot{x}\int_0^{\dot{x}} e^{-\alpha^2}\,d\alpha\right)$. Its equation of motion is

[✓ (a) $\ddot{x} + x = 0$] (b) $\ddot{x} - x = 0$ (c) $\ddot{x} + x^2 = 0$ (d) $\ddot{x} + \dot{x}^2 = 0$

8. Lagrangian of a system is $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 2\omega(x\dot{y} - y\dot{x}) + \omega^2(x^2 + y^2)$ corresponding Hamiltonian $H(x,y,z,p_x,p_y,p_z)$ is

(a) $\frac{p_x^2 + p_y^2 + p_z^2}{2m} + \omega(xp_y - yp_x)$ (b) $\frac{p_x^2 + p_y^2 + p_z^2}{2m} + \omega(xp_y + yp_x)$

[✓ (c) $\frac{p_x^2 + p_y^2 + p_z^2}{2m} - \omega(xp_y - yp_x)$] (d) $\frac{p_x^2 + p_y^2 + p_z^2}{2m} - \omega(xp_y + yp_x)$

9. Lagrangian of a system is $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{qB}{2}(x\dot{y} - y\dot{x})$ for this system value of Poisson bracket $[\dot{x}, \dot{y}]$ is equal to

(a) $\frac{\text{qB}}{2\text{m}^2}$ [✓ (b) $\frac{\text{qB}}{\text{m}^2}$] (c) $\frac{2\text{qB}}{\text{m}^2}$ (d) zero

10. Four Lagrangian function are given to be

$L_1 = f(x,\dot{x}),\ L_2 = f(x,\dot{x}) + A(x\dot{y} - y\dot{x})$

$L_3 = f(x,\dot{x}) + A(x\dot{x} + y\dot{y}),\ L_4 = f(x,\dot{x}) + A(x\dot{y} + y\dot{x})$

which of these Lagrangian functions give same equation of motion

(a) $L_1$ and $L_2$ (b) $L_1$ and $L_3$ (c) $L_1$ and $L_4$ (d) $L_2$ and $L_3$

11. Lagrangian of a system is $L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + qEy - \frac{qB}{c}y\dot{x}$. Which of the following is not correct.

(a) $m\dot{x} - \frac{qBy}{c} = \text{constant}$ (b) $m\dot{z} = \text{constant}$

(c) $m\dot{y} + \frac{qBx}{c} + qEt = \text{constant}$ (d) $m\dot{y} + \frac{qBx}{c} - qEt = \text{constant}$

12. Lagrangian of a system is

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + 2m\omega\left(x\dot{y} - y\dot{x}\right) + m\omega^2\left(x^2 + y^2\right)$$

what $x$ component of force on the system.

(a) $2m\omega\dot{y} - 2m\omega x$ (b) $2m\omega\dot{x} - 2m\omega y$ (c) $2m\omega\dot{x} + 2m\omega x$ (d) $2m\omega\dot{y} + 2m\omega x$

13. A system is governed by the Hamiltonian $H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$

where a and b are constants and $p_x$, $p_y$ are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved?

(a) $a = -3,\ b = 2$ (b) $a = 3,\ b = -2$ (c) $a = 2,\ b = -3$ (d) $a = -2,\ b = 3$

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14. The Hamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length $l$ is $H = \frac{p_\theta^2}{2m\ell^2} + mg\ell(1-\cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is :

[✓ (a) $-\frac{2g}{\ell}p_\theta \sin\theta$] (b) $-\frac{g}{\ell}p_\theta \sin 2\theta$ (c) $\frac{g}{\ell}p_\theta \cos\theta$ (d) $\ell\, p_\theta^2 \cos\theta$

15. If the Lagrangian of a particle moving in one dimensions is given by $L = \frac{\dot{x}^2}{2x} - V(x)$, the Hamiltonian is:

(a) $\frac{1}{2}xp^2 + V(x)$ (b) $\frac{\dot{x}^2}{2x} + V(x)$ (c) $\frac{1}{2}\dot{x}^2 + V(x)$ (d) $\frac{p^2}{2x} + V(x)$

16. A thin rod of length of mass 'M' and length L is suspended from its one end. The rod is free to rotate about its one end. If motion of the rod is not confined in one plane, its Lagrangian in spherical polar coordinate is

(a) $\frac{ML^2}{3}\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + \frac{MgL}{2}\cos\theta$ (b) $\frac{ML^2}{6}\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + \frac{MgL}{2}\cos\theta$

(c) $\frac{ML^2}{2}\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + \frac{MgL\cos\theta}{2}$ (d) $\frac{ML^2}{2}\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + MgL\cos\theta$

17. Lagrangian of a system is $L = L_0 + L_1 + L_2 + L_3 + .....$ where $L_i$ is a homogeneous function of degree 'i' in $\dot{q}_i$ with coefficients being any function of $q_i$. Hamiltonian of the system is

(a) $H = L_0 + L_1 + 2L_2 + 3L_3........$ (b) $H = L_0 - L_1 + L_2 - L_3........$

(c) $H = -L_0 + L_2 + 2L_3 + 3L_4........$ (d) $H = -L_0 + L_1 + 2L_2 + 3L_3 + ........$

18. Lagrangian of a system is $L = \frac{1}{2}m\dot{x}^2\left[1 + \left(\frac{df}{dx}\right)^2\right] - mgf(x)$, its Hamiltonian is

(a) $\frac{p_x^2}{2m}\left[1 + \left(\frac{df}{dx}\right)^2\right] + mg\ f(x)$ (b) $\frac{p_x^2}{2m}\left(\frac{df}{dx}\right)^2 + mg\ f(x)$

(c) $\frac{p_x^2}{2m\left[1 + \left(\frac{df}{dx}\right)^2\right]} + mg\ f(x)$ (d) $\frac{p_x^2}{2m\left(\frac{df}{dx}\right)^2} + mg\ f(x)$

19. A block of mass ‘M’ is attached to a spring of mass ‘m’ and spring constant ‘$k$’. If ‘$x$’ be displacement of block from mean position. Lagrangian of the system is

(a) $\frac{1}{2}M\dot{x}^2 - \frac{1}{2}kx^2$ (b) $\frac{1}{2}(M+m)\dot{x}^2 - \frac{1}{2}kx^2$

(c) $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ (d) $\frac{1}{2}\left(M+\frac{m}{3}\right)\dot{x}^2 - \frac{1}{2}kx^2$

20. Lagrangian of a system is of $a\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + b\dot{\psi}^2 + c\cos\theta$. The constants of motion are

(a) $\dot{\theta}$ and $\dot{\phi}$ (b) $\dot{\phi}\sin^2\theta$ and $\dot{\psi}$ (c) $\dot{\theta}$ and $\dot{\psi}$ (d) $\dot{\phi}$ and $\dot{\psi}$

21. A bead of mass ‘m’ is constrained to move undergravity along a planar rigid wire that has parabolic shape $y = \frac{x^2}{\ell}$ where ‘$x$’ and ‘$y$’ are respectively horizontal and vertical coordinates. Lagrangian of the system is

(a) $\frac{1}{2}m\left(1+\frac{x^2}{\ell^2}\right)\dot{x}^2 + \frac{mgx^2}{\ell}$ (b) $\frac{1}{2}m\left(1+\frac{2x^2}{\ell^2}\right)\dot{x}^2 + \frac{mgx^2}{\ell}$

(c) $\frac{1}{2}m\left(1+\frac{x^2}{\ell^2}\right)\dot{x}^2 - \frac{mgx^2}{\ell}$ (d) $\frac{1}{2}m\left(1+\frac{4x^2}{\ell^2}\right)\dot{x}^2 - \frac{mgx^2}{\ell}$

22. A particle is moving under a potential $V = Ar^k$. If equation of motion of the particle is invariant under the transformation $\vec{r} \to \vec{r}\,' = \alpha\vec{r}$ and $t \to t' = \alpha^s t$, then value of ‘$s$’ must be equal to

(a) $\frac{k}{2}$ (b) $2k$ (c) $2-k$ (d) $\frac{2-k}{2}$

23. A particle of charge $q$ is moving in an electromagnetic potential $(\phi, \vec{A})$. Rate of change of canonical momentum is equal to

(a) $-q\vec{\nabla}\phi$ (b) $q\vec{\nabla}(\vec{v}.\vec{A})$ (c) $-q\vec{\nabla}\phi + q\vec{\nabla}(\vec{v}.\vec{A})$ (d) $-q\vec{\nabla}\phi + q\vec{\nabla}.\vec{A}$

24. A relativistic particle of rest mass $m_0$ and charge $q$ is moving in an electromagnetic potential $(\phi, \vec{A})$. Lagrangian of the particle is

(a) $\frac{1}{2}m_0v^2 - q\phi + q\vec{A}.\vec{v}$ (b) $\frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2 - q\phi + q\vec{A}.\vec{v}$

(c) $-m_0c^2\sqrt{1-v^2/c^2} - q\phi + q\vec{A}.\vec{v}$ (d) $-m_0c^2\sqrt{1-v^2/c^2} + q\phi - q\vec{A}.\vec{v}$

25. A particle of mass ‘m’ and charge ‘q’ moves in an electromagnetic field $(\phi, \vec{A})$, canonical momentum of the particle is

(a) $m\vec{v}$ (b) $m\vec{v} + q\vec{A}$ (c) $m\vec{v} - q\vec{\nabla}\phi$ (d) $m\vec{v} - q\vec{A}$

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CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-11: Lagrangian and Hamiltonian Formulation

### Answer Key

1. (c) 2. (b) 3. (b) 4. (b) 5. (b) 6. (b) 7. (a)
8. (c) 9. (b) 10. (c) 11. (c) 12. (d) 13. (d) 14. (a)
15. (a) 16. (b) 17. (c) 18. (c) 19. (d) 20. (b) 21. (d)
22. (d) 23. (c) 24. (c) 25. (b)

## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-12: CT & PB(Classical Mechanics)

1. Which of the following statements is NOT correct
   (a) In restricted (time independennt) CT new hamiltonian is equal to old hamilton
   (b) In CT Jacobian of transformation is equal to one
   (c) In CT new coordinates and momenta always have same dimension as the old ones.
   (d) CT satisfies group properties.

2. Generating function of an identity transformation is
   (a) $F_1 = Q_i q_i$ (b) $F_2 = q_i p_i$ (c) $F_1 = -Q_i q_i$ (d) $F_2 = -q_i p_i$

3. Which of the following generating function represents rotation in phase space through an angle $\theta$.
   (a) $qQ\cot\theta - \frac{\left(q^2+Q^2\right)}{2}\operatorname{cosec}\theta$ (b) $qQ\cos\theta - \left(\frac{q^2+Q^2}{2}\right)\sin\theta$
   (c) $qQ\operatorname{cosec}\theta - \frac{\left(q^2+Q^2\right)}{2}\cot\theta$ (d) $qQ\sin\theta - \frac{\left(q^2+Q^2\right)}{2}\cos\theta$

4. If the following transformation is canonical
   $$Q = q\tan\alpha p, \quad P = 2\ln\sin\beta p$$
   values of $\alpha$ and $\beta$ are respectively.
   (a) (1, 1) (b) (1, ½) (c) (½, 1) (d) ½, ½

5. Hamiltonian of a system is $H = \frac{p^2}{Qm} + mgq$. Consider a CT generated by G.F.
   $F(q,Q,t) = -\frac{1}{3m^2 g}\left[2m^2 g\left(Q-q\right)\right]^{3/2}$, new hamiltonian of the system is
   (a) $mgP$ (b) $mgQ$ (c) $mg\left(P+Q\right)$ (d) $mg\left(P-Q\right)$

6. Consider a CT $Q_i = p_i\tan t,\ P_i = q_i\tan t$. Generating function for the transformation is
   (a) $-Q_i\, q_i\cot 2t$ [✓ (b) $-2Q_i q_i\cot 2t$] (c) $-2Q_i q_i\tan t$ (d) $-2Q_i q_i\tan 2t$

7. If the transformation $Q = aq + bp,\ P = cq + dp$ is canonical then
   (a) ab + cd = 1 (b) ac + bd = 1 (c) ad – bc = 1 (d) ab – cd = 1

8. If $\vec{A}$ be a function of $p$ and $r$ and $\vec{L}$ is angular momentum. Poisson bracket $\left[\vec{A},\vec{L}\right]$ is
   (a) $\vec{L}\times\vec{A}$ (b) $2\vec{L}\times\vec{A}$ (c) $2\vec{r}\times\vec{A}$ (d) $2\vec{p}\times\vec{A}$

9. Values of 'a' and 'b' for which following transformation $Q=(2q)^a\cos^b p, P=(2q)^a\sin^b p$ is canonical are

(a) $a=\frac{1}{2}, b=1$ (b) $a=2, b=\frac{1}{2}$ (c) $a=1, b=1$ (d) $a=\frac{1}{2}, b=\frac{1}{2}$

10. Canonical transformation defined by the generating function, $F(Q,p)=-\left(e^{Q}-1\right)^2\tan p$ is

(a) $Q=\ln\left[\sqrt{2}\cos p+1\right], P=2\sqrt{2}\left[\sqrt{q}\cos p+1\right]\sin p$

(b) $Q=2\sqrt{q}\left[\sqrt{2}\cos p+1\right]\sin p, P=\sqrt{q}\left[\sqrt{q}\cos p+1\right]$

(c) $Q=\ln\left[\sqrt{q}\sin p+1\right], P=\sqrt{q}\left[\sqrt{q}\sin p+1\right]\cos p$

(d) $Q=\ln\left[\sqrt{q}\sin p+1\right], P=2\sqrt{q}\left[\sqrt{q}\sin p+1\right]\cos p$

11. The frequency function $F(p,q)$ for the transformation $p=\frac{1}{Q}, q=PQ^2$ is

(a) $\sqrt{pq}$ (b) $-\sqrt{pq}$ (c) $2\sqrt{pq}$ (d) $-2\sqrt{pq}$

12. If $H$ be the Hamiltonian of a system. Which of the given options is correct

(a) $\frac{dH}{dt}=0$ (b) $\frac{\partial H}{\partial t}=0$

(c) $\frac{dH}{dt}=\frac{\partial H}{\partial t}$ (d) None

13. If $q_k$ and $p_k$ be generalised co-ordinate and generalized momenta then $\{(q_k+p_k,H)\}$ is equal to

(a) $\dot{q}_k$ (b) $\dot{p}_k$ (c) $\dot{q}_k+\dot{p}_k$ (d) zero

14. Which one of the following options is correct

(a) $\{q_i,q_j\}=0,\{p_i,p_j\}=\delta_{ij},\{q_i,p_j\}=\delta_{ij}$ (b) $\{q_i,q_j\}=0,\{p_i,p_j\}=0,\{q_i,p_j\}=\delta_{ij}$

(c) $\{q_i,q_j\}=\delta_{ij},\{p_i,p_j\}=0,\{q_i,p_j\}=0$ (d) $\{q_i,q_j\}=\delta_{ij},\{p_i,p_j\}=\delta_{ij},\{q_i,p_j\}=0$

15. If $F$ and $G$ are two dynamical variables and $\dot{F}=0$ and $\dot{G}=0$, that is the two are constants of motion then

(a) $[F,\dot{G}]\neq 0$ (b) $[\dot{F},\dot{G}]\neq 0$ (c) $\frac{d}{dt}[F,G]=0$ (d) $[\dot{F},G]\neq 0$

16. If $\vec{F}$ is a vector and $p_j$ is generalized momentum then $[\vec{F},p_j]$ is equal to

(a) $\frac{\partial \vec{F}}{\partial p_j}$ (b) $\frac{\partial \vec{F}}{\partial q_j}$ (c) $\frac{\partial F}{\partial p_j}\dot{q}_i\delta_{ij}$ (d) zero

17. If $L_z$ be $z$ component of angular momentum and $p_y$ be $y$ component of linear momentum than $[L_z, xp_y]$ is equal to

(a) $yp_y+xp_x$ (b) $yp_y-xp_x$ (c) $xp_x-yp_y$ (d) zero

18. Poisson bracket $[L_x, yL_z]$ is equal to

(a) $zL_z$ (b) $p_z L_z$ (c) $zL_z - yL_y$ (d) $zL_z + yL_y$

19. $[p_x, e^{\alpha x^2}]$ is equal to

(a) $e^{\alpha x}$ (b) $2\alpha x e^{\alpha x^2}$ (c) $2\alpha x p_x e^{\alpha x^2}$ (d) $-2\alpha x e^{\alpha x^2}$

20. Poisson bracket $\{e^{\alpha q_i}, e^{\beta p_j}\}$ is equal to

(a) $e^{\alpha q_i} e^{\beta q_j}$ (b) $\alpha\beta e^{\alpha q_i} e^{\beta q_j}$ (c) $\alpha\beta\, e^{\alpha q_i} e^{\beta q_j} \delta_{ij}$ (d) zero

21. If $L_i$ is component of angular momentum then $[L_i, \vec{r}.\vec{p}]$ is equal to

(a) $\varepsilon_{ijk} r_j p_k$ (b) zero (c) $r_j p_k \delta_{jk}$ (d) $r_j p_k$

22. If $\vec{L}$ is angular momentum and $\phi$ is a scalar than $[\vec{L}, \phi]$ is equal to

(a) $\vec{\nabla}_{\vec{r}}\, \phi$ (b) $\vec{\nabla}_{\vec{p}}\, \phi$ (c) $\vec{\nabla}_{\vec{r}}.\vec{\nabla}_{\vec{p}}\, \phi$ (d) zero

23. $[\vec{r}, \phi]$ is equal to

(a) $\nabla_{\vec{r}}\, \phi$ (b) $\vec{\nabla}_{\vec{p}}\, \phi$ (c) $\vec{\nabla}_{\vec{r}}.\vec{\nabla}_{\vec{p}}\, \phi$ (d) zero

24. $\vec{L}, \vec{A}$ and $\hat{n}$ be angular momentum vector, a vector and a unit vector then $[\vec{A}, \vec{L}.\hat{n}]$ is equal to

(a) $\vec{A}.\hat{n}$ (b) $-\vec{A}.\hat{n}$ (c) $-\hat{n}\times\vec{A}$ (d) $(\vec{L}\times\vec{A}).\hat{n}$

25. $[\vec{p}, \phi]$ is equal to

(a) zero (b) $-\vec{\nabla}_{\vec{r}}\phi$ (c) $-\vec{\nabla}_{\vec{p}}\, \phi$ (d) $\phi\vec{r}.\vec{p}$

26. For a system of Lagrangian $L = L(\dot{q}_i, \dot{q}_j)$ and Hamiltonian $H = H(q_i, p_j)$ with $p_j = m\dot{q}_j$ $\frac{dL}{dt}$ is equal to

(a) $\frac{p_j}{m}\dot{p}_j$ (b) $q_j p_j$ (c) $\frac{2p_j\dot{p}_j}{m}$ (d) $2q_j\dot{q}_j$

27. If $\vec{r}$ and $\vec{v}$ be position vector and velocity of a particle of mass *m* then value of $[\vec{v}, \vec{r}\cdot\hat{n}]$ is { $\hat{n}$ is a constant unit vector}

(a) $-m\vec{n}$ (b) $-\frac{\vec{n}}{m}$ (c) zero (d) $\frac{\vec{n}}{m}$

28. In spherical polar co-ordinates value of $[\dot{\phi}, \phi]$ for a free particle of mass '*m*' is

(a) $-\frac{1}{m}$ (b) $-\frac{1}{mr^2}$ (c) $-\frac{1}{mr^2\sin^2\theta}$ (d) $-1$

29. In cylindrical co-ordinate value of $\frac{d}{dt}(\dot{r}\,\dot{z})$ for a free particle of mass *m* is

(a) $\dot{z}\,\dot{\phi}^2\, r$ (b) $\frac{\dot{z}\,\dot{r}\,\dot{\phi}}{2}$ (c) $2\,\dot{z}\,\dot{r}\,\dot{\phi}$ (d) $-\dot{z}\,\dot{r}\,\dot{\phi}$

30. Value of poisson bracket $[\vec{a}.\vec{L},\vec{b}.\vec{r}]$ is

(a) $(\vec{a}\times\vec{b}).\vec{r}$  (b) $(\vec{b}\times\vec{a}).\vec{r}$  (c) $(\vec{a}\times\vec{b}).\vec{p}$  (d) $(\vec{b}\times\vec{a}).\vec{p}$

31. If Hamiltonian is constant of motion and a quantity F is also a constant of motion then

(a) $[F,H]=0$  (b) $\frac{\partial H}{\partial t}=0$  (c) $\left[\frac{\partial F}{\partial t},H\right]=0$  (d) $\frac{\partial F}{\partial t}$ = constant

## ANSWER KEY

**1. (c)** **2. (b)** **3. (c)** **4. (d)** **5. (b)** **6. (b)** **7. (c)**
**8. (b)** **9. (a)** **10. (a)** **11. (c)** **12. (c)** **13. (c)** **14. (b)**
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**29. (a)** **30. (b)** **31. (d)**

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## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-12: CT & PB(Classical Mechanics)

1. Which of the following statements is NOT correct
(a) In restricted (time independennt) CT new hamiltonian is equal to old hamilton
(b) In CT Jacobian of transformation is equal to one
(c) In CT new coordinates and momenta always have same dimension as the old ones.
(d) CT satisfies group properties.

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(a) $F_1 = Q_i q_i$ (b) $F_2 = q_i P_i$ (c) $F_1 = -Q_i q_i$ (d) $F_2 = -q_i p_i$

3. Which of the following generating function represents rotation in phase space through an angle $\theta$.
(a) $qQ\cot\theta - \frac{(q^2+Q^2)}{2}\operatorname{cosec}\theta$ (b) $qQ\cos\theta - \left(\frac{q^2+Q^2}{2}\right)\sin\theta$
(c) $qQ\operatorname{cosec}\theta - \frac{(q^2+Q^2)}{2}\cot\theta$ (d) $qQ\sin\theta - \frac{(q^2+Q^2)}{2}\cos\theta$

4. If the following transformation is canonical
$$Q = q\tan\alpha p, \quad P = 2\ln\sin\beta p$$
values of $\alpha$ and $\beta$ are respectively.
(a) (1, 1) (b) (1, ½) (c) (½, 1) (d) ½, ½

5. Hamiltonian of a system is $H = \frac{p^2}{2m} + mgq$. Consider a CT generated by G.F. $F(q,Q,t) = -\frac{1}{3m^2g}\left[2m^2g(Q-q)\right]^{3/2}$, new hamiltonian of the system is
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6. Consider a CT $Q_i = p_i\tan t$, $P_i = q_i\tan t$. Generating function for the transformation is
(a) $-Q_i q_i \cot 2t$ [✓ (b) $-2Q_iq_i\cot 2t$] (c) $-2Q_iq_i\tan t$ (d) $-2Q_iq_i\tan 2t$

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(a) $\vec{L}\times\vec{A}$ (b) $2\vec{L}\times\vec{A}$ (c) $2\vec{r}\times\vec{A}$ (d) $2\vec{p}\times\vec{A}$

No option Machiny.

9. Values of 'a' and 'b' for which following transformation $Q=(2q)^a\cos^b p,\ P=(2q)^a\sin^b p$ is canonical are

(a) $a=\frac{1}{2}, b=1$ (b) $a=2, b=\frac{1}{2}$ (c) $a=1, b=1$ (d) $a=\frac{1}{2}, b=\frac{1}{2}$

10. Canonical transformation defined by the generating function, $F(Q,p)=-\left(e^Q-1\right)^2\tan p$ is

(a) $Q=\ln\left[\sqrt{2}\cos p+1\right], P=2\sqrt{q}\left[\sqrt{q}\cos p+1\right]\sin p$

(b) $Q=2\sqrt{q}\left[\sqrt{2}\cos p+1\right]\sin p, P=\sqrt{q}\left[\sqrt{q}\cos p+1\right]$

(c) $Q=\ln\left[\sqrt{q}\sin p+1\right], P=\sqrt{q}\left[\sqrt{q}\sin p+1\right]\cos p$

(d) $Q=\ln\left[\sqrt{q}\sin p+1\right], P=2\sqrt{q}\left[\sqrt{q}\sin p+1\right]\cos p$

11. The ~~frequency~~ Generating function $F(p,q)$ for the transformation $p=\frac{1}{Q}, q=PQ^2$ is

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12. If $H$ be the Hamiltonian of a system. Which of the given options is correct

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(c) $\frac{dH}{dt}=\frac{\partial H}{\partial t}$ (d) None

13. If $q_k$ and $p_k$ be generalised co-ordinate and generalized momenta then $\{(q_k+p_k,H)\}$ is equal to

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14. Which one of the following options is correct

(a) $\{q_i,q_j\}=0,\{p_i,p_j\}=\delta_{ij},\{q_i,p_j\}=\delta_{ij}$ (b) $\{q_i,q_j\}=0,\{p_i,p_j\}=0,\{q_i,p_j\}=\delta_{ij}$

(c) $\{q_i,q_j\}=\delta_{ij},\{p_i,p_j\}=0,\{q_i,p_j\}=0$ (d) $\{q_i,q_j\}=\delta_{ij},\{p_i,p_j\}=\delta_{ij},\{q_i,p_j\}=0$

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(a) $[F,\dot{G}]\neq 0$ (b) $[\dot{F},\dot{G}]\neq 0$ (c) $\frac{d}{dt}[F,G]=0$ (d) $[\dot{F},G]\neq 0$

16. If $\vec{F}$ is a vector and $p_j$ is generalized momentum then $[\vec{F},p_j]$ is equal to

(a) $\frac{\partial\vec{F}}{\partial p_j}$ (b) $\frac{\partial\vec{F}}{\partial q_j}$ (c) $\frac{\partial F}{\partial p_j}\dot{q}_i\delta_{ij}$ (d) zero

17. If $L_z$ be $z$ component of angular momentum and $p_y$ be $y$ component of linear momentum than $[L_z,xp_y]$ is equal to

(a) $yp_y+xp_x$ (b) $yp_y-xp_x$ (c) $xp_x-yp_y$ (d) zero

18. Poisson bracket $[L_x, yL_z]$ is equal to

(a) $zL_z$ (b) $p_zL_z$ (c) $zL_z - yL_y$ (d) $zL_z + yL_y$

19. $[p_x, e^{\alpha x^2}]$ is equal to

(a) $e^{\alpha x}$ (b) $2\alpha x e^{\alpha x^2}$ (c) $2\alpha x p_x e^{\alpha x^2}$ (d) $-2\alpha x e^{\alpha x^2}$

20. Poisson bracket $\{e^{\alpha q_i}, e^{\beta p_j}\}$ is equal to

(a) $e^{\alpha q_i} e^{\beta q_j}$ (b) $\alpha\beta e^{\alpha q_i} e^{\beta q_j}$ (c) $\alpha\beta\, e^{\alpha q_i} e^{\beta q_j} \delta_{ij}$ (d) zero

21. If $L_i$ is component of angular momentum then $[L_i, \vec{r}.\vec{p}]$ is equal to

(a) $\varepsilon_{ijk}\, r_j p_k$ (b) zero (c) $r_j p_k \delta_{jk}$ (d) $r_j p_k$

22. If $\vec{L}$ is angular momentum and $\phi$ is a scalar than $[\vec{L}, \phi]$ is equal to

(a) $\vec{\nabla}_{\vec{r}}\, \phi$ (b) $\vec{\nabla}_{\vec{p}}\, \phi$ (c) $\vec{\nabla}_{\vec{r}}.\vec{\nabla}_{\vec{p}}\, \phi$ (d) zero

23. $[\vec{r}, \phi]$ is equal to

(a) $\nabla_{\vec{r}}\, \phi$ (b) $\vec{\nabla}_{\vec{p}}\, \phi$ (c) $\vec{\nabla}_{\vec{r}}.\vec{\nabla}_{\vec{p}}\, \phi$ (d) zero

24. $\vec{L}, \vec{A}$ and $\hat{n}$ be angular momentum vector, a vector and a unit vector then $[\vec{A}, \vec{L}.\hat{n}]$ is equal to

(a) $\vec{A}.\hat{n}$ (b) $-\vec{A}.\hat{n}$ (c) $-\hat{n}\times\vec{A}$ (d) $(\vec{L}\times\vec{A}).\hat{n}$

25. $[\vec{p}, \phi]$ is equal to

(a) zero (b) $-\vec{\nabla}_{\vec{r}}\phi$ (c) $-\vec{\nabla}_{\vec{p}}\, \phi$ (d) $\phi\vec{r}.\vec{p}$

26. For a system of Lagrangian $L = L(\dot{q}_i, \dot{q}_j)$ and Hamiltonian $H = H(q_i, p_j)$ with $p_j = m\dot{q}_j$ $\frac{dL}{dt}$ is equal to

(a) $\frac{p_j}{m}\dot{p}_j$ (b) $q_j p_j$ (c) $\frac{2p_j\dot{p}_j}{m}$ (d) $2q_j\dot{q}_j$

27. If $\vec{r}$ and $\vec{v}$ be position vector and velocity of a particle of mass *m* then value of $[\vec{v}, \vec{r}\cdot\hat{n}]$ is {$\hat{n}$ is a constant unit vector}

(a) $-m\vec{n}$ (b) $-\frac{\vec{n}}{m}$ (c) zero (d) $\frac{\vec{n}}{m}$

28. In spherical polar co-ordinates value of $[\dot{\phi}, \phi]$ for a free particle of mass '*m*' is

(a) $-\frac{1}{m}$ (b) $-\frac{1}{mr^2}$ (c) $-\frac{1}{mr^2\sin^2\theta}$ (d) $-1$

29. In cylindrical co-ordinate value of $\frac{d}{dt}(\dot{r}\,\dot{z})$ for a free particle of mass *m* is

(a) $\dot{z}\,\dot{\phi}^2\, r$ (b) $\frac{\dot{z}\,\dot{r}\,\dot{\phi}}{2}$ (c) $2\,\dot{z}\,\dot{r}\,\dot{\phi}$ (d) $-\dot{z}\,\dot{r}\,\dot{\phi}$

30. Value of poisson bracket $[\vec{a}.\vec{L}, \vec{b}.\vec{r}]$ is

(a) $(\vec{a}\times\vec{b}).\vec{r}$ (b) $(\vec{b}\times\vec{a}).\vec{r}$ (c) $(\vec{a}\times\vec{b}).\vec{p}$ (d) $(\vec{b}\times\vec{a}).\vec{p}$

31. If Hamiltonian is constant of motion and a quantity F is also a constant of motion then

(a) $[F, H] = 0$ (b) $\frac{\partial H}{\partial t} = 0$ (c) $\left[\frac{\partial F}{\partial t}, H\right] = 0$ (d) $\frac{\partial F}{\partial t}$ = constant

## ANSWER KEY

**1. (c)** **2. (b)** **3. (c)** **4. (d)** **5. (b)** **6. (b)** **7. (c)**

**8. (b)** **9. (a)** **10. (a)** **11. (c)** **12. (c)** **13. (c)** **14. (b)**

**15. (c)** **16. (b)** **17. (b)** **18. (c)** **19. (d)** **20. (c)** **21. (b)**

**22. (d)** **23. (b)** **24. (c)** **25. (b)** **26. (c)** **27. (b)** **28. (c)**

**29. (a)** **30. (b)** **31. (d)**

# CAREER ENDEAVOUR
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## CSIR-UGC-NET/JRF | GATE - PHYSICS
## Unit Test : CLASSICAL MECHANICS

**Time : 01:00 Hour** **Date : 20-09-2014**
**M.M. : 60**

**Instructions:**

1. **There are Two Parts. Part-A contains 10 objective type questions, each question carry 3.5 marks and Part-B contains 5 objective type questions, each question carry 5 marks.**
2. **There is negative marking, @ 25% will be deducted for each wrong answer.**
3. **Attempt all the questions, use of calculator is not allowed.**

## PART-A

1. A car is moving with speed $\frac{3c}{5}$ on a horizontal road. A person on the car throws a ball in vertically upward direction with speed $\frac{c}{4}$. Speed of the ball as seen from ground is

(a) $\frac{c}{\sqrt{5}}$ (b) $\frac{2c}{\sqrt{5}}$ (c) $\sqrt{\frac{2}{5}}c$ (d) $\frac{c}{\sqrt{3}}$

2. Life time of a μ meson in its rest frame is $2\times10^{-6}$ sec. Rest mass of μ meson is $m_0$ and it is found to travel a distance of 450 meter in lab frame. Energy of μ meson is

(a) $\frac{3m_0c^2}{5}$ (b) $\frac{4m_0c^2}{5}$ (c) $\frac{2m_0c^2}{5}$ (d) $\frac{5}{2}m_0c^2$

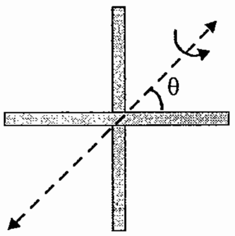
3. Proper life time of a particle is 10μs and its life time measured in lab frame is 20μs. What distance it can travel in lab.

(a) $2.6\times10^3\ m$ (b) $5.2\times10^3\ m$ (c) $1.3\times10^3\ m$ (d) $6.5\times10^2\ m$

4. From a circular disc of mass M and radius R, a circular portion of radius $\frac{R}{2}$ is removed. The diameter of removed portion lies along the radius of complete disc. Moment of inertia of remaining portion of the disc about a perpendicular axis through the centre of the complete disc is

(a) $\frac{5MR^2}{16}$ (b) $\frac{11MR^2}{32}$ (c) $\frac{13MR^2}{32}$ (d) $\frac{7MR^2}{16}$

5. Two thin rods each of mass M and length L are joined at their centres as shown in figure. Moment of inertia of system about the axis shown in the figure is



(a) $\frac{ML^2}{6}$ (b) $\frac{ML^2}{6}\sin^2\theta$

(c) $\frac{ML^2}{6}\cos^2\theta$ (d) $\frac{ML^2}{12}$

6. A solid cylinder is rolling on a horizontal surface. Ratio of its rotational kinetic energy to total kinetic energy is

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{1}{3}$ ✓

7. A hollow sphere of mass M rolls down an inclined plane of inclination $\theta$. Friction force on the sphere is

(a) $\frac{2}{3}g\sin\theta$ (b) $\frac{2}{5}g\sin\theta$ (c) $\frac{2}{3}g\cos\theta$ (d) $\frac{2}{5}g\cos\theta$

8. A particle of mass m moves under one dimensional potential $V(x)=bx^4-ax^3$. Angular frequency of oscillation about stable equilibrium point is

(a) $\frac{2}{3}\sqrt{\frac{a^2}{bm}}$ (b) $\frac{3}{2}\sqrt{\frac{a^2}{bm}}$ ✓ (c) $\sqrt{\frac{a^2}{bm}}$ (d) none of these

9. A particle is moving under one dimensional force. The potential energy of the particle is $V(x)=\frac{1}{2}\left(x^2-2\right)^2$, the maximum force on the particle is

(a) $\sqrt{\frac{128}{27}}$ ✓ (b) $\sqrt{\frac{64}{27}}$ (c) $\frac{8}{3}$ (d) $\sqrt{\frac{32}{27}}$

10. From a solid sphere of mass M and radius R a solid cube of maximum possible volume is cut. Moment of inertia of cube about a perpendicular axis through its face centre is

(a) $\frac{2MR^2}{3\sqrt{3}\pi}$ (b) $\frac{4MR^2}{9\pi}$ (c) $\frac{9MR^2}{32\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

## PART-B

11. A particle of rest mass $4m_0$ initially at rest decays into two particles each of rest mass $m_0$. Relative velocity of the two particle is

(a) $\frac{\sqrt{3}c}{2}$ (b) $\frac{4\sqrt{3}c}{7}$ ✓ (c) $\frac{4c}{7}$ (d) $\frac{\sqrt{3}c}{4}$

12. A rod of proper length $L_0$ is moving with speed $\frac{3c}{5}$ parallel to its length. A particle of rest mass $m_0$ is moving in opposite direction. Total energy of the particle is $\frac{5m_0c^2}{3}$. What is the length of the rod in the rest frame of the particle?

(a) $\frac{12L_0}{35}$ (b) $\frac{12L_0}{37}$ ✓ (c) $\frac{3L_0}{5}$ (d) $\frac{4L_0}{5}$

13. A thin rod of length L suspended from a point on its length such that it can oscillate about a horizontal axis, through point of suspension. If time period of oscillation be $2\pi\sqrt{\frac{7L}{12g}}$, then distance of a point of suspension from centre of the rod is

(a) $\frac{L}{3}$ ✓ (b) $\frac{2L}{3}$ (c) $\frac{L}{4}$ (d) $\frac{2L}{5}$

14. An atom of rest mass M can be excited to a state of rest mass 2M by a photon capture. The frequency of photon which can cause this transition is

(a) $\frac{Mc^2}{h}$ (b) $\frac{2Mc^2}{3h}$ [✓ (c) $\frac{3Mc^2}{2h}$] (d) $\frac{5Mc^2}{2h}$

15. Two point masses each equal to M are fixed at a seperation 2d. When a particle of mass 'M' is placed at the mid point and displaced slightly along the line perpendicular to the line joining the two fixed masses it executes S.H.M. with angular frequency $\sqrt{\frac{GM}{nd^3}}$. Which of the following is correct?

(a) $n=2$ [✓ (b) $n=\frac{1}{2}$] (c) $n=\frac{1}{3}$ (d) $n=\frac{1}{4}$

# CAREER ENDEAVOUR

ACADEMY PRIVATE LIMITED

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**CSIR-UGC-NET/JRF | GATE - PHYSICS**

**Unit Test: CLASSICAL MECHANICS**

**Time : 01:00 Hour** **Date : 18-08-2014**

**M.M. : 45**

**Instructions:**

1. **Question Paper contains 15 objective type questions, each question carry 3 marks.**
2. **There is negative marking, 1 mark will be deducted for each wrong answer.**
3. **Attempt all the questions, use of calculator is not allowed.**

---

1. A particle of unit mass moves under a potential $V(x) = 2x^3 - 3x^2$. Its kinetic energy at $x = \frac{1}{2}$ is 1 joule. Speed of the particle when it reaches $x = 1$ will be

(a) $\sqrt{2}\ m/s$ (b) $\sqrt{3}\ m/s$ (c) 1 m/s (d) 2 m/s

2. A particle of mass 'm' is placed on the top of a smooth sphere of radius R. Particle is slightly displaced due to which it slides on the sphere before leaving contact with the sphere. Kinetic energy of the particle at the moment it leaves contact with the sphere is

(a) $mgR$ (b) $\frac{mgR}{2}$ (c) $\frac{mgR}{3}$ (d) $\frac{mgR}{4}$

3. A person stands on a weighing machine kept in a lift. The lift accelerates in upward direction and then deaccelerates. If accelertion and deacceleration have same constant value and maximum and minimum reading of weighing machine are 100kg and 60kg then true weight of the person and acceleration of lift are respectively

(a) 90 kg and 2 m/s$^2$ (b) 80 and 2.5 m/s$^2$ (c) 90 and 2.5 m/s$^2$ (d) 80 and 2 m/s$^2$.

4. A circular disc rotates with constant angular velocity about its diameter. An insect moves along the periphery of the disc with constant speed relative to the disc. Which of the following statements about coriolis force on the insect is true.

(a) It is zero
(b) It is constant
(c) It increases as insect moves towards the axis of rotation
(d) It decreases as insect moves towards the axis of the rotation.

5. A particle is moving along a circle with constant speed. If $\vec{v}$ and $\vec{a}$ be velocity and acceleration of the particle then which of the following statements is correct.

(a) $\vec{v}$ = constant, $\vec{a}$ = constant (b) $\vec{v} \neq$ constant, $\vec{a}$ = constant

(c) $\vec{v} \neq$ constant, $\vec{a} \neq$ constant (d) $\langle\vec{v}\rangle \neq 0, \langle\vec{a}\rangle \neq 0$

6. An object is thrown upward with speed u. If returns with speed $v$. If in addition to gravity a constant resistive force F also acts on particle $u/v$ is

(a) $\frac{g - F/m}{g + F/m}$ (b) $\frac{g + F/m}{g - F/m}$ (c) $\sqrt{\frac{g - F/m}{g + F/m}}$ (d) $\sqrt{\frac{g + F/m}{g - F/m}}$

7. A body is thrown horizontally with velocity 500 m/s to the east at latitude 30°N, from top of a tower of height 19.6m Southward deflection of body as it striker the ground is
(a) 2.5 cm (b) 5.2 cm (c) 7.3 cm (d) 10.6 cm

8. A body of mass 0.3 kg starts sliding down from the top of a smooth sphere of radius 1m. The sphere rotates with constant angular velocity 6 rad/sec about vertical axis through its centre. In the frame fixed to the sphere what is centrifugal force on the particle at the moment it leaves contact with the sphere
(a) 2N (b) 4N (c) 6N (d) 8 N

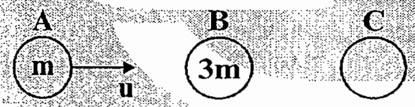
9. A train of mass 2000 tons moves at a place where latitude is 60°N. What should be speed of the train so that net pseudo forces (inertial forces) becomes zero
(a) 116 m/s E to W (b) 116 m/s N to S (c) 210 m/s W to E (d) 315 m/s E to W

10. A block of mass 100kg is placed on an inclined plane of inclination 30°. A horizontal force F = 100 pushes the block against inclined plane. If coefficient of static and kinetic friction are 0.2 and 0.18 respectively, then friction force on the block is
(a) 179 N (b) 161.8 N (c) 173.2 N (d) 155.9 N

11. A particle of mass ‘m’ falling under gravity is acted upon by a resistive force b$v$, where b is a constant. Time taken by particle to reach terminal velocity is
(a) $\frac{b}{mg}$ (b) $\frac{2b}{mg}$ (c) $\frac{b}{2mg}$ (d) infinity

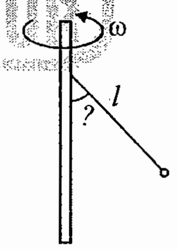
12. Two particles of masses $m_1$ and $m_2$ are placed ‘2d’ distance apart. Due to gravitational attraction the masses move towards each other what is ratio of K.E. to magnitude of P.E. when separation between them becomes ‘d’
(a) 1 (b) 1/2 (c) 2 (d) 1/3

13. Three blocks A, B and C are placed on smooth surface as shown in figure. A is given a speed ‘u’ towards B. If after collision A, B and C stick to each other and final speed of system becomes 4/5 then value mass of block ‘C’ is



(a) m (b) 2m (c) 4m (d) 6m

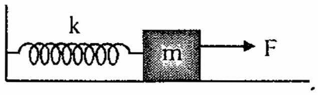
14. A simple pendulum is suspended from a vertical axis which is rotating with constant angular velocity ‘$\omega$’ as shown in the figure. Angle made by the string of pendulum with axis is



(a) $\tan^{-1}\frac{g}{\omega^2\ell}$ (b) $\sin^{-1}\frac{g}{\omega^2\ell}$

(c) $\cos^{-1}\frac{g}{\omega^2\ell}$ (d) $\tan^{-1}\frac{\omega^2\ell}{g}$

15. A block of mass ‘m’ attached to a spring of spring constant ‘k’. The spring is pulled by a horizontal force of constant magnitude F. Potential energy of spring at the moment when block momentarily comes to rest is



(a) $\frac{F^2}{k}$ (b) $\frac{2F^2}{k}$ (c) $\frac{F^2}{2k}$ (d) $\frac{2F^2}{3k}$

# CSIR-UGC-NET/JRF | GATE

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# Assignment

## Quantum Physics

## PHYSICAL SCIENCES

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**South Delhi Centre:**
28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## Assignment – 1: Fundamental of Quantum Mechanics

1. The de Broglie wavelength of particles of mass m with average momentum p at a temperature T in three dimensions is given by

(a) $\lambda = \dfrac{h}{\sqrt{2mk_BT}}$ (b) $\lambda = \dfrac{h}{\sqrt{3mk_BT}}$ (c) $\lambda = \dfrac{h}{\sqrt{2k_BT}}$ (d) $\lambda = \dfrac{h}{\sqrt{2m}}$

2. The angular frequency $\omega$ of deep water waves varies as the inverse square root of the wavelength $\lambda$, i.e., $\omega(\lambda) \propto 1/\sqrt{\lambda}$. Which of the following is the relation between its group velocity $v_g$ and $v_p$?

(a) $v_g = v_p/2$ (b) $v_g = v_p$ (c) $v_g = 2v_p$ (d) $v_g = \lambda v_p$

3. The energy of a 200 nm photon is:

(a) 0.01 eV (b) 100 eV (c) 10 eV (d) 1 eV

4. Phase velocity and group velocity for a wave will be identical

(a) If the phase velocity is independent of wavelength.
(b) If the phase velocity is dependent on wavelength.
(c) If the group velocity is dependent on wavelength.
(d) If the group velocity is dependent on refractive index of the medium.

5. An electron in an X-ray tube is accelerated through a voltage V and strikes a tungsten target. The shortest wavelength photon emitted will have

(a) $\lambda = \dfrac{\hbar c}{eV}$ (b) $\lambda = \dfrac{eV}{hc}$ (c) $\lambda = \dfrac{hc}{eV}$ (d) $\lambda = \dfrac{eV}{\hbar c}$

6. The dispersion relation for a certain type of excitation is given by $\omega = Ak^a$, where A and $a > 0$ are constants. For these excitations, the group velocity is less than the phase velocity when

(a) $a = 1$ (b) $a = \sqrt{2}$ (c) $a < 1$ (d) $a \geq 2$

7. A small object is viewed with a microscope, using light of wavelength $\lambda$ to illuminate the object. Let $\alpha$ be the half-angle subtended by the objective of the microscope at the position of the object being examined. The uncertainty $\Delta x$ in the position of the object is of the order of

(a) $\lambda/\sin\alpha$ (b) $\lambda/\cos\alpha$ (d) $\lambda\sin\alpha$ (d) $2\lambda\sin\alpha$

8. The angular frequency $\omega$ of deep water waves varies as the inverse square root of the wavelength $\lambda$, i.e., $\omega(\lambda) \propto 1/\sqrt{\lambda}$. Which of the following is the relation between its group velocity $v_g$ and $v_p$?

(a) $v_g = v_p/2$ (b) $v_g = v_p$ (c) $v_g = 2v_p$ (d) $v_g = \lambda v_p$

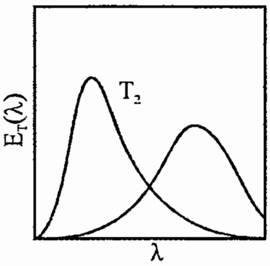
9. If a particle of mass $m = 5 \times 10^{-28}$ kg is confined to move in a space of size $10^{-15}$ m, the minimum value of its kinetic energy will approximately be

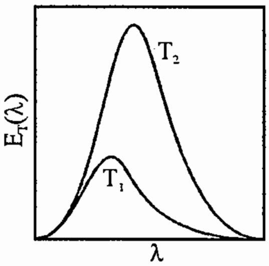
(a) $10^{-9}$ kg m$^2$/s$^2$ (b) $6.6 \times 10^{-19}$ kg m$^2$/s$^2$
(c) $10^{-12}$ kg m$^2$/s$^2$ (d) $6.6 \times 10^{-15}$ kg m$^2$/s$^2$

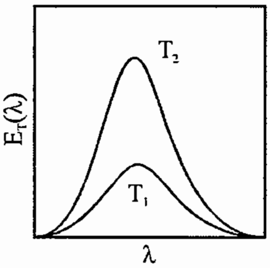
10. The velocity of an electron in the ground state of a hydrogen atom is $v_H$. If $v_p$ be the velocity of an electron in the ground state of postironium, then

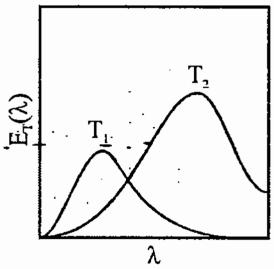
(a) $v_p = v_H$ (b) $v_p = 2v_H$ (c) $v_p = \frac{v_H}{2}$ (d) $v_p = \sqrt{2}v_H$

11. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength $\lambda$ at two temperatures $T_1$ and $T_2$ ($T_1 < T_2$)?

(a)


(b)


(c)


(d)


**[ANSWERS]**

**1. (b)** **2. (a)** **3. ( )** **4. (a)** **5. (c)** **6. (c)** **7. (a)**

**8. (a)** **9. ( )** **10. ( )** **11. (c)**

## Assignment – 2: Operators

1. $\hat{A}$ and $\hat{B}$ represents two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A}\hat{B}$ to be Hermitian, it is sufficient that

(a) $\hat{B}$ is Hermitian
(b) $\hat{B}$ is anti-Hermitian
(c) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute
(d) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ is anti-commute.

2. If $[x, p] = i\hbar$, the value of $\left[x^3, p\right]$ is :

(a) $2i\hbar x^2$
(b) $-2i\hbar x^2$
(c) $3i\hbar x^2$
(d) $-3i\hbar x^2$

3. The commutator $\left[x, P^2\right]$, where x and P are position and momentum operators respectively, is

(a) $2i\hbar P$
(b) $-i\hbar P$
(c) $2i\hbar \, x \, P$
(d) $-2i\hbar \, x \, P$

4. The commutator $\left[x, p_x^2\right]$ of the quantum mechanical operators x and $p_x^2$ is given by

(a) $2i\hbar$
(b) $2i\hbar p_x$
(c) $-2i\hbar p_r$
(d) $2i\hbar x$

5. Two Hermitian operators A and B commute (on a finite dimensional linear vector space). This implies that

(a) every eigenvector of A is necessarily an eigenvector of B.
(b) a complete set of simultaneous eigenvector of A and B exist.
(c) every vector is necessarily an eigenvector of A and B.
(d) all eigenvectors of A are always orthogonal to those of B.

6. An electron moving in one dimension $(-\infty < x < \infty)$ in a potential of period L is in eigenstate of its Hamiltonian. Its wavefunction is of the form $u_k(x)\exp(ikx)$. The $u_k(x+L)$ is equal to

(a) $Lu_k(x)$
(b) $u_k(x-L)$
(c) $(1/L)u_k(x)$
(d) $u_k(x)$

7. Let A and B be the Hermitian operators correspoinding to two physical observables of a system, such that $[A, B] \neq 0$. We may conclude in general that

(a) The uncertainty product is always $(\Delta A)(\Delta B) > \frac{h}{2}$
(b) The system can never be in a state in which $(\Delta A)(\Delta B) = 0$
(c) Neither A nor B can have any eigenstates.
(d) A and B may have one or more common eigenstate(s), but not a complete set of these.

8. Let $N = b^\dagger b$, where the operators b and $b^\dagger$. satisfy the relations $bb^\dagger + b^\dagger b = 1$ and $b^2 = 0 = \left(b^\dagger\right)^2$. The eigenvalues of N are

(a) Positive and negative integers
(b) all positive integers
(c) ±1 and 0 only
(d) 0 and 1 only

9. The commutator $\left[\hat{x}, [\hat{x}, \hat{p}]\right]$ is

(a) $i\hbar\hat{x}$
(b) $-i\hbar\hat{x}$
(c) $2i\hbar\hat{x}$
(d) 0

10. An operator $\hat{F}$ is such that $\hat{F}\psi(x)=\psi(x+a)$. Here $\psi(x)$ is any wave function in the position representation and a is a constant. If $\hat{x}$ and $\hat{p}$ are position and momentum operators, then $\hat{F}$ is given by

(a) $\exp\left(\frac{-ia\hat{p}}{\hbar}\right)$ (b) $\exp\left(\frac{ia\hat{x}}{\hbar}\right)$ (c) $\exp\left(\frac{-ia\hat{x}}{\hbar}\right)$ (d) $\exp\left(\frac{ia\hat{p}}{\hbar}\right)$

11. If the momentum operator $\hat{p}$ is represented by multiplication by p on a wave function of a particle in one dimension, the position operator $\hat{x}$ is given by:

(a) $\hat{x}=i\hbar\frac{\partial}{\partial p}$ (b) $\hat{x}=-i\hbar\frac{\partial}{\partial p}$ (c) $\hat{x}=\frac{\partial}{\partial p}$ (d) $\hat{x}=x$ (multiplication by x)

12. Three operators X, Y and Z satisfy the commutation relations

$$[X,Y]=i\hbar Z,[Y,Z]=i\hbar X \text{ and } [Z,X]=i\hbar Y$$

The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is

(a) $\{0,\pm1,\pm2,\pm3,.......\}$ (b) $\left\{\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2},.......\right\}$

[✓ (c) $\left\{0,\pm\frac{1}{2},\pm1,\pm\frac{3}{2},\pm2,\pm\frac{3}{2},........\right\}$] (d) $\left\{-\frac{1}{2},+\frac{1}{2}\right\}$

13. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $\left[\hat{A},\hat{B}\right]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $\left[\left[\hat{A},\hat{B}\right],\left[\hat{B},\hat{A}\right]\right]$ is equal to

(a) $\hat{A}\hat{B}\hat{A}\hat{B}-\hat{B}\hat{A}\hat{B}\hat{A}$ (b) $\hat{A}\left(\hat{A}\hat{B}-\hat{B}\hat{A}\right)-\hat{B}\left(\hat{B}\hat{A}-\hat{A}\hat{B}\right)$ (c) zero (d) $\left(\left[\hat{A},\hat{B}\right]\right)^2$

14. x and p are two operators which satisfy $c[x,p]=i$. the operators X and P are defined as $X=x\cos\phi+p\sin\phi$ and $Y=-x\sin\phi+p\cos\phi$ for $\phi$ real. Then [X, Y] equals

(a) 1 (b) −1 (c) i (d) −i

15. For any operator $A, i\left(A^{+}-A\right)$ is:

(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

16. The Hamiltonian of a three-level system is:

$$H=a\begin{pmatrix}2&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

The energy eigenvalues are

(a) $2a, a-a$ (b) $2a, a, 0$ (c) $2a, 0, 0$ (d) $2a, a, a$

17. The operators x and $p_x$ are represented as $p_x\to\hat{p}_x, x\to i\hbar\frac{\partial}{\partial p_x}$. The value of $\left[\hat{x},\hat{p}_x\right]$ is:

(a) zero (b) $-i\hbar c$ (c) $i\hbar$ (d) $\hbar$

18. Define $\sigma_x=\left(f^{\dagger}+f\right)$, and $\sigma_y=-i\left(f^{\dagger}-f\right)$, where the $\sigma$'s are Pauli spin matrices and $f,f^{\dagger}$ obey anticommutations relations $\{f,f\}=0\{f,f^{\dagger}\}=1$. then $\sigma_z$ is given by

(a) $f^{\dagger}f-1$ (b) $2f^{\dagger}f-1$ (c) $2f^{\dagger}f+1$ (d) $f^{\dagger}f$

19. The commutator $\left[x^2, p^2\right]$ is:

(a) $2\,i\hbar\,xp$ (b) $2i\hbar(xp+px)$ (c) $2i\hbar px$ (d) $2i\hbar(xp-px)$

20. Which of the following is a self-adjoint operator in the spherical polar coordinate system $(r, \theta, \varphi)$?

(a) $-\frac{i\hbar}{\sin^2\theta}\frac{\partial}{\partial\theta}$ (b) $-i\hbar\frac{\partial}{\partial\theta}$ (c) $-\frac{i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$ (d) $-i\hbar\sin\theta\frac{\partial}{\partial\theta}$

21. Given the usual canonical commutation relations, the commutator $[A,B]$ of $A = i\left(xp_y - yp_x\right)$ and $B = \left(yp_z + zp_y\right)$ is:

(a) $\hbar\left(xp_z - p_xz\right)$ (b) $-\hbar\left(xp_z - p_xz\right)$ (c) $\hbar\left(xp_z + p_xz\right)$ (d) $-\hbar\left(xp_z + p_xz\right)$

22. A proton is confined to a cubic box, whose sides have length $10^{-12}$ *m*. What is the minimum kinetic energy of the proton? The mass of proton is $1.67 \times 10^{-27}$ *kg* and Planck's constant is $6.63 \times 10^{-34}$ *Js*.
(a) $1.1 \times 10^{-17}\,J$ (b) $3.3 \times 10^{-17}\,J$ (c) $9.9 \times 10^{-17}\,J$ (d) $6.6 \times 10^{-17}\,J$

23. The operator $\left(\frac{d}{dx} - x\right)\left(\frac{d}{dx} + x\right)$ is equivalent to

(a) $\frac{d^2}{dx^2} - x^2$ (b) $\frac{d^2}{dx^2} - x^2 + 1$

(c) $\frac{d^2}{dx^2} - x\frac{d}{dx}x^2 + 1$ (d) $\frac{d^2}{dx^2} - 2x\frac{d}{dx} - x^2$

24. The hermitian conjugate of the operator $(-\partial/\partial x)$ is
(a) $\partial/\partial x$ (b) $-\partial/\partial x$ (c) $i\partial/\partial x$ (d) $-i\partial/\partial x$

25. If the operators A and B satisfy the commutation relation [A, B] = I, where I is the identity operator, then
(1) $[e^A, B] = e^A$ (2) $[e^A, B] = [e^B, A]$
(3) $[e^A, B] = [e^{-B}, A]$ (4) $[e^A, B] = 1$

26. If A, B and C are non-zero Hermitian operators, which of the following relations must be false?
(a) [A, B] = C (b) AB + BA = C (c) ABA = C (d) A+ B = C

## [ANSWERS]

|  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|
| **1. (c)** | **2. (c)** | **3. (a)** | **4. (b)** | **5. (b)** | **6. (d)** | **7. (b)** |
| **8. (d)** | **9. (d)** | **10. (d)** | **11. (a)** | **12. (c)** | **13. (c)** | **14. (c)** |
| **15. (a)** | **16. (a)** | **17. (c)** | **18. (b)** | **19. (b)** | **20. (c)** | **21. (c)** |
| **22. (c)** | **23. ( )** | **24. ( )** | **25. ( )** | **26. (a)** |  |  |

## Assignment – 3: Schrodinger

1. A particle of mass m is represented by the wavefunction $\psi(x) = Ae^{ikx}$, where k is the wavevector and A is a constant. The magnitude of the probability current density of the particle is

(a) $|A|^2 \frac{\hbar k}{m}$  (b) $|A|^2 \frac{\hbar k}{2m}$  (c) $|A|^2 \frac{(\hbar k)^2}{m}$  (d) $|A|^2 \frac{(\hbar k)^2}{2m}$

2. If the probability that x lies between x and x+dx is $p(x)dx = ae^{-ax}$, where $0 < x < \infty, a > 0$, then the probability that x lies between $x_1$ and $x_2\ (x_2 > x_1)$ is:

(a) $\left(e^{-ax_1} - e^{-ax_2}\right)$  (b) $a\left(e^{-ax_1} - e^{-ax_2}\right)$  (c) $e^{-ax_2}\left(e^{-ax_1} - e^{-ax_2}\right)$  (d) $e^{-ax_1}\left(e^{-ax_1} - e^{-ax_2}\right)$

3. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$, with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigenstates of the system, requires that the constants $c_1$ and $c_2$ must satisfy the condition

(a) $|c_1|.|c_2| = 1$  (b) $|c_1| + |c_2| = 1$  (c) $(|c_1| + |c_2|)^2 = 1$  (d) $|c_1|^2 + |c_2|^2 = 1$

4. A normalized state of a particle moving in a potential $V(x)$ is given by

$$\psi(x,t) = \sum_{n=0}^{\infty} C_n \phi_n(x) e^{-F_n t/7}$$

where $\phi_n(x)$'s are the normalized eigenfunctions of the particle corresponding to the energies $E_n$'s. Then

(a) $\sum_{n=0} |C_n|^2 = 1$

(b) The average energy of the particle in the state $\psi(x,t)$ is $\sum_{n=\infty}^{\infty} |C_n| E_n$

(c) $\psi(x,t)$ is an eigenfunction of the Hamiltonian of the particle.

(d) $\psi(x,t)$ is an eigenfunction of the momentum operator.

5. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$.

(a) $\varphi(x) = A \tan x, A > 0$  (b) $\varphi(x) = B \cos x$, B real

(c) $\varphi(x) = C \exp(-D/x^2), C > 0, D < 0$  (d) $\varphi(x) = E\, x \exp(-Fx^2), E, F > 0$

6. Probability currentdensity in quantum mechanics is represented by the operator.

(a) $\frac{i\hbar}{2m}\left(\psi\bar{\nabla}\psi^* - \psi^*\bar{\nabla}\psi\right)$  (b) $\frac{\hbar}{2m}\left(\psi\bar{\nabla}\psi^* - \psi^*\bar{\nabla}\psi\right)$

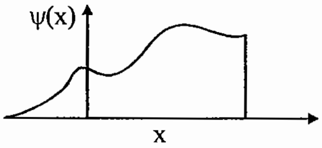
(c) $\frac{\hbar}{2m}\left(\psi\bar{\nabla}\psi^* + \psi^*\bar{\nabla}\psi\right)$  (d) $\frac{i\hbar}{2m}\left(\psi\bar{\nabla}\psi^* + \psi^*\bar{\nabla}\psi\right)$

7. Let the hermitian operator H be the Hamiltonian of a quantum mechanical particle, $E_0$ its ground state energy, and $\psi(\vec{r})$ an arbitrary normalized trial wavefunction. The quantity $\int \psi^*(\vec{r}) H \psi(\vec{r}) dV$
(a) must necessarily be less than or equal to $E_0$.
(b) must necessarily be greater than or equal to $E_0$.
(c) may be any real number — greater than, equal to, or less than $E_0$.
(d) is purely imaginary.

8. The wave function $\psi(\vec{r})$ of a particle moving in three-dimensional space has the physical dimensions of

(a) $(\text{Length})^{-3/2}$ (b) $(\text{Length})^{3/2}$ (c) $(\text{Length})^{-1}$ (d) $(\text{Length})$

9. The plot of a particular function is as shown



(a) This is not acceptable as a bound state wave function because it is not zero at the origin.
(b) This is not acceptable as a bound state wave function because it is discontinuous.
(c) This is acceptable as a bound state wave function.
(d) This is not acceptable as a bound state wave function but acceptable as a scattering state wave function.

10. Let $\psi = c_1\phi_1 + c_2\phi_2 + c_3\phi_3$, $\phi_1, \phi_2, \phi_3$ being (orthogonal) eigen functions of H then,
(a) $\psi$ is always an eigenfunction of H. (b) $\psi$ can never be an eigen function of H
(c) $\psi$ is an eigen function only if $\phi_1, \phi_2, \phi_3$ are degenerate.
(d) $\psi$ is an eigen function if any two of $\phi_1, \phi_2, \phi_3$ are degenerate.

11. A is an operator which does not vary with time, and $\langle A \rangle$ the expection value of A in the state $|\psi\rangle$. Which of the following equation is correct in the Schrodinger representation.

(a) $\frac{\partial}{\partial t}\langle A \rangle = \frac{1}{i\hbar}\langle [A,H] \rangle$ (b) $\frac{d}{dt}\langle A \rangle = +\frac{1}{i\hbar}\langle [A,H] \rangle$

(c) $\frac{\partial}{\partial t}\langle A \rangle = -\frac{1}{i\hbar}\left\langle \frac{1}{H-A} \right\rangle$ (d) $\frac{d}{dt}\langle A \rangle = 0$

12. The Schordinger equation for a quantum system is a linear differential equation of the type
(a) First order in time and first order in space
(b) First order in space and second order in time.
(c) Second order in space and first order in time.
(d) Second order in space and second order in time.

13. The wave function of a particle at time t = 0 is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$, where $|u_1\rangle$ and $|u_2\rangle$ are the normalized eigenstates with eigenvalues $E_1$ and $E_2$ respectively, $(E_2 > E_1)$. The shortest time after which $|\psi(t)\rangle$ will become orthogonal to $|\psi(0)\rangle$ is

(a) $\frac{-\hbar\pi}{2(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{E_2 - E_1}$ (c) $\frac{\sqrt{2}\,\hbar\pi}{E_2 - E_1}$ (d) $\frac{2\hbar\pi}{E_2 - E_1}$

**Common data for Q. 14 and Q. 15**

The wavefunction of a particle moving in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$

14. The energy of the particle is

(a) $\frac{5\hbar^2k^2}{2m}$ (b) $\frac{3\hbar^2k^2}{4m}$ (c) $\frac{\hbar^2k^2}{2m}$ (d) $\frac{\hbar^2k^2}{m}$

15. The probability current density for the real part of the wavefunction is

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

**Common data for Q.16 and Q.17.**

A free particle of mass m moves along the x direction. At t = 0, the normalized wave function of the particle is given by $\psi(x,0) = \frac{1}{(2\pi\alpha)^{1/4}} \exp\left[-\frac{x^2}{4\alpha^2} + ix\right]$, where $\alpha$ is a real constant.

16. The expectation value of the momentum, in this state is:

(a) $\hbar\alpha$ (b) $\hbar\sqrt{\alpha}$ (c) $\alpha$ (d) $\hbar/\sqrt{\alpha}$

17. The expectation value of the particle energy is:

(a) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{3/2}}$ (b) $\frac{\hbar^2}{2m}\alpha^2$ (c) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{3/2}}$ (d) $\frac{\hbar^2}{8m\alpha^{3/2}}$

18. A particle is in the normalized state $|\psi\rangle$ which is superposition of the energy eigenstates $|E_0 = 10\text{ eV}\rangle$ and $|E_1 = 30\text{ eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV. The state $|\psi\rangle$ is given by

(a) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle + \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0 = 10\text{ eV}\rangle + \sqrt{\frac{2}{3}}|E_1 = 30\text{ eV}\rangle$

(c) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle - \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0 = 10\text{ eV}\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\text{ eV}\rangle$

19. The wave function of a particle, moving in a one-dimensional time-independent potential V(x), is given by $\psi(x) = e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form

(a) $V(x) \propto x$ (b) $V(x) \propto x^2$ (c) $V(x) = 0$ (d) $V(x) \propto e^{-ax}$

20. The wavefunction of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t=0) = \frac{1}{\sqrt{15}}\left[2\psi_0(x) - \psi_1(x)\right]$$

where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies $E_0$ and $E_1$. The wavefunction of the particle at a time 't' is:

(a) $\frac{1}{\sqrt{5}}e^{i(E_0E_i)t/2\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (b) $\frac{1}{\sqrt{5}}e^{-iE_0t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$

(c) $\frac{1}{\sqrt{5}}e^{-iE_0t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (d) $\frac{1}{\sqrt{15}}\left[2\psi_0(x)e^{-iE_0t/\hbar} - \psi_1(x)e^{-iE_1t/\hbar}\right]$

21. Which of the following is an allowed wavefunction for a particle in a bound state? N is a constant and $\alpha, \beta > 0$

(a) $\psi = N\dfrac{e^{-\alpha r}}{r^3}$ (b) $\psi = N\left(1-e^{-\alpha r}\right)$

(c) $\psi = Ne^{-\alpha x}e^{-\beta\left(x^2+y^2+z^2\right)}$ (d) $\psi = \begin{cases} \text{non-zero constant} & \text{if } r < R \\ 0 & \text{if } r > R \end{cases}$

22. Which of the following wave function is acceptable as the solution of the Schrodinger equation for all values of x?

(a) $\psi(x) = A\sec x$ (b) $\psi(x) = A\tan x$ (c) $\psi(x) = Ae^{x^2}$ (d) $\psi(x) = Ae^{-x^2}$

23. Which of the following wavefunctions can be bound-state solution of the one-dimensional Schrodinger equation for all values of x?

(a) $\psi = A\sec x$ (b) $\psi = A\,e^{x}$ (c) $\psi = A\,e^{x^2}$ (d) $\psi = A\,e^{-x^2}$

24. The ground state (apart form normalization) of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $\exp\left(-x^2/2\right)\cosh\left(\sqrt{2}.x\right)$. The potential $V(x)$, in suitable units so that $\hbar = 1$, is (up to an additive constant).

(a) $x^2/2$ (b) $x^2/2-\sqrt{2}\,x\tanh\left(\sqrt{2}\,x\right)$

(c) $x^2/2-\sqrt{2}\,x\tan\left(\sqrt{2x}\right)$ (d) $x^2/2-\sqrt{2}\,x\coth\left(\sqrt{2}\,x\right)$

25. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \dfrac{\sqrt{15}\left(a^2-x^2\right)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty $\Delta p$ in its momentum is

(a) $\dfrac{2\hbar}{5a}$ (b) $\dfrac{5\hbar}{2a}$ (c) $\dfrac{\sqrt{10}\,\hbar}{a}$ (d) $\dfrac{\sqrt{5}\,\hbar}{\sqrt{2}\,a}$

26. Consider the wave function $Ae^{ikr}(r_0/r)$, where $A$ is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k/m)$, is _______.

27. If the expectation value of the momentum is $\langle p\rangle$ for the wavefunction $\psi(x)$, then the expectation value of momentum for the wavefunction $e^{ikx/\hbar}\psi(x)$ is

(a) $k$ (b) $\langle p\rangle - k$ (c) $\langle p\rangle + k$ (d) $\langle p\rangle$

## Assignment -3: Schrodinger

### [ANSWERS]

1. (a) 2. (a) 3. (d) 4. (a,c) 5. (d) 6. (a) 7. (b)

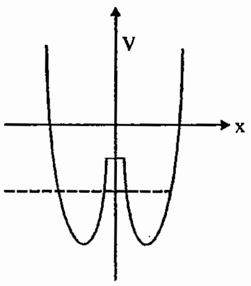
8. (a) 9. (c) 10. (c) 11. (a) 12. (c) 13. (b) 14. (c)

15. (d) 16. (b) 17. (c) 18. (d) 19. (c) 20. (d) 21. (c)

22. (d) 23. (d) 24. (b) 25. (d) 26. (0.25) 27. ( )

## Assignment – 4: One-dimensional Potential

1. A particle with energy E is in a time-independent double well potential as shown in the figure. Which of the following statements about the particle is NOT correct?



(a) The particle will always be in a bound state.
[✓ (b)] The probability of finding the particle in one well will be time-dependent.
(c) The particle will be confined to any one of the wells.
(d) The particle can tunnel from one well to the other, and back.

2. A one-dimensional harmonic oscillator is in the state

$$\psi(x)=\frac{1}{\sqrt{14}}\left[3\psi_0(x)-2\psi_1(x)+\psi_2(x)\right]$$

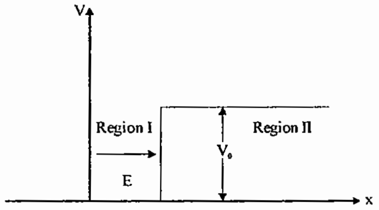
Where $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is:

(a) 0 (b) $\frac{3}{\sqrt{14}}$ [✓ (c)] $\frac{9}{14}$ (d) 1

3. A free particle is moving in +x direction with a linear momentum p. The wavefunction of the particle normalised in a length L is:

(a) $\frac{1}{\sqrt{L}}\sin\frac{p}{h}x$ (b) $\frac{1}{\sqrt{L}}\cos\frac{p}{h}x$ (c) $\frac{1}{\sqrt{L}}e^{-i\frac{p}{h}x}$ [✓ (d)] $\frac{1}{\sqrt{L}}e^{i\frac{p}{h}x}$

4. A partricle is incident with a constant energy E on a one-dimensional potential barrier as shown in the figure.



The wavefunctions in regions I and II are respectively
(a) decaying, oscillatory
(b) oscillatory, oscillatroy
[✓ (c)] oscillatory, decaying
(d) decaying, decaying.

5. A particle of energy E is incident on a rectangular potential barrier of height $V_0$ and width b.
[✓ (a)] The reflection coefficient for $E<V_0$ tends to unity as b tends to infinity.
(b) The reflection coefficient is zero for some specific values of $E > V_0$.
(c) The transmission coefficient is always zero for $E < V_0$.
(d) The transmission coefficient is zero for some specific values of $E>V_0$.

6. In the first excited state of a one-dimensional harmoic oscillator with angular frequency $\omega$, the energy eigenvalue is given by

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ [✓ (c)] $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$

7. For the ground state of a particle moving freely in a one-dimensional box $0 \le x \le L$ with rigid reflecting end-points, the uncertainty product $(\Delta x)(\Delta p)$ is:

(a) $h/2$ (b) $\sqrt{2}\,h$ (c) $> h/2$ (d) $h/\sqrt{3}$

8. The ground state energy of a particle in an infinite square-well potential of width L is E. If the width of the wall is reduced to L/2, then the ground state energy becomes
(a) 2E (b) E/2 [✓ (c) 4E] (d) E/4

9. Consider the operators a and $a^+$, satisfying the commutation relation $\left[a, a^+\right] = I$, the unit operator. There are no normalisable eigenstates of the operator
(a) $a^+a$ [✓ (b) $a$] [✓ (c) $a^+$] (d) $\left(a^+a\right)^2$

10. The first excited state of a one dimensional harmonic oscillator has eigenfunction $\phi(x) = Nxe^{-\alpha^2x^2/2}$ where N is:
(a) $\sqrt{2\alpha^3/\sqrt{\pi}}$ (b) $2\alpha^3/\sqrt{\pi}$ [✓ (c) $\sqrt{\alpha^2/\pi}$] (d) $\alpha/\pi$

11. The expectation value of the momentum in the ground state of one - dimensional harmonic oscillator of mass m and frequency $\omega$ is:
(a) $\frac{1}{\sqrt{2m\hbar\omega}}$ (b) $\sqrt{2m\hbar\omega}$ [✓ (c) 0] (d) $\infty$

12. The unnormalized ground state wave function of a partial confined to a one dimensional box of length L is:
(a) $A\sin\left(\frac{2\pi x}{L}\right)$ [✓ (b) $A\sin\left(\frac{\pi x}{L}\right)$] (c) $A\sin\left(\frac{L}{\pi x}\right)$ (d) $A\sin\left(\frac{L}{2\pi x}\right)$

13. A stream of particles of mass m and energy E encounter a potential step of height W (<E). In the limit W approaching E, the fraction of particles reflected from the barrrier will
(a) Appraoch zero [✓ (b) Appraoch unity] (c) Diverge to infinity (d) Approach half.

14. Consider one dimensional attractive potential of arbitrary shape. Which of the following statement is true?
(a) Only one bound state exists.
[✓ (b) It has at least one bound state.]
(c) It may have bound state only if the depth of the potential is greater than some critical depth.
(d) It has no bound state.

15. For a one-dimensional harmonic oscillator, let $\psi_n(x)$ denote the eigenstate of the $n^{th}$ quantum state. Which one of the statements is correct?
[✓ (a) Parity of $\psi_n(x)$ is even when n is even.] (b) Parity of $\psi_n(x)$ is odd when n is even.
(c) Parity of $\psi_n(x)$ is even where n is odd (d) $\psi_n(x)$ has mixed parity.

16. The wavefunction of a particle is given by $\psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\phi_1\right)$, where $\phi_0$ and $\phi_1$ are the normalized eigenfictions with energies $E_0$ and $E_1$ corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state $\psi$ is:
(a) $\frac{E_0}{2} + E_1$ (b) $\frac{E_0}{2} - E_1$ (c) $\frac{E_0 - 2E_1}{3}$ [✓ (d) $\frac{E_0 + 2E_1}{3}$]

17. A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $u(x) = u_0x$. The mean position of the particle at temperature T is:
[✓ (a) $\frac{k_BT}{u_0}$] (b) $(k_BT)^2/u_0$ (c) $\sqrt{\frac{k_BT}{u_0}}$ (d) $u_0k_BT$

18. A Particle in a one-dimensional potential has the wavefunction

$$\psi(x)=\frac{1}{\sqrt{a}}\exp\left(\frac{-|x|}{a}\right)$$

Where 'a' is a constant. It follows that for a positive constant $V_0$, the potential $V(x)=$

(a) $V_0x^2$ (b) $V_0|x|$ (c) $-V_0\delta(x)$ (d) $-V_0/|x|$

19. Consider a praticle in a one dimensional potential that satisfies $V(x)=V(-x)$. Let $|\psi_0\rangle$ and $|\psi_1\rangle$ denote the ground and the first excited states, respectively, and let $|\psi\rangle=\alpha_0|\psi_0\rangle+\alpha_1|\psi_1\rangle$ be a normalized state with $\alpha_0$ and $\alpha_1$ being real constants. The expectation value $\langle x\rangle$ of the position operator x in the state $\langle\psi|$ is given by

(a) $\alpha_0^2\langle\psi_0|x|\psi_0\rangle+\alpha_1^2\langle\psi_1|x|\psi_1\rangle$ (b) $\alpha_0\alpha_1\left[\langle\psi_0|x|\psi_1\rangle+\langle\psi_1|x|\psi_0\rangle\right]$

(c) $\alpha_0^2+\alpha_1^2$ (d) $2\alpha_0\alpha_1$

20. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and first excited state of a one dimensional harmonic oscillator. The uncertainty $\Delta p$ in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, is:

(a) $\Delta p=\sqrt{\hbar m\omega}/2$ (b) $\Delta p=\sqrt{\hbar m\omega/2}$

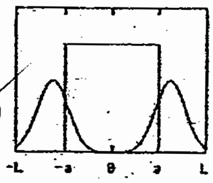
(c) $\Delta p=\sqrt{\hbar m\omega}$ (d) $\Delta p=\sqrt{2\hbar m\omega}$

21. The schematic form of the ground state wave-function in the one-dimensional potential.

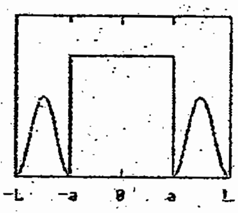
$$V(x)=\begin{cases}V_0, & \text{for } a\geq|x|\\ 0, & \text{for } a<|x|<L\\ \infty, & \text{for } |x|\geq L\end{cases}$$

where $V_0$, a positive constant, is given by

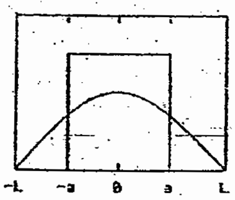
(a)



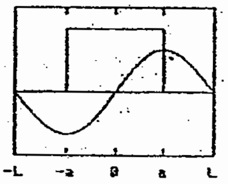
(b)



(c)



(d)



22. A particle of mass m in one dimension moves in a potential $V(x)=\frac{k}{2a}\left(1-e^{-ax^2}\right)$, where k and $\alpha$ are positive constants. If the particle executes small oscillations round x= 0, its quantum mechanical zero-point energy is

(a) $\frac{1}{2}\sqrt{\frac{\hbar^2 k}{m}}$ (b) $\frac{1}{2}\sqrt{\frac{\hbar^2 k}{m\alpha}}$ (c) $\frac{1}{2}\sqrt{\frac{\hbar^2 m}{k}}$ (d) $\frac{1}{2}\sqrt{\frac{\hbar^2 m\alpha}{k}}$

23. A particle is confined to a one dimensional harmonic oscillator potential in the region $0 < x < \infty$. At x = 0 there is n infinite barrier. The energy levels of the particle are separated by

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$

24. A function of the form $(x^2-2)e^{-ax^2}$ can represent the state of a one dimensional quantum mechanical system in

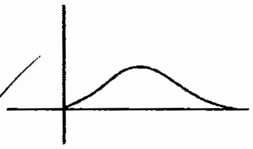
(a) its ground state (b) its first excited state

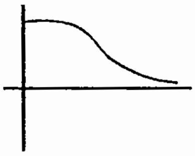
(c) its second exited state (d) a superposition of first and second excited state

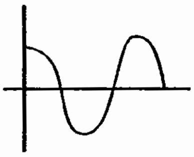
25. A system is in the state $\psi=\frac{1}{2}\left(\psi_1+\sqrt{3}\,\psi_2\right)$ where $\psi_1$ and $\psi_2$ denote energy eigenstates corresponding to the eigenvalues $E_1$ and $E_2$ respectively. When a measurement of energy is made, the probability of obtaining the value $E_1$ is

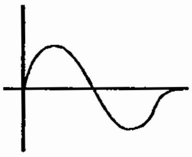
(a) $\sqrt{3}/2$ (b) $3/4$ (c) $1/2$ (d) $1/4$

26. Consider a particle in a one dimensional potential $V(x)=\begin{cases} x^2 \text{ for } x>0 \\ \infty \text{ for } x\le 0\end{cases}$. The schematic form of the ground state wave function is as follows”

(a)


(b)


(c)


(d)


27. $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the simple harmonic oscillator wave functions corresponding to energies $\frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega$ and $\frac{5}{2}\hbar\omega$ respectively At t = 0 a particle is in he state

$$\psi(x)=\frac{i}{3}\psi_0(x)-\frac{\sqrt{5}}{3}\psi_1(x)+\frac{1}{\sqrt{3}}\psi_2(x).$$

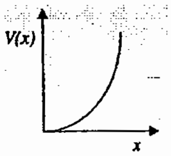
If an energy measurement is made the most probable outcome is

(a) $\frac{5}{2}\hbar\omega$ (b) $\frac{1}{2}\hbar\omega$ (c) $\frac{9}{2}\hbar\omega$ (d) $\frac{3}{2}\hbar\omega$

28. One-dimensional barrier is given by the potential $V(x)=\begin{cases}U \text{ for } |x|<1\\ 0 \text{ for } |x|>1\end{cases}$ where U is a positive constant. Particles arriving from the left with an energy $E<U$ impinge on the barrier. The general form of the wave function in the region $|x|<1$ is

[✓ (a) $a\,e^{kx}+b\,e^{-kx}$] (b) $a\,e^{-ikx}+b\,e^{-kx}$ (c) $a\,e^{kx}+b\,e^{-ikx}$ (d) $a\,e^{ikx}+b\,e^{-ikx}$

29. A particle is constrained to move in a truncated harmonic potential well ($x>0$) as shown in the figure. Which one of the following statements is CORRECT ?

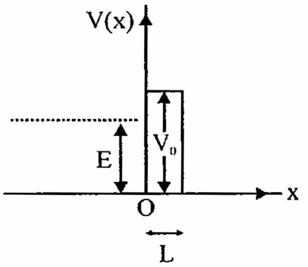


(a) The parity of the first excited state is even (b) The parity of the ground state is even

(c) The ground state energy is $\frac{1}{2}\hbar\omega$ [✓ (d) The first excited state energy is $\frac{7}{2}\hbar\omega$]

**Common data for Q.30, Q.31 and Q.32.**

A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0<x<L$) and height $V_0$ as shown in the figure $(E<V_0)$.



For $x>L$, there is tunneling with a transmission coefficient $T>0$. Let $A_0$, $A_R$ and $A_T$ denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

30. The ratio of the reflected to the incident amplitude $\frac{A_R}{A_0}$ is:

(a) $1-\frac{A_R}{A_0}$ (b) $\sqrt{1-T}$ in magnitude (c) A real negative number [✓ (d) $\sqrt{\left(1-\left|\frac{A_T}{A_0}\right|^2\right)\frac{E}{V_0-E}}$]

31. Throughout $0<x<L$, the wave -function
(a) Can be chosen to be real [✓ (b) Is exponentially decaying.]
(c) Is generally complex (d) Is zero.

32. Let the probability current associated with the incident wave be $S_0$. Let R be the reflection coefficient. Then
(a) The probabiltiy current vanishes in the classically forbidden region.
[✓ (b) The probability current is $TS_0$ for $x>L$.]
(c) For, $x<0$, the probability current is $S_0(I+R)$
(d) For $x>L$, the probability current is complex.

33. An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x = x_0$. [The SHO has enery levels $E_n$ $(n = 0,1,2,....)$ and associated normalized wavefunctions $\psi_n$].

Subssequently, an exact measurement of energy E is made, using the the general notation $\Pr(E = E')$ denoting the probability that a result $E'$ is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

(a) $\Pr(E = E_0) = 0$ (b) $\Pr(E = E_n) = 1$ for some value of n.

(c) $\Pr(E = E_n) \propto \psi_n(x)$ (d) $\Pr(E > E'') > 0$ for any E''

34. A particle is placed in a one dimensional box of size L along the x-axis (0<x<L). Which of the following is true?

(a) In the ground state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half.

(b) In the first excited state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half. This also holds for states with n = 4, 6, 8, ......

[✓ (c)] For an arbitory state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half.

(d) In the ground state, the particle has a definite momentum.

**Common data for Q.35, Q.36 and Q.37.**

A particle of mass m is confined in the ground state of a one-diemensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$. Where $\psi_0$ is a constant.

35. The normalization factor $\psi_0$ of this wavelength is:

(a) $\sqrt{\frac{2}{L}}$ (b) $\sqrt{\frac{1}{4L}}$ [✓ (c)] $\sqrt{\frac{1}{2L}}$ (d) $\sqrt{\frac{1}{L}}$

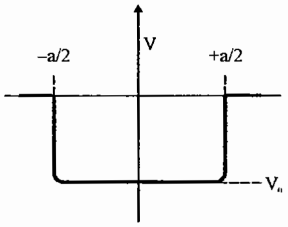
36. The energy eigenvalue corresponding to this state is:

(a) $\frac{\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{4mL^2}$ (c) $\frac{\hbar^2\pi^2}{16mL^2}$ [✓ (d)] $\frac{\hbar^2\pi^2}{32mL^2}$

37. The expectation value of $p^2$ (p is the momentum operator) in this state is:

(a) 0 (b) $\frac{\hbar^2\pi^2}{32L^2}$ [✓ (c)] $\frac{\hbar^2\pi^2}{16L^2}$ (d) $\frac{\hbar^2\pi^2}{8L^2}$

38. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth $V_0$ of the potential satisfies



[✓ (a)] $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$ (d) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

39. The energy levels of a particle of mass m in a potential of the form

$$V(x)=\infty, \qquad x\le 0$$
$$=\frac{1}{2}m\omega^2x^2, \qquad x>0$$

are given, in terms of quantum number $n=0,1,2,3$.........., by

(a) $\left(n+\frac{1}{2}\right)\hbar\omega$ (b) $\left(2n+\frac{1}{2}\right)\hbar\omega$ [✓ (c) $\left(2n+\frac{3}{2}\right)\hbar\omega$] (d) $\left(n+\frac{3}{2}\right)\hbar\omega$

**Common data for Q. 40 and Q. 41.**

A particle is confined to the region $0<x<L$ in one dimension.

40. If the particle is in the first excited state, then the probability of finding the particle is maximum at

(a) $x=\frac{L}{6}$ (b) $x=\frac{L}{2}$ (c) $x=\frac{L}{3}$ [✓ (d) $x=\frac{L}{4}$ and $\frac{3L}{4}$]

41. If the particle is in the lowest energy state, then the probability of finding the particle in the region 0 to $\frac{L}{4}$ is:

[✓ (a) $\frac{1}{4}-\frac{1}{(2\pi)}$] (b) $\frac{1}{4}$ (c) $\frac{1}{4}+\frac{1}{(2\pi)}$ (d) $\frac{1}{2}$

**Common data for Q. 42 and Q. 43.**

The normalized wave function $\psi_1$ and $\psi_2$ correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1=\psi_2$ and $\hat{A}\psi_2=\psi_1$

42. The expectation value of A for the state $\psi=(3\psi_1+4\psi_2)/5$ is:

(a) –0.32 (b) 0.0 (c) 0.75 [✓ (d) 0.96]

43. Which of the following are eigenfunctions of $\hat{A}^2$ ?

[✓ (a) $\psi_1$ and $\psi_2$] (b) $\psi_2$ and not $\psi_1$ (c) $\psi_1$ and not $\psi_2$ (d) neither $\psi_1$ and $\psi_2$

44. The energy density of states of an electron in a one dimensional potential well of infinitely high walls is (the symbols have their usual meaning)

(a) $L\sqrt{m}/\left[\pi\hbar\sqrt{(2E)}\right]$ (b) $Lm/\left(\pi\hbar\sqrt{E}\right)$

(c) $Lm/\left[\pi\hbar\sqrt{(2E)}\right]$ (d) $L\sqrt{m}/(2\pi\hbar E)$

45. A free particle with energy E whose wave-function is a plane wave with wavelength $\lambda$ enters a regionof constant potential V>0 where the wavelength of the particle is $2\lambda$. The ratio (V/E) is:

[✓ (a) 1/2] (b) 2/3 (c) 3/4 (d) 4/5

46. If the wave function of a particle trapped in space between x = 0 and x = L is given by $\psi(x)=A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum?

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) $\frac{L}{6}$ and $\frac{L}{3}$ [✓ (d) $\frac{L}{4}$ and $\frac{3L}{4}$]

47. Let a particle move in a potential field given by

$$V(x)=\begin{Bmatrix}\left(\frac{1}{2}\right)m\omega^2x^2 & \text{for } x>0\\ \infty & \text{for } x\le 0\end{Bmatrix}$$

The allowed energies of this particle are

(a) $(n+1/2)\hbar\omega$ (b) $(2n+3/2)\hbar\omega$ (c) $(2n+1/2)\hbar\omega$ (d) $(n+5/2)\hbar\omega$

where 'n' is a non-negative integer.

48. At t = 0, a one dimensional harmonic oscillator of frequency $\omega$ is in a state given by

$$\psi(x,0)=\frac{1}{2}\mu_0(x)+i\sqrt{3}/2\,\mu_1(x)$$

where $u_0$ and $u_1$ are the first two normalized eigenfunctions.

(a) The probability of finding the oscillator in state $u_1$ is $\frac{3}{4}$.

(b) The probability of finding the oscillator in the state $u_0$ is $\frac{1}{2}$.

(c) The average energy of the oscillator is $2\,\hbar\omega$.

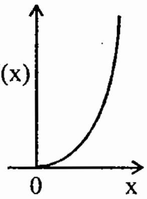
(d) The average energy of the oscillator is $(5\hbar)$.

49. A particle of mass m is confined in the potential, $V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0,\\ \infty & \text{for } x\le 0\end{cases}$

Let the wavefunction of the particle be given by

$$\psi(x)=-\frac{1}{\sqrt{5}}\psi_0+\frac{2}{\sqrt{5}}\psi_1,$$

where $\psi_0$ and $\psi_1$ are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is:



(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$ (c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

**Statement for Linked Answer type Q. 50 and Q. 51.**

Positronium is an atom formed by an electron and a positron. The mass of a positron is the same as that of an electron and its charge is equal in magnitude but opposite in sign to that of an electron. The positronium atom is thus similar to the hydrogen atom with the positron replacing the proton.

50. The binding energy of a positronium atom is:

(a) 13.6 eV (b) 6.8 eV (c) 27.2 eV (d) 3.4 eV

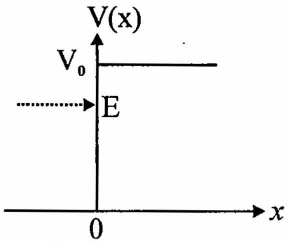
51. If a positronium atom makes a transition from the state with n = 3 to a state with n = 2, the energy of the photon that is emitted in this transition is closest to

(a) 1.88 eV (b) 0.94 eV (c) 1.13 eV (d) 2.27 eV

52. An electron with energy E is incident from left on a potential barrier, given by

$$V(x)=0 \quad \text{for} \quad x<0$$
$$=V_0 \text{ for } x>0$$

as shown in the figure



For $E < V_0$, the space part of the wavefunction for $x > 0$ is of the form

(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$ (c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

where $\alpha$ is a real positive quantity.

**Statement for Linked Answer type Q.53 and Q.54.**

In a one-dimensional harmonic oscillator, $\varphi_0, \varphi_1$ and $\varphi_2$ are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. $\psi_1$ and $\psi_2$ are two states defined by $\psi_1 = \varphi_0 - 2\varphi_1 + 3\varphi_2$; $\psi_2 = \varphi_0 - \varphi_1 + \alpha\varphi_2$. Where $\alpha$ is a constant.

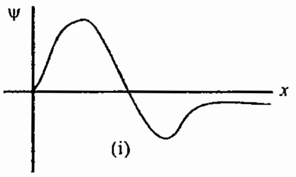
53. The value of $\alpha$ for which $\psi_2$ is orthogonal to $\psi_1$ is:

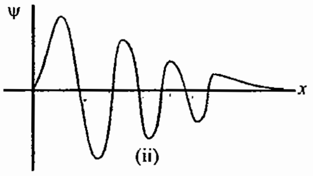
(a) 2 (b) 1 (c) –1 (d) –2

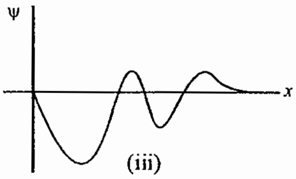
54. For the value of $\alpha$ determined in Q.13, the expectation value of energy of oscillator in the state $\psi_2$ is:

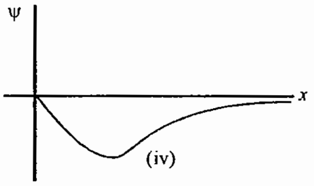
(a) $\hbar\omega$ (b) $3\hbar\omega/2$ (c) $3\hbar\omega$ (d) $9\hbar\omega/2$

55. A quantum particle in one dimension x moves under the influence of a potential that supports bound states. Plotted below are the wave functions $\psi$ vs x corresponding to four eigenstates of energy. Identify the wave functions corresponding to the ground state and the highest energy state among those shown:

(a)


(b)


(c)


(d)


(a) (i) corresponds to the ground state and (iii) to the highest energy state.
(b) (ii) corresponds to the ground state and (iv) to the highest energy state.
[✓ (c)] (iv) corresponds to the ground state and (ii) to the highest energy state.
(d) None of these is the ground state and (ii) corresponds to the highest energy state.

56. The ground state energy of a particle in the one-dimensional potential

$$V(x) = \begin{cases} \frac{1}{2} m w^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x < 0 \end{cases} \quad \text{is equal to}$$

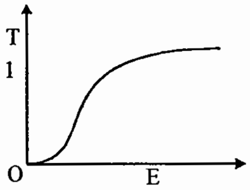
(a) $\hbar\omega$ (b) $\frac{3}{2}\hbar\omega$ (c) $2\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

57. The particle in a box has a ground state wavefunction given by $\phi(x)=\frac{1}{\sqrt{a}}\cos\left(\frac{\pi x}{2a}\right)$. The expectation value of x is:

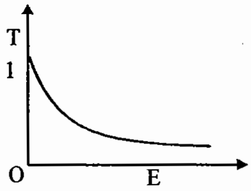
(a) 0 (b) $\frac{a^2}{3}$ (c) $\frac{a^2}{\pi^2}$ (d) 1

58. The variation of transmission probability with energy E, of the particles hitting a potential barrier of finite height, is represented as

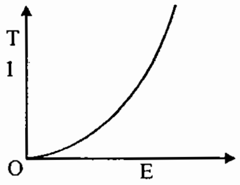
(a)



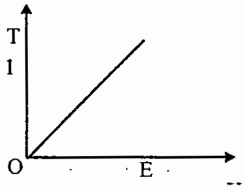
(b)



(c)



(d)



59. Consider a particle trapped as a potential well. If an observer makes a measurement to detect the particle just outside the well then
(a) He will always find the particle (b) He will never find the particle
(c) He will find the particle half the timer
(d) He will find the particle only if the measurement is performed for sufficiently long time.

60. The matter wave corresponding to a free particle moving in one dimension satisfies the time-dependent Schrodinger wave equation.

$$i\hbar\frac{\delta\psi(x,t)}{\delta t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

If $\hbar k$ is the momentum, then the wavepacket formed out of the matter waves moves with a group velocity $V_g$ equal to

(a) $\frac{\hbar}{2m}k^2$ (b) $\frac{\hbar^2}{2m}k^2$ (c) $\frac{\hbar^2}{2m}k$ (d) $\frac{\hbar}{2m}k$

61. A particle limited to the x-axis has the wave function $\psi=\sqrt{3}\,x$ between x = 0 and $x=1$; $\psi=0$ elsewhere. The expectation value $\langle x\rangle$ of the particle's position is:

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{4}$

62. The ground state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency $\omega$ is

(a) $(15/2)\hbar\omega$ (b) $(13/2)\hbar\omega$ (c) $(1/2)\hbar\omega$ (d) $5\hbar\omega$

63. The wave function of free particle in one dimension is given by $\psi(x)=A\sin x+B\sin 3x$. Then $\psi(x)$ is an eigenstate of
(a) the position operator (b) the Hamiltonian
(c) the momentum operator (d) the party operator

64. A particle in one-dimension is in the potential

$$V(x)=\begin{cases}\infty & \text{if } x<0\\ -V_0 & \text{if } 0\le x\le \ell\\ 0 & \text{if } x>\ell\end{cases}$$

If there is at least one bound state, the minimum depth of the potential is:

(a) $\frac{\hbar^2\pi^2}{8m\ell^2}$ (b) $\frac{\hbar^2\pi^2}{2m\ell^2}$ (c) $\frac{2\hbar^2\pi^2}{m\ell^2}$ (d) $\frac{\hbar^2\pi^2}{m\ell^2}$

65. The displacement of a one dimensional oscillator is given by $x = A\cos(\omega t+\phi)$. The probability that the displacement lies between x and x + dx is

(a) $\frac{1}{\pi\sqrt{A^2-x^2}}dx$ (b) $\frac{\pi\sqrt{A^2-x^2}}{A}dx$ (c) $\frac{A}{\pi\left(A^2-x^2\right)}dx$ (d) $\frac{1}{\pi A}dx$

66. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is

(a) $14\pi^2\hbar^2/mL^2$ (b) $91\pi^2\hbar^2/mL^2$ (c) $7\pi^2\hbar^2/mL^2$ (d) $3\pi^2\hbar^2/mL^2$

67. The energies in the ground state and first excited state of a particle of mass $m=\frac{1}{2}$ in a potential V(x) are –4 and –1, respectively, (in units in which $\hbar=1$). If the corresponding wavefunctions are related by $\psi_1(x)=\psi_0(x)\sinh x$, then the ground state eigenfunction is

(a) $\psi_0(x)=\sqrt{\operatorname{sech} x}$ (b) $\psi_0(x)=\operatorname{sech} x$ (c) $\psi_0(x)=\operatorname{sech}^2 x$ (d) $\psi_0(x)=\operatorname{sech}^3 x$

68. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty $\Delta x$ in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ is:

(a) $\Delta x=\sqrt{\hbar/2m\omega}$ (b) $\Delta x=\sqrt{\hbar/m\omega}$ (c) $\Delta x=\sqrt{2\hbar/m\omega}$ (d) $\Delta x=\sqrt{\hbar/4m\omega}$

69. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator: $|\psi\rangle=b_1|0\rangle+b_2|1\rangle$ where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and $b_1$ and $b_2$ are real constants. The expectation value of the displacement x in the state $|\psi\rangle$ will be a minimum when

(a) $b_2=0$, $(b_1=1)$ (b) $b_2=\frac{1}{\sqrt{2}}b_1$ (c) $b_2=\frac{1}{2}b_1$ (d) $b_2=b_1$

70. A quantum mechanical particle in a harmonic oscillator potential has the initial wavefunction $\psi_0(x)+\psi_1(x)$, where $\psi_0$ and $\psi_1$ are the real wavefunction in the ground and first excited states of the harmonic oscillator Hamiltonian. For convenience we take $m=\hbar=\omega=1$ for oscillator. What is the probability density of finding the particle at $x$ at time $t=\pi$?

(a) $(\psi_1(x)-\psi_0(x))^2$ (b) $(\psi_1(x))^2-(\psi_0(x))^2$

(c) $(\psi_1(x)+\psi_0(x))^2$ (d) $(\psi_1(x))^2+(\psi_0(x))^2$

71. A particle is in the ground state of an infinite square well potential given by,

$$V(x)=\begin{cases}0 & \text{for } -a\le x\le a\\ \infty & \text{otherwise}\end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{\pi}$ (c) $\frac{1}{2}-\frac{1}{\pi}$ (d) $\frac{1}{\pi}$

72. A one dimensional harmonic oscillator is in the superposition of number state, $|n\rangle$, given by

$$|\psi\rangle=\frac{1}{2}|2\rangle+\frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _______ $\hbar\omega$.

73. A particle in the infinite square well

$$V(x)=\begin{cases}0 & 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x)=\begin{cases}A\sin^3\left(\frac{\pi x}{a}\right) & 0<x<a\\ 0 & \text{otherwise}\end{cases}$$

The expectation value of the energy of the particle is

(a) $\frac{5\hbar^2\pi^2}{2ma^2}$ (b) $\frac{9\hbar^2\pi^2}{2ma^2}$ (c) $\frac{9\hbar^2\pi^2}{10ma^2}$ (d) $\frac{\hbar^2\pi^2}{2ma^2}$

## Assignment -4: One-dimensional Potential

### [ANSWERS]

1. (b) 2. (c) 3. (d) 4. (c) 5. (a) 6. (c) 7. (d)

8. (c) 9. (b, c) 10. (c) 11. (c) 12. (b) 13. (b) 14. (b)
15. (a) 16. (d) 17. (a) 18. (c) 19. (b) 20. (c) 21. (a)

22. ( ) 23. (d) 24. (c) 25. (d) 26. (d) 27. (d) 28. (a)

29. (d) 30. (d) 31. (b) 32. (b) 33. (b) 34. (c) 35. (c)
36. (d) 37. (c) 38. (a) 39. (c) 40. (d) 41. (c) 42. (d)
43. (a) 44. (a) 45. (c) 46. (d) 47. (b) 48. (a) 49. (a)

50. (b) 51. (b) 52. (b) 53. (c) 54. (b) 55. (c) 56. (b)

57. (a) 58. (b) 59. (b) 60. (d) 61. (d) 62. (b) 63. (d)

64. (a) 65. ( ) 66. (a) 67. (c) 68. (a) 69. (d) 70. ()

71. (b) 72. (3.2 to 3.3)W 73. (c)

## Assignment – 5: Angular Momentum Alzebra

1. $\sigma_i \left(i=1,2,3\right)$ represents the Pauli spin matrices. Which one of the following is NOT true?

   (a) $\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}$ (b) $\mathrm{Tr}\left(\sigma_i\right)=0$

   (c) The eigenvalues of $\sigma_i$ are $\pm 1$ (d) $\det\left(\sigma_i\right)=1$

2. Let $\left|\psi_0\right\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below

   (a) $\left[L_x, L_y\right]\left|\psi_0\right\rangle = 0$ (b) $J^2\left|\psi_0\right\rangle = 0$ (c) $\left[\vec{L},\vec{S}\right]\left|\psi_0\right\rangle \neq 0$ (d) $\left[S_x, S_y\right]\left|\psi_0\right\rangle = 0$

3. The commutator $\left[L_x, y\right]$, where $L_x$ is the x-component of the angular momentum operator and y is the y-component of the position operator, is equal to:

   (a) 0 (b) $i\hbar x$ (c) $i\hbar y$ (d) $i\hbar z$

4. Which one of the following relations is true for Pauli matrices

   (a) $\sigma_x\sigma_y = \sigma_y\sigma_x$ (b) $\sigma_x\sigma_y = \sigma_z$ (c) $\sigma_x\sigma_y = i\sigma_z$ (d) $\sigma_y\sigma_x = i\sigma_z$

5. For a spin 1/2 particle, the expectation value of $s_x s_y s_z$. where $s_x, s_y$ and $s_z$ are spin operators, is

   (a) $\dfrac{i\hbar^3}{8}$ (b) $-\dfrac{i\hbar^3}{8}$ (c) $\dfrac{i\hbar^3}{16}$ (d) $-\dfrac{i\hbar^3}{16}$

6. The commutator, $\left[L_z, Y_{\ell m}\left(\theta,\phi\right)\right]$ where $L_z$ is the z component of the orbital angular momentum and $Y_{\ell m}\left(\theta,\phi\right)$ is a spherical harmonic, is:

   (a) $\ell\left(\ell+1\right)\hbar$ (b) $-m\hbar$ (c) $m\hbar$ (d) $+\ell\hbar$

7. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $\left|+\hat{Z}\uparrow\right\rangle$ state. Which of the following statements is true?

   (a) The particle has a definite value of the y-component of the spin angular momentum.
   (b) The particle has a definite value of the square of the spin angular momentum.
   (c) The particle has a definite value of x-components of spin angular momentum.
   (d) The particle has definite values of x- and y-components of spin angular momentum.

8. Let $\vec{L} = \left(L_x, L_y, L_z\right)$ denote the orbital angular momentum operators of a particle and let $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$. The particle is in an eigenstate of $L^2$ and $L_z$ eigenvalues $\ell\left(\ell+1\right)\hbar^2$ and $\hbar\ell$ respectively. The expectation value of $L_+ L_-$ in this state is:

   (a) $\ell\hbar^2$ (b) $2\ell\hbar^2$ (c) 0 (d) $\ell\hbar$

9. A spin ½ particle with g>0, is subjected to a magnetic field $\vec{H} = H_0\hat{x}$. If the quantization axis is along $+\hat{z}$, then the minimum energy eigenstate is given by,

   (a) $\left|\downarrow\right\rangle$ (b) $\dfrac{1}{\sqrt{2}}\left|\uparrow\right\rangle - \left|\downarrow\right\rangle$ (c) $\left|\uparrow\right\rangle$ (d) $\dfrac{1}{\sqrt{2}}\left|\uparrow\right\rangle + \left|\downarrow\right\rangle$

10. If $L_x$, $L_y$ and $L_z$ are respectively the x, y and z components of angular momentum operator L, the commutator $\left[L_x L_y, L_z\right]$ is equal to

(a) $i\hbar\left(L_x^2+L_y^2\right)$ (b) $2i\hbar L_z$ (c) $i\hbar\left(L_x^2-L_y^2\right)$ (d) 0

11. The classical definition of orbital angular momentum of a particle is $\vec{L}=\vec{r}\times\vec{p}$. The corresponding quantum mechanical definition for the orbital angular momentum operator (taking into account the fact that $\vec{r}$ and $\vec{p}$ do not commute with each other) is:

(a) $\vec{L}=\vec{r}\times\vec{p}$ (b) $L=\frac{1}{2}\left[\left(\vec{r}\times\vec{p}\right)+\left(\vec{p}\times\vec{r}\right)\right]$

(c) $\vec{L}=\frac{1}{2i}\left[\left(\vec{r}\times\vec{p}\right)-\left(\vec{p}\times\vec{r}\right)\right]$ (d) $L=\frac{r}{r}\left(\vec{p}\times\vec{r}\right)$

12. Consider $L_x, L_y$ and $L_z$ to be the components of the angular momentum operator $\vec{L}$. If $L_+$ is the raising operator, the commutator $\left[L_+, L_z\right]$

(a) $\hbar L_+$ (b) $-\hbar L_x$ (c) $-\hbar L_y$ (d) $-\hbar L_+$

13. $J_x, J_y, J_z$ are the operators corresponding to the components of the angular momentum $\vec{J}$ and $\psi_{jm}$ is the eigenfunction of $J^2$ and $J_z$ with eigenvalues.

$$J^2\psi_{jm}=j(j+1)\hbar^2\psi_{jm}, \qquad J_z\psi_{jm}=m\hbar\psi_{jm}$$

Then the eigenvalue relation for $J_+=J_x+iJ_y$ is:

(a) $J+\psi_{jm}=\sqrt{j(j+1)+m(m+1)}\,\hbar\psi_{jm}$ (b) $J+\psi_{jm}=\sqrt{j(j+1)-m(m+1)}\,\hbar\psi_{jm}$

(c) $J+\psi_{jm}=\sqrt{(j-m)}\,\hbar\psi_{jm}$ (d) $J+\psi_{jm}=\left[\sqrt{j(j+1)\hbar}-\sqrt{m(m+1)\hbar}\right]\psi_{jm+1}$

14. P is a beam of atoms with spin quantum number $\frac{1}{2}$ and zero orbital angular momentum, with all angular momenta being along x axis. Q is a similar beam which is unpolarized. These beams can be differentiated experimentally by passing P and Q separately through Stern-Gerlach apperatus with

(a) Magnetic field along X axis (b) Magnetic field along Y axis.
(c) Magnetic field along Z axis (d) Magnetic field along YZ plane.

15. The expression $\exp i(\vec{\sigma}.\vec{n})\theta$ can be rewritten as

(a) $(\vec{\sigma}.\vec{n})\cos\theta+i\sin\theta$ (b) $\sin\theta+i(\vec{\sigma}.\vec{n})\cos\theta$

(c) $\cos\theta-i(\vec{\sigma}.\vec{n})\cos\theta$ (d) $\cos\theta+i(\vec{\sigma}.\vec{n})\sin\theta$

16. The eigenvalues of the pauli spin matrix $\sigma_x$ are
(a) i, –i (b) –i, i (c) 1, –1 (d) 1, 1

17. The Clebsch-Gordon coefficient $\langle j_1m_1 : j_2m_2 \mid j_1j_2jm\rangle$ is zero unless

(a) $j=j_1+j_2$ (b) $j=j_1-j_2$ (c) $m=m_1+m_2$ (d) $m=m_1-m_2$

18. Pauli's spin operators $\sigma_x, \sigma_y, \sigma_z$ are represented by $2\times2$ matrices. These are

(a) All anti-hermitian and traceless
(b) All orthogonal and traceless
(c) All Hermitian and traceless
(d) Only $\sigma_z$ is hermitian and traceless

19. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}.\vec{B}$ where $\mu$ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the $2\times2$ unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to

(a) $I\cos\frac{\omega t}{2} + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\frac{\omega t}{2}$
(b) $I\cos\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\omega t$
(c) $I\sin\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos\omega t$
(d) $I\sin 2\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos 2\omega t$

20. The orbital angular momentum eigenstate $\psi_{\ell,m}(r,\theta\ \phi)$ with $l = 0, m = 0$ (in standard notation) is an eigenstate of the following operators

(a) $L_x, L_y, L_z$ and $L^2$
(b) $L_x$ and $L^2$ only
(c) $L_y$ and $L^2$ only
(d) $L_z$ and $L^2$ only

21. The Dirac Hamiltonian for the relativistic Hydrogen atom has a coupling $\vec{L}.\vec{S}$ between the orbital angular momentum and the spin of the electron. Let $s_z$, $l_z$ and $j_z$ denote the z-components of the spin, orbit and total angular momenta respectively. For the electron in an eigenstate of energy

(a) $j_z$ and $s_z$ (but not $l_z$) can be measured precisely
(b) $j_z$ (but not $s_z$ and $l_z$) can be measured precisely
(c) $j_z$ and $l_z$ (but not $s_z$) can be measured precisely
(d) $l_z$ (but not $l_z$ and $s_z$) can be measured precisely

22. which of the following describes the same physical state of a spin-1/2 particle as $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$?

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$
(b) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix}$
(c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$
(d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\ -1\end{pmatrix}$

23. If $J_+|j,m\rangle = 0$ where '$J_+$' denotes the angular momentum raising operator, $J_+ = J_x + iJ_y$ the value of m is

(a) $-j+1$
(b) $-j$
(c) $j-1$
(d) $j$

24. The wavefunction of a particle is of the form $\psi(r,\theta,\phi) = u(r,\theta)\cos(\phi)$. If the z-component of the angular momentum $L_z$ is measured, then the possible outcome is $+\hbar$ with probability

(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) $\frac{1}{\sqrt{2}}$

25. The spin function of a free particle, in the basis in which $S_z$ is diagonal, can be written as $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ with eigenvalues $+\frac{\hbar}{2}$ and $\frac{\hbar}{2}$, respectively. In the given basis, the normalized eigenfunction of $S_y$ with eigenvalue $-\frac{\hbar}{2}$

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\i\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\1\end{pmatrix}$

26. Two spin -1/2 fermions having spins $\vec{S}_1$ and $\vec{S}_2$ interact via a potential $V(r)=\vec{S}_1.\vec{S}_2V_0(r)$. The contributions of this potential in the singlet and triplet states, respectively, are

(a) $-\frac{3}{2}V_0(r)$ and $\frac{1}{2}V_0(r)$ (b) $\frac{1}{2}V_0(r)$ and $-\frac{3}{2}V_0(r)$

(c) $\frac{1}{4}V_0(r)$ and $-\frac{3}{4}V_0(r)$ (d) $-\frac{3}{4}V_0(r)$ and $\frac{1}{4}V_0(r)$

**Common data for Q.27, Q. 28 and Q.29.**

A system consists of three spin-half particles, the z components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value $+1/2$ and $-1/2$. The total spin of the system is $S_z=S_z(1)+S_z(2)+S_z(3)$

27. The total number of possible micro-states of this system is:
(a) 3 (b) 6 (c) 7 (d) 8

28. The total number of micro-states with $S_z=1/2$ is:
(a) 3 (b) 5 (c) 6 (d) 7

29. Consider an ensemble of systems where each microstate has equal probability. The ensemble average of $S_z$ is:
(a) $-1/2$ (b) 0 (c) 1/2 (d) 3/2

30. A spin half particle is in the state $S_z=\hbar/2$. The expectation values of $S_x, S_x^2, S_y, S_y^2$ are given by

(a) $0,0,\hbar^2/4,\hbar^2/4$ (b) $0,\hbar^2/4,\hbar^2/4,0$ (c) $0,\hbar^2/4,0,\hbar^2/4$ (d) $\hbar^2/4,\hbar^2/4,0,0$

31. An electron is in a state spin wave function $\phi_s=\begin{pmatrix}\sqrt{3}/2\\1/2\end{pmatrix}$ in the $S_z$ representation. What is the probability of finding the z-componenet of its spin along the $-\hat{z}$ direction?
(a) 0.75 (b) 0.50 (c) 0.35 (d) 0.25

32. If a spin I particle is in the state $|m=0\rangle$ with respect to a quantization axis $\hat{n}$, which of the following is correct?

(a) $\langle\vec{S}\rangle=0$ (b) $\langle\vec{S}\rangle=\hat{n}$ (c) $\langle\vec{S}\rangle=\sqrt{2}\hat{n}$ (d) $\langle\vec{S}\rangle=-\hat{n}$

33. The total spin-angular momentum of a system of three free electrons is:

(a) $S_{tot}=\frac{1}{2}$ and $S_{tot}=\frac{3}{2}$ (b) $S_{tot}=\frac{1}{2}$ only

(c) $S_{tot}=\frac{3}{2}$ only (d) $S_{tot}=0\ S_{tot}=1$

**Statement for Linked Answer type Q. 34 and Q. 35.**

The state $|m>$, $m = -1, 0, 1$ are the eigenstates of $S_z$, the z component of the spin angular momentum of a particle with $S = 1.\left[S_z|m\rangle = m\hbar|m\rangle\right]$

34. The expectation value of $S_z$ in the state $|\psi\rangle = \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle + \frac{1}{\sqrt{2}}|-1\rangle$ is:

(a) $\hbar/4$ (b) $-\hbar/2$ (c) $\hbar/2$ (d) $-\hbar/4$

35. Consider the states $|\psi_1\rangle = \frac{1}{\sqrt{6}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{3}}|-1\rangle$ and $|\psi_2\rangle = -\frac{1}{\sqrt{6}}|1\rangle + a|0\rangle - \frac{1}{\sqrt{3}}|-1\rangle$. The value of a for which these states are orthogonal is:

(a) $1/\sqrt{3}$ (b) $1/2$ (c) $1/\sqrt{2}$ (d) $-1/\sqrt{2}$

**Statement for Linked Answer type Q. 36 and Q. 37.**

An electron is placed in an uniform magnetic field $\vec{H}$ that points in the $\hat{x}$ direction. The Hamiltonian of the system is $H = -K\vec{S}.\vec{H}$ where $k > 0$ is a constant and $\vec{S}$ is the electron spin operator.

36. The energy of the ground state of the Hamiltonian is:

(a) $-KS\hbar$ (b) $\frac{1}{2}KH\hbar$ [✓ (c) $-\frac{1}{2}KH\hbar$] (d) 0

37. The expectation value of $S_z$ in the ground state of this Hamiltonian is:

(a) $-\hbar/2$ (b) 0 (c) $\hbar/2$ (d) $\hbar$

38. The eigenstates of $\hat{S}_z$ of a quantum particle with spin $\frac{1}{2}$ are $|\uparrow\rangle = \left|s = \frac{1}{2}, s_z = \frac{1}{2}\right\rangle$ and $|\downarrow\rangle = \left|s = \frac{1}{2}, s_z = -\frac{1}{2}\right\rangle$.

Which of the following statements about eigenstates of the operator $\hat{S}_x$ is true?

(a) $|\uparrow\rangle$ is an eigenstate of $\hat{S}_x$. (b) $|\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(c) $|\uparrow\rangle + |\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(d) No linear combination of $|\uparrow\rangle$ and $|\downarrow\rangle$ can be an eigenstate of $\hat{S}_x$

39. Consider a system of two spin-half particles, in a state with total spin quantum number $S = 0$. The eigenvalue of the spin Hamiltonian $H = A\,\vec{S}_1.\vec{S}_2$ (A is a positive constant) in this state is:

(a) $A\hbar^2/4$ (b) $-A\hbar^2/4$ (c) $3A\hbar^2/4$ [✓ (d) $-3A\hbar^2/4$]

40. The eigenvalues of the Pauli spin matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ are

(a) $+1, +1$ (b) $+1, -1$ (c) $-i, +i$ (d) $+i, +i$

41. In a system consisting of two spin $\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)} = \frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)} = \frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators. Here $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ and $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices.

**A.** In the standard basis the matrices for the operators $S_x^{(1)}S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ respectively,

(a) $\frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ [✓ (b)] $\frac{\hbar^2}{4}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$

(c) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & -i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & -i & 0\\ 0 & i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}$ (d) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

**B.** These two operators satisfy the relation

(a) $\left\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right\} = S_z^{(1)}S_z^{(2)}$ (b) $\left\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right\} = 0$

(c) $\left[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right] = i\,S_z^{(1)}S_z^{(2)}$ (d) $\left[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right] = 0$

42. Consider a spin- 1/2 particle in the presence of a homogeneous magnetic field of magnitude B long along z-axis which is prepared initially in a state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$ at time t = 0. At what time t will the particle be in the state $-|\psi\rangle$ ($\mu_B$ is Bhor magneton) ?

(a) $t = \frac{\pi\hbar}{\mu_B B}$ (b) $t = \frac{2\pi\hbar}{\mu_B B}$ (c) $t = \frac{\pi\hbar}{2\mu_B B}$ (d) Never

43. The quantum state $\sin x|\uparrow\rangle + \exp(i\phi)\cos x|\downarrow\rangle$, where $\langle\uparrow|\downarrow\rangle = 0$ and $x, \phi$ are real is orthogonal to :

(a) $\sin x|\uparrow\rangle$ (b) $\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$

(c) $-\cos x|\uparrow\rangle - \exp(i\phi)\sin x|\downarrow\rangle$ (d) $-\exp(-i\phi\cos x)|\uparrow\rangle + \sin x|\downarrow\rangle$

44. The component along an arbitrary direction $\hat{n}$, with direction cosines $(n_x, n_y, n_z)$, of the spin of a spin$-\frac{1}{2}$ particle is measured. The result is:

(a) 0 (b) $\pm\frac{\hbar}{2}n_z$ (c) $\pm\frac{\hbar}{2}(n_x, n_y, n_z)$ (d) $\pm\frac{\hbar}{2}$

45. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings.

(a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = i\hbar L_z$ (c) $[L_z, L_+] = \hbar L_+$ (d) $[L_z, L_-] = \hbar L_-$

46. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigen states of $\sigma_z$ then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here, symbols have their usual meanings ________

47. In a basis in which the z-component $S_z$ of the spin is diagonal, an electron is in a spin state

$$\psi = \begin{pmatrix} (1+i)/\sqrt{6} \\ \sqrt{2/3} \end{pmatrix}$$

The probabilities that a measurement of $S_z$ will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively.

(a) 1/2 and 1/2 (b) 2/3 and 1/3 (c) 1/4 and 3/4 (d) 1/3 and 2/3

48. If $J_x, J_y, J_z$ are angular momentum operators, the possible ~~eigen~~values of the operator $(J_x + J_y)/\hbar$ are

(a) real and discrete with rational spacing (b) real and discrete with irrational spacing

(c) real and continuous (d) not all real

49. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the $L_z$ basis of states with $m = +1, 0, -1$. If $L_z^2$ is measured in this state yielding a result 1, what is the state after the measurement?

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

50. What are the eigenvalues of the operator $H = \vec{\sigma}\cdot\vec{a}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{a}$ is a vector?

(a) $a_x + a_y$ and $a_z$ (b) $a_x + a_z \pm i a_y$ (c) $\pm(a_x + a_y + a_z)$ (d) $\pm|\vec{a}|$

51. The expectation value of the x-component of the orbital angular momentum $L_x$ in the state

$\psi = \frac{1}{5}\left[3\psi_{2,1,-1} + \sqrt{5}\psi_{2,1,0} - \sqrt{11}\psi_{2,1,+1}\right]$ (where $\psi_{nlm}$ are the eigenfunctions in usual notation), is

(a) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11}-3)$ (b) 0

(c) $\frac{\hbar\sqrt{10}}{25}(\sqrt{11}+3)$ (d) $\hbar\sqrt{2}$

52. A particle is prepared in a simultaneous eigenstate of $L^2$ and $L_z$. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of $L^2$ and $L_z$, then the expectation value $\langle L_x^2 \rangle$ of the particle in this state satisfies

(a) $\langle L_x^2 \rangle = 0$ (b) $0 \le \langle L_x^2 \rangle \le \ell^2\hbar^2$

(c) $0 \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{3}$ (d) $\frac{\ell\hbar^2}{2} \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{2}$

53. $\psi_1$ and $\psi_2$ are two orthogonal states of a spin ½ system. it is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represents the spin-up and spin-down states, respectively. When the system is in the state $\psi_2$, its probability to be in the spin-up state is ________________

## [ANSWERS]

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (d)** | **2. (a)** | **3. (d)** | **4. (c)** | **5. (a)** | **6. (c)** | **7. (d)** |
| **8. (b)** | **9. (b)** | **10. (c)** | **11. (a)** | **12. (a)** | **13. (d)** | **14. (a)** |
| **15. (d)** | **16. (c)** | **17. (c)** | **18. (c)** | **19. (b)** | **20. (d)** | **21. (b)** |
| **22. (d)** | **23. (d)** | **24. (c)** | **25. (d)** | **26. (d)** | **27. (d)** | **28. (a)** |
| **29. (b)** | **30. (c)** | **31. (d)** | **32. (a)** | **33. (a)** | **34. (d)** | **35. (c)** |
| **36. (c)** | **37. (b)** | **38. (c)** | **39. (d)** | **40. (b)** | **41_A(b)** | **41_B(b)** |
| **42. (a)** | **43. (d)** | **44. (d)** | **45. (d)** | **46. (7.6)** | **47. (d)** | **48. ( )** |
| **49. ( )** | **50. ( )** | **51. (a)** | **52. (d)** | 53. (0.66 to 0.68) | | |

## Assignment – 6: 3-D Potentials

1. The radial wave function of the electrons in the state of n = 1 and l = 0 in a hydrogen atom is $R_{10} = \frac{2}{a_0^{3/2}} \exp\left(-\frac{r}{a_0}\right)$, $a_0$ is the Bohr radius. The most probable value of r for an electron is:

[✓ (a) $a_0$] (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

2. An atomic state of hydrogen is represented by the following wave function:

$$\psi(r,\theta,\phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where $a_0$ is a constant. The quantum number of the state are

(a) $\ell = 0, m = 0, n = 1$ (b) $\ell = 1, m = 1, n = 2$ [✓ (c) $\ell = 1, m = 0, n = 2$] (d) $\ell = 2, m = 0, n = 3$

3. The expectation value of the z coordinate (z), in the ground state of the hydrogen atom (wavefunction: $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and $a_0$ is the Bohr radius), is

(a) $a_0$ (b) $\frac{a_0}{2}$ (c) $\frac{a_0}{4}$ [✓ (d) 0]

4. The degeneracy of the n = 2 level for a three dimensional isotropic oscillator is

(a) 4 [✓ (b) 6] (c) 8 (d)

5. Bound state eigenfunctions of an attractive finite range smooth potential behave for larger r as:

[✓ (a) $\exp(-r/r_0)$ where $r_0$ is a positive constant.] (b) $(1/r^n)$ where n>0

(c) constant (d) $\exp(ikr)/r$

6. Given that binding energy of hydrogen atom in the ground state is 13.6 eV, the binding energy for n = 2 state of the positronium atom is

(a) 13.6 eV (b) 6.8 eV (c) 3.4 eV [✓ (d) 1.7 eV]

7. Degeneracy of the first excited state of an isolated hydrogen atom is:

(a) 2 (b) 4 (c) 6 (d) 8

8. The ground state wave function $\psi$ of hydrogen atom in spherical polar co-ordinates has no angular dependence but only radial dependence. Then $\psi$ is an eigen function of

[✓ (a) $L_x, L_y$ and $L_z$ simultaneously] (b) $L_z$ but not of $L_x$ and $L_y$

(c) none of $L_x, L_y$ and $L_z$ (d) $p_z$

9. The Hamiltonian of the electron in the hydrogen atom is

$$H = p^2/(2m) - e^2/(4\pi\varepsilon_0 r)$$

where the symbols have their usual meanings. Let $|\psi\rangle$ denote the ground state of H, with $\langle\psi|\psi\rangle = 1$

(a) $|\psi\rangle$ is an eigen state of $p^2/(2m)$ (b) $\langle\psi|r|\psi\rangle$ is zero

(c) $\langle\psi|(1/r)|\psi\rangle$ is infinite (d) $\langle\psi|H|\psi\rangle < 0$

10. The ground state energy of the Hydrogen atom is –13.6 eV. The energy of the second excited state is:
(a) –27.2 eV (b) –6.8 eV (c) –1.5 eV (d) –4.5 eV

11. The degeneracy of energy for a hydrogen-like atom with principal quantum number n and orbital quantum number $l$ is:
(a) n (b) $n^2$ (c) $2n^2$ (d) $l(l+1)$

12. In a central force field, the angular part of the solution of the Schrodinger equation is in general
(a) Independent of $\theta$ and $\phi$ (b) A constant
(c) A spherical harmonic (d) A Bessel function

13. Consider 1s and 2s states for hydrogen atom, then $\int \psi_{100}(\vec{r})\psi_{200}(\vec{r})\, d^3r$, is
(a) –ve (b) +ve (c) zero (d) 1.0

14. The wave function $\psi(\vec{r}) = e^{i\vec{p}.\vec{r}}$ is an eigen function for a Hamiltonian with
[✓ (a)] A constant potential $V \geq 0$
(b) A zero potential only
(c) A potential V(r) (not constant) which is spherically symmetric.
(d) Any potential which is periodic with periodicity $\vec{R}$.

15. The azimuthal wave function for the hydrogen atom is: $\phi(\varphi) = Ae^{im\phi}$. Where m is the magnetic quatum number. If $\phi(\varphi)$ is normalized, the value of the normalization constant A is:
(a) $\frac{1}{2\sqrt{\pi}}$ (b) $\frac{1}{2\sqrt{m\pi}}$ (c) $\frac{1}{\sqrt{2\pi}}$ (d) $\frac{1}{\sqrt{2m\pi}}$

16. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$ are given by $E_{n\ell m} \propto -1/n^2$, where n is the principal quantum number, and the corresponding wave functions are given by $\psi_{n\ell m}$, where $\ell$ is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement?
(a) There are exactly $(2\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.
(b) There are $\ell(\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.
(c) $E_{n\ell m}$ does not depend on $\ell$ and m for the Coulomb potential.
(d) There is a unique wave function $\psi_{n\ell m}$ and $E_{n\ell m}$.

17. Consider the high excited states of a Hydrogen atom corresponding to large values of the principal quantum number $(n \gg 1.)$. The wavelength $\lambda$ of a photon emitted due to an electron undergoing transition between two such states with consecutive values of n (i.e., $\psi_{n+1} \rightarrow \psi_n$) is related to the wavelenth $\lambda_\alpha$ of the $K_\alpha$ line of Hydrogen by $\alpha$
(a) $\lambda = n^3\lambda_\alpha/8$ (b) $\lambda = 3n^3\lambda_\alpha/8$ (c) $\lambda = n^2\lambda_\alpha$ (d) $\lambda = 4\lambda_\alpha/n^2$

18. The normalized wavefunctions of a Hydrogen atom are denoted by $\psi_{n,l,m}(\vec{x})$, where n, $l$ and m are, respectively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\psi(\vec{x})=\frac{1}{3}\psi_{1,0,0,}(\vec{x})+\frac{2}{3}\psi_{2,1,0}(\vec{x})+\frac{2}{3}\psi_{3,2,-2}(\vec{x})$$

The expectation value $\langle E\rangle$ of the energy of this electron, in electron-Volts (eV) will be approximately

(a) −1.5 (b) −3.7 [✓ (c) −13.6] (d) −80.1 (e) +13.6

19. The energy of the first excited quantum state of a particle in the two-dimensional potential $V(x,y)=\frac{1}{2}m\omega^2(x^2+4y^2)$ is:

(a) $2\hbar\omega$ (b) $3\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

20. If $\psi_{n,\ell,m}(\vec{r})$ denote normalised eigenfunctions of the Hamiltonian H of the hydrogen atom, then the wave function $\frac{1}{2}\left[\psi_{2,1,1}(\vec{r})+\psi_{2,1,-1}(\vec{r})\right]$ is an eigenfunction of

[✓ (a) $L^2$ and $L_z$] (b) H and $L_z$ (c) H and $L^2$ (d) $L_z$ alone

21. An electron in the hydrogen atom is in a state whose energy is −3.4 eV. The degeneracy of the state is

(a) 4 (b) 8 (c) 2 (d) 1

22. The quantum states of a particle in a Coulomb potential is labelled by the principal, orbital and azimuthal quantum numbers n, $l$ and m respectively. From the rotational symmetry of the Hamiltoian alone we may conclude that the energy eigenvalues are

(a) independent of n only (b) independent of $l$ only
(c) independent of $l$ and m (d) independent of m only

23. The degeneracy of the state of energy $E_n=\hbar\omega(n+1)$, where n is an integer, in a two-dimensional isotopic harmocnic oscillator with potential $v(x,y)=\frac{1}{2}m\omega^2(x^2+y^2)$ is

(a) $\frac{n(n-1)}{2}$ (b) $n+1$ (c) $n-1$ (d) n

24. A measurement of the z-omponent of angular momentum $L_z$ is made for an electrons moving in the Coulomb field of a proton with wave function $\frac{1}{4}\left[\psi_{1,0,0}(\vec{r})+3\psi_{2,1,1}(\vec{r})-\sqrt{6}\,\psi_{2,1,-1}(\vec{r})\right]$ where $\psi_{n,\ell,m}(\vec{r})$ are Hydrogen atom states. The expectation value of $L_z$ is:

(a) $\frac{3}{16}\hbar$ (b) $\frac{3-\sqrt{6}}{4}\hbar$ (c) $\frac{3+\sqrt{6}}{4}\hbar$ (d) $\frac{3-\sqrt{6}}{16}\hbar$

25. If $E_H$ is the binding energy of the electron in the Bohr model of the Hydrogen atom, and both the charge and mass of the electron are doubled, the binding energy becomes

(a) $32\,E_H$ (b) $2\,E_H$ (c) $4\,E_H$ (d) $16\,E_H$

26. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by

$$V(x,y) = 0 \text{ for } -a < x < a \text{ and } -a < y < a$$
$$= \infty \text{ else where}$$

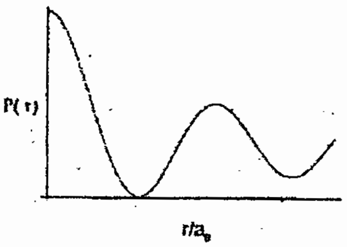
The energy of the first excited state for this particle is given by,

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

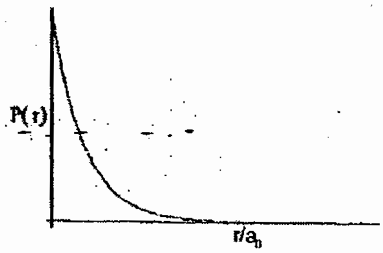
27. The ground state wavefunction for the hydrogen atom is given by $\Psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where $a_0$ is the Bohr radius.

The plot of the radial probability density, P(r) for the hydrogen atom in the ground state is

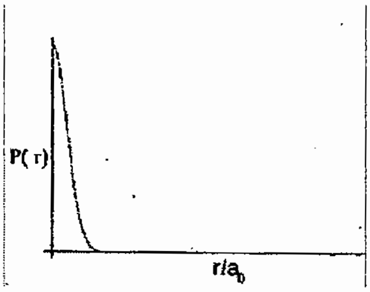
(a)



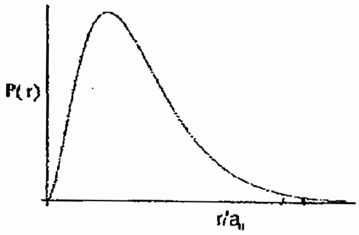
(b)



(c)



(d)



**Linked Answer for Q.28 and Q.29:**

The one-electron states for non-interacting electrons confined in a cubic box of side a are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc.

28. The energy of the lowest state is:

(a) zero (b) $\frac{\hbar^2\pi^2}{2ma^2}$ (c) $\frac{\hbar^2\pi^2}{ma^2}$ (d) $\frac{3\hbar^2\pi^2}{2ma^2}$

29. The degeneracy (including spin) of the level $\varepsilon_3$ is:

(a) 2 (b) 4 (c) 6 (d) 8

**Common data for Q.30 and Q.31.**

A particle is located in a three diamensional cubic well of width L with impenetrable walls

30. The sum of he energies of the third and the fourth levels is:

(a) $10\pi^2\hbar^2/mL^2$ (b) $10\pi^2\hbar^2/3\,mL^2$ (c) $11\pi^2\hbar^2/2mL^2$ (d) $15\pi^2\hbar^2/2mL^2$

31. The degeneracy of the fourth level is given by

(a) 1 (b) 2 (c) 3 (d) 4

32. A quantum particle of mass m is confined to a square region in xoy-plane whose vertices are given by (0, 0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for l, m, n positive integers)?

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$ (b) $\frac{2}{L}\sin\left(\frac{\ell\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$ (d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\ell\pi y}{L}\right)$

33. The ground state wave function of the hydrogen atom S given by $\psi(r) = \left(1\sqrt{\pi}\, a^{3/2}\right) e^{-r/a}$ where a is a constant. If $P(r)dr$ is the probability of finding the electron between r and $r + dr$, then

(a) $P(r) = 0$ at $r = 0$ (b) $P(r)$ is maximum at $r = 0$

(c) $P(r)$ is maximum at $r = a$ (d) $\int_0^\infty P(r)dr = 1$

34. A system is known to be in a state described by the wave function, $\psi(\theta,\phi) = \frac{1}{\sqrt{30}}\left(5Y_4^0 + Y_6^0 - 2Y_6^3\right)$ where $Y_\ell^m(\theta,\phi)$ are the spherical harmonics. The probability of finding the system in a state with $m = 0$ is:

(a) zero (b) 2/15 (c) 1/4 (d) 13/15

35. A quantum particle of mass m moves in two dimensions in an anistropic harmonic oscillator potential $V(x,y) = \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2$. The energy eigenvalues are (n is a positive integer or zero)

(a) $\hbar\omega(2n+1)$ (b) $\hbar\omega(n+1)$ (c) $2\hbar\omega(n+1)$ (d) $\hbar\omega\left(n+\frac{3}{2}\right)$

**Statement for Linked Answer type Q. 36 and Q. 37.**

Consider a three dimensional harmonic oscillator with Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2\left(x^2 + y^2 + z^2\right)$$

36. The number of distinct eigenvalues with energy eigenvalue $\frac{5}{2}\hbar\omega$ is:

(a) 5 (b) 3 (c) 2 (d) 1

37. The eigenvalue of $L^2$ (where $\bar{L}$ is the angular momentum operator) in the ground state is:

(a) $\hbar^2$ (b) $2\hbar^2$ (c) 0 (d) $6\hbar^2$

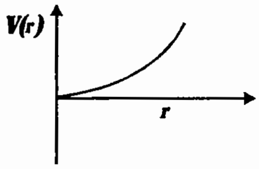
38. The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\left\langle \frac{1}{r^2} \right\rangle$ is:

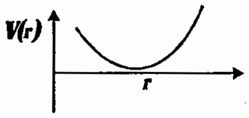
(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

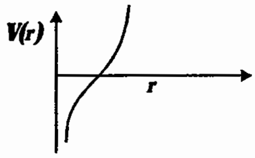
39. The number of nodes of the radial wave function for an atomic electron, $R(r)=A\left[27-18r+2r^2\right]e^{-r/3}$ is:

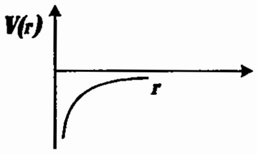
(a) 0 (b) 1 (c) 2 [✓ (d) 3]

40. Which of the following figure represents the potential, $V(r)=-\frac{A}{r}+Br^2$

(a)


(b)


(c)


(d)


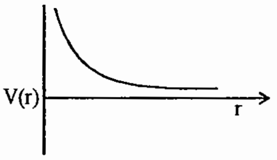
41. The electronic configuration of the ground state of a nitrogen is $1s^2\,2s^2\,2p^3$. The degeneracy of this configuration is:

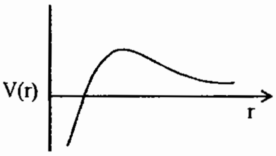
(a) 3 (b) 20 (c) 6 (d) 0

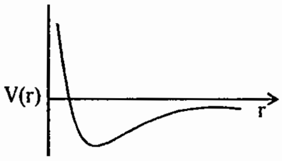
42. The effective radial potential for the motion of the electron in the hydrogen atom is of the form

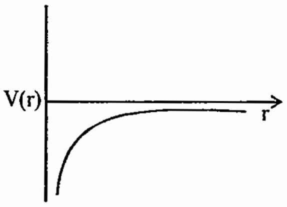
$$V(r)=-\frac{K}{r}+\frac{\ell(\ell+1)\hbar^2}{2mr^2}$$

Wheren K is strength of interaction. $\ell$ is angular momentum quantum number and m is mass of the particle. The variation of V(r) with r is given by

(a)


(b)


(c)


(d)


43. Consider a quantum particle of mass m in a three-dimensional isotropic simple harmonic potential $V(x,y,z)=\frac{1}{2}m\omega^2\left(x^2+y^2+z^2\right)$. It is known that the particle is in an energy eigenstate with eigenvalue $E=7\hbar\omega/2$. Which of the following cannot be the wave function of the particle? (In the following $\alpha=\sqrt{m\omega/\hbar}$ and $H_n(\xi)$ is the n^th^ hermite polynomial).

(a) $H_2(\alpha x)\exp\left(-\alpha\left(y^2+z^2\right)\right)$ (b) $H_2(\alpha x)\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)$

(c) $H_1(\alpha y)eH_1(\alpha z)\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)$ (d) $H_1(\alpha x)H_1(\alpha z)\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)$

44. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately

(a) 0.68 (b) 0.48 (c) 0.28 (d) 0.91

45. Consider a particle of mass m moving inside a two- dimensional square box whose sides are described by the equations $x = 0, x = L, y = 0, y = L$. What is the lowest eigenvalue of an eigenvalue which changes sign under the exchange of x and y?

(a) $\hbar^2/(mL^2)$ (b) $3\hbar^2/(2mL^2)$ (c) $5\hbar^2/(2mL^2)$ (d) $7\hbar^2/(2mL^2)$

46. A particle of mass m is in a cubic box of size a. The potential inside the box. $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinte outside. If the particle is in an eigenstate of energy $E = \dfrac{14\pi^2 h^2}{2ma^2}$, its wave function is:

(a) $\psi = \left(\dfrac{2}{a}\right)^{3/2} \sin\dfrac{3\pi x}{a} \sin\dfrac{5\pi y}{a} \sin\dfrac{6\pi z}{a}$ (b) $\psi = \left(\dfrac{2}{a}\right)^{3/2} \sin\dfrac{7\pi x}{a} \sin\dfrac{4\pi y}{a} \sin\dfrac{3\pi z}{a}$

(c) $\psi = \left(\dfrac{2}{a}\right)^{3/2} \sin\dfrac{4\pi x}{a} \sin\dfrac{8\pi y}{a} \sin\dfrac{2\pi z}{a}$ (d) $\psi = \left(\dfrac{2}{a}\right)^{3/2} \sin\dfrac{\pi x}{a} \sin\dfrac{2\pi y}{a} \sin\dfrac{3\pi z}{a}$

47. Let $\Psi_{n\ell m}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential V(r). The wavefunction $\Psi = \dfrac{1}{4}\left[\Psi_{210} + \sqrt{5}\Psi_{21-1} + \sqrt{10}\Psi_{213}\right]$ is an eigenfunction only of

(a) H, $L^2$ and $L_z$ (b) H and $L_z$ (c) H and $L^2$ (d) $L^2$ and $L_z$

48. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}$$

where $\psi_{n\ell m}$ denotes the normalized eigen function of the state with quantum numbers n, $l$ and m in the usual notation. The expectation value of $L_z$ in the state $\psi$ is:

(a) $\dfrac{15\hbar}{16}$ (b) $\dfrac{11\hbar}{16}$ (c) $\dfrac{3\hbar}{8}$ (d) $\dfrac{\hbar}{8}$

49. A muon $(\mu^-)$ from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be

(a) 5.62 Å (b) 6.67 Å (c) 3.75 Å (d) 13.3 Å

50. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by $\psi(\bar{r}) = z f(r)$ where f(r) is a function of the radial variable r. The eigenvalue of the operator $\underline{L}^2$ (namely the square of the orbital angular momentum) is

(a) $\hbar^2/4$ (b) $\hbar^2/2$ (c) $\hbar^2$ (d) $2\hbar^2$

51. If $\psi_{nlm}$ denotes the eigenfunction of the Hamiltonian with a potential $V = V(r)$ then the expectation value of the operator $L_x^2 + L_y^2$ in the state $\psi = \dfrac{1}{5}[3\psi_{211} + \psi_{210} - \sqrt{15}\psi_{21-1}]$ is

(a) $39\hbar^2/25$ (b) $13\hbar^2/25$ (c) $2\hbar^2$ (d) $26\hbar^2/25$

52. If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue $E = 0$, the potential $V(x)$ (in units where $\hbar = 2m = 1$) is

(a) $12x^2$ (b) $16x^6$ (c) $16x^6 + 12x^2$ (d) $16x^6 - 12x^2$

53. An electron in the ground state of the hydrogen atom has the wave function

$$\psi(\vec{r})\frac{1}{\sqrt{\pi a_0^3}}e^{-(r/a_0)}$$

where $a_0$ is constant. The expectation value of the operator $\hat{Q}=z^2-r^2$, where $z=r\cos\theta$ is (Hint: $\int_0^\infty e^{-ar}r^n dr=\frac{\Gamma(n)}{\alpha^{n+1}}=\frac{(n-1)!}{\alpha^{n+1}}$)

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$ (c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

54. A particle of mass 'm' is subjected to a potential,

$$V(x,y)=\frac{1}{2}m\omega^2\left(x^2+y^2\right),\quad -\infty\le x\le\infty,-\infty\le y\le\infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of g is _________

55. A hydrogen atom is in the state

$$\psi=\sqrt{\frac{8}{21}}\psi_{200}-\sqrt{\frac{3}{7}}\psi_{310}+\sqrt{\frac{4}{21}}\psi_{321}$$

where $n,\ell,m$ in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of $L^2$ is ____________ $\hbar^2$.

56. A particle of mass $m$ in three dimensions is in the potential

$$V(r)=\begin{cases}0 & r<a\\ \infty & r\ge a\end{cases}$$

Its ground state energy is

[✓ (a)] $\frac{\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

57. Given that $\hat{p}_r=-i\hbar\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$, the uncertainty $\Delta p_r$ in the ground state

$$\psi_0(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$

of the hydrogen atom is

[✓ (a)] $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$ (c) $\frac{\hbar}{2a_0}$ (d) $\frac{2\hbar}{a_0}$

58. A particle of mass m in the potential $V(x,y)=\frac{1}{2}m\omega^2\left(4x^2+y^2\right)$, is in an eigenstate of energy $E=\frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is

(a) $y\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$ (b) $x\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$ (d) $xy\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$

59. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to

(a) 0.60 (b) 0.90 (c) 0.16 (d) 0.32

**[ANSWERS]**

**1. (a) 2. (c) 3. (d) 4. (b) 5. (a) 6. (d) 7. (d)**
**8. (b) 9. (d) 10. (c) 11. (c) 12. (c) 13. (c) 14. (a)**
**15. (c) 16. (c) 17. (b) 18. (b) 19. (d) 20. (c) 21. (b)**
**22. (c) 23. (b) 24. (a) 25. (a) 26. (c) 27. (d) 28. (d)**
**29. (c) 30. (a) 31. (c) 32. (c) 33. (c) 34. (d) 35. (d)**
**36. (b) 37. (c) 38. (d) 39. (c) 40. (c) 41. (b) 42. (c)**
**43. (a) 44. (a) 45. (c) 46. (d) 47. (c) 48. (d) 49. (b)**
**50. (d) 51. (d) 52. (d) 53. (d) 54. (3.99 to 4.01)**
**55. (1.99 to 2.01) 56. (a) 57. (a) 58. (a) 59. (d)**

## Assignment – 7: Identical Particles

1. A system consists of two identical particles, each of which can be in any one of the two quantum states. The allowed states are

[✓ (a) 3 symmetric and antisymmetric] (b) 1 symmetric and 3 antisymmetric
(c) 1 symmetric and 1 antisymmetric (d) 3 symmetric and 3 antisymmetric.

2. Consider a one dimensional infinite square well potential $V(x)=\begin{cases}0, \text{ for } 0\le x\le L\\ \infty, \text{ otherwise}\end{cases}$.

If two identical non- interacting bosons occupy the lowest two energy levels, the unnormalized wavefunction of the combined system is given by

(a) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(b) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right]$

[✓ (c)] $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right]$

(d) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)\right]$

3. Consider two collision processes one involving two identical $\frac{1}{2}$ fermions and the other involving two identical spin 1 bosons. If in both cases the particles are in relative orbital angular momentum state $l=1$, the spinoral parts of wave functions must be

[✓ (a) odd for fermions and even for bosons] (b) odd in both cases
(c) even in both cases [✓ (d) even for fermions and odd for bosons]

4. Consider the wavefunction $\psi=\psi(\vec{r}_1,\vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by

$$\psi(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left[\phi_1(\vec{r}_2)\phi_2(\vec{r}_2)+\phi_2(\vec{r}_1)\phi_1(\vec{r}_2)\right]$$

Where $\phi_1$ and $\phi_2$ are single particle states. The spin part $\chi_s$ of the wavefunction with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta+\beta\alpha)$ [✓ (b) $\frac{1}{\sqrt{2}}(\alpha\beta-\beta\alpha)$] (c) $\alpha\alpha$ (d) $\beta\beta$

5. The wavefunctions of two identical particles in state n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two-particle system is expressed as

(a) $\phi_n(r_1)+\phi_s(r_2)$
(b) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)+\phi_s(r_2)+\phi_n\phi_s(r_1)\right]$
(c) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)+\phi_s(r_2)-\phi_n\phi_s(r_1)\right]$
(d) $\phi_n(r_1)\phi_s(r_2)$

6. The eigenfunction for the Hamiltonian of a particle of mass m in a one dimensional potential V(x) is given by $\psi(x)=A\exp\left(-bx^2/2\right)$, where A and b are constants. It follows that

(a) V(x) = constant
(b) $V(x)\propto x^3$
(c) $V(x)\propto x^2$
(d) $V(x)\propto 1/x$

7. A free particle in one dimension is in a state described by

$$\psi(x,t)=A\exp\left((ipx-iEt)/h\right)+B\exp\left((-ipx-iEt)/h\right)$$

The probability current density is given by

(a) $p/m$
(b) $\left(|A|-|B|\right)p/m$
(c) $\left(|A|^2-|B|^2\right)p/m$
(d) $\left(|A|^2+|B|^2\right)p/m$

8. Consider a one-dimensional infinite square well potential

$$V(x)=\begin{cases}0 & \le x\le L\\ \infty & \text{otherwise}\end{cases}$$

If each of the lowest two energy levels are occupied by identical non-interacting bosonic particles (one in each level), then the unnormalized wave function of the combined system is:

(a) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(b) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)$

(c) $\psi(x_1,x_2)=\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

9. Lithium has three electrons in the state $\psi_1(\vec{r}_1);\psi_2(\vec{r}_2)$ and $\psi_3(\vec{r}_3)$.
The wavefunction of the three electrons (together) can be written as

(a) $\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)$
(b) $\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_1)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3)\right)$
(c) $\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_1)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3)\right)$
(d) $\frac{1}{\sqrt{6}}\begin{vmatrix}\psi_1(\vec{r}_1) & \psi_1(\vec{r}_2) & \psi_1(\vec{r}_3)\\ \psi_2(\vec{r}_1) & \psi_2(\vec{r}_2) & \psi_3(\vec{r}_3)\\ \psi_3(\vec{r}_1) & \psi_3(\vec{r}_2) & \psi_3(\vec{r}_3)\end{vmatrix}$

10. Two electrons are confined in a one dimensional box of length $L$. The one-electron states are given by $\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$. What would be the ground state wave function $\psi(x_1, x_2)$ if both electrons are arranged to have the same spin state?

(a) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) + \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

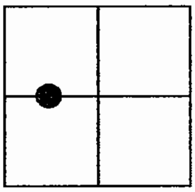
(b) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(c) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

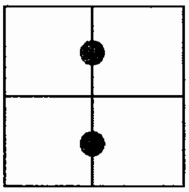
(d) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

11. In a Stern-Gerlach experiment with spin-½ particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another Stern-Gerlach apparatus but with its magnets rotated by 90 degrees counter-clockwise about the axis of the beam direction. Which of the following shows what happens on the second screen?

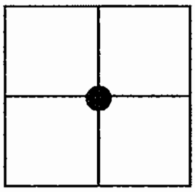
(a)



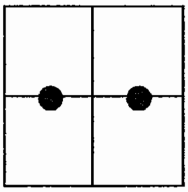
(b)



(c)



(d)



12. The ground state and the first excited state wave functions of a one dimensional infinite potential well are $\psi_1$ and $\psi_2$, respectively. When two spin-up electrons are placed in this potential, which one of the following, with $x_1$ and $x_2$ denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)\right]$

13. Which one of the following is a fermion?

(a) $\alpha$ – particle (b) ${}_4Be^7$ nucleus (c) Hydrogen atom (d) Deuteron

14. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does not commute with

(a) $S_z$ (b) $L^2$ (c) $S^2$ (d) $\left(\vec{L}+\vec{S}\right)^2$

15. Consider a system of two non-interacting identical fermions, each of mass $m$ in an infinite square well potential of width $a$. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy

$$E=\frac{5\pi^2\hbar^2}{2ma^2}$$

is

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)-\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)+\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right)-\sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right)-\sin\left(\frac{\pi x_2}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

**[ANSWERS]**

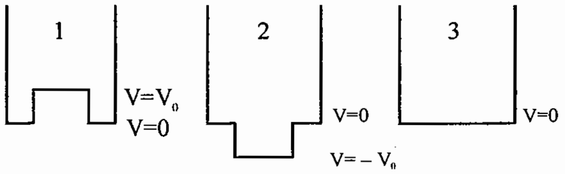
**1. (a)** **2. (c)** **3. (a)** **4. (b)** **5. (d)** **6. (c)** **7. (c)**
**8. (b)** **9. (d)** **10. ( )** **11. (d)** **12. (b)** **13. (b)** **14. (a)**
**15. (a)**

## Assignment – 8: Pertubation Theory

1. A quantum harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda\left(a^{\dagger}a\right)^{2}$ acts on the particle, where $\lambda$ is a constant of suitable dimensions and a and $a^{\dagger}$ are lowering and raising operators respectively. Then the first order energy shift is gien by

(a) $\lambda n$ (b) $\lambda^2 n$ (c) $\lambda n^2$ (d) $(\lambda n)^2$

2. Let $E_1, E_2, E_3$ be the respective ground state energies of the following potentials: Which one of the following is correct?



(a) $E_1<E_2<E_3$ (b) $E_3<E_1<E_2$ [✓ (c) $E_2<E_3<E_1$] (d) $E_2<E_1<E_3$.

3. The validity of Fermi's golden rule which connects two states with energy difference $\Delta E$, implies that it is true for

(a) All times t (b) $t\geq\frac{\hbar}{\Delta E}$ (c) $t\leq\frac{\hbar}{\Delta E}$ (d) $\Delta E=0$

4. If the perturbation $H'=ax$, where a is a constant; is added to the infinite square well potential

$$V(x)=\begin{cases}0 \text{ for} & 0\leq x\leq\pi\\ \infty & \text{otherwise}\end{cases}$$

The first order correction to ground state energy is:

[✓ (a) $\frac{a\pi}{2}$] (b) $a\pi$ (c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

5. The perturbation $H'=bx^2$, where 'b' is a constant, is added to the one dimensional harmonic oscillator potential $V(x)=\frac{1}{2}m\omega^2x^2$. Which of the following denotes the correction to the ground state energy to first order in b ?

[Hint : The normalized ground state wave function of the one dimensional harmonic oscillator potential is $\psi_0=\left(\frac{m\omega}{\hbar\pi}\right)^{1/4}e^{-m\omega x^2/2\hbar}$. You may use the following integral $\int_{-\infty}^{\infty}x^{2n}e^{-ax^2}dx=a^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$

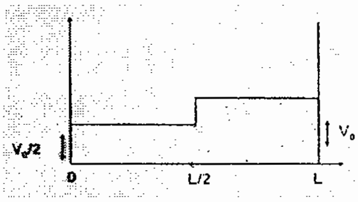
(a) $\frac{3b\hbar^2}{4m^2\omega^2}$ [✓ (b) $\frac{3b\hbar^2}{2m^2\omega^2}$] (c) $\frac{3b\hbar^2}{2\pi m^2\omega^2}$ (d) $\frac{15b\hbar^2}{4m^2\omega^2}$

6. A constant perturbation as shown in the figure below acts on a particle of mass *m* confined in a infinite potential well between 0 and *L*.

The first-order correction to the ground state energy of the particle is

(a) $\frac{V_0}{2}$ (b) $\frac{3V_0}{4}$

(c) $\frac{V_0}{4}$ (d) $\frac{3V_0}{2}$



7. A one-dimensional harmonic oscillator whose Hamiltonian is $H_0 = \frac{1}{2m}\hat{p}^2 + \frac{1}{2m}\omega^2\hat{x}^2$ is perturbed such that the total Hamiltonian is $H = H_0 + \varepsilon\hat{x}$. The first order correction to the ground state is

(a) $2\varepsilon\hbar\omega$ (b) $\varepsilon\hbar\omega/2$ (c) $\varepsilon\hbar\omega$ (d) 0

8. The first-order correction to the eigenvalues of the Hamiltonian $\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$ due to a perturabation $\hat{H}_1 = \alpha\hat{x}$ are

(a) 0 (b) $\alpha/k$ (c) $2\alpha/k$ (d) $-2\alpha/k$

9. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$, where $\delta << 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are

(a) 1 and $(1+2\delta)$ (b) $(1+\delta)$ and $(1-\delta)$ (c) $(1+2\delta)$ and $(1-2\delta)$ (d) $(1+\delta)$ and $(1-2\delta)$

**Common data for Q.10, Q.11 and Q.12.**

An unperturbed two-level system has energy eigenvalues E1 and E2, and eigen functions $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

When perturbed, its Hamiltonian is represented by $\begin{pmatrix} E_1 & A \\ A^* & E_2 \end{pmatrix}$

10. The first-order correction to $E_1$ is:

(a) 4A (b) 2A (c) A (d) 0

11. The second-order correction to $E_1$ is

(a) 0 (b) A (c) $\frac{A^2}{E_2 - E_1}$ (d) $\frac{A^2}{E_1 - E_2}$

12. The first-order correction to the eigenfunction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is:

(a) $\begin{pmatrix} 0 \\ A^*/(E_1 - E_2) \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} A^*/(E_1 - E_2) \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

13. The wave function of a one-dimensional harmonic oscillator is $\psi_0 = A\exp\left(\frac{-\alpha^2 x^2}{2}\right)$ for the ground state $E_0(\alpha x/10)^4$, the first order change in the ground state energy is: [Given: $\left[(x+1)=\int_0^\infty t^x \exp(-t)\,dt\right]$

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(\underset{4}{3}E_0)10^{-4}$ (c) $(4E_0)10^{-4}$ (d) $(E_0)10^{-4}$

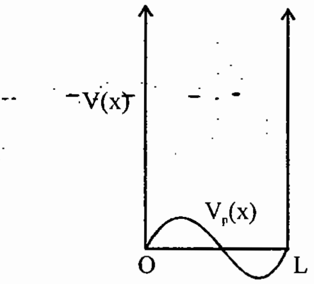
14. Let $E'_n\ (n=0,1,2,\ldots)$ be the energy eigenvalues for a particle of mass m placed in an anharmonic potential $V(x)=\frac{1}{2}m\omega^2x^2+ax^4,\ (a>0)$. Let $E_n \equiv \left(n+\frac{1}{2}\right)\hbar\omega$. Then according to the first order perturbation theory:

(a) $E'_0 = E_0$ (b) $E'_0 > E_0$ (c) $E'_0 < E_0$ (d) $E'_0 < E_0$ for all n

15. A particle of mass m is confined in an infinite potential well:

$$V(x)=\begin{cases}0 & \text{if } 0<x<L,\\ \infty & \text{otherwise}\end{cases}$$

It is subjected to a perturbing potential $V_p(x)=V_0\sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in $V_0$, respectively. Which of the following are true?



(a) $E^{(1)}=0;\ E^{(2)}<0$ (b) $E^{(1)}>0;\ E^{(2)}=0$

(c) $E^{(1)}=0;\ E^{(2)}$ depends on the sign of $V_0$ (d) $E^{(1)}<0;\ E^{(2)}<0$

16. Consider an electron in a box of lenght L with periodic boundary condition $\psi(x)=\psi(x+L)$. If the electron is in the $\psi_k(x)=\frac{1}{\sqrt{L}}e^{jkx}$ with energy $\varepsilon_k=\frac{\hbar^2k^2}{2m}$, what is the correction to its energy, to second order of perturbation theory, when it is subjected to weak periodic potential $V(x)=V_0\cos gx$, where g is an integral multiple of the $2\pi/L$?

(a) $V_0^2\,\varepsilon_g/\varepsilon_k^2$ (b) $-\frac{mV_0^2}{2\hbar^2}\left(\frac{1}{g^2+2kg}+\frac{1}{g^2-2kg}\right)$

(c) $V_0^2(\varepsilon_k-\varepsilon_g)/\varepsilon_g^2$ (d) $V_0^2/(\varepsilon_k+\varepsilon_g)$

17. The energy eigenvalues of a particle in the potential $V(x)=\frac{1}{2}m\omega^2x^2-ax$ are

(a) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-\frac{a^2}{2m\omega^2}$ (b) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega+\frac{a^2}{2m\omega^2}$

(c) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-\frac{a^2}{m\omega^2}$ (d) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$

18. The perturbation

$$H' = \begin{cases} b(a-x) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

acts on a particle of mass 'm' confined in an infinite square well potential

$$V(x) = \begin{cases} 0 & -a < x < a \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to the ground state energy of the particle is

(a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$ (c) 2ba (d) ba

**Common Data for Q. 19 and Q. 20:**

To the given unperturbed Hamiltonian $\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

we add a small perturbation given by $\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

where $\varepsilon$ is a small quantity.

19. The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$ (c) (0, 0, 1) (d) (1, 0, 0)

20. A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3+2\varepsilon, 7+2\varepsilon$ (b) $3+2\varepsilon, 2+\varepsilon$ (c) $3, 7+2\varepsilon$ (d) $3, 2+2\varepsilon$

21. Consider a two-dimensional infinite square will $V(x,y) = \begin{cases} 0 & 0<x<a, \quad 0<y<a \\ \infty & \text{otherwise} \end{cases}$. Its normalized eigenfunctions are $\psi_{n_x, n_y}(x,y) = \frac{2}{a}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{a}\right)$ where $n_x, n_y = 1, 2, 3, \ldots$ If a perturbation $H' = \begin{cases} V_0 & 0<x<\frac{a}{2}, \quad 0<y<\frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$ is applied, then the correction to the energy of the first excited state to order $V_0$ is

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{4}\left[1 \pm \frac{64}{9\pi^2}\right]$ (c) $\frac{V_0}{4}\left[1 \pm \frac{16}{9\pi^2}\right]$ (d) $\frac{V_0}{4}\left[1 \pm \frac{32}{9\pi^2}\right]$

22. A particle of mass *m* is contained in a one-dimensional infinite well extending from $x = -L/2$ to $x = L/2$. The particle is in its ground state given by $\psi_0(x) = \sqrt{2/L}\cos(\pi x / L)$. The walls of the box are moved suddenly to form a box extending from $x = -L$ to $x = L$. What is the probability that the particle will be in the ground state after this sudden expansion?

(a) $(8/3\pi)^2$ (b) 0 (c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

[illegible]
B day 29

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### Assignment – 9: Scattering Theory

1. If $\sigma$ is the total cross-section and $f(\theta)$, $\theta$ being the angle of scattering is the scattering amplitude for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction.

   (a) $\sigma = |f(\theta)|^2$  (b) $\sigma = \frac{4\pi}{k}|f(\theta=0)|^2$

   (c) $\sigma = \frac{4\pi}{k} \times$ imaginary part of $[f(\theta=0)]$  (d) $\sigma = \frac{4\pi}{k}|f(\theta)|^2$

2. For the scattering under a central force field, the asymptotic form of the total wave function after scattering is:

   (a) $\psi(r) = e^{ikr} + e^{-ikr}$  (b) $\psi(r) = f(0)\frac{e^{-ik.r}}{r}$

   (c) $\psi(r) = e^{ik.r} + f(0)e^{ik.r.}$  (d) $\psi(r) = e^{ik.r} + f(\theta)\frac{e^{ik.r}}{r}$

3. Born approximation is justified for
   (a) Particle moving very slowly.  (b) Very strong potential fields.
   (c) Fast particles in a weak potential.  (d) Hydrogen atom.

4. A beam of quantum mechanical particles of momentum k moving along the z-direction scatters off a spherically symmetric potential

$$V(r) = \begin{cases} V_0, & \text{for } 0 \le r \le R \\ 0, & \text{for } r > R \end{cases}$$

   The leading term of the wave function of the scattered particles is of the form

   (a) $e^{ikr}$  (b) $e^{ikz}$  (c) $\frac{e^{ikr}}{r}$  (d) $\frac{e^{ikr}}{r^2}$

**Common data for Q. 5 and Q. 6.**

A particle is scattered by a spherically symmetric potential. In the centre of mass (CM) frame the wavefunction of the incoming particle is $\psi = Ae^{ikz}$ where k is the wavevector and A is a constant.

5. If $f(\theta)$ is an angular function then in the asymptotic region the scattered wavefunction has the form

   (a) $\frac{Af(\theta)e^{ikr}}{r}$  (b) $\frac{Af(\theta)e^{-ikr}}{r}$  (c) $\frac{Af(\theta)e^{ikr}}{r^2}$  (d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

6. The differential scattering cross section $\sigma(0)$ in CM frame is:

   (a) $\sigma(\theta) = |A|^2\frac{|f(\theta)|^2}{r^2}$  (b) $\sigma(\theta) = |A|^2|f(\theta)|^2$

   (c) $\sigma(\theta) = |f(\theta)|^2$  (d) $\sigma(\theta) = |A||f(\theta)|$

7. Let k be the wave number of the incident plane wave in a scattering experiment. If the scattering is purely a p-wave with the phase shift $\delta_1 = \pi/4$, then the total scattering cross-section is:

(a) $2\pi/k^2$ (b) $6\pi/k^2$ (c) 0 (d) $k^2$

8. For a spherically symmetric potential, $V(r) = g\delta(\vec{r})$ where $\delta(\vec{r})$ is a three dimensional $\delta-$ function and g is a constant. The scattering amplitude f(0) in the Born approximation then varies as

(a) g (b) $g^2$ (c) 1/g (d) $1/g^2$.

9. A beam of particles of linear momentum $\hbar k$ is scattered by a short-range potential of range $a_1$ where $ak << 1$. Which of the following statements is correct?

(a) All phase shifts are zero (b) There is no scattering

(c) The scattering is spherically symmetric (d) All phase shifts are equal.

10. Consider the scattering of two identical fermions. If the two particles are in a spin singlet state, the differential cross section relates to the scattering amplitude as

(a) $\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |f(\pi-\theta)|^2$ (b) $\frac{d\sigma}{d\Omega} = |f(\theta)|^2 - |f(\pi-\theta)|^2$

(c) $\frac{d\sigma}{d\Omega} = |f(\theta) - f(\pi-\theta)|^2$ (d) $\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2$

11. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential $V(r) = \begin{cases} V_0 & \text{if } r \le R \\ 0 & \text{if } r > R \end{cases}$ if $V_0$ is changes to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the born approximation,

(a) increases to four times the original value (b) increases to twice the original value

(c) decreases to half the original value (d) decreases to one fourth the original value

12. In the Born approximation, the scattering amplitude $f(\theta)$ for the Yukawa potential $V(r) = \frac{\beta e^{-\mu r}}{r}$ is given by: (in the following $b = 2k \sin\frac{\theta}{2}$, $E = \hbar^2 k^2/2m$)

(a) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$ (b) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)}$ (c) $-\frac{2m\beta}{\hbar^2\sqrt{\mu^2+b^2}}$ (d) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^3}$

**[ANSWERS]**

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (c)** | **2. (d)** | **3. (c)** | **4. (c)** | **5. (a)** | **6. (c)** | **7. (b)** |
| **8. (a)** | **9. (c)** | **10. (a)** | **11. (a)** | **12. (b)** | | |

## Assignment – 10: Miscellaneous

1. Let $E_T$ be the energy obtained variationally by using trial function $Ae^{-\alpha r^2}$ (with $\alpha$ as a variational parameter) for Hydrogen atom. If the exact ground state is $E_H$ then,

(a) $E_T = E_H$ (b) $E_T > E_H$ (c) $E_T \geq E_H$ (d) $E_T < E_H$

2. The dispersion relation for travelling waves admitted by the wave equation $\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial x^2} + \alpha\frac{\partial^4\psi}{\partial x^4} = 0$

($\alpha$ : parameter) is:

(a) $\omega = \pm\sqrt{k^2+\alpha k^4}$ (b) $\omega = \pm\sqrt{k+\alpha k^2}$

(c) $\omega = \pm\left(k^2+\alpha k^4\right)$ (d) $\omega = \pm\left(k^2+\alpha k^2\right)$

3. Given the Hamiltonian $\hat{H} = \hat{H}_0 + i\hat{T}$ where $\hat{H}_0$ and $\hat{T}$ are hermitian, which of the following statements are correct?

(a) $\hat{H}$ is a hermitian operator and the total probability is consrved.

(b) $\hat{H}$ is a hermitian and the total probability is not conserved.

(c) $\hat{H}$ is not a hermitian operator and the total probability is conserved.

(d) $\hat{H}$ is not a hermitian operator and the total probability is not conserved.

4. A particle in one dimension moves under the influence of a potential $V(x) = ax^6$, where a is a real constant. For large n the quantized energy level $E_n$ depends on n as:

(a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$ (c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

5. A variational calculation is done with the normalized trial wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}\left(a^2 - x^2\right)$ for the one dimensional potential well $V(x) = \begin{cases} 0 & \text{if } |x| \leq a \\ \infty & \text{if } |x| > a \end{cases}$

The ground state energy is estimated to be

(a) $\frac{5\hbar^3}{3ma^2}$ (b) $\frac{3\hbar^2}{2ma^2}$ (c) $\frac{3\hbar^2}{5ma^2}$ (d) $\frac{5\hbar^2}{4ma^2}$

6. For a harmonic oscillator of mass $m = \frac{1}{2}$ and energy E, moving in a potential $V(x) = \frac{1}{2}kx^2$, the WKB quantization condition reads

(a) $\int_{-\sqrt{2E/k}}^{\infty} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$ (b) $\int_{\infty}^{\sqrt{2E/k}} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$

(c) $\int_{-\sqrt{2E/k}}^{\sqrt{2E/k}} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$ (d) $\int_{-\infty}^{\infty} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$

7. What would be the ground state energy of the Hamiltonian

$$H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-\alpha\delta(x)$$

if vibrational principle is used to estimate it with the trial wavefunction $\psi(x)=A\,e^{-bx^2}$ with b as the variational parameter?

[Hint: $\int\limits_{-\infty}^{\infty} x^{2n}e^{-2bx^2}dx=(2b)^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$]

(a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

8. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-\alpha\delta(x)$$

using the variational principle with the trial wavefunction $\psi(x)=A\exp(-bx^2)$ is

[Note: $\int\limits_{0}^{\infty} e^{-t}t^a dt=\Gamma(a+1)$]

(a) $-m\alpha^2/4\pi\hbar^2$ (b) $-m\alpha^2/2\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $-m\alpha^2/\sqrt{5}\pi\hbar^2$

9. A spin $-\frac{1}{2}$ particle is in the state $\chi=\frac{1}{\sqrt{11}}\begin{pmatrix}1+i\\3\end{pmatrix}$ in the eigenbasis of $S^2$ and $S_z$. If we measure $S_z$ the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$ (c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

10. Let $\psi_{nlm}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The expectation value of $L_z$ in the state

$$\psi=\frac{1}{6}\left[\psi_{200}+\sqrt{5}\,\psi_{210}+\sqrt{10}\,\psi_{21-1}+\sqrt{20}\,\psi_{211}\right]\text{ is}$$

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$ (c) $\hbar$ (d) $\frac{5}{18}\hbar$

11. The probability function for a variable $x$ which assumes only positive values is

$$f(x)=x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda>0$. The ratio $\langle x\rangle/\hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable $x$, is

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

12. A particle moving in a one dimension has the un-normalised wave function

$$\psi(x) = x\exp\left(-\frac{x^2}{\lambda^2}\right)$$

where $\lambda$ is a real constant.The expectation value of its momentum is $\langle p \rangle =$

(a) $\frac{\hbar}{\lambda}\exp\left(-\frac{x^2}{\lambda^2}\right)$ (b) $\frac{\hbar^2}{\lambda^2} - 2\frac{\hbar}{\lambda}$ (c) $\frac{\hbar}{\lambda}\exp(-1)$ (d) zero

13. A particle of mass 'm' and charge 'e' is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

(a) Make a transition to any harmonic oscillator state with $x = -eE/k$ as origin without emitting any photon.

(b) Make a transition to any harmonic oscillator state with $x = 0$ as origin and absorb a photon.

(c) Settle into the harmonic oscillator ground state with $x = 0$ as origin after absorbing a photon.

(d) Oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

14. Consider the Hamiltonian

$$H = f\vec{\sigma}.\vec{x}$$

Here $\vec{x}$ is the position vector, $f$ is a constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices. The energy eigenvalues are

(a) $f\left(\sqrt{x^2+y^2} \pm z\right)$ (b) $f(x \pm iy)$ (c) $\pm f\sqrt{x^2+y^2+z^2}$ (d) $\pm f(x+y+z)$

15. A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta, \varphi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\varphi$$

where $\theta$ and $\varphi$ are the usual polar angles. If two successive measurements of $L_z$ are made on this rotator, the probability that the second measurement will yield the value $+\hbar$ is

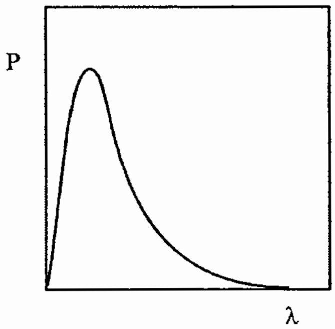
(a) 0.25 (b) 0.33 (c) 0.5 (d) negligible

16. A particle in the 2s state of hydrogen has the wave function

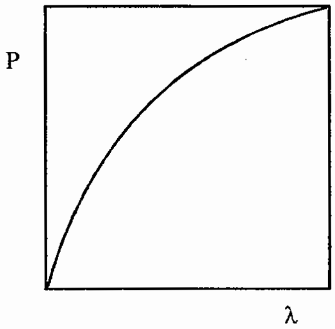
$$\psi_{2s}(r)=\frac{1}{4\sqrt{2}\,\pi}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

where 'r' is the radial coordinate w.r.t. the nucleus as origin and $a_0$ is the Bohr radius. The probability P of finding the electron somewhere inside a sphere of radius $\lambda a_0$ centred at the nucleus, is best described by the graph

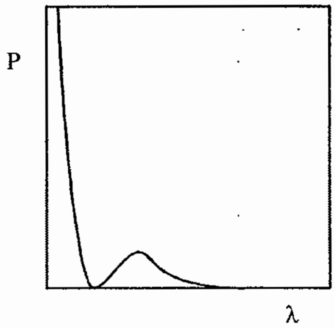
(a)



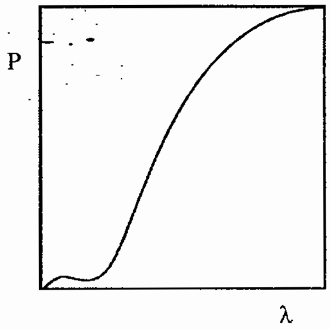
(b)



(c)



(d)



17. Which one of the following is not true for a hydrogen atom?
(a) The components of angular momentum do not commute with each other
(b) The components of angular momentum commute with the total angular momentum
(c) The total energy is independent of principle quantum number *n*
(d) The energy level corresponding to the principal quantum number *n* has $n^2$ fold degeneracy

18. If a proton were ten times lighter, the ground state energy of the electron in a hydrogen atom would be

(a) less (b) more (c) the same
(d) less, more or equal depending on the electron mass

19. The recoil momentum of an atom is pA when it emits an infrared photon of wavelength 1500 nm, and it is pB when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is:

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

### Assignment -10: Miscellaneous

### [ANSWERS]

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (c)** | **2. (a)** | **3. (d)** | **4. (a)** | **5. (d)** | **6. (c)** | **7. (c)** |
| **8. (c)** | **9. ( )** | **10. ( )** | **11. (a)** | **12. (d)** | **13. (b)** | **14. (c)** |
| **15. (c)** | **16. (d)** | **17. (b)** | **18. ( )** | **19. (c)** | | |

# Quantum Mechanics Assignments

## Assignment – 1: Fundamental of Quantum Mechanics [ANSWERS]

1. (b) 2. (a) 3. ( ) 4. (a) 5. (c) 6. (c) 7. (a)
8. (a) 9. ( ) 10. (a) 11. (c)

## Assignment -2: Operators. [ANSWERS]

1. (c) 2. (c) 3. (a) 4. (b) 5. (b) 6. (d) 7. (b)
8. (d) 9. (d) 10. (d) 11. (a) 12. (c) 13. (c) 14. (c)
15. (a) 16. (a) 17. (c) 18. (b) 19. (b) 20. (c) 21. (c)
22. (c) 23. (b) 24. (a) 25. (a) 26. (a)

## Assignment -3: Schrodinger
## [ANSWERS]

1. (a) 2. (a) 3. (d) 4. (a,c) 5. (d) 6. (a) 7. (b)
8. (a) 9. (b) 10. (c) 11. (a) 12. (c) 13. (b) 14. (c)
15. (d) 16. (b) 17. (c) 18. (d) 19. (c) 20. (d) 21. (c)
22. (d) 23. (d) 24. (b) 25. (d) 26. (0.25) 27. (c)

## Assignment -4: One-dimensional Potential [ANSWERS]

1. (b) 2. (c) 3. (d) 4. (c) 5. (a) 6. (c) 7. (d)
8. (c) 9. (b, c) 10. (c) 11. (c) 12. (b) 13. (b) 14. (b)
15. (a) 16. (d) 17. (a) 18. (c) 19. (b) 20. (c) 21. (a)
22. (a) 23. (d) 24. (c) 25. (d) 26. (a) 27. (d) 28. (a)
29. (d) 30. (b) 31. (b) 32. (b) 33. (b) 34. (c) 35. (c)
36. (d) 37. (c) 38. (a) 39. (c) 40. (d) 41. (c) 42. (d)
43. (a) 44. (a) 45. (c) 46. (d) 47. (b) 48. (a) 49. (a)
50. (b) 51. (b) 52. (b) 53. (c) 54. (b) 55. (c) 56. (b)
57. (a) 58. (b) 59. (b) 60. (d) 61. (d) 62. (b) 63. (d)
64. (a) 65.( ) 66. (a) 67. (c) 68. (a) 69. (d) 70. (a)
71. (b) 72. (3.2 to 3.3)W 73. (c)

## Assignment – 5: Angular Momentum Alzebra [ANSWERS]

1. (d) 2. (a) 3. (d) 4. (c) 5. (a) 6. (c) 7. (d)

8. (b) 9. (b) 10. (c) 11. (a) 12. (a) 13. (d) 14. (a)

15. (d) 16. (c) 17. (c) 18. (c) 19. (b) 20. (a) 21. (b)

22. (d) 23. (d) 24. (c) 25. (d) 26. (d) 27. (d) 28. (a)

29. (b) 30. (c) 31. (d) 32. (a) 33. (a) 34. (d) 35. (c)

36. (c) 37. (b) 38. (c) 39. (d) 40. (b) 41_A(b) 41_B(b)

42. (a) 43. (d) 44. (d) 45. (d) 46. (7.6) 47. (d) 48. ( )

49. (b) 50. (d) 51. (a) 52. (d) 53. (0.66 to 0.68)

## Assignment – 6: 3-D Potentials [ANSWERS]

1. (a) 2. (c) 3. (d) 4. (b) 5. (a) 6. (d) 7. (d)

8. (a) 9. (d) 10. (c) 11. (c) 12. (c) 13. (c) 14. (a)

15. (c) 16. (c) 17. (b) 18. (b) 19. (d) 20. (c) 21. (b)

22. (c) 23. (b) 24. (a) 25. (a) 26. (c) 27. (d) 28. (d)

29. (c) 30. (a) 31. (c) 32. (c) 33. (a,b,c) 34. (d) 35. (d)

36. (b) 37. (c) 38. (d) 39. (c) 40. (c) 41. (b) 42. (c)

43. (a) 44. (a) 45. (c) 46. (d) 47. (c) 48. (d) 49. (b)

50. (d) 51. (d) 52. (d) 53. (d) 54. (3.99 to 4.01)

55. (1.99 to 2.01) 56. (a) 57. (a) 58. (a) 59. (d)

## Assignment – 7: Identical Particles [ANSWERS]

1. (a) 2. (c) 3. (a) 4. (b) 5. (d) 6. (c) 7. (c)

8. (b) 9. (d) 10. (b) 11. (d) 12. (b) 13. (b) 14. (a)

15. (a)

## Assignment – 8: Pertubation Theory [ANSWERS]

1. (c) 2. (c) 3. (d) 4. (a) 5. (d) 6. (b) 7. (d)

8. (a) 9. (a) 10. (d) 11. (d) 12. (a) 13. (c) 14. (b)

15. (a) 16. (b) 17. (a) 18. (d) 19. (c) 20. (c) 21. (b)

22. (a)

## Assignment – 9: Scattering Theory [ANSWERS]

1. (c) 2. (d) 3. (c) 4. (c) 5. (a) 6. (c) 7. (b)

8. (a) 9. (c) 10. (d) 11. (a) 12. (b)

## Assignment -10: Miscellaneous [ANSWERS]

1. (b) 2. (a) 3. (d) 4. (a) 5. (d) 6. (c) 7. (c)

8. (c) 9. (d) 10. (d) 11. (a) 12. (d) 13. (b) 14. (c)

15. (c) 16. (d) 17. (b) 18. (a) 19. (c)

CAREER ENDEAVOUR
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### *Subject: PHYSICAL SCIENCES (NET-JRF/GATE)*

### *Section: QUANTUM MECHANICS*

### *Assignment 1: ORIGIN OF QUANTUM MECHANICS*

1. *The number of photons emitted per second by a 100W source of monochromatic light of wavelength* $5893A^0$ *is*

   *(a)* $2.96\times10^{20}$ *(b)* $2.96\times10^{24}$ *(c)* $8.24\times10^{24}$ *(d)* $8.24\times10^{20}$

2. *Light of wavelength* $3500A^0$ *is incident on two metals A and B having work functions 4.2 eV and 1.9 eV respectively. Which metal will yeild photoelectrons?*

   *(a) Only A* *(b) Only B* *(c) Both A and B* *(d) None of them*

3. *In an experiment, tungsten cathode having a threshold wavelength* $2300A^0$ *is irradiated by a light of wavelength* $1800A^0$. *The maximum energy of the emitted photoelectrons is*

   *(a) 0.89 eV* *(b) 1.23 eV* *(c) 1.49 eV* *(d) 1.97 eV*

4. *A beam of light of wavelength* $4200A^0$ *has the same intensity as that beam of light of wavelength* $6300A^0$. *The first beam, falling perpendicularly on a certain area of a wall, deposits* $12\times10^{16}$ *photons per second per square meter of the wall. The number of the photons that will be deposited on the wall per second per square meter by the second beam is*

   *(a)* $6\times10^{16}$ *(b)* $8\times10^{16}$ *(c)* $12\times10^{16}$ *(d)* $18\times10^{16}$

5. *When an e.m radiation* $E=A(1+\sin\omega t)\cos\omega_0 t$ *is incident on the surface of lithium having work function* $W$, *the maximum kinetic energy of the emitted photoelectrons is*

   *(a)* $\hbar(\omega_0+\omega)-W$ *(b)* $\hbar(\omega_0-\omega)-W$ *(c)* $\hbar\omega_0-W$ *(d)* $\hbar\omega_0+W$

6. *A beam of* $\lambda$ *is incident on a metal having work function* $W$, *placed in a magnetic field B. The most energetic electrons of mass* $m_e$ *and charge q, emitted perpendicular to the magnetic field, are bent in circular arcs of radius R. The value of the magnetic field is*

   *(a)* $\frac{\sqrt{2m_e\left(\frac{hc}{\lambda}+W\right)}}{qR}$ *(b)* $\frac{\sqrt{2m_e\left(\frac{hc}{\lambda}-W\right)}}{qR}$ *(c)* $\sqrt{\frac{2m_e\left(\frac{hc}{\lambda}-W\right)}{qR}}$ *(d)* $\sqrt{\frac{2m_e\left(\frac{hc}{\lambda}+W\right)}{qR}}$

7. *Light of wavelength* $\lambda$ *and power P is incident on a caesium surface having work function* $W$. *If 0.5% of the incident photons produce photoelectrons, then the photoelectric current will be*

   *(a)* $0.5\frac{eP\lambda}{hc}$ *(b)* $0.005\frac{eP\lambda}{hc}$ *(c)* $0.005\frac{hc}{eP\lambda}$ *(d)* $0.5\frac{hc}{eP\lambda}$

8. *X-rays of wavelength 1$A^0$ from a block at an angle $90^0$ w.r.t the incident beam. The kinetic energy of the recoiling electron is*
(a) 296 eV (b) 232 eV (c) 312 eV (d) 344 eV

9. *X-rays of wavelength 10 pm are scattered from the target. The maximum wavelength that are present in the scattered x-rays, is*
(a) 13.2 pm (b) 14.9 pm (c) 15.7 pm (d) 16.9 pm

10. *Consider a photon that sactters at an angle $60^0$ from one electron at rest. If the compton shift is observed to be 3 times of the wavelength of the incident photons, then wavelength of the incident photon is*
(a) 0.403 pm (b) 0.516 pm (c) 0.646 pm (d) 0.718 pm

11. *A photon of frequency $\nu$ is scattered by an electron (rest mass 'm') initially at rest. The maximum kinetic energy of the recoil electron is*

(a) $\dfrac{\left(2h^2\nu^2/mc^2\right)}{1+\left(2h^2\nu^2/mc^2\right)}$ (b) $\dfrac{\left(2h^2\nu^2/mc^2\right)}{1-\left(2h^2\nu^2/mc^2\right)}$ (c) $\dfrac{\left(2h^2\nu^2/mc^2\right)}{1-\left(2h\nu/mc^2\right)}$ (d) $\dfrac{\left(2h^2\nu^2/mc^2\right)}{1+\left(2h\nu/mc^2\right)}$

12. *In a Compton experiment, the ultraviolet ray of wavelength 2000 $A^0$ is scattered from an electron at rest. If the observation of the scattered x-rays is made at $90^0$ w.r.t the direction of incident light, then to measure the compton shift, the minimum resolving power of an optical instrument that will be needed is*
(a) 62300 (b) 82440 (c) 89530 (d) 98600

13. *The de-broglie wavelength for (i) a proton of kinetic energy 70 MeV, (ii) a 10 gm ball moving at 900m/s are approximately*

(a) $10^{-15}\ m, 10^{-34}\ m$ (b) $10^{-10}\ m, 10^{-30}\ m$ (c) $10^{-20}\ m, 10^{-30}\ m$ (d) $10^{-15}\ m, 10^{-30}\ m$

14. *The speed of an electron, whose de-broglie wavelength is equal to Compton wavelength is*

(a) $\dfrac{c}{2}$ (b) c (c) $\dfrac{c}{\sqrt{2}}$ (d) $\dfrac{c}{\sqrt{3}}$

15. *To have a wavelength equal to that of green colour (550 nm), the electron is to be accelerated through a potential difference of*
(a) 4 μV (b) 5 μV (c) 6 μV (d) 7 μV

16. *The relation between wave number and angular frequency for a given wave is $\omega^2 = \alpha k + \beta k^3$. The wave number for which phase velocity equal to group velocity is*

(a) $\sqrt{\dfrac{\alpha}{\beta}}$ (b) $\dfrac{\alpha}{\beta}$ (c) $\dfrac{\alpha^2}{\beta}$ (d) $\dfrac{\alpha}{\beta^2}$

17. *The phase velocity of ocean waves is $\sqrt{\dfrac{g\lambda}{2\pi}}$, where g is the gravitational acceleration of gravity.*

*The group velocity of the ocean waves is equal to*

(a) $\dfrac{3}{2}\sqrt{\dfrac{g\lambda}{2\pi}}$ (b) $\dfrac{5}{2}\sqrt{\dfrac{g\lambda}{2\pi}}$ (c) $\dfrac{1}{2}\sqrt{\dfrac{g\lambda}{2\pi}}$ (d) $\sqrt{\dfrac{g\lambda}{2\pi}}$

18. *The phase velocity of the de-broglie waves of electron having kinetic energy 500 keV is*
(a) 1.16c (b) 1.21c (c) 1.39c (d) 1.52c

19. *The dispersion relation of a certain wave is* $\omega^2 = c^2k^2 + m_0^{\ 2}$. *The limiting value of group velocity* $v_g$ *as* $k \to 0$ *and* $k \to \infty$ *are*

*(a) 0, c/2* *(b) 0, c* *(c) c/2, c* *(d) c/3, c/2*

20. *A particle having rest mass '*$m_0$*' and charge 'q' is accelerated through a potential difference 'V'. Taking relativistic correction into account, the de-broglie wavelength associated with the particlecan be expressed as*

(a) $\lambda_{rel} = \dfrac{h}{\sqrt{2qVm_0\left(1+\dfrac{qV}{2m_0c^2}\right)}}$ (b) $\lambda_{rel} = \dfrac{h}{\sqrt{2qVm_0\left(1-\dfrac{qV}{2m_0c^2}\right)}}$

(c) $\lambda_{rel} = \dfrac{h}{\sqrt{qVm_0\left(1-\dfrac{qV}{2m_0c^2}\right)}}$ (d) $\lambda_{rel} = \dfrac{h}{\sqrt{qVm_0\left(1+\dfrac{qV}{2m_0c^2}\right)}}$

21. *The phase velocity of the de-broglie waves asscocitaed with a particle of rest mass 'm' and wavelength '*$\lambda$*' is*

(a) $c\sqrt{1+\left(\dfrac{hc}{m\lambda}\right)^2}$ (b) $c\sqrt{1-\left(\dfrac{hc}{m\lambda}\right)^2}$ (c) $c\sqrt{1-\left(\dfrac{mc\lambda}{h}\right)^2}$ (d) $c\sqrt{1+\left(\dfrac{mc\lambda}{h}\right)^2}$

22. *Assume that the uncertainty in the location of the particle A is equal to its de-broglie wavelength. The uncertainty in the measurement of velocity 'v' of the particle is*

(a) $\dfrac{v}{2\pi}$ (b) $\dfrac{v}{4\pi}$ (c) $\dfrac{v}{\pi}$ (d) $\dfrac{v}{8\pi}$

23. *A proton is confined to a nucleus of radius* $5\times10^{-15}\,m$. *The minimum value of the kinetic energy of the proton is*

*(a) 46 keV* *(b) 54 keV* *(c) 74 keV* *(d) 82 keV*

24. *The average time that the atomic system remains in a energy state whose spectral line width is* $10^{-4}A^0$ *for* $\lambda = 4000A^0$, *is equal to*

*(a) 1.23 ns* *(b) 2.71 ns* *(c) 4.24 ns* *(d) 5.45 ns*

25. *Consider an electron and proton is confined within a 10F box. The ratio of uncertainities of their velocities will be*

*(a) 1* *(b) 1000* *(c) 1836* *(d) 10000*

## Answer Key

**1.(a)** **2.(b)** **3.(c)** **4.(d)** **5.(a)** **6.(b)** **7.(b)**
**8.(a)** **9.(b)** **10.(a)** **11.(d)** **12.(b)** **13.(a)** **14.(c)**
**15.(b)** **16.(a)** **17.(c)** **18.(a)** **19.(b)** **20.(a)** **21.(d)**
**22.(b)** **23.(a)** **24.(c)** **25.(c)**

### *Subject: PHYSICAL SCIENCES (NET/JRF|GATE)*

### *Section: QUANTUM MECHANICS*

### *Assignment 2: CONCEPT OF WAVE FUNCTION*

1. *The wave function of a particle is given by* $\psi(x) = Ce^{-\alpha^2x^2}\ (-\infty < x < \infty)$ *, where c and α are constants. The probability of finding the particle in the region* $0 < x < \infty$ *is equal to*

*(a) 1/3* *(b) 1/2* *(c) 3/5* *(d) 4/7*

2. *An one dimensional wave function is given by,*

$$\psi(x) = \begin{cases} A\sin\left(\dfrac{\pi x}{l}\right); & 0 < x < l \\ 0 \qquad ; & otherwise \end{cases}.$$

*The value of the normalization constant 'A' is*

*(a)* $\sqrt{1/l}$ *(b)* $\sqrt{2/3l}$ *(c)* $\sqrt{2/l}$ *(d)* $\sqrt{1/3l}$

3. *A particle is represented by the wave function* $\psi(x) = e^{-|x|}\sin\beta x$. *The probability that the position of the particle will to the right of the point x = 1 is*

*(a)* $\dfrac{e^{-2}}{2\beta^2}\left[1+\beta^2-\cos 2\beta+\beta\sin 2\beta\right]$ *(b)* $\dfrac{e^{-2}}{4(1-\beta^2)}\left[1-\beta^2-\cos 2\beta+\beta\sin 2\beta\right]$

*(c)* $\dfrac{e^{-2}}{4\beta^2}\left[1-\beta^2-\sin 2\beta+\beta\cos 2\beta\right]$ *(d)* $\dfrac{e^{-2}}{4(1+\beta^2)}\left[1+\beta^2-\sin 2\beta+\beta\cos 2\beta\right]$

4. *The wave function of a particle, constraint to move along x* $(-\infty < x < \infty)$ *at certain instant is given by* $\psi(x) = Ae^{-\frac{x^2}{a^2}+ibx}$ *[a,b are real constants]. The normalization constant 'A' is*

*(a)* $\sqrt{\dfrac{\sqrt{2}}{a\sqrt{\pi}}}$ *(b)* $\sqrt{\dfrac{2\sqrt{2}}{a\sqrt{\pi}}}$ *(c)* $\dfrac{2\sqrt{2}}{\sqrt{\pi a}}$ *(d)* $\dfrac{2\sqrt{2}}{a\sqrt{\pi}}$

5. *A one dimensional wave function corresponding a particle is given by* $\psi(x) = \sqrt{a}e^{-a|x|}$. *The probability of finding the particle between x=1/a to x=2/a is equal to*

*(a)* $e^{-2}-e^{-4}$ *(b)* $\dfrac{e^{-2}-e^{-4}}{2}$ *(c)* $\dfrac{e^{-2}-e^{-4}}{4}$ *(d)* $\dfrac{e^{-2}+e^{-4}}{4}$

6. *The normalized ground state wave function of an electron in a hydrogen atom is given by* $\psi(r)=\left[\frac{1}{\pi a_0^3}\right]^{1/2} e^{-r/a_0}$, *where* $a_0$ *is the first Bohr radius. The probability of finding the electron in the ground state, at a distance greater than* $a_0$ *from the nucleus is*

(a) $\frac{1}{e^2}$ (b) $\frac{3}{e^2}$ (c) $\frac{5}{e^2}$ (d) $\frac{7}{e^2}$

*The wave function corresponding to a particle is given by*

$$\psi(x)=\frac{1}{\sqrt{a}}e^{-|x|/a}$$

7. *The probability of finding the particle in the region* $-a < x < a$ *is*

(a) $\frac{1}{e^2}$ (b) $1-\frac{1}{e^2}$ (c) $\frac{3}{e^2}$ (d) $1-\frac{3}{e^2}$

8. *The value of 'b' such that probability of finding the particle between* $-b < x < b$ *is 0.5 will be*

(a) $\frac{2a}{\ln 2}$ (b) $\frac{a\ln 2}{2}$ (c) $2a\ln 2$ (d) $a\ln 2$

9. *The wave function of the particle at a certain instant is given as* $\psi(x)=A\exp\left(-\frac{x^2}{a^2}+ikx\right)$. *If* $P_1$ *and* $P_2$ *denote the probabilities of finding the particle in the range a to a+da and 2a to 2a+da respectively. The ratio* $P_1/P_2$ *is*

(a) $e^2$ (b) $e^3$ (c) $e^6$ (d) $e^8$

10. *Which of the following represent a realistic wave function of a particle?*

(a) $\psi(x)=A\exp\left(\frac{x^2}{a^2}+ikx\right)$ (b) $\psi(x)=Ae^{-|x|/a}$

(c) $\psi(x)=\begin{Bmatrix}0 & \text{for}\,|x|>a\\ c & \text{for}\,|x|\le a\end{Bmatrix}$ (d) $\psi(x)=\frac{A\sin kx}{x^2}$

11. *Consider a system whose state is given in terms of an orthonormal set of three vectors:*

$$|\psi\rangle=\frac{\sqrt{3}}{3}|\phi_1\rangle+\frac{2}{3}|\phi_2\rangle+\frac{\sqrt{2}}{3}|\phi_3\rangle$$

*Take an ensemble of 1800 identical system each one of them is in the state* $|\psi\rangle$. *If measurements are done, how many systems will be in the states* $|\phi_1\rangle,|\phi_2\rangle,|\phi_3\rangle$?

(a)600,800,400 (b) 800,600,400 (c) 200,600,1000 (d) 300,900,600

12. *Let* $|\alpha\rangle,|\beta\rangle,|\gamma\rangle$ *represents three ortho-normalized wave functions and two state is defined:*

$$|\psi\rangle=c_1\left[|\alpha\rangle+2|\beta\rangle+(1+i)|\gamma\rangle\right]$$
$$|\phi\rangle=c_2\left[|\alpha\rangle-i|\beta\rangle+|\gamma\rangle\right]$$

*The real, positive values of* $c_1$ *and* $c_2$ *for which* $|\psi\rangle$ and $|\phi\rangle$ *are normalized are*

(a) $\frac{1}{\sqrt{7}},\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{7}}$ (c) $\frac{1}{\sqrt{5}},\frac{1}{\sqrt{7}}$ (d) $\frac{1}{\sqrt{5}},\frac{1}{\sqrt{3}}$

13. *The momentum space representation* $\phi(p)$ *of the position space wave function*

$$\psi(x) = Ae^{i\frac{p_o x}{\hbar}}$$

(a) $\psi(x) = \frac{Ap_o}{\hbar} e^{i\frac{p_o x}{\hbar}}$ (b) $\psi(x) = \frac{Ap_o}{\hbar} e^{-i\frac{p_o x}{\hbar}}$

(c) $\psi(x) = A\sqrt{2\pi\hbar}\,\delta(p - p_0)$ (d) $\psi(x) = \frac{A}{\sqrt{2\pi\hbar}}\delta(p - p_0)$

14. *The momentum space representation of the following position space wave function*

$$\psi(x) = \begin{Bmatrix} Ae^{ip_o x} & ; |x| < a \\ 0 & ; |x| > a \end{Bmatrix}$$

(a) $\frac{A}{\sqrt{2\pi\hbar}} \frac{\sin\left(p_0 - \frac{p}{\hbar}\right)a}{\left(p_0 - \frac{p}{\hbar}\right)}$ (b) $\frac{2A}{\sqrt{2\pi\hbar}} \frac{\sin\left(p_0 - \frac{p}{\hbar}\right)a}{\left(p_0 - \frac{p}{\hbar}\right)}$

(c) $\frac{2A}{\sqrt{2\pi\hbar}} \sin\left(p_0 - \frac{p}{\hbar}\right)a$ (d) 0

*The wave function of a particle is given as:* $\psi(x) = \begin{Bmatrix} \frac{2}{a^{3/2}} xe^{-x/a} ; & (x > 0) \\ 0 & ; (x \le 0) \end{Bmatrix}$

15. *The corresponding momentum space wave function will be*

(a) $\sqrt{\frac{2a\hbar}{\pi}} \frac{\hbar}{(\hbar + iap)^2}$ (b) $\sqrt{\frac{a\hbar}{\pi}} \frac{\hbar}{(\hbar + iap)^2}$ (c) $\sqrt{\frac{a\hbar}{\pi}} \frac{\hbar^2}{(\hbar + iap)^2}$ (d) $\sqrt{\frac{a\hbar}{\pi}} \frac{\hbar}{p^2}$

16. *The most probable value of momentum will be*

(a) $\hbar a$ (b) 0 (c) $\frac{\hbar a}{2}$ (d) $\frac{\hbar a}{4}$

17. *The wave function of a particle is given as* $\psi(x) = \begin{Bmatrix} \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right); & (0 < x < a) \\ 0 & ; (otherwise) \end{Bmatrix}$

*The probability of its momentum being in the range* $\frac{2h}{a}$ *to* $\frac{2h}{a} + \frac{h}{50a}$ *is*

(a) $\frac{8}{5625\pi^2}$ (b) $\frac{16}{5625\pi^2}$ (c) $\frac{32}{5625\pi^2}$ (d) $\frac{4}{5625\pi^2}$

## Answer Key

**1.(b)** **2.(c)** **3.(a)** **4.(a)** **5.(b)** **6.(c)** **7.(b)**

**8.(b)** **9.(c)** **10.(b)** **11.(a)** **12.(a)** **13.(c)** **14.(b)**

**15.(a)** **16.(b)** **17.(d)**

### *Subject:* *PHYSICAL SCIENCES (NET/JRF|GATE)*

### *Section:* *QUANTUM MECHANICS*

## *Assignment 3:* GENERAL FORMALISM OF QUANTUM MECHANICS

1. *The value of the commutator bracket* $\left[\hat{x}^2, \hat{p}_x{}^2\right]$ *(where* $\hat{x}$ *and* $\hat{p}_x$ *are position and linear momentum operators respectively) is*

   (a) $2i\hbar\left(\hat{x}\hat{p}_x + \hat{p}_x\hat{x}\right)$ (b) $2i\hbar\left(\hat{x}\hat{p}_x - \hat{p}_x\hat{x}\right)$ (c) $4i\hbar\hat{x}\hat{p}_x$ (d) $4i\hbar\hat{p}_x\hat{x}$

2. *The value of the commutator bracket* $\left[\hat{x},\left[\hat{x},\hat{H}\right]\right]$ *(where and are position and Hamiltonian operators respectively) is*

   (a) $\dfrac{i\hbar^2}{m}$ (b) $-\dfrac{i\hbar^2}{m}$ (c) $-\dfrac{\hbar^2}{m}$ (d) $\dfrac{\hbar^2}{m}$

3. *Which of the following is not a hermitian operator?*

   (a) $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$ (b) $\dfrac{\partial}{\partial x}$ (c) $\hat{x}$ (d) $\hat{p}_x$

4. *If* $\psi(x) = Ae^{ikx}$ *is an eigen function of the kinetic energy operator, then corresponding kinetic energy eigenvalue is*

   (a) $\dfrac{\hbar^2 k^2}{2m}$ (b) $\dfrac{\hbar^2 k^2}{m}$ (c) $\dfrac{3\hbar^2 k^2}{2m}$ (d) $\dfrac{3\hbar^2 k^2}{m}$

5. *The value of the commutator brackets* $\left[\hat{x}\hat{p}_x + \hat{p}_x\hat{x}, \hat{p}_x\right]$ *and(where and are position and linear momentum operators respectively) are*

   (a) $2i\hbar\hat{p}_x, -2i\hbar\hat{x}$ (b) $2i\hbar\hat{p}_x, 2i\hbar\hat{x}$ (c) $-2i\hbar\hat{p}_x, 2i\hbar\hat{x}$ (d) $-2i\hbar\hat{p}_x, -2i\hbar\hat{x}$

6. *The value of the commutator bracket* $\left[x, \dfrac{\partial^2}{\partial x^2}\right]$ *is*

   (a) $-2\dfrac{\partial}{\partial x}$ (b) $2\dfrac{\partial}{\partial x}$ (c) $\dfrac{\partial}{\partial x}$ (d) $-\dfrac{\partial}{\partial x}$

7. *If a wave function* $\psi(x)$ *satisfies the following relation* $x\psi(x) = -\left(\dfrac{ia^2}{\hbar}\right)p_x\psi(x)$ *('a' is real positive quantity), then* $\psi(x)$ *can be written as*

   (a) $\psi_0 e^{-x/2}$ (b) $\psi_0 e^{x/2}$ (c) $\psi_0 e^{x^2/2}$ (d) $\psi_0 e^{-x^2/2}$

8. *A particle is subjected to the potential energy* $V(x)=\frac{1}{2}kx^2$. *At a particular time, it has the wave function* $\psi(x)=Axe^{-x^2/a^2}$. *If the particle has a definite total mechanical energy, then the possible value of 'a' is*

(a) $\left[\frac{4\hbar^2}{mk}\right]^{1/4}$ (b) $\left[\frac{4\hbar^2}{mk}\right]^{1/2}$ (c) $\left[\frac{4\hbar}{mk}\right]^{1/2}$ (d) $\left[\frac{4\hbar}{mk}\right]^{1/4}$

9. *Let,* $N=\hat{b}^\dagger\hat{b}$ *where the operator* $\hat{b}$ *satisfies the relation* $\hat{b}\hat{b}^\dagger+\hat{b}^\dagger\hat{b}=1$ *and* $\hat{b}^2=\left(\hat{b}^\dagger\right)^2=0$, *then the eigenvalues of operator N are*

*(a) Positive & negative integers* *(b) All positive integers*
*(b) 0, 1, -1 only* *(d) 0, 1 only*

10. *The components of the initial state* $|\psi_i\rangle$ *of a quantum system are given in a complete and orthonormal basis of three states* $|\phi_1\rangle,|\phi_2\rangle,|\phi_3\rangle$ *by* $\langle\phi_1|\psi_i\rangle=\frac{i}{\sqrt{3}},\langle\phi_2|\psi_i\rangle=\sqrt{\frac{2}{3}},\langle\phi_3|\psi_i\rangle=0$. *The probability of finding the system in a state* $|\psi_f\rangle$, *whose components are given in the same basis by* $\langle\phi_1|\psi_f\rangle=\frac{1+i}{\sqrt{3}},\langle\phi_2|\psi_f\rangle=\frac{1}{\sqrt{6}},\langle\phi_3|\psi_f\rangle=\frac{1}{\sqrt{6}}$, *will be*

*(a) 1/9* *(b) 4/9* *(c) 5/9* *(d) 7/9*

11. *Let* $|\phi_1\rangle$ *and* $|\phi_2\rangle$ *be two orthonormal state vectors, check that if* $|\phi_1\rangle\langle\phi_2|+|\phi_2\rangle\langle\phi_1|$ *is a projection operator or not?*

12. *(i) Show that, the operator* $i\left(\hat{x}^2+1\right)\frac{d}{dx}+i\hat{x}$ *is hermitian in nature.*

*(ii) Find the state for which* $\hat{A}\psi(x)=0$ *and then find its normalized form.*

*(iii) Calculate the probability of finding the particle in the region* $-1\le x\le 1$.

13. *A system which is initially in the state,* $|\psi_0\rangle=\left[\sqrt{2}|\phi_1\rangle+\sqrt{3}|\phi_2\rangle+|\phi_3\rangle+|\phi_4\rangle\right]/\sqrt{7}$, *where* $|\phi_n\rangle$ *are eigenstates of the system's Hamiltonian such that* $\hat{H}|\phi_n\rangle=n^2\varepsilon_0|\phi_n\rangle$

*(i) If energy is measured what values will be obtained and with what probabilities?*

*(ii) Consider an operator* $\hat{A}$ *for which* $\hat{A}|\phi_n\rangle=n(n+1)a_0|\phi_{n+1}\rangle$. *Suppose that a measurement of energy yields* $9\varepsilon_0$. *If we measure immediately afterwards, what value will be obtained?*

14. *A particle of mass 'm' is confined in a 1-D box from x=-2L to x=2L. The wave function of the particle in this state is* $\psi(x)=\psi_0\cos\left(\frac{\pi x}{4L}\right)$. *The expectation value of* $p^2$ *in this state is*

(a) $\frac{\pi^2\hbar^2}{2L^2}$ (b) $\frac{\pi^2\hbar^2}{4L^2}$ (c) $\frac{\pi^2\hbar^2}{16L^2}$ (d) $\frac{\pi^2\hbar^2}{32L^2}$

15. *Let, wave function corresponding to two particles is given as* $\psi_1(x)=\left(\frac{2}{\pi a^2}\right)^{1/4} e^{-x^2/a^2}$ *and* $\psi_2(x)=\left(\frac{32}{\pi a^6}\right)^{1/4} xe^{-x^2/a^2}$*, then the value of* $\langle\psi_2|p_x|\psi_1\rangle$ *is*

(a) $\frac{i\hbar}{a}$ (b) $-\frac{i\hbar}{a}$ (c) $-\frac{\hbar}{a}$ (d) $\frac{\hbar}{a}$

16. *An one-dimensional wave function of a particle is given as* $\psi(x)=e^{-x^2/a^2+ikx}$*. The expectation value of position and momentum of the particle is equal to*

(a) $0, \hbar k$ (b) $a, \hbar k$ (c) $a/2, \hbar k$ (d) $a/2, \hbar k/2$

17. *Which of the following is an eigenfunction the kinetic energy operator?*

(a) $Ae^{-x/a}$ (b) $Ae^{x/a}$ (c) *Both of them* (d) *None of these*

18. *A particle of mass 'm' is represented by wave function* $\psi(x)=Ae^{ikx}$ *( k is wave number and A is constant). The magnitude of probability current density is*

(a) $|A|^2\frac{\hbar k}{m}$ (b) $|A|^2\frac{\hbar k}{2m}$ (c) $|A|^2\frac{\hbar^2k^2}{2m}$ (d) $|A|^2\frac{\hbar^2k^2}{m}$

19. *The wave function of a particle moving in a free space is* $\psi(x)=e^{ikx}+2e^{-ikx}$*. The magnitude of probability current density for the real part of the wave function is*

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

20. *A beam of electrons and a beam of protons are described by the same wave function* $\psi(x)=Ae^{ikx}$*. The ratio of the flux of electron beam and flux of proton beam is*

(a) 1 (b) 1836 (c) $k$ (d) $k^2$

21. *The wave nature corresponding to a particle is represented by a gaussian wave packet* $\psi(x)=A\exp\left[-\frac{x^2}{a^2}+i\frac{p_0}{\hbar}x\right]$*. The magnitude of probability current density corresponding to that particle is*

(a) $\frac{p_0}{2m}|A|^2 e^{-\frac{2x^2}{a^2}}$ (b) $\frac{p_0}{m}|A|^2 e^{-\frac{2x^2}{a^2}}$ (c) $\frac{2p_0}{m}|A|^2 e^{-\frac{2x^2}{a^2}}$ (d) *None of these*

22. *The probability current density for three dimensional wave function* $\psi(r)=\frac{A}{r}e^{ikr}$ *corresponding to a particle is*

(a) $\frac{\hbar k}{m}\frac{|A|^2}{r^2}\hat{r}$ (b) $\frac{\hbar k}{m}\frac{|A|^2}{r}\hat{r}$ (c) $\frac{\hbar k}{2m}\frac{|A|^2}{r}\hat{r}$ (d) $\frac{\hbar k}{2m}\frac{|A|^2}{r^2}\hat{r}$

*The wave function of a beam of particle in a certain region is described by*

$$\psi(x) = A\left[e^{ikx} + 4e^{-ikx}\right]$$

23. *The probability current density for this wave function is*

*(a)* $30|A|^2 \frac{\hbar k}{m}$ *(b)* $-30|A|^2 \frac{\hbar k}{m}$ *(c)* $15|A|^2 \frac{\hbar k}{m}$ *(d)* $-15|A|^2 \frac{\hbar k}{m}$

24. *The ratio of the number of particle going in the positive x-direction and the number of particles going in the negative x-direction is*

*(a) 16:1* *(b) 1:16* *(c) 15:1* *(d) 1:15*

25. *The wave function of a particle moving under a one-dimensional potential* $V(x)$ *is* $\psi(x) = e^{-iax+b}$. *The one-dimensional potential will be of the form*

*(a)* $V(x) \infty x^2$ *(b)* $V(x) \infty x$ *(c)* $V(x) = 0$ *(d) None of these*

26. *The wave function of a spinless particle moving under a one-dimensional potential* $\psi(x) = Ae^{-\alpha^2 x^2}$, *corresponding to the energy eigenvalue* $E_0 = \frac{\alpha^2\hbar^2}{m}$. *The one-dimesional potential will be of the form*

*(a)* $2E_0\alpha^2 x^2$ *(b)* $E_0\alpha^2 x^2$ *(c)* $\frac{3E_0\alpha^2 x^2}{2}$ *(d) None of these*

27. *The wave function of a particle in given as* $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_0\rangle + i|\phi_1\rangle$ *where* $|\phi_0\rangle$ *and* $|\phi_1\rangle$ *are the normalized eigenstates with energies* $E_0, E_1$ *respectively. The energy of the particle in the state* $|\psi\rangle$ *is*

*(a)* $\frac{E_0}{2} + E_1$ *(b)* $\frac{E_0}{2} - E_1$ *(c)* $\frac{E_0 - 2E_1}{3}$ *(d)* $\frac{E_0 + 2E_1}{3}$

28. *Consider a system whose initial state at t = 0 is given in terms of three orthonormal state vectors* $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ *as*

$$|\psi(x,0)\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + A|\phi_2\rangle + \frac{1}{\sqrt{5}}|\phi_3\rangle \quad \text{(A is real constant)}$$

*(i) Find the value of* $A$ *such that* $|\psi(x,0)\rangle$ *is normalized.*

*(ii) What are the probabilities regarding the system to be in the states* $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$.

*(iii) Find the state of the system at any time 't'.*

29. *The eigenstates of the Hamiltonian of a quantum system are* $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ *and the corresponding energies are* $E_0, 2E_0, 3E_0$ *respectively. At time t = 0, the state* $|\psi\rangle$ *has the following scalar products*

$$\langle\phi_1|\psi\rangle = \frac{3}{5}, \langle\phi_2|\psi\rangle = \frac{2\sqrt{2}}{5}, \langle\phi_3|\psi\rangle = \frac{2\sqrt{2}}{5}$$

*Find these scalar products at time* $t = \frac{\pi\hbar}{E_0}$.

30. *The wave function at* $t = 0$ *is given by*

$$|\psi(x,0)\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$$

*where* $|\phi_1\rangle$ *and* $|\phi_2\rangle$ *are normalized eigenstates of the Hamiltonian with energy eigenvalues* $E_1, E_2\ (E_2 > E_1)$ *respectively. Calculate the shortest time after which* $|\psi(x,0)\rangle$ *will become orthogonal to* $|\psi(x,t)\rangle$ *?*

## Answer Key

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.(a)** | **2.(c)** | **3.(b)** | **4.(a)** | **5.(a)** | **6.(b)** | **7.(d)** |
| **8.(a)** | **9.(d)** | **10.(c)** | **14.(c)** | **15.(a)** | **16.(a)** | **17.(d)** |
| **18.(a)** | **19.(d)** | **20.(b)** | **21.(b)** | **22.(a)** | **23.(d)** | **24.(b)** |
| **25.(c)** | **26.(a)** | **27.(d)** | | | | |

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## CSIR-UGC-NET/JRF|GATE-PHYSICAL SCIENCES

## Test-1: Quantum Mechanics

**Date: 24-09-2014**
**Time: 01 : 00 Hour** **Max. Marks: 85**

**Instructions:-**

- There are two parts, part **A** and part **B**.
- Part A contains **10** objective type questions. Each question carries **3.5** marks.
- Part B contains **10** objective type questions. Each question carries **5** marks.
- There will be **negative** marking in both parts at the rate of **25%**.
- If you feel any mistake in statement or option of any question, please mention in the corresponding question.

### Part A

1. A particle has the wavefunction, $\psi(r) = Ne^{-\alpha r}$

Where N is a normalization factor and $\alpha$ is a known real parameter. The expectation value of x-component of position is [illegible]

a. $\frac{3}{2\alpha}$
b. $\frac{3\alpha}{2}$
c. $\alpha$ ✓
[✓ d. 0]

2. Consider the one-dimensional normalized wave functions $\psi_0(x)$, $\psi_1(x)$ with the properties
$\psi_0(-x) = \psi_0(x) = \psi_0^*(x)$, $\psi_1(x) = N\frac{d\psi_0(x)}{dx}$
Consider also the linear combination
$\psi(x) = c_1\psi_0(x) + c_2\psi_1(x)$
with $|c_1|^2 + |c_2|^2 = 1$. The constants $N$, $c_1$, $c_2$ are considered as known. Which option is correct?

a. $\psi_0$ and $\psi_1$ are orthogonal and $\psi$ is normalized. ✓
b. The expectation value of $x$ in the states $\psi_0$, $\psi_1$ and $\psi$ is zero.
c. The expectation value of $p$ in the states $\psi_0$, $\psi_1$ and $\psi$ is zero.
d. All are correct.

3. A green photon collides with a stationary electron. After the the collision the photon color is
a. unchanged.
[✓ b. shifted toward red.]
c. shifted toward blue.
d. Photon is absorbed by electron.

4. A wave function is written in terms of orthonormal basis as

$$|\psi\rangle = \frac{1}{3}|u_1\rangle + B|u_2\rangle + \sqrt{\frac{5}{6}}|u_3\rangle$$

The probability that the system is found in the state $|u_2\rangle$ is

a. $1/3$

[✓ b.] $1/18$

c. $1/9$

d. $\sqrt{1/18}$

5. If $\hat{A}$ and $\hat{B}$ are two Hermitian operators then among the following which is /are Hermitian.

a. $\hat{A}\hat{B}\hat{A}$

b. $i[\hat{A},\hat{B}]$

c. $(1+i)\hat{A}\hat{B} + (1-i)\hat{B}\hat{A}$

[✓ d.] All

6. Consider that $|\phi_1>$ and $|\phi_2>$ are orthonormal basis vectors. The value of $\alpha$ so that the states

$$|\psi> = \alpha|\phi_1> + 5|\phi_2> \qquad and \qquad |\chi> = 3\alpha|\phi_1> - 4|\phi_2>$$

are orthogonal, is

a. 0

[✓ b.] $\sqrt{20/3}$

c. 3/20

d. 20/3

7. The Hamiltonian of a system is given by $\widehat{H} = \widehat{H_0} + i\widehat{V}$, where $\widehat{H_0}$ and $\widehat{V}$ are hermitian operators, P and S represent position probability and probability current density. Which option is correct ?

a. The total probability is conserved.

[✓ b.] $\frac{\partial P}{\partial t} + \nabla.\mathbf{S} = 0$

c. $\nabla.\mathbf{S} = 0$

d. The total probability is not conserved.

8. A simple harmonic oscillator is initially (at $t$=0) in a state with wave function

$$\psi(x,0) = N\sum_{n=0}^{\infty} c^n \psi_n(x)$$

where $\psi_n(x)$ are the harmonic oscillator energy eigenfunction and c is a complex parameter. The value of normalization constant N is

[✓ a.] $\sqrt{1-|c|^2}$

b. $\sqrt{1-c}$

c. $\sqrt{c}$

d. 1

9. In the position space the position operator $\hat{x}$ is represented by multiplication of $x$ on the position space wave function and momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ and $\hat{p}^2 = -\hbar^2\frac{\partial^2}{\partial x^2}$ .If in the momentum space the momentum operator is represented by multiplication of momentum space wave function then $\hat{x}$ and $\hat{x}^2$ defined as respectively

   a. $\hat{x} = \frac{\partial}{\partial p}$ and $\hat{x}^2 = \frac{\partial^2}{\partial p^2}$

   b. $\hat{x} = -i\hbar\frac{\partial}{\partial p}$ and $\hat{x}^2 = -\hbar^2\frac{\partial^2}{\partial p^2}$

   c. $\hat{x} = i\hbar\frac{\partial}{\partial p}$ and $\hat{x}^2 = \hbar^2\frac{\partial^2}{\partial p^2}$

   [✓ d. $\hat{x} = i\hbar\frac{\partial}{\partial p}$ and $\hat{x}^2 = -\hbar^2\frac{\partial^2}{\partial p^2}$]

10. An operator $\hat{A}$ is defined as $\hat{A}\psi(t) = \psi(t + t_0)$

    where $\psi(t)$ is the function of time and $t_0$ is any arbitrary time . Then the operator $\hat{A}$ is given by

    a. $\exp\left(\frac{it_0\hat{H}}{\hbar}\right)$ b. $\exp\left(\frac{it_0\hat{p}}{\hbar}\right)$ c. $\exp\left(-\frac{it_0\hat{p}}{\hbar}\right)$ [✓ d. $\exp\left(-\frac{it_0\hat{H}}{\hbar}\right)$]

## Part B

11. The Hermitian conjugate of the operator $\hat{B} = -i\frac{\partial}{\partial r}$ is

    a. $-i\frac{\partial}{\partial r}$ b. $+i\frac{\partial}{\partial r}$ [✓ c. $-i\frac{1}{r^2}\frac{\partial}{\partial r}r^2$] d. $-i\frac{1}{r}\frac{\partial}{\partial r}r^2$

12. A particle of mass m moving in one-dimension is represented by the wave function

    $$\psi(x, t=0) = Ae^{-\frac{x^2}{a^2}}e^{ik_0x}$$

    where $a, k_0$ are constants and $A$ is normalization constant. The uncertainty in propagation vector $\Delta k$ is

    a. $\Delta k = 0$ b. $\Delta k = \frac{2}{a}$ [✓ c. $\Delta k = \frac{1}{2a}$] d. $\Delta k = \sqrt{\frac{1}{a^2} - k_0^2}$

13. The wavefunction of the harmonic oscillator at time t=0 is $\psi(x,0) = \sqrt{2}\phi_1 + \frac{1}{\sqrt{2}}\phi_2 + \phi_3$

    where $\phi_n$ is the stationary eigen function of the oscillator for the nth state and A is a normalization constant. The expectation values of energy at times t=π/ω, is

    a. $\frac{19}{4}\hbar\omega$

    b. $\frac{19}{14}\hbar\omega$

    [✓ c. $\frac{31}{14}\hbar\omega$]

    d. 0

14. A particle of mass m is in a harmonic oscillator potential $V(x) = 2kx^2$. Given that $\psi(x) = A exp(-ax^2)$ is an eigenfunction of energy. What is the value of constant a?

[✓ a. $\frac{\sqrt{mk}}{\hbar}$]
b. $\hbar\sqrt{\frac{2k}{m}}$
c. $\frac{\sqrt{mk}}{2\hbar}$
d. $\hbar\sqrt{\frac{k}{m}}$

15. Consider that $|\phi_1>$ and $|\phi_2>$ are orthonormal basis vectors of two-dimensional space. If an operator $\hat{B} = |\phi_1> < \phi_2|$ ,then

a. $\hat{B}$ is hermitian.
b. $\hat{B}^2 \neq 0$
c. $\hat{B}\hat{B}^\dagger - \hat{B}^\dagger\hat{B}$ is unitary.
[✓ d. $\hat{B}\hat{B}^\dagger$ and $\hat{B}^\dagger\hat{B}$ are projection operators.] ✓

16. The components of the initial state $|\psi_i\rangle$ of a quantum system are given in a complete and orthonormal basis of three state vectors $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ by $\langle\phi_1|\psi_i\rangle = \frac{i}{\sqrt{3}}, \langle\phi_2|\psi_i\rangle = \sqrt{\frac{2}{3}}$ and $\langle\phi_3|\psi_i\rangle = 0$. If system evolves in time , the probability of finding the system in a state $|\psi_f\rangle$, whose components are given in the same basis by $\langle\phi_1|\psi_f\rangle = \frac{1+i}{\sqrt{3}}$, $\langle\phi_2|\psi_f\rangle = \frac{1}{\sqrt{6}}$ and $\langle\phi_3|\psi_f\rangle = \frac{1}{\sqrt{6}}$, will be

a. 1　b. $\frac{4}{9}$　[✓ c. $\frac{5}{9}$]　d. $\frac{7}{9}$

17. Two states of a particle with definite energy $E_1$ and $E_2$ are represented by the following normalized, orthogonal solutions of the Schroedinger equation:
$\psi_1(x,t) = \phi_1(x,)e^{-iE_1t/\hbar}$ and $\psi_2(x,t) = \phi_2(x)e^{-iE_2t/\hbar}$
The uncertainty in energy ΔE for the state $\psi(x,t) = \frac{1}{2}\phi_1(x,)e^{-iE_1t/\hbar} + \frac{\sqrt{3}}{2}\phi_2(x)e^{-iE_2t/\hbar}$ is

a. $|E_1 - E_2|$
b. $\frac{1}{2}|E_1 - E_2|$
c. $\frac{1}{2}|E_1 - \sqrt{3}E_2|$
[✓ d. $\frac{\sqrt{3}}{4}|E_1 - E_2|$]

18. If $\hat{x}$ and $\hat{p}_x$ are quantum mechanical operators corresponding to position and momentum respectively. For any positive constant $a$ the value of $\left[\hat{x}, exp\left(\frac{i\hat{p}_x a}{\hbar}\right)\right]$ is

a. $i\hbar\, exp\left(\frac{i\hat{p}_x a}{\hbar}\right)$

b. $-a\, exp\left(\frac{i\hat{p}_x a}{\hbar}\right)$

c. $-i\hbar\hat{x}\, exp\left(\frac{i\hat{p}_x a}{\hbar}\right)$

d. 0

19. If $\psi = xe^{-\nu x^2}$ represent the state of a system then the value of $\langle px\rangle$ in this state is

a. 0 b. 1 c. $\frac{2\nu\hbar^2}{m}$ d. $\frac{-i\hbar}{2}$

20. Consider a state which is given in terms of three orthonormal vectors $|\phi_1>, |\phi_2>$ and $|\phi_3>$ as follows:

$$|\psi> = \frac{1}{\sqrt{15}}|\phi_1> + \frac{1}{\sqrt{3}}|\phi_2> + \frac{1}{\sqrt{5}}|\phi_3>,$$

where $|\phi_n>$ are eigenstates to an operator $\hat{B}$ such that : $\hat{B}\ |\phi_n> =(3n^2-1)\ |\phi_n>$ with n=1,2,3. The expectation value of $\hat{B}$ is

a. 0

b. 145/15

c. 145/9

d. None of these

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## Test-2: Quantum Mechanics

**Date: 16-10-2014**
**Time: 01 : 00 Hour** **Max. Marks: 85**

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**Instructions:-**

- There are two parts, part A and part B.
- Part A contains **10** objective type questions. Each question carries 3.5 marks.
- Part B contains **10** objective type questions. Each question carries 5 marks.
- There will be **negative** marking in both parts at the rate of **25%.**

## Part A

1. The number of energy states for a cubical box of side $a$ in energy range $0 < E < 15\pi^2\hbar^2/2ma^2$ is

a. 6 b. 11 c. 15 [✓ d.] 17

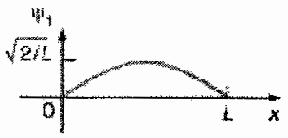
2. A mesic atom is formed by a muon of mass 207 times the electron mass, charge $-e$ and hydrogen nucleus, the radius of first orbit of mesic atom is

[✓ a.] 283 fm b. 0.51 $A^0$ c. 0.49 $A^0$ d. 347 fm

3. An operator $f$ describing the interaction of two spin-1/2 particles has the form $f = a + b\sigma_1 . \sigma_2$ where a and b are constant, $\sigma_1$ and $\sigma_2$ are Pauli spin operators. The total spin angular momentum is $S = \frac{\hbar}{2}(\sigma_1 + \sigma_2)$.

a. Only $S^2$ and $f$ have simultaneous eigen-function.
b. Only $S_z$ and $f$ have simultaneous eigen-function.
c. Only $S^2$ and $S_z$ have simultaneous eigen-function.
[✓ d.] $S^2$ , $S_z$ and $f$ have simultaneous eigen-function.

4.



The lowest energy state for a particle in a box looks like this. How many electrons can have a wave-function that looks like this?

a. 0
b. 1
[✓ c.] 2
d. Any number

5. A particle in a central potential has an orbital angular momentum quantum number 2 $\hbar$ and spin quantum number $\hbar$. A spin –orbit interaction is represented by Hamiltonian $H = A\,\mathbf{L.S}$. The energy levels are given by

a. $2A\hbar^2, 0, -2A\hbar^2$
b. $2A\hbar, -A\hbar, -3A\hbar$
c. $2A\hbar^2, A\hbar^2, -3A\hbar^2$
[✓ d.] $2A\hbar^2, -A\hbar^2, -3A\hbar^2$

6. A spin½ particle is in a state described by the
$A\binom{1+i}{2}$
where $A$ → normalised constant. The probability of finding
$S_z = -\frac{\hbar}{2}$ the particle with spin projection $S_z = -\frac{\hbar}{2}$ is

(a) $\frac{1}{6}$
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
[✓ (d)] $\frac{2}{3}$

7. $\hat{a}$ and $\hat{a}^\dagger$ are lowering and raising operators defined as

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} + \frac{i\hat{p}}{\sqrt{2m\hbar\omega}},$$

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} - \frac{i\hat{p}}{\sqrt{2m\hbar\omega}}$$

The expectation value of $\hat{a}\hat{a}^\dagger$ for the state $|n\rangle$ is

a. 0
[✓ b.] n+1
c. n
[✓ d.] n-1

8. A quantum particle of mass m moves in two dimensions in an anisotropic harmonic oscillator potential $V(x,y) = \frac{1}{2}m\omega^2x^2 + 8m\omega^2y^2$. The enrgy eigenvalues are (n =0,1,2,3,4……..)

a. $(n+1)\hbar\omega$
b. $\left(2n+\frac{3}{2}\right)\hbar\omega$
c. $(n+\frac{3}{2})\hbar\omega$
[✓ d.] $(n+\frac{5}{2})\hbar\omega$

9. Consider a particle of mass m moving in one-dimensional potential that consists of two Dirac-delta functions: $V(x) = -V_0[\delta(x-a) + \delta(x+a)]$ , where $V_0$ and $a$ are positive constants, $\beta^2 = -\frac{2mE}{\hbar^2}$. The wave function of the particle in the region $-a < x < +a$ is

Symmetric Potential.

a. $Ae^{-\beta x}$
b. $Ae^{+\beta x}$
c. $Ae^{\pm i\beta x}$
[✓ d.] $A(e^{\beta x} \pm e^{-\beta x})$

10. At time t=0 , the wave function for Hydrogen atom is

$$\psi(r,0) = \frac{1}{\sqrt{10}}(\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1})$$

where the subscripts are values if the quantum numbers n,l,m. Ignore spin and radiative transitions. The expectation values of the energy of the system is

a. 0
b. -3.40 eV
c. -7.47 eV
d. -34.0 eV

## Part B

11. An electron is described by a Hamiltonian that does not depend on spin. The electron's spin wave function is an eigenstate of $S_z$ with eigenvalue $+\hbar/2$. The operator $\hat{n}.S$ represents the spin projection along a direction $\hat{n}$.We can express this direction as

$\hat{n} = \sin\theta(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}) + \cos\theta\hat{z}$.

What is the probability of finding the electron in eigenstate of $\hat{n}.S$ with negative eigenvalue?

a. 1/2
b. 1/3
c. $cos^2(\theta/_2)$
[✓ d.] $sin^2(\theta/_2)$

$\begin{pmatrix}1\\0\end{pmatrix} = C_1(\quad) + C_2(\quad)$

$C_2 \Rightarrow (C_2)^2 - (1^{st}$ element ka $|\,|^2) \to$ ans

12. Consider a system of spin ½ .The eigenvalues of the operator $\hat{O} = A\,\hat{S}_y + B\,\hat{S}_z$ ,where A and B are constant, are

$O = A^2 S_y^2 + B^2 S_z^2$

a. 0 and $\pm 1$
b. $\pm 1$
[✓ c.] $\pm\frac{\hbar}{2}\sqrt{A^2+B^2}$
d. $\pm\frac{\hbar}{2}\sqrt{A+B}$

13. If a lowering operator defined as

$$a = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i\hat{p}}{\sqrt{2\hbar m\omega}}$$

and $f(x)$ is any function of x ,Then the value of $[a, f(x)]$ is

[✓ a.] $\sqrt{\frac{\hbar}{2m\omega}}f'(x)$ $[a, f(x)]$

[✓ b.] $-\sqrt{\frac{\hbar}{2m\omega}}f'(x)$

c. $\sqrt{\frac{\hbar}{2m\omega}}(x+f'(x))$

d. 0

14. A linear harmonic oscillator is in a state $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_0\rangle + \frac{i}{\sqrt{2}}|\phi_1\rangle$

Where $|\phi_0\rangle\ and\ |\phi_1\rangle$ are ground and first excited energy eigenstates respectively? Then expectation value of momentum in this state is

a. 0

b. $\sqrt{\hbar m\omega}$

[✓ c.] $\sqrt{\frac{\hbar m\omega}{2}}$

d. $\sqrt{\frac{\hbar m\omega}{4}}$

$$= \frac{1}{\sqrt{2}}\left[\cancelto{0}{\langle\phi_0|p|\phi_0\rangle} + \cancelto{0}{\langle\phi_1|p|\phi_1\rangle} + \langle\phi_0|p|\phi_1\rangle + \langle\phi_1|p|\phi_1\rangle.\right.$$

15. Consider a particle in a central potential. Given that $|l,m\rangle$ is an eigenstate of $L^2$ and $L_z$. If $\Delta L_x$ and $\Delta L_y$ represent root mean square deviation in $x$ and $y$ component of orbital angular momentum respectively. Then the value of $(\Delta L_x)^2 + (\Delta L_y)^2$ is

a. 0

[✓ b.] $\hbar^2[l(l+1) - m^2]$

c. $2\hbar^2[l(l+1) - m^2]$

d. $4\hbar^2[l(l+1) - m^2]$

16. A spin-less particle of mass m and charge q is constrained to move in a circle of radius R as shown in figure

Then allowed energy levels are given by

a. $\frac{\hbar^2}{2mR^2}l(l+1)$, with $l = 0,1,2,3$………

b. $\frac{\hbar^2\pi^2}{2mR^2}n(n+1)$, with $n = 1,2,3$………

[✓ c.] $\frac{\hbar^2 n^2}{2mR^2}$, with $n = 0, \pm1, \pm2, \pm3$………

d. $\frac{\hbar^2\pi^2n^2}{2mR^2}$, with $n = 1,2,3$………

17. A particle of mass constrained to move in one dimensional potential

$$V(x) = \begin{cases} 0, & if\ -\frac{L}{2} < x < +\frac{L}{2} \\ \infty, & otherwise \end{cases}$$

The period of time evolution of the state $|\psi(0)\rangle = \alpha|n\rangle + \beta|n+1\rangle$, where $\alpha, \beta$ are constant, is

a. $\frac{4mL^2}{3\hbar}$

b. $\frac{8mL^2}{3h}$

[✓ c.] $\frac{8mL^2}{(2n+1)h}$

d. $\frac{8mL^2}{(2n+1)\hbar}$

18. Let us consider a system whose Hamiltonian is

$$\hat{H} = k\frac{L_+L_-}{\hbar}$$

where $L_\pm$ are the raising and lowering operators for the z component of the orbital angular momentum and $k$ is constant of suitable dimension. At time $t=0$ the system is described by the following wave- function

$$\psi(\theta,\phi) = A\sin\theta\sin\phi$$

where $\theta, \phi$ are usual spherical polar co-ordinates. The expectation value of energy of the system at $t=0$, is

a. $2\hbar k$

b. $3\hbar k$

[✓ c.] $\hbar k$

d. $-\hbar k$

19. The dispersion for the 2p state of hydrogen atom is

find $\sqrt{\langle r^2\rangle - \langle r\rangle^2}$ for 2P state.

a. 4 $a_0$

[✓ b.] $\sqrt{5}$ $a_0$

c. 5 $a_0$

d. 0

20. A particle is in the ground state of the harmonic oscillator with frequency $\omega$, when spring constant quadruples suddenly. What is the probability that a measurement of energy would give the value $\frac{\hbar\omega}{2}$?

$k' = 4k$

$\omega' = \sqrt{\frac{k'}{m}} = \sqrt{\frac{4k}{m}} = 2\omega$

$A^+$ $A^-$

$A$

a. 1

b. $\frac{1}{2}$

c. $\frac{2\sqrt{2}}{3}$

[✓ d.] 0

$E_n' = (n+\frac{1}{2})\hbar\omega'$

$= (n+\frac{1}{2})\,\cancel{2}\,\omega\hbar$

$= \hbar\omega$

$=$

Ground state $n = 0$

## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-1: QUANTUM MECHANICS

### LEVEL-1

1. Which of the following is *not* a true statement about the Compton Effect (photon scattering off a particle initially at rest)?

(a) Wavelength of a scattered photon is greater than of the incident one. ✓

(b) Effect cannot be explained within a purely classical wave picture of electro-magnetic radiation.

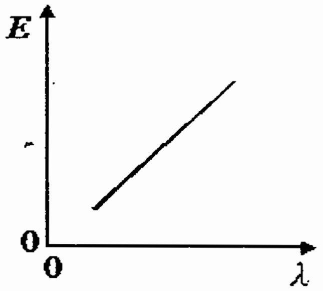
(c) Effect is weaker for scattering off protons than off electrons.

[✓ (d)] Effect is most pronounced in the visible range of photon energies.

(e) Compton shift does not depend on the wavelength of the incident photon.

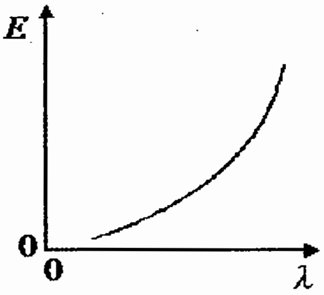
2. Which of the following graphs best shows how photon energy $E$ varies with the wavelength ë of the light?

(a)

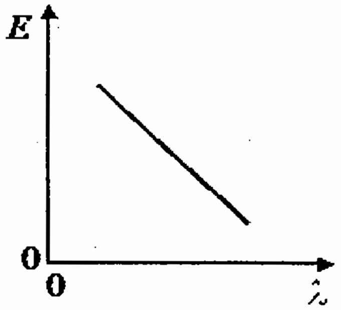


✓

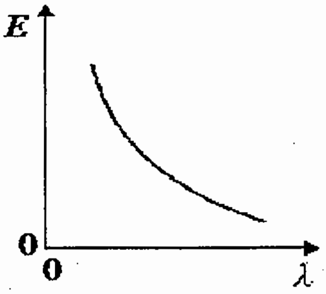
(b)



(c)



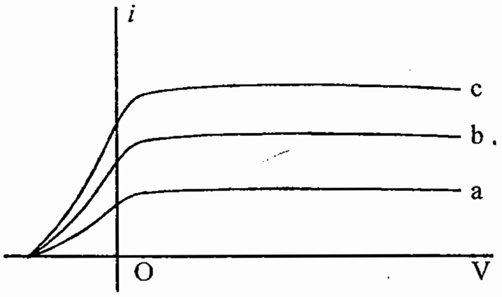
[✓ (d)]



3. The ratio of the de Broglie wavelengths of two electrons moving with speeds of 0.8c and 0.6c respectively is

(a) $\left(\frac{3}{4}\right)^2$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$

4. Three sets of photoemission measurements, a, b, and c, are performed for the same metal surface, electric current vs applied potential difference, i(V). The following statements about the intensity I of the incident radiation and its wavelength *l* are made.



[✓ (1)] $I_b > I_a$, $\lambda_a = \lambda_b$ (2) $I_b = I_a$, $\lambda_a > \lambda_b$

(3) $I_a = I_b$, $\lambda_a < \lambda_b$ (4) $\lambda_a < \lambda_b < \lambda_c$

(5) $\lambda_c < \lambda_a = \lambda_b$ (6) $\lambda_c > \lambda_a = \lambda_b$

5. X-rays of wavelength 10 pm are scattered from the target. The maximum wavelength that are present in the scattered x-rays, is

(a) 13.2 pm [✓ (b) 14.9 pm] (c) 15.7 pm (d) 16.9 pm

### LEVEL-2

6. A particle of mass m is confined in an infinite potential box of length $a$ and the corresponding ground state wave function inside the well is given by ✓

$$\psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

The probability of finding the particle in the interval $0 \le x < \frac{a}{3}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ [✓ (c) $\frac{1}{3} - \frac{\sqrt{3}}{4\pi}$] (d) $\frac{1}{3} - \frac{1}{4\pi}$

(e) $\frac{1}{3} - \frac{1}{2\pi}$ (f) none of the above.

7. A particle is confined to a region $0 \le x \le a$ and has a wave function of the form,

$\Psi(x) = Nx(a - x)$, where $N$ is a constant,

The standard deviation in momentum is

(a) 0 [✓ (b) $\frac{10\hbar^2}{a^2}$] (c) $\frac{\sqrt{5}\hbar}{a}$ (d) $\frac{5\hbar^2}{a^2}$

8. The phase velocity of the de-broglie waves of electron having kinetic energy 500 keV is

[✓ (a) 1.16c] [✓ (b) 1.21c] (c) 1.39c (d) 1.52c

9. A particle is represented by the wave function $\psi(x) = e^{-|x|}\sin\beta x$. The probability that the position of the particle will to the right of the point x = 1 is

(a) $\frac{e^{-2}}{2\beta^2}\left[1+\beta^2-\cos 2\beta+\beta\sin 2\beta\right]$ (b) $\frac{e^{-2}}{4\left(1-\beta^2\right)}\left[1-\beta^2-\cos 2\beta+\beta\sin 2\beta\right]$

(c) $\frac{e^{-2}}{4\beta^2}\left[1-\beta^2-\sin 2\beta+\beta\cos 2\beta\right]$ (d) $\frac{e^{-2}}{4\left(1+\beta^2\right)}\left[1+\beta^2-\sin 2\beta+\beta\cos 2\beta\right]$

10. The state of a free particle is described by the following wave function

$$\begin{aligned}\psi(x) &= 0 \quad \text{for } x < -3a\\ &= c \quad \text{for } -3a < x < a\\ &= 0 \quad \text{for } x > a\end{aligned}$$

The variance in position is given by

(a) $a^2$ (b) $\frac{7a^2}{3}$ (c) $\frac{4a^2}{3}$ (d) $\sqrt{\frac{4}{3}}a$ => for uncertainty condition, it is Right

**LEVEL-3**

11. Consider a photon that scatters at an angle 60° from an electron at rest. If the Compton shift is observed to be three times of the incident wavelength of the incident photon, find the wavelength of the incident photon.

12. Normalize the wave function $\psi(x) = e^{-|x|}\sin(\alpha x)$.

13. If $v_g$ and $v_p$ are the group and phase velocities in a medium of refractive index *n*. Show that the relation between group and phase velocities are given by $v_g = v_p\left(1+\frac{\lambda}{n}\frac{dn}{d\lambda}\right)$.

14. The wave function of a particle at a certain instant is $\psi(x) = \frac{A}{\sqrt{x^2+a^2}}e^{ikx}$. Find the real positive value of $A$ so that $\psi(x)$ is normalized to unity.

15. The wave function of a particle at $t = 0$ is given by

$$\psi(x) = \begin{cases} A\dfrac{x}{a}, & \text{if } 0 \le x \le a \\ A\dfrac{(b-x)}{(b-a)}, & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Find the probability of finding the particle left to $x = a$.

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## CSIR-UGC-NET/JRF|GATE PHYSICS

## Assignment-1: QUANTUM MECHANICS

### LEVEL-1

1. Which of the following is *not* a true statement about the Compton Effect (photon scattering off a particle initially at rest)?

(a) Wavelength of a scattered photon is greater than of the incident one.

(b) Effect cannot be explained within a purely classical wave picture of electro-magnetic radiation.

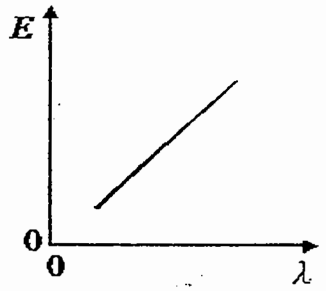
(c) Effect is weaker for scattering off protons than off electrons.

(d) Effect is most pronounced in the visible range of photon energies.

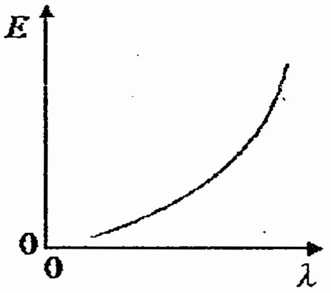
(e) Compton shift does not depend on the wavelength of the incident photon.

2. Which of the following graphs best shows how photon energy $E$ varies with the wavelength ë of the light?

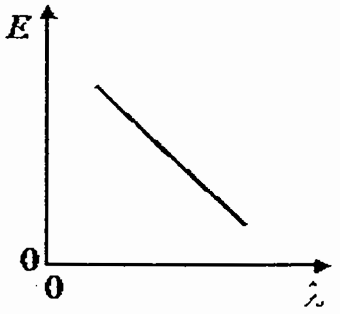
(a)



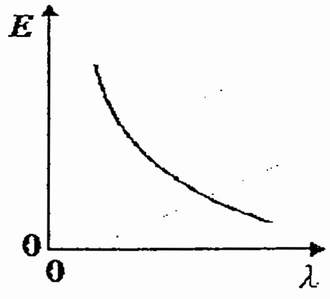
(b)



(c)



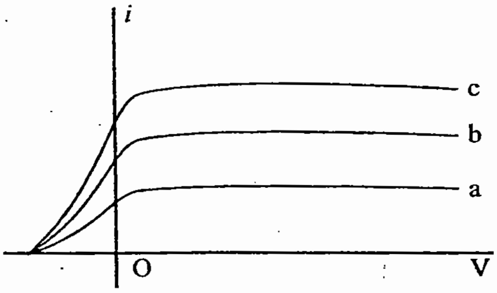
(d)



3. The ratio of the de Broglie wavelengths of two electrons moving with speeds of 0.8c and 0.6c respectively is

(a) $\left(\frac{3}{4}\right)^2$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$

4. Three sets of photoemission measurements, a, b, and c, are performed for the same metal surface, electric current vs applied potential difference, i(V). The following statements about the intensity I of the incident radiation and its wavelength *l* are made.



(1) $I_b > I_a$, $\lambda_a = \lambda_b$ (2) $I_b = I_a$, $\lambda_a > \lambda_b$

(3) $I_a = I_b$, $\lambda_a < \lambda_b$ (4) $\lambda_a < \lambda_b < \lambda_c$

(5) $\lambda_c < \lambda_a = \lambda_b$ (6) $\lambda_c > \lambda_a = \lambda_b$

5. An atomic state of hydrogen is represented by the following wave function:

$$\psi(r,\theta,\phi)=\frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where $a_0$ is a constant. The quantum number of the state are

(a) $\ell=0, m=0, n=1$ (b) $\ell=1, m=1, n=2$ (c) $\ell=1, m=0, n=2$ (d) $\ell=2, m=0, n=3$

## LEVEL-2

6. A particle of mass m is confined in an infinite potential box of length *a* and the corresponding ground state wave function inside the well is given by

$$\psi(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

The probability of finding the particle in the interval $0 \le x < \frac{a}{3}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{3}-\frac{\sqrt{3}}{4\pi}$ (d) $\frac{1}{3}-\frac{1}{4\pi}$

(e) $\frac{1}{3}-\frac{1}{2\pi}$ (f) none of the above.

7. A particle is confined to a region $0 \le x \le a$ and has a wave function of the form,

$\Psi(x)=Nx(a-x)$, where $N$ is a constant,

The standard deviation in momentum is

(a) 0 (b) $\frac{10\hbar^2}{a^2}$ (c) $\frac{\sqrt{5}\hbar}{a}$ (d) $\frac{5\hbar^2}{a^2}$

8. For a spin 1/2 particle, the expectation value of $s_x s_y s_z$. where $s_x, s_y$ and $s_z$ are spin operators, is

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$ (c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

9. A particle is in the normalized state $|\psi\rangle$ which is superposition of the energy eigenstates $|E_0 = 10\,\text{eV}\rangle$ and $|E_1 = 30\,\text{eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV. The state $|\psi\rangle$ is given by

(a) $\frac{1}{2}|E_0 = 10\,\text{eV}\rangle + \frac{\sqrt{3}}{4}|E_1 = 30\,\text{eV}\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0 = 10\,\text{eV}\rangle + \sqrt{\frac{2}{3}}|E_1 = 30\,\text{eV}\rangle$

(c) $\frac{1}{2}|E_0 = 10\,\text{eV}\rangle - \frac{\sqrt{3}}{4}|E_1 = 30\,\text{eV}\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0 = 10\,\text{eV}\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\,\text{eV}\rangle$

10. The state of a free particle is described by the following wave function

$$\begin{aligned}\psi(x) &= 0 \quad \text{for } x < -3a\\ &= c \quad \text{for } -3a < x < a\\ &= 0 \quad \text{for } x > a\end{aligned}$$

The variance in position is given by

(a) $a^2$ (b) $\frac{7a^2}{3}$ (c) $\frac{4a^2}{3}$ (d) $\sqrt{\frac{4}{3}}a$

## LEVEL-3

11. Consider a photon that scatters at an angle $60^0$ from an electron at rest. If the Compton shift is observed to be three times of the incident wavelength of the incident photon, find the wavelength of the incident photon.

12. Normalize the wave function $\psi(x) = e^{-|x|}\sin(\alpha x)$.

13. If $v_g$ and $v_p$ are the group and phase velocities in a medium of refractive index $n$. Show that the relation between group and phase velocities are given by $v_g = v_p\left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$.

14. The wave function of a particle at a certain instant is $\psi(x) = \frac{A}{\sqrt{x^2 + a^2}}e^{ikx}$. Find the real positive value of $A$ so that $\psi(x)$ is normalized to unity.

15. The wave function of a particle at $t = 0$ is given by

$$\psi(x) = \begin{cases} A\frac{x}{a}, \text{ if } 0 \le x \le a \\ A\frac{(b-x)}{(b-a)}, \text{ if } 0 \le x \le a \\ 0\ , \quad \text{otherwise} \end{cases}$$

Find the probability of finding the particle left to $x = a$.

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**CSIR-UGC-NET/JRF | GATE-PHYSICAL SCIENCES**

**Test-2: Quantum Mechanics**

**Date: 16-10-2014**

**Time: 01 : 00 Hour** **Max. Marks: 85**

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**Instructions:-**

- There are two parts, part **A** and part **B**.
- Part A contains **10** objective type questions. Each question carries 3.5 marks.
- Part B contains **10** objective type questions. Each question carries **5** marks.
- There will be **negative** marking in both parts at the rate of **25%**.

## Part A

1. The number of energy states for a cubical box of side $a$ in energy range $0 < E < 15\pi^2\hbar^2 / 2ma^2$ is

a. 6 b. 11 c. 15 d. 17

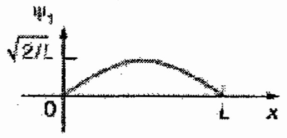
2. A mesic atom is formed by a muon of mass 207 times the electron mass, charge $-e$ and hydrogen nucleus, the radius of first orbit of mesic atom is

a. 283 fm b. 0.51 $A^0$ c. 0.49 $A^0$ d. 347 fm

3. An operator $f$ describing the interaction of two spin-1/2 particles has the form $f = a + b\sigma_1 . \sigma_2$ where a and b are constant, $\sigma_1$ and $\sigma_2$ are Pauli spin operators. The total spin angular momentum is $S = \frac{\hbar}{2}(\sigma_1 + \sigma_2)$.

a. Only $S^2$ and $f$ have simultaneous eigen-function.
b. Only $S_z$ and $f$ have simultaneous eigen-function.
c. Only $S^2$ and $S_z$ have simultaneous eigen-function.
d. $S^2$, $S_z$ and $f$ have simultaneous eigen-function.

4.



The lowest energy state for a particle in a box looks like this. How many electrons can have a wave-function that looks like this?

a. 0
b. 1
c. 2
d. Any number

5. A particle in a central potential has an orbital angular momentum quantum number 2 $\hbar$ and spin quantum number $\hbar$. A spin –orbit interaction is represented by Hamiltonian $H = A\,\mathbf{L}.\mathbf{S}$ .The energy levels are given by

a. $2A\hbar^2, 0, -2A\hbar^2$
b. $2A\hbar, -A\hbar, -3A\hbar$
c. $2A\hbar^2, A\hbar^2, -3A\hbar^2$
d. $2A\hbar^2, -A\hbar^2, -3A\hbar^2$

6. A Spin $\frac{1}{2}$ particle is in a state described by the

$$A\binom{1+i}{2}$$

where A is normalization constant, the probability of finding the particle with projection $S_z = -\frac{\hbar}{2}$ is

a) $\frac{1}{6}$
b) $\frac{1}{2}$
c. $\frac{1}{3}$
d. $\frac{2}{3}$

7. $\hat{a}$ and $\hat{a}^\dagger$ are lowering and raising operators defined as

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} + \frac{i\hat{p}}{\sqrt{2m\hbar\omega}},$$

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} - \frac{i\hat{p}}{\sqrt{2m\hbar\omega}}$$

The expectation value of $\hat{a}\hat{a}^\dagger$ for the state $|n\rangle$ is

a. 0
b. n+1
c. n
d. n-1

8. A quantum particle of mass m moves in two dimensions in an anisotropic harmonic oscillator potential $V(x,y) = \frac{1}{2}m\omega^2x^2 + 8m\omega^2y^2$.The enrgy eigenvalues are (n =0,1,2,3,4........)

a. $(n+1)\hbar\omega$
b. $\left(2n+\frac{3}{2}\right)\hbar\omega$
c. $(n+\frac{3}{2})\hbar\omega$
d. $(n+\frac{5}{2})\hbar\omega$

9. Consider a particle of mass m moving in one-dimensional potential that consists of two Dirac-delta functions: $V(x) = -V_0[\delta(x-a) + \delta(x+a)]$ , where $V_0$ and $a$ are positive constants, $\beta^2 = -\frac{2mE}{\hbar^2}$. The wave function of the particle in the region $-a < x < +a$ is

a. $Ae^{-\beta x}$
b. $Ae^{+\beta x}$
c. $Ae^{\pm i\beta x}$
d. $A(e^{\beta x} \pm e^{-\beta x})$

10. At time t=0 , the wave function for Hydrogen atom is

$$\psi(r,0) = \frac{1}{\sqrt{10}}(\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1})$$

where the subscripts are values if the quantum numbers n,*l*,m. Ignore spin and radiative transitions. The expectation values of the energy of the system is

a. 0
b. -3.40 eV
c. -7.47 eV
d. -34.0 eV

## Part B

11. An electron is described by a Hamiltonian that does not depend on spin. The electron's spin wave function is an eigenstate of $S_z$ with eigenvalue $+\hbar/2$. The operator $\hat{\boldsymbol{n}}.\boldsymbol{S}$ represents the spin projection along a direction $\hat{\boldsymbol{n}}$. We can express this direction as

$$\hat{\boldsymbol{n}} = \sin\theta(\cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}}) + \cos\,\hat{\mathbf{z}}.$$

What is the probability of finding the electron in eigenstate of $\hat{\boldsymbol{n}}.\boldsymbol{S}$ with negative eigenvalue?

a. 1/2
b. 1/3
c. $cos^2(\theta/2)$
d. $sin^2(\theta/2)$

12. Consider a system of spin ½ .The eigenvalues of the operator $\hat{O} = A\,\hat{S}_y + B\,\hat{S}_z$ ,where A and B are constant, are

a. 0 and $\pm 1$
b. $\pm 1$
c. $\pm\frac{\hbar}{2}\sqrt{A^2+B^2}$
d. $\pm\frac{\hbar}{2}\sqrt{A+B}$

13. If a lowering operator defined as

$$a = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i\hat{p}}{\sqrt{2\hbar m\omega}}$$

and $f(x)$ is any function of x ,Then the value of $[a, f(x)]$ is

a) $\sqrt{\frac{\hbar}{2m\omega}} f'(x)$

b) $-\sqrt{\frac{\hbar}{2m\omega}} f'(x)$

c. $\sqrt{\frac{\hbar}{2m\omega}} (x + f'(x))$

d. 0

14. A linear harmonic oscillator is in a state $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_0\rangle + \frac{i}{\sqrt{2}}|\phi_1\rangle$

Where $|\phi_0\rangle$ *and* $|\phi_1\rangle$ are ground and first excited energy eigenstates respectively? Then expectation value of momentum in this state is

a. 0

b. $\sqrt{\hbar m\omega}$

c. $\sqrt{\frac{\hbar m\omega}{2}}$

d. $\sqrt{\frac{\hbar m\omega}{4}}$

15. Consider a particle in a central potential. Given that $|l, m\rangle$ is an eigenstate of $L^2$ and $L_z$. If $\Delta L_x$ and $\Delta L_y$ represent root mean square deviation in $x$ and $y$ component of orbital angular momentum respectively. Then the value of $(\Delta L_x)^2 + (\Delta L_y)^2$ is

a. 0

b. $\hbar^2[l(l+1) - m^2]$

c. $2\hbar^2[l(l+1) - m^2]$

d. $4\hbar^2[l(l+1) - m^2]$

16. A spin-less particle of mass m and charge q is constrained to move in a circle of radius R as shown in figure

Then allowed energy levels are given by

a. $\frac{\hbar^2}{2mR^2} l(l+1)$, with $l = 0,1,2,3 \dots\dots\dots$

b. $\frac{\hbar^2\pi^2}{2mR^2} n(n+1)$, with $n = 1,2,3 \dots\dots\dots$

c. $\frac{\hbar^2 n^2}{2mR^2}$, with $n = 0, \pm1, \pm2, \pm3 \dots\dots\dots$

d. $\frac{\hbar^2\pi^2 n^2}{2mR^2}$, with $n = 1,2,3 \dots\dots\dots$

17. A particle of mass constrained to move in one dimensional potential

$$V(x) = \begin{cases} 0, \ if -\frac{L}{2} < x < +\frac{L}{2} \\ \infty, \quad otherwise \end{cases}$$

The period of time evolution of the state $|\psi(0)\rangle = \alpha|n\rangle + \beta|n+1\rangle$, where $\alpha, \beta$ are constant, is

a. $\frac{4mL^2}{3\hbar}$

b. $\frac{8mL^2}{3h}$

c. $\frac{8mL^2}{(2n+1)h}$

d. $\frac{8mL^2}{(2n+1)\hbar}$

18. Let us consider a system whose Hamiltonian is

$$\hat{H} = k\frac{L_+L_-}{\hbar}$$

where $L_\pm$ are the raising and lowering operators for the z component of the orbital angular momentum and $k$ is constant of suitable dimension. At time $t=0$ the system is described by the following wave- function

$$\psi(\theta,\phi) = A\sin\theta\sin\phi$$

where $\theta, \phi$ are usual spherical polar co-ordinates. The expectation value of energy of the system at $t=0$, is

a. $2\hbar k$

b. $3\hbar k$

c. $\hbar k$

d. $-\hbar k$

19. The dispersion for the 2p state of hydrogen atom is

a. 4 $a_0$

b. $\sqrt{5}$ $a_0$

c. 5 $a_0$

d. 0

20. A particle is in the ground state of the harmonic oscillator with frequency $\omega$, when spring constant quadruples suddenly. What is the probability that a measurement of energy would give the value $\frac{\hbar\omega}{2}$?

a. 1

b. $\frac{1}{2}$

c. $\frac{2\sqrt{2}}{3}$

d. 0

# CSIR-UGC-NET/JRF | GATE

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# Assignment

# Mathematical Physics

## PHYSICAL SCIENCES

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**South Delhi Centre:**
28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T: 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## CSIR-UGC-NET/JRF | GATE Physics

## MATHEMATICAL PHYSICS

### Assignment I: Vector Alzebra & Vector Analysis

1. $\nabla^2\left(\frac{1}{r}\right)$ is:

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$

2. $\iiint \nabla^2\left(\frac{1}{r}\right)dV \quad r \neq 0$ is:

(a) 0 (b) $-4\pi$ (c) $4\pi$ (d) 1

3. $\text{div}\left(\hat{i}+\hat{j}+\hat{k}\right)$ is

(a) 3 (b) 2 (c) 1 (d) 0

4. The value of t for which three vevtors $[(1-t),0,0],[1,(1-t),0]$ & $[1,1,(1-t)]$ are linearly dependent is

(a) 1 (b) 0 (c) 2 (d) −1

5. For what value of n vector $r^n\vec{r}$ is solenoidal.

(a) 0 (b) 1 (c) −1 (d) −3

6. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by

(a) $\frac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{a^2}$ (b) $\frac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{b^2}$ (c) $\frac{\left(\vec{a}.\vec{b}\right)\vec{b}}{b^2}$ (d) $\frac{\left(\vec{b}.\vec{a}\right)\vec{a}}{a^2}$

7. A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z axes, is:

(a) $L^3$ (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

8. If $\vec{r} = x\hat{i} + y\hat{j}$, then:

(a) $\vec{\nabla}\bullet\vec{r} = 0$ and $\vec{\nabla}|\vec{r}| = \vec{r}$ (b) $\vec{\nabla}\bullet\vec{r} = 2$ *and* $\vec{\nabla}|\vec{r}| = \vec{r}$

(c) $\vec{\nabla}\bullet\vec{r} = 2$ *and* $\vec{\nabla}|\vec{r}| = \frac{\vec{r}}{r}$ (d) $\vec{\nabla}\bullet\vec{r} = 3$ *and* $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

9. Cosnider a vector $\vec{p} = 2\hat{i}+3\hat{j}+2\hat{k}$ in the coordinate system $\left(\hat{i},\hat{j},\hat{k}\right)$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $\left(\hat{i}',\hat{j}',\hat{k}'\right)$ is:

(a) $\left(1-\sqrt{3}\right)\hat{i}'+3\hat{j}'+\left(1+\sqrt{3}\right)\hat{k}'$ (b) $\left(1+\sqrt{3}\right)\hat{i}'+3\hat{j}'+\left(1-\sqrt{3}\right)\hat{k}'$

(c) $\left(1-\sqrt{3}\right)\hat{i}'+\left(3+\sqrt{3}\right)\hat{j}'+2\hat{k}'$ (d) $\left(1-\sqrt{3}\right)\hat{i}'+\left(3-\sqrt{3}\right)\hat{j}'+2\hat{k}'$

10. The value of $\oint \vec{A}\cdot\vec{d\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is:

(a) 0 (b) 2LA (c) 4LA (d) $L^2A$

11. The value of $\oint\limits_S \dfrac{\vec{r}\cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is:

(a) 0 (b) $\pi$ (c) $4\pi$ (d) $8\pi$

12. If a vector field $\vec{F} = x\hat{i}+2y\hat{j}+3z\hat{k}$, then $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)$ is

(a) 0 (b) $\hat{i}$ (b) $2\hat{j}$ (d) $3\hat{k}$

13. Given the four vectors, $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}, u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}, u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$, the linearly dependent pair is:

(a) $u_1, u_2$ (b) $u_1, u_3$ (c) $u_1, u_4$ (d) $u_3, u_4$

14. The unit normal to the curve $x^3y^2+xy=17$ at the point (2, 0) is:

(a) $\dfrac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

15. For the function $\phi = x^2y+xy$ the value of $\left|\vec{\nabla}\phi\right|$ at $x=y=1$ is

(a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

**Common data for Q.16 and Q.17.**

Consider the vector $\vec{V} = \dfrac{\vec{r}}{r^3}$

16. The surface integral of this vector over the surface of a cube of size and centered at the origin.
(a) 0 (b) $2\pi$ (c) $2\pi a^3$ (d) $4\pi$

17. Which one of the following is NOT correct?
(a) Value of the line integral of this vector around any closed curve is zero
(b) This vector can be written as the gradient of some scalar function
(c) The line integral of this vector from point P to point Q is independent of the path taken.
(d) This vector can represent the magnetic field of some current distribution

18. The curl of the vector $\vec{A} = z\,\hat{i}+x\,\hat{j}+y\,\hat{k}$ is given by:

(a) $\hat{i}+\hat{j}+\hat{k}$ (b) $\hat{i}-\hat{j}+\hat{k}$ (c) $\hat{i}+\hat{j}-\hat{k}$ (d) $-\hat{i}-\hat{j}-\hat{k}$

19. The two vectors $\vec{P} = \hat{i}, \vec{q} = (\hat{i}+\hat{j})\ /\ \sqrt{2}$ are:
(a) related by a rotation
(b) related by a reflection through the xy-plane
(c) related by an inversion
(d) not linearly independent

20. A vector $\vec{A} = (5x+2y)\hat{i} + (3y-z)\hat{j} + (2x-az)\hat{k}$ is solenoidal if the constant a has a value:
(a) 4
(b) –4
(c) 8
(d) –8

21. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$, where a and b (a≠b) are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors?
(a) $-b\hat{i}+a\hat{j}$
(b) $-a\hat{i}+b\hat{j}$
(c) $-a\hat{i}-b\hat{j}$
(d) $-b\hat{i}-a\hat{j}$

22. The unit vector normal to the surface $3x^2 + 4y = z$ at the ponit (1, 1, 7) is:
(a) $\left(-6\hat{i}+4\hat{j}+\hat{k}\right)/\sqrt{53}$
(b) $\left(4\hat{i}+6\hat{j}-\hat{k}\right)/\sqrt{53}$
(c) $\left(6\hat{i}+4\hat{j}-\hat{k}\right)/\sqrt{53}$
(d) $\left(4\hat{i}+6\hat{j}+\hat{k}\right)/\sqrt{53}$

23. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the positive vector, then the value of the following integral $\iint_S \vec{r}.\hat{n}dS$ is:
(a) V
(b) 2V
(c) 0
(d) 3V

24. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$
(a) The three vectors are orthonormal
(b) The three vectors are linearly independent
(c) The three vectors cannot form a basis in a three-dimensional real vector space.
(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$.

25. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?
(a) $\frac{d\vec{A}}{dt} = 0$
(b) $\frac{d^2\vec{A}}{dt^2} = 0$
(c) $\frac{d\vec{A}}{dt} \bullet \vec{A} = 0$
(d) $\frac{d\vec{A}}{dt} \times \vec{A} = 0$

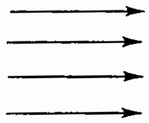
26. The function $r^n\ \hat{r}$ $(r > 0)$, where r refers to spherical coordinate system, is:
(a) an irrotational vector for $n = -2$
(b) a solenoidal vector for $n = -2$
(c) an irrotational vector for $n = -3$
(d) a solenoidal vector for $n = -3$

27. Two sets of vectors, one with m elements and the other with n elements $(m < n)$ span the same linear vector space. If k is the dimension of the vector space, then:
(a) $k \leq m$
(b) $k \geq m$
(c) $k = m + n$
(d) $k = n$

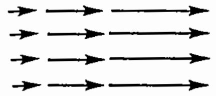
28. Given $\phi = x^2 - yz$, $\nabla\phi$ at (3, 4, 1) will be:
(a) $6\hat{i}-4\hat{j}+\hat{k}$
(b) $6\hat{i}-\hat{j}+4\hat{k}$
(c) $4\hat{i}-6\hat{j}-\hat{k}$
(d) $6\hat{i}-3\hat{j}-2\hat{k}$

29. Identify the vector field given below which has a finite curl

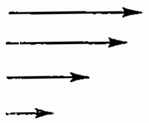
(a)



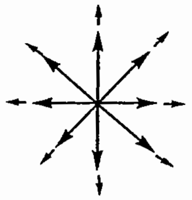
(b)



(c)



(d)



30. If $\vec{A}$ and $\vec{B}$ are two unit vectors and $\theta \neq 0$ is the angle between them, then

(a) $\sin\theta = \frac{1}{2}\left|\vec{A}+\vec{B}\right|$ (b) $\sin\theta = \frac{1}{2}\left|\vec{A}-\vec{B}\right|$

(c) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{A}-\vec{B}\right|$ (d) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{A}+\vec{B}\right|$

31. Given any three non-zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$, their triple product $\vec{A}.\left(\vec{B}\times\vec{C}\right)$ vanishes if

(a) They are perpendicular to each other. (b) Any two of them are perpendicular

(c) Any two of them are parallel. (d) They are non-coplanar.

32. The unit vector normal to the surface $x^2 + 4y = z$ at the point (1, 1, 0) is:

(a) $\frac{1}{\sqrt{21}}\left(2\hat{i}+4\hat{j}-\hat{k}\right)$ (b) $\frac{1}{\sqrt{21}}\left(-2\hat{i}+4\hat{j}-\hat{k}\right)$ (c) $\frac{1}{\sqrt{21}}\left(2\hat{i}-4\hat{j}-\hat{k}\right)$ (d) $\frac{1}{\sqrt{21}}\left(2\hat{i}+4\hat{j}+\hat{k}\right)$

33. The necessary and sufficient condition that $\oint_c \vec{A}.d\vec{r} = 0$, for any closed curve C is

(a) $\vec{\nabla}.\vec{A}=0$ (b) $\vec{\nabla}\times\vec{A}=0$ (c) $\vec{\nabla}.\vec{\nabla}\times\vec{A}=0$ (d) $\vec{\nabla}\times\vec{\nabla}\times\vec{A}=0$

34. Any arbitrary vector in a three dimensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors:

(a) Arbitrary number (b) 1 (c) 2 (d) 3

35. Which one of the following vectors is normal to the surface $x^2y + z^3 + xz^2 = 10$?

(a) $x^2y\hat{i} + z^3\hat{j} + xz^2\hat{k}$ (b) $2xy\hat{i} + 2xz\hat{k}$

(c) $2y\hat{i} + \left(6z+2x\right)\hat{k}$ (d) $\left(2xy+z^2\right)\hat{i} + x^2\hat{j} + \left(3z^2+2xz\right)\hat{k}$

36. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to

(a) Curl of a vector function (b) Gradient of a scalar function

(c) Gradient of a vector function (d) Zero

37. When the fluid is incompressible, the equation of continuity can be reduced to $\frac{\partial \rho}{\partial t} = 0 = \rho\left(\vec{\nabla}.\vec{v}\right)$. Since the density $\rho$ is constant in this case. Here $\vec{v}$ is the velocity of a typical particle of the fluid. Further, if the flow is irrotational, then the equation can be rewritten as

(a) $\vec{\nabla}.\vec{\nabla}\phi = 0$ (b) $\vec{\nabla}\times\vec{\nabla}\phi = 0$ (c) $\vec{\nabla}\phi = 0$ (d) $\vec{v} = \text{constant}$

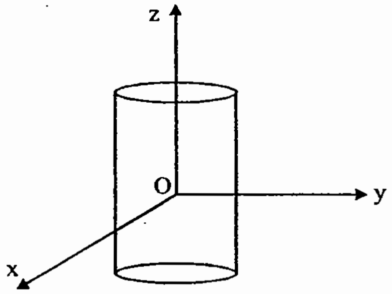
38. The vector perpendicular to $3\hat{i}+4\hat{j}-5\hat{k}$ is:

(a) $-2\hat{i}+4\hat{j}-2\hat{k}$ (b) $2\hat{i}+4\hat{j}-2\hat{k}$ (c) $2\hat{i}-4\hat{j}+2\hat{k}$ (d) $2\hat{i}-4\hat{j}-2\hat{k}$

39. The unit vector normal to the surface $x^2+y^2-z=1$ at the point P(1, 1, 1) is:

(a) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

40. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x+\hat{j}y+\hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is:



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) Zero.

41. Given the vector $\vec{A}(y,-x,0)$, the line integral $\oint_c \vec{A}.\vec{dl}$, where C is a circle of radius 5 units with its centre at the origin, (correct to the first decimal place) is

(a) 172.8 (b) 157.1 (c) −146.3 (d) 62.8

42. The value of the integral I = $\int_s \vec{r}.d\vec{s}$ where S is the surface enclosing the volume V is

(a) 3 (b) V (c) 3V (d) 0

43. The cross product of $\left(\hat{i}+\hat{j}\right)\times\left(\hat{j}+\hat{k}\right)$ where $\hat{i}, \hat{j}, \hat{k}$ are orthonormal unit vectors in 3-dimensions, is

(a) $\hat{i}-\hat{k}$ (b) $\hat{i}+\hat{k}$ (c) $\hat{i}+\hat{j}+\hat{k}$ (d) $\hat{k}-\hat{j}+\hat{i}$

44. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$

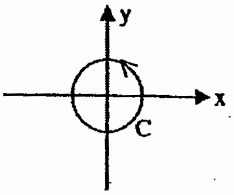
(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

45. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0\left(\hat{i} + \hat{j} + k\right)$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is,



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

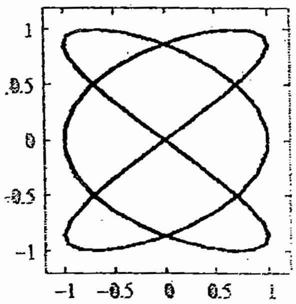
46. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is

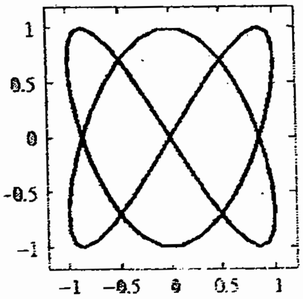
(a) $\vec{i} + \vec{j}$ (b) $\vec{j} + \vec{k}$ (c) $\vec{i} + \vec{j} + \vec{k}$ (d) $2\vec{i} + 3\vec{j} + 5\vec{k}$

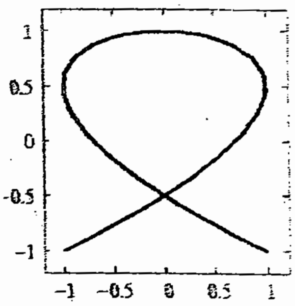
47. A two-dimensional vector

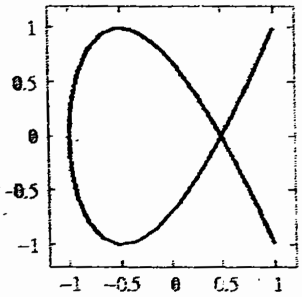
$$\vec{A}(r) = \hat{i}\sin 2t + \hat{j}\cos 3t$$

Which of the following graphs best describes the locus of the tip of the vector, as 't' is varied from 0 to $2\pi$?

(a)


(b)


(c)


(d)


48. Consider the surface corresponding to the equation

$$4x^2 + y^2 + z = 0$$

A possible unit tangent to this surface at the point $(1, 2, -8)$ is

(a) $\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$ (b) $\frac{1}{5}\hat{j} - \frac{4}{5}\hat{k}$ (c) $\frac{4}{9}\hat{i} - \frac{8}{9}\hat{j} + \frac{1}{9}\hat{k}$

49. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, is

(a) $\frac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$ (b) $\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (c) $\frac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (d) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

50. If $A = \hat{i}yz + \hat{j}xz + \hat{k}xy$, then the integral $\oint_C A.dl$ (where $C$ is along the perimeter of a rectangular area bounded by $x = 0, x = a$ and $y = 0, y = b$) is

(a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$ (c) $\pi(a^3 + b^3)$ (d) 0

51. If $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by $z = 1$, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.\vec{d\ell}$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{\pi}{4}$ (d) 0

## Vector Alzebra and Vector Analysis_Answer Key

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (c)** | **2. (a)** | **3. (d)** | **4. (a)** | **5. (d)** | **6. (a)** | **7. (a)** |
| **8. (c)** | **9. (a)** | **10. (a)** | **11. (c)** | **12. (a)** | **13. (d)** | **14. (d)** |
| **15. (d)** | **16. (d)** | **17. (d)** | **18. (a)** | **19. (a)** | **20. (c)** | **21. (a)** |
| **22. (c)** | **23. (d)** | **24. (b)** | **25. (c)** | **26. (b)** | **27. (b)** | **28. (b)** |
| **29. (c)** | **30. (c)** | **31. (c)** | **32. (a)** | **33. (b)** | **34. (d)** | **35. (d)** |
| **36. (b)** | **37. (b)** | **38. (d)** | **39. (d)** | **40. (b)** | **41. (b)** | **42. (c)** |
| **43. (d)** | **44. (a)** | **45. (c)** | **46. (c)** | **47. (b)** | **48. (a)** | **49. (a)** |
| **50. (d)** | **51. (d)** | | | | | |

## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS

### Assignment II: Matrix Alzebra

1. The inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ is :

(a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) does not exist.

2. Which one of the following is not hermitian?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$

3. Eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(a) 2, 1, 0 (b) 1, 1, 1 (c) 3, 0, 0 (d) 1, 2, 1

4. The eigenvalues of a matrix are i, -2i and 3i. The matrix is:
(a) unitary (b) anti-unitary (c) hermitian (d) anti-hermitian

5. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are:

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

6. The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigenvalue is:
(a) $-1$ (b) 0 (c) 1 (d) 2

7. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\bar{x}$ from a three-dimensional real space to a two-dimensional real space. The transformation matrix T is:

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

**Common data for Q. 8 and Q. 9:**

One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5.

8. The other two eigen values are:
(a) 0 and 0 (b) 1 and 1 (c) 1 and –1 (d) –1 and –1

9. The normalized eigen vector corresponding to the eigen value 5 is:
(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

10. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are:
(a) –2, –1, 1, 2 (b) –1, 1, 0, 2 (c) 1, 0, 2, 3 (d) –1, 1, 0, 3

11. The determinant of a $3 \times 3$ real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvlaue is:
(a) 4 (b) 6 (c) 8 (d) 9

12. A real traceless $4 \times 4$ unitary matrix has two eigen values - 1 and 1 . The other eigenvalues are:
(a) zero and +2 (b) –1 and +1 (c) zero and +1 (d) +1 and +1

13. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are:
(a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) –1 and +1

14. A $3 \times 3$ matrix has eigenvalues $0, 2 + i$ and $2 - i$. Which one of the following statements is correct?
(a) The matrix is Hermitian (b) The matrix is unitary
(c) The inverse of the matrix exists (d) The determinant of the matrix is zero

15. If two matrices A and B can be diagonalized simultaneously, which of the following is true?
(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$

16. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?
(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

17. For any operator A, $i(A^+ - A)$ is:
(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

18. A square matrix A is unitary if:
(a) $A^\dagger = A$ (b) $A^\dagger = A^{-1}$ (c) $\mathrm{Tr}(A) = 1$ (d) $\det(A) = 1$

19. If A is an antisymmetric matrix and $A^T$ be the transpose of this matrix, then which one of the following relations does not hold good?

(a) $A^T = -A$
(b) $AA^T = A^TA$
(c) $A^2$ is an antisymmetric matrix
(d) $A^2$ is a symmetirc matrix

20. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are given by $\lambda_1$, $\lambda_2$ and $\lambda_3$. Which one of th following statement is NOT TRUE?

(a) $\lambda_1 + \lambda_2 + \lambda_3 = 0$
(b) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$
(c) All eigenvalues are real
(d) Product of the eigenvalues is 1

21. The matrix $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is:

(a) orthogonal
(b) hermitian
(c) symmetric
(d) anti-symmetric

22. A given (n×n) nilpotent matrix A satisfies the equation $A^k = 0$ for $1 < k < n$. Therefore,

(a) Exactly k eigenvalues of A must be zero.
(b) Exactly (n–k) eigenvalue of A must be zero.
(c) Every eigenvalue of A is zero.
(d) A can have (n–1) non-zero eigenvalues.

23. Consider the (n×n) matrix A with every element equal to unity. Which of the following statement is correct?

(a) The eigenvalues of A are all equal to unity.
(b) All the eigenvalues of A are zero.
(c) The largest eigenvalue of A is n.
(d) A cannot be diagonalised by a similarly transformation.

24. The eigenvalues of the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are

(a) 0, 0
(b) 1, 0
(c) 1, –1
(d) 1, 1

25. If L(x) is a linear differential operator and $y_1(x)$, $y_2(x)$ are two arbitrary functions. Then

(a) $L(x)(y_1 + y_2) = Ly_1 + Ly_2$
(b) $L(x)(y_1 + y_2) = y_1Ly_2 + y_2Ly_1$
(c) $L(x)(y_1y_2) = Ly_1 + Ly_2$
(d) $L(x)(y_1 - y_2) = Ly_1 + Ly_2$

26. For which of the following values of a, the inverse of the matrix, $M = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 3 & 5 \\ 0 & 4 & a \end{pmatrix}$, does not exist?

(a) –4
(b) 4
(c) 0
(d) 1

27. The eigenvalues of the matrix $M = \begin{pmatrix} 1 & 0 \\ 1 & i \end{pmatrix}$ are

(a) 1, 1
(b) 1, i
(c) 1, 0
(d) Cannot be determined.

28. Which one of the following matrices is orthogonal?

(a) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

29. The eigenvalues of a skew-Hermitian matrix are

(a) Real and positive (b) Real and negative

(c) Either zero or pure imaginary (d) Complex with nonzero real parts.

30. The number of independent real parameters of a most general hermitian matrix of order 4 is:

(a) 4 (b) 8 (c) 16 (d) 32

31. The eigen values of the matrix, $\begin{bmatrix} 0 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & 0 \end{bmatrix}$ are

(a) Purely imaginary (b) Complex (c) Real (d) Zero

32. If a matrix A satisfies the condition $A^2 = I$ and $\mathrm{Tr}(A) = 0$ (where I is a $n \times n$ identity matrix), then

(a) The determinant of A must be 0 (b) The determinant of A must be +1

(c) A must be odd dimensional (d) A must be even dimensional

33. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is **NOT TRUE**?

(a) Det A = Det B (b) Trace of A = Trace of B

(c) A and B have the same eigenvectors (d) A and B have the same eigenvalues.

34. A $3\times3$ matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:

(a) 18 (b) 12 (c) 9 (d) 6

35. The matrix A, defined by $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$ is orthogonal if

(a) $a = 1, b = -1$ (b) $a = 1/\sqrt{2}, b = -1/\sqrt{2}$

(c) $a = 1/\sqrt{2}, b = -i/\sqrt{2}$ (d) $a = 1,\ b = 1$

36. Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

A. The eigenvalues of M are

(a) 0, 1, 2 (b) 0, 0, 3 (c) 1, 1, 1 (d) −1, 1, 3

**B.** The exponential of M simplifies to (I is the 3×3 identity matrix)

(a) $e^M = I + \left(\frac{e^3-1}{3}\right)M$ (b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$ (d) $e^M = (e-1)M$

37. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(a) 3,0, 1 (b) 3,0, 0 (c) 1,2, 0 (d) 1,1, 1

38. If A and B are two $n \times n$ matrices, then the trace of $C = [A, B]$ is

(a) $\mathrm{Tr}(AB)$ (b) $\mathrm{Tr}(A)\ \mathrm{Tr}(B)$ (c) 0 (d) $\mathrm{Tr}(A) + \mathrm{Tr}(B)$

39. Consider the matrix $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a and c are real and b may be complex. If M can be diagonalized by a matrix S, then S is

(a) diagonal (b) symmetric (c) orthogonal (d) unitary

40. The trace and the determinant of a 2×2 matrix are denoted by T and D. The eigenvalues of the matrix are given by

(a) $\frac{T \pm \sqrt{T^2 - 4D}}{2}$ (b) $\pm\frac{\sqrt{T^2 - 4D}}{2}$ (c) $\frac{T \pm D}{2}$ (d) $\pm\frac{T}{2}$

41. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0,1,1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

42. For an N×N matrix consisting of all ones,

(a) all eigenvalues = 1 (b) all eigenvalues = 0

(c) the eigenvalues are 1,2, ..., N (d) one eigenvalue = N, the others = 0

43. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

(a) (1, 4, 9) (b) (0, 7, 7) (c) (0, 1, 13) (d) (0, 0, 14)

44. The eigenvalues of the antisymmetric matrix, $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where $n_1, n_2$ and $n_3$ are the components of a unit vector, are

(a) $0, i, -i$ (b) $0, 1, -1$ (c) $0, 1+i, -1, -i$ (d) $0, 0, 0$

45. A $2\times2$ matrix A has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = 1$ is:

(a) 20 (b) 30 (c) 60 (d) 120

46. Given a $2\times2$ unitary matrix U satisfying $U'U = UU' = 1$ with det $U = e^{i\varphi}$, one can construct a unitary matrix $V(V'V = VV' = 1)$ with det $V = 1$ from it by

(a) Multiplying U by $e^{-i\varphi/2}$

(b) Multiplying any single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column of U by $e^{-i\varphi/2}$

(d) Multiplying U by $e^{-i\varphi}$.

47. Consider an $n \times n (n > 1)$ matrix A, in which $A_{ij}$ is the product of the indices $i$ and $j$ (namely $A_{ij} = ij$). The matrix A

(a) has one degenerate eigenvalue with degeneracy $(n-1)$

(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) has one degenerate eigenvalue with degeneracy $n$

(d) does not have any degenerate eigenvalue

48. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that

(a) $\{M,N\}=0$ (b) $[M,N]=0$ (c) $M^{\dagger}=N$ (d) $M^{\dagger}=N^{-1}$

49. The coordinate transformation $x' = 0.8x + 0.6y$, $y' = 0.6x - 0.8y$ represents

(a) a translation (b) a proper rotation

(c) a reflection (d) none of the above

50. A 3 × 3 matrix M has $\mathrm{Tr}[M]=6, \mathrm{Tr}[M^2]=26$ and $\mathrm{Tr}[M^3]=90$. Which of the following can be a possible set of eigenvalues of M ?

(a) $\{1,1,4\}$ (b) $\{-1,0,7\}$ (c) $\{-1,3,4\}$ (d) $\{2,2,2\}$

51. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

(a) 5, 2, −2 (b) −5, −1, −1 (c) 5, 1, −1 (d) −5, 1, 1

52. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are

(a) −5, −2, 7 (b) −7, 0, 7 (c) −4i, 2i, 2i (d) 2, 3, 6

53. The matrices

$$A=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\qquad B=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \text{ and } C=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

satisfy the commutation relations

(a) $[A,B]=B+C,[B,C]=0,[C,A]=B+C$

(b) $[A,B]=C,[B,C]=A,[C,A]=B$

(c) $[A,B]=B,[B,C]=0,[C,A]=A$

(d) $[A,B]=C,[B,C]=0,[C,A]=B$

## Matrix Alzebra_Answer Key

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (b)** | **2. (d)** | **3. (c)** | **4. (d)** | **5. (a)** | **6. (a)** | **7. (a)** |
| **8. (c)** | **9. (d)** | **10. (a)** | **11. (b)** | **12. (b)** | **13. (c)** | **14. (d)** |
| **15. (c)** | **16. (c)** | **17. (a)** | **18. (a)** | **19. (c)** | **20. (c)** | **21. (a)** |
| **22. (c)** | **23. (c)** | **24. (c)** | **25. (a)** | **26. (b)** | **27. (b)** | **28. (b)** |
| **29. (c)** | **30. (a)** | **31. (c)** | **32. (d)** | **33. (c)** | **34. (d)** | **35. (b)** |
| **36. (b,c)** | **37. (b)** | **38. (c)** | **39. (d)** | **40. (a)** | **41. (b)** | **42. (d)** |
| **43. (d)** | **44. (a)** | **45. (c)** | **46. (a)** | **47. (a)** | **48. (a)** | **49. (c)** |
| **50. (c)** | **51. (c)** | **52. (b)** | **53. (d)** | | | |

### CSIR-UGC-NET/JRF | GATE Physics
### MATHEMATICAL PHYSICS

## Assignment III: Complex Analysis

1. The residue of $\frac{1}{1+z^2}$ at $z=i$ is :

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{2i}$ (d) $\frac{1}{-2i}$

2. The value of $\oint_C \frac{dz}{(z-3)}$ if C is the circle $|z-2|=5$ is

(a) $\pi i$ (b) $2\pi i$ (c) 0 (d) $2\pi$

3. The complex function $f(z)=e^{\frac{+1}{(z-1)^2}}$ has singular point at z=1 which is:

(a) a pole of order 1 (b) a pole of order 2
(c) an isolated essential singularity (d) a non-isolated essential singularity

4. The contour integral $\oint \frac{dz}{z^2+a^2}$ is to be evaluated on a circle of radius 2a centered at the origin. It will have contributions only from the points.

(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (b) ia and $-$ia

(c) ia, $-$ia, $\frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$ (d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

5. The value of $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where c is a circle defined by $|z|=3$, is:

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$ (c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^{2}$

6. Which of the following functions of the complex variable z is NOT analytic everywhere?

(a) $e^z$ (b) $|z|^3$ (c) $z^3$ (d) $\frac{|z|}{3}$

7. The value of the integral $\int_C \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z|=4$, is:

(a) 0 (b) $\pi i$ (c) $2\pi i$ (d) $4\pi i$

8. The inverse of the complex number $\frac{3+4i}{3-4i}$ is:

(a) $\frac{7}{25}+i\frac{24}{25}$ (b) $-\frac{7}{25}+i\frac{24}{25}$ (c) $\frac{7}{25}-i\frac{24}{25}$ (d) $-\frac{7}{25}-i\frac{24}{25}$

9. The value of $\int_C \frac{dz}{(z^2+a^2)}$, where C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is:

(a) $\pi$ for $a=2$ (b) zero for $a=\frac{1}{2}$ (c) $4\pi$ for $a=2$ (d) $\frac{\pi}{2}$ for $a=\frac{1}{2}$

10. The value of the integral $\int_C dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is:

(a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$

11. If a function $f(z)=u(x,y)+iv(x,y)$ of the complex variable $z=x+iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true?

(a) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 v}{\partial x\partial y}$

12. The value of the integral $\int_C z^{10}dz$, where C is the unit circle with the origin as the centre is:

(a) 0 (b) $z^{11}/11$ (c) $2\pi iz^{11}/11$ (d) 1/11

13. If $Z_1$ and $Z_2$ are complex numbers, which of the following is always true?

(a) $Z_1^*Z_2+Z_2^*Z_1=2Z_1^*Z_2$ (b) $Z_1^*Z_2+Z_2^*Z_1=2Z_1Z_2$

(c) $Z_1^*Z_2+Z_2^*Z_1\le 2|Z_1||Z_2|$ (d) $Z_1^*Z_2+Z_2^*Z_1\ge 2|Z_1||Z_2|$

14. Which of the following inequalities are valid in case of complex numbers?

(a) $|Z_1+Z_2|\le|Z_1|+|Z_2|$ (b) $|Z_1+Z_1|>|Z_2|+|Z_2|$

(c) $|Z_1-Z_2|\ge|Z_1|-|Z_2|$ (d) $|Z_1-Z_2|<|Z_1|-|Z_2|$

15. The function tan (1/z) does not have a pole at z =

(a) $\pi/2$ (b) $2/\pi$ (c) $2/(3\pi)$ (d) $-2/(3\pi)$

16. The real part of a complex function f(z), analytic in a region is given by $u(x,y)=x^2-y^2$
If the function vainshes at $z=0$, the imaginary part of the function is

(a) 2xy (b) –2xy (c) $x^2+y^2$ (d) $y^2-x^2$

17. The residue of the function, $f(x)=\frac{z}{(2z+1)(5-z)}$ at $z=-1/2$ is:

(a) 1/11 (b) –1/11 (c) 1/22 (d) –1/22

18. If $z_1$ and $z_2$ are two complex numbers then:

(a) $|z_1+z_2|<|z_1|+|z_2|$ (b) $|z_1+z_2|=|z_1|+|z_2|$

(c) $(z_1+z_2)^2>0$ (d) $|z_1+z_2|^2=(z_1+z_2)(z_1+z_2)$

19. Two complex numbers $Z_1$ and $Z_2$ are given to satisfy $|Z_1|=|Z_2|$ necessarily follows that:

(a) $Z_1=Z_2$ (b) $Z_1=e^{i\phi}Z_2$ (c) $Z_1=Z_2$ (d) $Z_1=Z_2^{-1}$

20. The continuous complex function $f(z)=z^*=x-iy$ is differentiable

(a) only at $z=0$ (b) only at $z=1$ (c) nowhere (d) everywhere

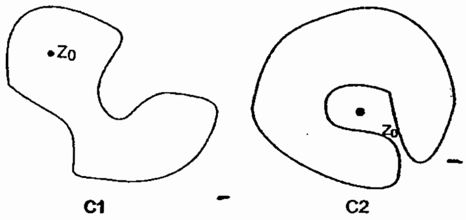
21. Two functions of a complex variable $z = x + iy$ are given as (i) $|z|$ and (ii) $z^2$. which of the following statements is correct?

(a) Both (i) and (ii) are analytic
(b) (i) is analytic and (ii) is nonanalytic
(c) Both are nonanalytic
(d) (i) is nonanalytic and (ii) is analytic

22. A function f(Z) of a complex variable Z is given as $f(Z) = Z\exp\left(\frac{1}{Z}\right)$. At $Z = 0$.

(a) It has no singularity
(b) It has a simple pole and residue is 1
(c) It has an essential sigularity and residue is 1/2
(d) It has a pole of order 2 and residue is 1/6

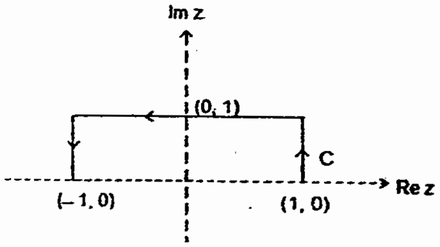
23. A function f(Z) of a complex variable Z is analytic everywhere in the complex plane except at $Z = Z_0$. Two contours C1 and C2 around $Z_0$ are shown in the figure.



The values of the integrals $I_1 = \oint_{C1} \frac{f(Z)dZ}{Z - Z_0}$, $I_2 = \oint_{C2} \frac{f(Z)dZ}{Z - Z_0}$ are

(a) $I_1 = 0, I_2 = 2\pi i f(Z_0)$
(b) $I_1 = 2\pi i f(Z_0), I_2 = 0$
(c) $I_1 = \pi i f(Z_0), I_2 = \pi i f(Z_0)$
(d) $I_1 = 2\pi i f(Z_0), I_2 = 2\pi i f(Z_0)$

24. The value of the integral $\int_C dz\, z^2\, e^z$, where C is an open contour in the complex z-plane as shown in the figure below, is:



(a) $\frac{5}{e} + e$
(b) $e - \frac{5}{e}$
(c) $\frac{5}{e} - e$
(d) $-\frac{5}{e} - e$

25. The residue of $e^{2/z^2}$ at $z = 0$ is:

(a) 0
(b) $\infty$
(c) $2\pi i$
(d) 2

26. The value of the integral $\oint_C \frac{z^3}{(z-1)^2}\,dz$, where C is the circle $|z|=4$ is

(a) $6\pi i$ (b) $-6\pi i$ (c) $2\pi i$ (d) Zero

27. The leading term in Laurent expansion of $f(z)=\frac{1}{z(1-z)^2}$ around z = 0 is

(a) $\frac{1}{z^3}$ (b) $\frac{1}{z^2}$ (c) $\frac{1}{z}$ (d) 1

28. The value of $\oint_c \frac{e^z(z+3)}{z-2}\,dz$ where c is a unit circle with centre at origin is:

(a) $5e^2$ (b) $\frac{5}{2\pi i}e^2$ (c) 0 (d) $10\pi i e^2$

29. If $f(z)=\frac{Z^2+a^2-2aZ}{(Z-a)^3(Z-b)}$, then Z = a is:

(a) Pole of order 1 (b) Pole of order 2 (c) Pole of order 3 (d) An essential singularity.

30. Which one of the following functions is not analytic?

(a) $f(z)=|z|$ (b) $f(z)=z^2$ (c) $f(z)=\cos z$ (d) $f(z)=\sin z$

31. The value of the integral $\oint_c \frac{\cos\pi z}{z(z+1)}$ where C is the circle $|z-1|=3$ is:

(a) 0 (b) $\pi i$ (c) $2\pi i$ (d) $4\pi i$

32. What is the type of singularity possessed by the function, $f(Z)=\exp\left[\frac{1}{(Z-1)^2}\right]$ at Z = 1?

(a) Simple pole (b) Branch point

(c) Pole of second order (d) Essential singularity

33. The residue of the function $f(z)=\frac{1}{z(z+2)^2}$ at z = −2 is:

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) $-\frac{\pi i}{2}$ (d) $4\pi i$

**Common data for Q. 34 and Q. 35.**

Consider a function $f(z)=\dfrac{z\sin z}{(z-\pi)^2}$ of a complex variable z.

34. Which of the following statements is TRUE for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z=\pi$

(c) $f(z)$ has a pole of order 2 at $z=\pi$ (d) $f(z)$ has a simple pole at $z=\pi$.

35. Consider a counterclockwise circular contour $|z|=1$ about the origin. The integral $\oint f(z)dz$ over this contour is:

(a) $-i\pi$ (b) Zero (c) $i\pi$ (d) $2i\pi$

36. The value of the contour integral, $\frac{1}{2\pi i}\oint_c dz\, f(z)$. Where $f(z)=\frac{z}{2}+\frac{1}{z}+\frac{2z}{z^2-1}$ and the contour 'C' is a circle of radius 2 centered at the origin, traversed in the counter-clockwise direction is:

(a) 2 (b) 1/2 (c) 1 (d) 3

37. Which of the following is an analytic function of the complex variable $z=x+iy$ in the domain $|z|<2$?

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

38. The first few terms in the Taylor series expansion of the function $f(x)=\sin x$ around $x=\frac{\pi}{4}$ are

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3\ldots\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3\ldots\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3\ldots\right]$ (d) $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{X^3}{3!}\ldots\right]$

39. The residue of the function $f(z)=\frac{1}{1+z^2}$ at z=i is

(a) $-i$ (b) $2i$ (c) $\frac{1}{2i}$ (d) 1

40. The value of the integral $\oint_C e^{1/z}dz$, using the contour C of circle with unit radius $|z|=1$ is:

(a) 0 (b) $1-2\pi i$ (c) $1+2\pi i$ (d) $2\pi i$

41. The value of the integral $\int_0^\infty \frac{\ell n x}{\left(x^2+1\right)^2}dx$ is:

(a) 0 (b) $-\pi/4$ (c) $-\pi/2$ (d) $\pi/2$

42. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \le |z| \le 2$, and around $z=1$ is

(a) $\frac{1}{2}\left[1+z+z^2+z^3+\ldots\ldots\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\ldots\ldots\right]$

(b) $\frac{1}{1-z}-z-(1-z)^2+(1-z)^3+\ldots\ldots\ldots$ (c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+\ldots\ldots\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+\ldots\ldots\right]$

(d) $2(z-1)+5(z-1)^2+7(z-1)^3+\ldots\ldots$

43. Let $u(x,y)=x+\frac{1}{2}\left(x^2-y^2\right)$ be the real part of an analytic function f(z) of the complex variable $z=x+iy$. the imaginary part of f(z) is

(a) $y+xy$ (b) $xy$ (c) $y$ (d) $y^2-x^2$

44. The value of the integral $\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}\cos\left(\frac{\pi t}{2R}\right)dt$ is

(a) $-2\pi/R$ (b) $-\pi/R$ (c) $\pi/R$ (d) $2\pi/R$

45. If $z=x+iy$ then the function,

$$f(x,y)=(1+x+y)(1+x-y)+a\left(x^2-y^2\right)-1+2iy(1-x-ax)$$

where 'a' is a real parameter, is analytic in the complex 'z' plane if $a=0$

(a) 1 (b) +1 (c) 0 (d) i

46. The Taylor expansion of the function $\ell n(\cosh x)$, where 'x' is real, about the point $x=0$ starts with the following terms:

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+\ldots\ldots\ldots\ldots$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+\ldots\ldots\ldots\ldots$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+\ldots\ldots\ldots\ldots$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+\ldots\ldots\ldots\ldots$

47. The value of the integral $\int_C \frac{z^3dz}{z^2-5z+6}$, where C is a closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is:

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

48. Compute $\lim\limits_{z\to 0}\frac{\mathrm{Re}(z^2)+\mathrm{Im}(z^2)}{z^2}$

(a) The limit does not exist (b) 1

(c) $-i$ (d) $-1$

49. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

(a) $x^2 y$ (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2 y - y - y^3$

50. The integral $\int_0^{\infty} \frac{dx}{4 + x^4}$

evalautes to

(a) $\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

51. Given that the integral $\int_0^{\infty} \frac{dx}{y^2 + x^2} = \frac{\pi}{2y}$, the value of $\int_0^{\infty} \frac{dx}{(y^2 + x^2)^2}$ is

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

52. The partial differential equation, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

upon change of variables to $z = x + iy$ $z^* = x - iy$ becomes

(a) $\frac{\partial u}{\partial z} + \frac{\partial u}{\partial z^*} = 0$ (b) $z\frac{\partial u}{\partial z^*} + z^*\frac{\partial u}{\partial z} = 0$ (c) $z\frac{\partial u}{\partial z} + z^*\frac{\partial u}{\partial z^*} = 0$ (d) $z\frac{\partial u}{\partial z} - z^*\frac{\partial u}{\partial z^*} = 0$

53. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral

$$\oint_C \frac{dz}{\sin^2 z}$$

is

(a) $\infty$ (b) $2\pi i$ (c) 0 (d) $\pi i$

54. The function $\Phi(x, y, z, t) = \cos(z - vt) + \mathrm{Re}(\sin(x + iy))$ satisfies the equation

(a) $\frac{1}{v^2}\frac{\partial^2 \Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$ (b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$ (d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

## Complex Analysis_Answer Key

1. (c) 2. (b) 3. (c) 4. (b) 5. (a) 6. (d) 7. (c)
8. (d) 9. (b) 10. (a) 11. (b) 12. (a) 13. (c) 14. (a,c)
15. (a) 16. (a) 17. (d) 18. (a) 19. (b) 20. (d) 21. (d)
22. (c) 23. (b) 24. (c) 25. (a) 26. (a) 27. (c) 28. (c)
29. (a) 30. (a) 31. (d) 32. (d) 33. (a) 34. (c) 35. (b)
36. (d) 37. (b) 38. (b) 39. (c) 40. (d) 41. (b) 42. (b)
43. (a) 44. (a) 45. (a) 46. (b) 47. (a) 48. (a) 49. (a)
50. (d) 51. (b) 52. (d) 53. (c) 54. (a)

## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS

### Assignment-IV: Differential Equations

1. Solution of $\frac{d^2y}{dx^2}-\frac{5dy}{dx}+6=0$ is

(a) $C_1e^{-3x}+C_2e^{-2x}$ (b) $C_1\cos x+C_2\sin x$

(c) $C_1\cosh x+C_2\sinh x$ (d) $C_1e^{3x}+C_2e^{2x}$

2. Solution of $\frac{d^2x}{dt^2}+\frac{2dx}{dt}+1=0$ is

(a) $x=C_1e^{-t}+C_2e^{-t}$ (b) $x=C_1e^{t}+C_2e^{t}$ (c) $x=C_1e^{-t}+C_2te^{-t}$ (d) none

3. The value of $P_3(x)$ is

(a) $\frac{1}{2}(3x^2-1)$ (b) $3x$ (c) $\frac{1}{2}(5x^3-3x)$ (d) $\frac{1}{2}(5x^3-1)$

4. If $f(x)=x^4-2=\sum_{n=0}^{\infty}a_nP_n(x)$, then $a_3$ is

(a) $\frac{1}{2}$ (b) 1 (c) $-1$ (d) 0

5. The value of $\frac{J_{1/2}(x)}{J_{-1/2}(x)}$ is:

(a) 0 (b) $\tan x$ (c) $\cot x$ (d) $\tanh x$.

6. The points, where the series solution of the Legendre differential equation $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\frac{3}{2}\left(\frac{3}{2}+1\right)y=0$ will diverge, are located at:

(a) 0 and 1 (b) 0 and $-1$ (c) $-1$ and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

**Statement for Linked Answer Q. 7 and Q. 8:**

For the differential equaiton $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$

7. One of the solutions is:

(a) $e^x$ (b) $\ln x$ (c) $e^{-x^2}$ (d) $e^{x^2}$

8. The second linearly independent solution is:
(a) $e^{-x}$ (b) $xe^{x}$ (c) $x^2e^{x}$ (d) $x^2e^{-x}$

9. Consider the differential equation $d^2x/dt^2 + 2\,dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by:
(a) $1/e$ (b) $2/e$ (c) 1 (d) $3/e$

10. The solution of the differential equation,

$$(1+x)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}-y(x)=0 \text{ is:}$$

(a) $Ax^2+B$ (b) $Ax+Be^{-x}$ (c) $Ax+Be^{x}$ (d) $Ax+Bx^2$
where A and B are constants.

11. The solution of the system of differential equations,

$$\frac{dy}{dx}=y-z \text{ and } \frac{dz}{dx}=-4y+z$$

is given by (for A and B are arbitrary constants)
(a) $y(x)=Ae^{3x}+Be^{-x}; (x)=-2Ae^{3x}+2Be^{-x}$ (b) $y(x)=Ae^{3x}+Be^{-x}; z(x)=2Ae^{3x}+2B^{-x}$
(c) $y(x)=Ae^{3x}+Be^{-x}; z(x)=2Ae^{3x}-2Be^{-x}$ (d) $y(x)=Ae^{3x}+Be^{-x}; z(x)=-2Ae^{3x}-2Be^{-x}$

12. Legendre's differential equation: $(1-x^2)\,d^2y/dx^2 - 2x\,dy/dx + n(n+1)y = 0$
has an ordinary point at x =
(a) $-1$ (b) 0 (c) 1 (d) $\infty$

13. For the differential equation

$$z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}+y=0$$

(a) $z=\infty$ is an ordinary point
(b) $z=0$ is a regular singular and $z=\infty$ is an irregular singular point
(c) $z=0$ and $z=\infty$ both are regular singular points
(d) $z=0$ is an irregular singular point

14. The Legendre polynomial $P_n(x)$, where $-1 \le x \le +1$

(a) Is singular at $x=\pm 1$ (b) Satisfies $\int_{-1}^{1} dx P_n^2(x) = 1$

(c) Satisfies $\int_{-1}^{1} dx P_n(x) = 0$ for $n \ge 1$ (d) Is always an even function of x.

15. For the Bessel equation, $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$, where n is an integer, the maximum number of linearly independent solutions, well-defined at $x = 0$ is
(a) zero (b) one (c) two (d) three

16. The general solution of the differential equation, $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ is:

(a) $C_1e^{2x}+C_2e^{-2x}$ (b) $C_1e^{2x}+C_2x\,e^{2x}$ (c) $C_1e^{-2x}+iC_2\,e^{2x}$ (d) $C_1e^{-2x}+C_2\,xe^{-2x}$

Here $C_1$ and $C_2$ are arbitrary constants.

17. In order to obtain the solution of the initial value problem of the equation of motion, $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+16x=0$.

How many initial conditions are required?

(a) 0 (b) 1 (c) 2 (c) 3

18. The general solution of the linear differential equation, $\frac{d^4u}{dx^4}=0$ is, u(x) equal to

(a) 0 (b) $c_3t+c_4$ (c) $c_2t^2+c_3t+c_4$ (d) $c_1t^3+c_2t^2+c_3t+c_4$

19. The regular singular points of the associated Legendre differential equation in the finite domain are

(a) 1, −1 (b) 0 (c) $\infty$ (d) 2

20. The Wronskian of the linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}+4y=0$ is:

(a) Constant (b) 0 (c) $\infty$ (d) Undetermined

21. The solutions to the differential equation $\frac{dy}{dx}=-\frac{x}{y+1}$ are a family of

(a) Circles with different radii (b) Circles with different centres.

(c) Straight lines with different slopes (d) Straight lines with different intercepts on the y-axis

22. Given the Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)\frac{3x^2-1}{2}$, then the polynomial $(3x^2+x-1)$ can be expressed as

(a) $P_2(x)-P_1(x)$ (b) $2P_2(x)+P_1(x)$

(c) $P_2(x)+P_1(x)$ (d) $2P_2(x)+P_1(x)+P_0(x)$

23. The generating function $F(x,t)\equiv\sum_{n=0}^{\infty}P_n(x)t^n$ for the Legendre polynomials $P_n(x)$ is $F(x,t)=(1-2xt+t^2)^{-1/2}$. The value of $P_3(-1)$ is:

(a) 5/2 (b) 3/2 (c) +1 (d) -1

24. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+f(t)x=0$,

and let $w(t)=x_1(t)\frac{dx_2(t)}{dt}-x_2(t)\frac{dx_1(t)}{dt}$ if $w(0)=1$, then $w(1)$ is given by

(a) 1 (b) $e^2$ (c) $1/e$ (d) $1/e^2$

25. If one expands $f(x)=x^4-2$ around $x=0$ in terms of the Legendre Polynomials $\left(f(x)=\sum_{n=0}^{\infty}a_n P_n(x)\right)$, the

coefficient $a_3$ is :

(a) 1 (b) 0 (c) 3 (d) $e^2$

26. The most general solution of the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6=0$ in terms of arbitrary constants

$C_1$ and $C_2$ is

(a) $c_1e^{-3x}+c_2e^{-2x}$ (b) $c_1\sin 3x+c_2\cos 2x$ (c) $c_1e^{3x}+c_2e^{2x}$ (d) $c_1e^{-3ix}+c_2e^{2ix}$

27. The solution of the differential equation $x\frac{dy}{dx}+y=1$, given the condition $y=0$ at $x=1$ is

(a) $y=(1-x^2)\ell n\, x$ (b) $y=\frac{x^2-1}{x}e^{-x}$ (c) $y=\frac{x-1}{x}$ (d) $y=\sinh(x)$

28. Let y(x) be a continuous real function in the range 0 and $2\pi$, satisfying the inhomogeneous differential equaltion:
$\sin x\frac{d^2y}{dx^2}+\cos x\frac{dy}{dx}=\delta\left(x-\frac{\pi}{2}\right)$. The value of dy/dx at the point $x=\pi/2$

(a) is continuous (b) has a discontinuity of 3

(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

29. The differential equation,

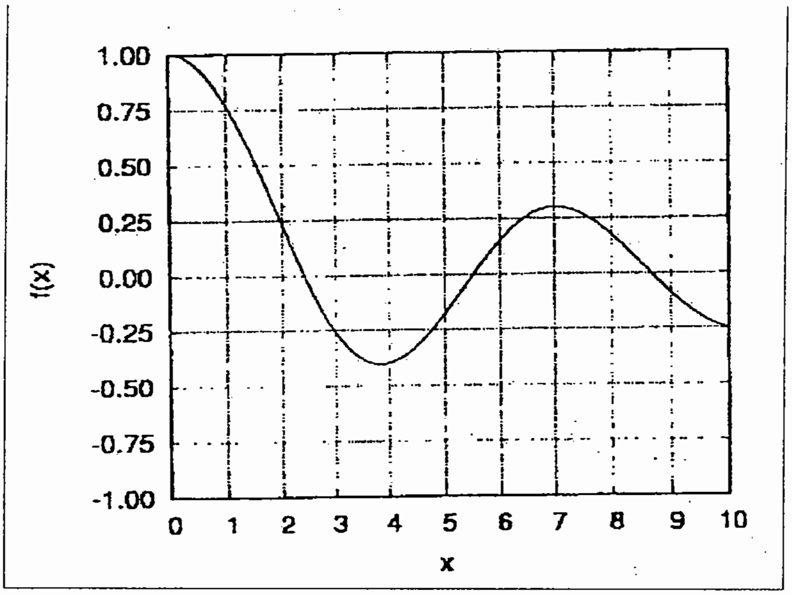
$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$$

has the complete solution, in terms of arbitary constants A and B,

(a) $A\exp x+B\,x\exp x$ (b) $A\exp x+B\exp(-x)$

(c) $A\exp x+B\,x\exp(-x)$ (d) $x\{A\exp x+B\exp(-x)\}$

30. The graph of the function f(x) as shown below is best described by



(a) The bessel function $J_0(x)$ (b) $\cos x$

(b) $e^{-x}\cos x$ (d) $\frac{1}{x}\cos x$

31. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2}-(3-4i)f=0$ and satisfies the conditions $f(0)=1$ and $f(x)\to 0$ as $x\to\infty$. The value of $f(\pi)$ is:

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi i}$

32. Consider the differential equation $\frac{dG(x)}{dx}+k\,G(x)=\delta(x)$, where $k$ is a constant. Which of the following statements is true?

(a) Both $G(x)$ and $G'(x)$ are continuous at $x=0$.

(b) $G(x)$ is continuous at $x=0$ but $G'(x)$ is not.

(c) $G(x)$ is discontinuous at $x=0$.

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x=0$ depend on the value of $k$.

33. Given,

$\sum_{n=0}^{\infty}P_n(x)t^n=\left(1-2xt+t^2\right)^{-1/2}$, for $|t|<1$, the value of $P_5(-1)$ is

(a) 0.26 (b) 1 (c) 0.5 (d) −1

34. Consider the differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$$

with the initial conditions $x(0)=0$ and $x(0)=1$. The solution $x(t)$ attains its maximum value when 't' is

(a) 1/2 (b) 1 (c) 2 (d) $\infty$

## Differential Equations_Answer Key

1. (d) 2. (c) 3. (c) 4. (d) 5. (b) 6. (d) 7. (a)
8. (b) 9. (b) 10. (b) 11. (a) 12. (b) 13. (a) 14. (c)
15. (c) 16. (b) 17. (c) 18. (d) 19. (a) 20. (a) 21. (a)
22. (b) 23. (d) 24. (d) 25. (b) 26. (c) 27. (c) 28. (d)
29. (a) 30. (a) 31. (c) 32. (c) 33. (d) 34. (b)

## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS

### Assignment V: Fourier Series, Fourier Transform & Laplace Transform

1. Inverse Laplace transform of $\frac{a}{s^2+a^2}$ is

(a) sin at (b) sinh at (c) cos at (d) cos h at

2. Using the fourier series of $f(x)=x^2, -\pi<x<\pi$. The value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{12}$ (d) $\pi^2$

3. Fourier transorm of Gaussian function $e^{-x^2}$ is:

(a) $\frac{1}{\sqrt{2}}e^{-\omega^2/4}$ (b) $\frac{1}{\sqrt{2\pi}}e^{-\omega^2/4}$ (c) $e^{-\omega^2}$ (d) $e^{-\omega^2/2}$

4. Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is:

(a) $\cos 2x+\frac{1}{2}\sin 2x$ (b) $\cos x+\frac{1}{2}\sin x$

(c) $\cosh x+\frac{1}{2}\sinh x$ (d) $\cosh 2x+\frac{1}{2}\sinh 2x$

5. The kth Fourier component of $f(x)=\delta(x)$ is:

(a) 1 (b) 0 (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

6. If $\bar{f}(s)$ is the Laplace transform of f(t) the Laplace transform of f(at), where a is a constnat, is

(a) $\frac{1}{a}\bar{f}(s)$ (b) $\frac{1}{a}\bar{f}(s/a)$ (c) $\bar{f}(s)$ (d) $\bar{f}(s/a)$

7. The Laplace transform of $f(t)=\sin\pi t$ is $F(s)=\frac{\pi}{(s^2+\pi^2)}$, $s>0$. Therefore, the Laplace transform of $t\sin\pi t$ is:

(a) $\frac{\pi}{s^2(s^2+\pi^2)}$ (b) $\frac{2\pi}{s^2(s^2+\pi^2)^2}$ (c) $\frac{2\pi s}{(s^2+\pi^2)^2}$ (d) $\frac{2\pi}{(s^2+\pi^2)^2}$

8. A periodic function f(x) = x for $-\pi < x < +\pi$ has the Fourier series representation $f(x)=\sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin nx$. Using this, one finds the sum $\sum_{n=1}^{\infty} n^{-2}$ to be:

(a) $2\ln 2$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) $\pi \ln 2$

9. The Fourier transform F(k) of a function f(x) is defined as:

$F(k)=\int_{-\infty}^{\infty} dx\, f(x)\exp(ikx)$. Then F(k) for $f(x) = \exp(-x^2)$ is:

[Given: $\int_{-\infty}^{\infty}\exp\left(-x^2\right)dx=\sqrt{\pi}$ ]

(a) $\pi\exp(-k)$ (b) $\sqrt{\pi}\exp\left(\frac{-k^2}{4}\right)$ (c) $\frac{\sqrt{\pi}}{2}\exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi}\exp\left(-k^2\right)$

10. The Fourier transform of the function f(x) is $F(k)=\int e^{ikx} f(x)\,dx$. The Fourier transform of df(x)/dx is:

(a) $dF(k)/dk$ (b) $\int F(k)/dk$ (c) $-ikF(k)$ (d) $ikF(k)$

11. Fourier transform of which of the following functions does not exist?

(a) $e^{-|x|}$ (b) $xe^{-x^2}$ (c) $e^{x^2}$ (d) $e^{-x^2}$

12. If f(x) be an even, period function of x which is continuous. Then
(a) It can be represented by a cosine series. (b) It can be represented by a sine series
(c) It cannot be represented by a Fourier series
(d) It needs both sine and cosine terms in its Fourier expansion

13. Laplace transform L{F(t)} of a function F(t) is f(s). Which of the following expression is FALSE?

(a) $\frac{d^n}{ds^n}f(s)=(-1)^n L\{t^nF(t)\}$ (b) $f(s+a)=L\{e^{-at}F(t)\}$

(c) $L\left\{\sum_{i=1}^{n}a_iF_i(t)\right\}=\sum_{i=1}^{n}a_iL\{F_i(t)\}$ (d) $L\{F(at)\}=\frac{1}{a}f(s)$

14. Which of the following is NOT a periodic function of $\theta$?

(a) $\sin\theta+\pi\cos\theta$ (b) $\sin(\pi\theta)+\cos(\pi\theta)$

(c) $\sin\theta+\cos(\pi\theta)$ (d) $\sin\theta+\cos(\theta+\pi)$

15. The Fourier series expansion of the function $f(x)=x(\pi-x), 0\le x\le t,$ is:

$\frac{\pi^2}{6}-\left(\frac{\cos 2x}{1^2}+\frac{\cos 4x}{2^2}+\frac{\cos 6x}{3^2}+\ldots\ldots\right)$. The value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is deduced as

(a) $\frac{\pi^2}{6}$ (b) $\frac{2\pi^2}{6}$ (c) $-\frac{\pi^2}{6}$ (d) Zero

16. The Fourier coefficients $a_n$ in the Fourier series expansion.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx,\text{ for } f(x)=\begin{cases}-1, & -\pi\le x\le 0\\ +1, & 0\le x\le \pi\end{cases}\text{, are}$$

(a) $\frac{2}{n\pi}\left[1-(-1)^n\right]$ (b) $\frac{4}{n\pi}$ (c) $\frac{2}{n\pi}\left[1+(-1)^n\right]$ (d) 0

17. Coefficient of cosx in the Fourier series expansion of x over 0 to $2\pi$ is:

(a) 0 (b) 1 (c) 2 (d) 1/2

18. The function $f(x)=x^2, 0<x<2$ has a Fourier series representation

$$x^2=\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos nx-\frac{4\pi}{n}\sin nx\right)$$

At x = 0 and $2\pi$, the series converges to $2\pi^2$. Then the value of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is:

(a) $2\pi^2$ (b) $\pi^2$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi^2}{6}$

19. Consider the Fourier series corresponding to the function

$$f(x)=\begin{cases}3, & 0<x<5\\ -3, & -5<x<0\end{cases}$$

Then the fourier series contains.

(a) Cosine terms only (b) Both cosine and sine terms

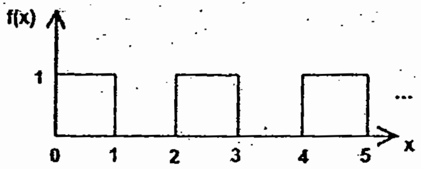
(c) Sine terms only (d) Sine terms and non-zero constant terms.

20. The constant coefficient in the Fourier series of the function

$$f(x)=\begin{cases}0 & -5<x<0\\ 3 & 0<x<5\end{cases}\text{ is:}$$

(a) 3 (b) $\frac{3}{2}$ (c) 0 (d) 5

21. The graph of the function $f(x)=\begin{cases}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{cases}$ (where $n=0,1,2,.......$) is shown below.



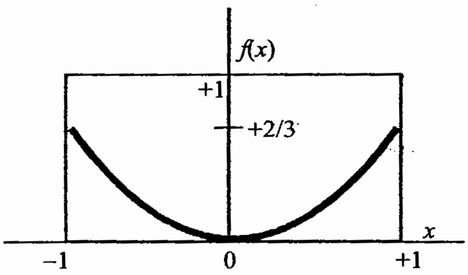
Its Laplace transform $\tilde{f}(s)$ is

(a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s(1+e^{-s})}$ (d) $\frac{1}{s(1-e^{-s})}$

22. Fourier transform of the derivative of the Dirac $\delta$-function, namely $\delta'(x)$, is proportional to

(a) 0 (b) 1 (c) sink (d) ik

23. A student is asked to find a series approximation for the function $f(x)$ in the domain $-1\le x\le +1$, as indicated by the thick line in the figure below.



The student represents the function by a sum of three terms

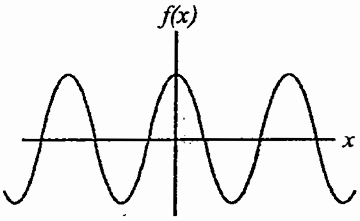
$$f(x)\approx a_0+a_1\cos\frac{\pi x}{2}+a_2\sin\frac{\pi x}{2}$$

Which of the following would be the best choices for the coefficients $a_0$, $a_1$ and $a_2$?

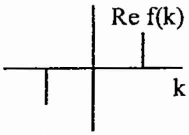
(a) $a_0=1, a_1=-\frac{1}{3}, a_2=0$ (b) $a_0=\frac{2}{3}, a_1=-\frac{2}{3}, a_2=0$

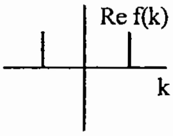
(c) $a_0=\frac{2}{3}, a_1=0, a_2=-\frac{2}{3}$ (d) $a_0=-\frac{1}{3}, a_1=0, a_2=-1$

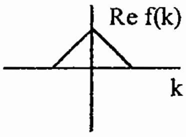
24. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below

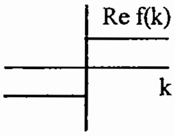


Which of the following graphs represents the real part of its Fourier transform?

(a)


(b)


(c)


(d)


## Fourier Series, Fourier Transform & Laplace Transform_Answer Key

**1. (a)** **2. (b)** **3. (a)** **4. (d)** **5. (c)** **6. (b)** **7. (c)**

**8. (c)** **9. (b)** **10. (c)** **11. (c)** **12. (a)** **13. (d)** **14. (c)**

**15. (a)** **16. (d)** **17. (a)** **18. (d)** **19. (c)** **20. ( b)** **21. (c)**

**22. (b)** **23. (b)** **24. (b)**

## CSIR-UGC-NET/JRF | GATE Physics

## Assignment VI: Probability

1. The probability of getting m heads out of n trials of a coin is

(a) $\frac{m!}{n!(n-m)!}\left(\frac{1}{2}\right)^m$ (b) $\frac{n!}{m!(n-m)!}\left(\frac{1}{2}\right)^n$ (c) $\frac{n!}{m!(n-m)!}\left(\frac{1}{2}\right)$ (d) $\frac{m!}{n!(n-m)!}\left(\frac{1}{2}\right)^n$

2. There are 3 cows and 4 bulls in a field, the probabiliy of taking birth of a calf by the combination of a cow and a bull is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

3. The probability of throwing more than 4 by one dice in a throw is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

4. An unbiased dice is thrown three times successively. The probability that the numbers of dots on the uppermost surface add up to 16 is

(a) $\frac{1}{16}$ (b) $\frac{1}{36}$ (c) $\frac{1}{108}$ (d) $\frac{1}{216}$

5. An unbiased coin is tossed n times. The probability that exactly m heads will come up is

(a) $\frac{1}{2^m}\frac{n!}{m!(n-m)!}$ (b) $\frac{1}{2^n}\frac{n!}{m!(n-m)!}$ (c) $\frac{n}{2^m}$ (d) $\frac{m}{2^n}$

6. If a three digit number is chosen at random, the probability for the middle digit to be 3 is

(a) $1/10$ (b) $(1/10)^3$ (c) $1/3$ (d) $1/30$

7. Suppose a team is equally likely to win or lose a match. In a series of two matches, if we know that it has won at least one match, what is the probability that it has won both matches?

(a) $3/4$ (b) $1/2$ (c) $1/3$ (d) $1/4$

8. A coin and a die are thrown independently. The probability that a 'head' appears on the coin and an even integer on the die (assuming both are unbiased )is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

9. An unbiased die is cast twice. The probability that the positive different (bigger– smaller) between the two numbers is 2 is

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

10. A ball is picked at random from one of two boxes that contain 2 black and 3 black and 4 white balls respectively. What is the probability that it is white?
(a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

11. In a series of five cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is
(a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

12. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to
(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

13. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the $x$ axis. The probability that they meet after $n$ steps is
(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

14. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one?
(a) 0.99 (b) 0.925 (c) 0.075 (d) 0.01

15. A loaded dice has the probabilities $\frac{1}{21},\frac{2}{21},\frac{3}{21},\frac{4}{21},\frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6, respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?
(a) $\frac{144}{441}$ (b) $\frac{225}{441}$ (c) $\frac{221}{441}$ (d) $\frac{220}{441}$

16. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0.255, 0.255, 0.245, and 0.245, respectively. After a large number of steps, N, the expected position of the child with respect to the starting point is at a distance
(a) $\sqrt{2}\times10^{-2}Na$ in the north-east direction (b) $\sqrt{2N}\times10^{-2}a$ in the north-east direction
(c) $2\sqrt{2}\times10^{-2}Na$ in the south-east direction (d) 0

17. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?
(a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

## Probability_Answer Key

**1. (b)** **2. (d)** **3. (b)** **4. (b)** **5. (b)** **6. (a)** **7. (d)**
**8. (a)** **9. (b)** **10. (b)** **11. (d)** **12. (d)** **13. (a)** **14. (c)**
**15. (b)** **16. (a)** **17. (a)**

## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS
### Assignment VII: Miscelleneous Problems

**(Tensors, Numerical Techniques, Group Theory, Beta-Gamma Function, Series, Dirac-Delta Function)**

1. Given that $T_{i,j}=\frac{1}{2}\left(x_iy_j-x_jy_i\right)$ and $S_{ij}=\frac{1}{2}\left(x_iy_j+x_jy_i\right)$. Then $T_{ij}\,S_{ij}$ is:

(a) 0 (b) $\sum x_i^4-y_i^4$ (c) $\sum x_i^2y_i^2$ (d) $\sum\left(x_i^2-y_i^2\right)$

2. The number of independent components of symmetric tensor of rank 2 in n-dimensional space is:

(a) 2n (b) $n^2$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

3. The number of independent components of antisymmetric tensor of rank 2 in n-dimensional space is:

(a) n (b) $n^2$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

4. $\int_0^\infty x\,e^{-x^2}dx$ is

(a) $\frac{1}{2}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\frac{2}{\pi}}$ (d) 1

5. The integral formula $\int_{x_0}^{x_0+nh} y\,dx=\frac{h}{3}\left[\left(y_0+y_n\right)+4\left(y_1+y_3++......+y\right)+2\left(y_2+y_4+.....+y_{n-2}\right)\right]$

(a) simpson's $\frac{3}{8}$ rule (b) simpson's $\frac{1}{3}$ rule (c) trapezoidal rule (d) weddle's rule.

6. The average value of the function $f(x)=4x^3$ in the interval 1 to 3 is:
(a) 15 (b) 20 (c) 40 (d) 80

7. The average of the function $f(x)=\sin x$ in the interval $(0,\pi)$ is:
(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$

8. $S_{ij}$ and $A_{ij}$ represent a symmetric and an antisymmetric real-valued tensor respectively in three dimensions. The number of independent components of $S_{ij}$ and $A_{ij}$ are:
(a) 3 and 6 respectively (b) 6 and 3 respectively
(c) 6 and 6 respectively (d) 9 and 6 respectively.

9. Consider the four statements given below about the function $f(x)=x^4-x^2$ in the range $-\infty<x<+\infty$. Which one of the following statements is correct?
P: the plot of f(x) versus x has two maxima and two minima
Q: the plot of f(x) versus x cuts the x axis at four points
R: the plot of f(x) versus x has three extrema

S: no part of the plot f(x) versus x lies in the fourth guadrant

Pick the right combination of correct choices from those given below.

(a) P and R (b) R only (c) R and S (d) P and Q

10. If u (x, y, z, t) = f(x + iβy − vt) + g (x − iβy − vt), where f and g are arbitrary and twice differentiable functions, is a solution of the wave equation, $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, then β is

(a) $\left(1-\frac{v}{c}\right)^{1/2}$ (b) $\left(1-\frac{v}{c}\right)$ (c) $\left(1-\frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1-\frac{v^2}{c^2}\right)$

11. The Dirac delta function δ(x) can be represented by:

(a) $\frac{1}{\pi}\lim_{N\to 0}\frac{\sin(Nx)}{x}$ (b) $\frac{1}{\pi}\lim_{\varepsilon\to 0+}\frac{\varepsilon}{x^2+\varepsilon^2}$

(c) $\frac{1}{\sqrt{x}}\lim_{\varepsilon\to+\infty}\frac{1}{\sqrt{\varepsilon}}\exp\left(-x^2/\varepsilon\right)$ (d) $\frac{1}{2p}\int_{-\infty}^{+\infty}\exp\left(i\left|x-x^4\right|u\right)du$

12. The error function is defined by $\operatorname{erf}(x)=\left(2/\sqrt{\pi}\right)\int_0^x \exp\left(-t^2\right)dt$

Choose the correct alternatives:

(a) Error function is an even function (b) Error function is an odd function

(c) $\operatorname{erf}(0)=1$ (d) $\operatorname{erf}(\infty)=1$

13. A wave f (r, z, t) = $J_0$ (αr) cos (ωt − kz), where r and z refer to cylindrical coordinate system, satisfies the wave equation

$$\nabla^2 f-\left(1/v^2\right)\left(\partial^2 f/\partial t^2\right)=0$$

when

(a) $v^2=\omega^2/k^2$ (b) $v^2=\omega^2/(k^2+\alpha^2)$ (c) $v^2=\omega^2/(k^2-\alpha^2)$ (d) $v^2=\omega^2/\alpha^2$

14. The volume element in the spherical polar coordinates is given by:

(a) $r^2\sin\theta\, d\theta\, d\phi dr$ (b) $r^2\sin\theta\sin\phi d\theta d\phi dr$

(c) $r\sin\theta d\theta d\phi dr$ (d) $r^2 d\theta d\phi dr$

15. From dimensional argument, the Fourier transform of Coulomb potential in three dimension is:

(a) $\frac{C}{k^3}$ (b) $\frac{C}{k}$ (c) C (d) $\frac{C}{k^2}$

where C is a constant, and k is the magnitude of the wave vector.

16. Which of the following is a valid solution of the differential equation, $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0$

(a) $x^2-y^2$ (b) $ax^2-by^2$ (c) $x^2+y^2$ (d) $x^4+y^4$

17. The Dirac delta function $\delta(x)$ is defined as $\int_{-\infty}^{\infty}\delta(x)dx=1$. Which of the following integration is **INCORRECTLY** evaluated?

(a) $\int_{0}^{2\pi}\cos x\,\delta(x-\pi)\,dx=-1$ (b) $\int_{-2}^{+2}x^3\delta(x+1)dx=-1$

(c) $\int_{-2}^{2}(2x+3)\delta(3x)dx=1$ (d) $\int_{0}^{2}(x^3+3x+2)\delta(x-1)dx=0$

18. The nonzero independent components of an antisymmetric tensor $A_{ij}$ of rank 2 in three dimensions are

(a) $A_{11}, A_{12}, A_{13}$ (b) $A_{12}, A_{13}, A_{22}$

(c) $A_{12}, A_{23}, A_{13}$ (d) $A_{13}, A_{23}, A_{33}$

19. Which of the following matrices is an element of the group SU(2)?

(a) $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}\frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}}\end{pmatrix}$ (c) $\begin{pmatrix}2+i & i\\ 3 & 1+i\end{pmatrix}$ (d) $\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$

20. The number of independent components of a symmetric and an antisymmetric tensor of rank 2(in 3–dimensions) are, respectively,

(a) 6, 6 (b) 9, 3 (c) 6, 3 (d) 3, 6

21. The trapezoidal, Simpson's 1/3 and Simpson's 3/8 rules are exact for polynomials of order

(a) 1, 2, 3 respectively (b) 1, 3, 3 respectively

(c) 1, 3, 4 respectively (d) 2, 3, 4 respectively.

22. The sum of $1-\frac{\pi^2}{2!4^2}+\frac{\pi^4}{4!4^4}-\frac{\pi^6}{6!4^6}+........$is

(a) Not convergent (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

23. The integral $\int_{-1}^{1}x^{-2}dx$ is:

(a) 0 (b) 1 (c) 2 (d) $\infty$

24. $\int_{-1}^{1}\sin mx\cos nx\,dx,\ (m\neq 0, n\neq 0)$ is:

(a) $\delta_{mn}$ (b) 2 (c) $2\,mn\,\pi$ (d) 0

25. Given $f(x)=\frac{1}{1-x^2}, x<1$, the series expansion for f(x) is:

(a) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+............$ (b) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+............$

(c) $1+x+x^2+x^3+......$ (d) $1+x^2+x^4+x^6+....$

26. Consider the function f such that $f(\alpha+\beta)=f(\alpha)+f(\beta)$ for any real $\alpha,\beta$. If m and n are integers, then $f\left(\frac{m}{n}\right)$ is

(a) $\frac{m}{n}f(1)$ (b) $\frac{m}{n}$ (c) $\frac{f(m)}{f(n)}$ (d) $f\left(\frac{1}{m}\right)+f\left(\frac{1}{n}\right)$

27. Given functions $\cos 2x, \sin^2 x, \cos^2 x$ are

(a) Linearly independent (b) Linearly dependent

(c) Orthogonal to each other (d) Unrelated to each other

28. The value of the integral $\int_0^\infty x\, e^{-x^2}\, dx$ is equal to

(a) $\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\pi}$ (c) 0 (d) $\frac{1}{2}$

29. Under a small rotation $\delta\theta$ about the x–axis, the component of a second rank tensor $\Gamma_{yz}$ transforms as

(a) $\Gamma'_{yz}=(1+\delta\theta)\Gamma_{yz}$ (b) $\Gamma'_{yz}=\Gamma'_{yz}+\delta\theta(\Gamma_{yy}-\Gamma_{zz})$

(c) $\Gamma'_{yz}=\Gamma_{yz}+\delta\theta(\Gamma_{yx}-\Gamma_{zx})$ (d) $\Gamma'_{yz}=\Gamma'_{yz}+\delta\theta\Gamma_{xx}$

30. In spherical polar coordinates $\vec{r}=(r,\theta,\varphi)$ the delta function $\delta(\vec{r}_1-\vec{r}_2)$ can be written as

(a) $\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$

(b) $\frac{1}{r_1^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$

(c) $\frac{1}{|\vec{r}_1-\vec{r}_2|^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$

(d) $\frac{1}{r_1^2\cos\theta_1}\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$

31. The character table of $C_{3v}$, the group of symmetries of an equilateral triangle is given below

| | $\chi^{(0)}$ | $\chi^{(1)}$ | $\chi^{(2)}$ |
|---|---|---|---|
| $1\mathcal{C}_1$ | 1 | 1 | b |
| $3\mathcal{C}_2$ | 1 | a | c |
| $2\mathcal{C}_3$ | 1 | 1 | d |

In the above $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ denotes the three classes of $C_{3v}$, containing 1, 3 and 2 elements repectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters o f the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of $C_{3v}$.

A. The entries a, b, c and d in this table are, respectively

(a) $2, 1, -1, 0$ (b) $-1, 2, 0, -1$ (c) $-1, 1, 0, -1$ (d) $-1, 1, 1, -1$

B. The reducible representation $\Gamma$ of $C_{3v}$ with character $\chi = (4, 0, 1)$ decomposes into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(2)}$

32. The function $f(x)$ represents the nearest integer less than $x$, e.g. $f(3.14) = 3$.

The derivative of this function (for arbitrary $x$) will be given in terms of the integers $n$ as $f'(x) =$

(a) 0 (b) $\sum_n \delta(x-n)$ (c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

33. Consider the integral, $\int_{-p^2}^{+p^2} \frac{dx}{\sqrt{x^2 - p^2}}$

where p is a constant. This integral has a real, nonsingular value if

(a) $p < -1$ (b) $p > 1$ (b) $p = 1$ (d) $p \to 0$

34. The value of the real integral $I = \int_{-3}^{+3} \frac{dx}{x^2 + 3x + 2}$ is:

(a) $\frac{3\pi}{2}$ (b) $\ell n\left(\frac{2}{5}\right)$ (c) $\infty$ (d) 0

35. Let $\varphi(x)$ be an arbitrary smooth function of x in $x_1 \le x \le x_2$. The condition that the integral $J = \int_{x_1}^{x_2} \left(\frac{d\varphi}{dx}\right)^2 dx$ has an extremum is given by

(a) $\frac{d^2\varphi}{dx^2} > 0$ (b) $\frac{d\varphi}{dx} = x$ (c) $\frac{d^2\varphi}{dx^2} < 0$ (d) $\frac{d^2\varphi}{dx^2} = 0$

36. Let $f(x) = e^{-x} \cos x$, what is the value of $f(x)$ at $x = 0.01$ up to four decimal places

(a) 0.0001 (b) 0.0101 (c) 0.9900 (d) 0.9999

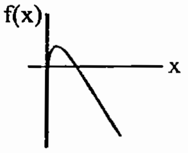
37. The value of the integral $\int_0^\infty x e^{-x^2} dx$ is equal to

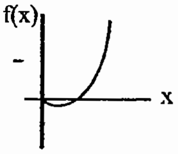
(a) $\sqrt{\frac{x}{2}}$ (b) $\sqrt{x}$ (c) $\frac{1}{2}$ (d) 0

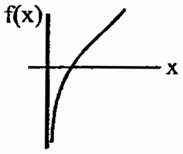
38. If $T_{ij}=\frac{1}{2}\left(x_iy_j-x_jy_i\right)$ and $S_{ij}=\frac{1}{2}\left(x_iy_j-x_jy_i\right)$ are components of tensor T and $S=\left(i,j=1,2,3\right)$ then $\sum_{i,j}T_{ij}S_{ij}$ is :

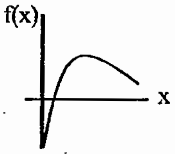
(a) 0 (b) $\sum_i\left(x_i^2-y_i^2\right)$ (c) $\sum_i x_i^2y_i^2$ (d) $\sum_i\left(x_i^4-y_i^4\right)$

39. Which of he following plots represents the function $f(x)=x\,\ell n\,x$ in the range $0\le x<\infty$ ?

(a)


(b)


(c)


(d)


40. The values of a function $f(x)$ are tabulated for three value of x below:

| x | $f(x)$ |
|---|---|
| 1.0 | 1.76 |
| 2.0 | 0.41 |
| 3.0 | −0.16 |

What is the approximate value of x at which $f(x)$ vanishes?

(a) 2.3 (b) 2.9 (c) 2.1 (d) 1.9

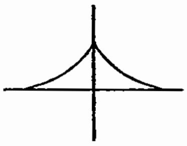
41. The number of independent components of symmetric second rank tensor $S_{ij},\ i,j=1,2,......,d$ is

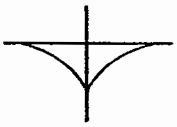
(a) 2 (b) $d^2$ (c) $d^2/2$ (d) $d(d+1)/2$

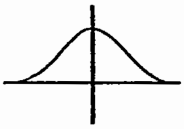
42. The sum of the infinite series $1+\frac{3}{2}+\frac{3.4}{2^2.2!}+\frac{3.4.5}{2^3.3!}+\frac{3.4.5.6}{2^4.4!}+.......$ is

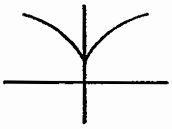
(a) 8 (b) $e^2$ (c) $2\pi$ (d) $\infty$

43. Which of the following plots represents the function $f(x)=e^{-|x|}$ for $-\infty<x<\infty$?

(a)


(b)


(c)


(d)


44. The maximum value of the function $f(x)=xe^{-x}$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) 1 (d) $e^2$

45. Given the integral $\int_0^\infty e^{-ax^2}dx=\frac{1}{2}\sqrt{\frac{\pi}{a}}$, the value of the integral $\int_0^\infty x^2e^{-2x^2}dx$ is

(a) $\frac{1}{4}\sqrt{\frac{\pi}{2}}$ (b) $2\sqrt{2\pi}$ (c) $2\sqrt{\pi}$ (d) 0

46. The numerical value of a function $f(x)$ at three points are tabulated below:

| $x$ | $f(x)$ |
|---|---|
| 0 | 0.0000 |
| 0.1 | 0.0998. |
| 0.2 | 0.1986 |

The approximate value $\frac{d^2f(x)}{dx^2}$ at $x=0.1$ is

(a) $-0.02$ (b) $-0.01$ (c) $-0.1$ (d) $+0.1$

47. The number of independent components of the symmetric tensor $A_{ij}$ with indices $i, j = 1, 2, 3$ is

(a) 1 (b) 3 (c) 6 (d) 9

48. As $x \to 1$, infinite series $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + .....$

(a) diverges (b) converges to unity

(c) converges to $\pi/4$ (d) none of the above

49. If $\lfloor x \rfloor$ denotes the greatest integer not exceeding x, then $\int_0^\infty \lfloor x \rfloor e^{-x} dx$ is

(a) $\frac{1}{e-1}$ (b) 1 (c) $\frac{e-1}{e}$ (d) $\frac{e}{e^2-1}$

50. What is the value of the following series ?

$$\left(1+\frac{1}{2!}+\frac{1}{4!}+...\right)^2 - \left(1+\frac{1}{3!}+\frac{1}{5!}+...\right)^2$$

(a) 0 (b) e (c) $e^2$ (d) 1

51. Which of the following limits exists?

(a) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}+\ln N\right)$ (b) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}-\ln N\right)$

(c) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{\sqrt{m}}-\ln N\right)$ (d) $\lim_{N\to\infty}\sum_{m=1}^{N}\frac{1}{m}$

52. A bag constain many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to

(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

53. The value of the integral

$$\int_0^\infty dx\, x^9 \exp\left(-x^2\right)$$

is

(a) 20160 (b) 12 (c) 18 (d) 24

54. The integral

$$\int_{-\infty}^{\infty} dx\, \delta\left(x^2-\pi^2\right)\cos x$$

evaluates to

(a) $-1$ (b) $0$ (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

55. What is the value of the following series? $\left(1-\frac{1}{2!}+\frac{1}{4!}-...\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}-...\right)^2$

(a) $0$ (b) $e$ (c) $e^2$ (d) $1$

56. If the distribution function of $x$ is $f(x)=xe^{-x/\lambda}$ over the interval $0<x<\infty$, the mean value of $x$ is

(a) $\lambda$ (b) $2\lambda$ (c) $\lambda/2$ (d) $0$

57. The expression

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$$ is proportional to

(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$ (d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

58. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij}=A_iB_j$

(a) reduces to a direct sum of three 3-dimensional representations

(b) is an irreducible 9-dimensional representation

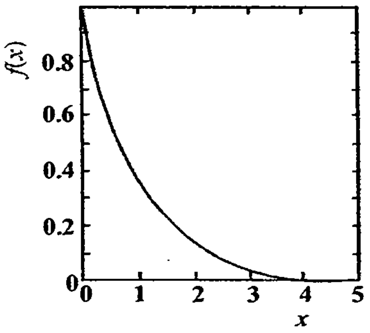
(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations

(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

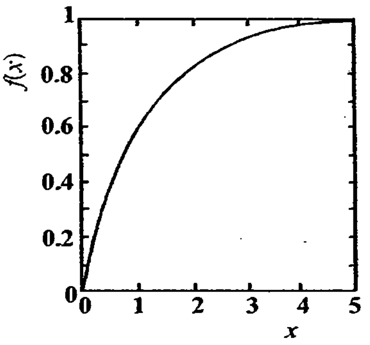
59. The solution of the integral equation $f(x)=x-\int_0^x dt\, f(t)$

has the graphical form

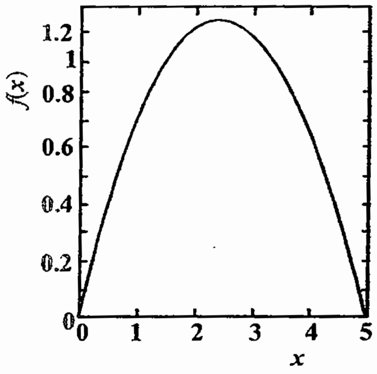
(a)



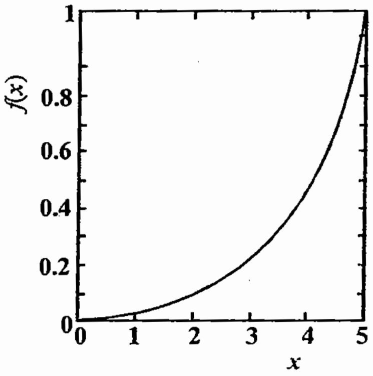
(b)



(c)



(d)



60. In the most general case, which one of the following quantities is NOT a second order tensor?
(a) Stress (b) Strain (c) Moment of inertia (d) Pressure

61. The following data is obtained in an expriment that measures the viscosity $\eta$ as a function of molecular weight M for a set of polymers.

| $M(Da)$ | $\eta(kPa-s)$ |
|---|---|
| 990 | $0.28 \pm 0.03$ |
| 5032 | $30 \pm 2$ |
| 10191 | $250 \pm 10$ |
| 19825 | $2000 \pm 200$ |

The relation that best describes the dependence of $\eta$ on M is
(a) $\eta \sim M^{4/9}$ (b) $\eta \sim M^{3/2}$ (c) $\eta \sim M^{2}$ (d) $\eta \sim M^{3}$

62. The integral $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is
(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

## Miscellaneous_Answer Key

1. (a) 2. (c) 3. (d) 4. (a) 5. (b) 6. (c) 7. (b)
8. (b) 9. (b) 10. (c) 11. (b) 12. (b,d) 13. (b) 14. (a)
15. (b) 16. (a) 17. (d) 18. (c) 19. (b) 20. (c) 21. (a)
22. (d) 23. (a) 24. (d) 25. (d) 26. (a) 27. (b) 28. (d)
29. (c) 30. (b) 31. (b), (c) 32. (b) 33. (d) 34. (d) 35. (d)
36. (c) 37. (c) 38. (a) 39. (b) 40. (b) 41. (d) 42. ( )
43. (a) 44. (a) 45. (a) 46. (d) 47. (c) 48. (c) 49. (a)
50. (d) 51. (b) 52. (a) 53. (b) 54. (d) 55. (d) 56. (b)
57. (b) 59. (b) 60. (d) 61. (d) 62. (c)

# CSIR-NET/JRF & GATE

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# Assignment

## Solid State Physics

## (I & II)

# PHYSICAL SCIENCES

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South Delhi Centre: 28-A/11, Jia Sarai, Near-IIT Hauz Khas, New Delhi-110 016
T: 011-26851008, 26861009, (0) 9873744231, 9899628311

North Delhi Centre: 48, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T: 011-27241845, 65462244, (0) 9540292991
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PHYSICAL SCIENCES (JRF-NET/GATE)

## Assignment – I: Solid State Physics

1. The packing fraction of a simple cubic lattice is approximately:
(a) 0.74 (b) 0.68 [✓ (c) 0.52] (d) 0.34

2. Given that r is the radius of the atoms and a is the lattice constant of a solid having a cubic structure, which one of the following relations is true for a body centred cubic structure?
(a) a = 2r (b) $a = 2r\sqrt{2}$ (c) $a = 2r\sqrt{3}$ [✓ (d) $a = 4r/\sqrt{3}$]

3. The crystal structure of diamond is:
[✓ (a) fcc with two atom basis of (000) and $\frac{a}{4}(\hat{i}+\hat{j}+\hat{k})$]
(b) simple cubic with two atom basis of (000) and $\frac{a}{2}(\hat{i}+\hat{j}+\hat{k})$
(c) fcc with two atoms basis of (000) and $\frac{a}{2}(\hat{i}+\hat{j}+\hat{k})$
(d) bcc with one atom basis

4. The origin of van der Waals interaction of molecular crystals is:
(a) nuclear (b) magnetic (c) ionic [✓ (d) fluctuating dipolar]

5. In the Meissner state the magnetic susceptibility of a super conductor in S.I. units is:
(a) $-\frac{1}{4\pi}$ (b) $\frac{1}{4\pi}$ [✓ (c) $-1$] (d) $-\frac{1}{\mu_a}$

6. Which of the following crystals be represented by interpenetrating lattices:
(a) Sodium (bcc structure) [✓ (b) Sodium chloride]
[✓ (c) Cesium chloride] [✓ (d) Diamond]

7. What is the second nearest-neighbour distance in a face centred cubic lattice whose conventional unit cell parameter is a?
(a) $a/\sqrt{2}$ (b) a/2 [✓ (c) a] (d) $\sqrt{2}/a$

8. Magnetic long range order is typically exhibited by:
(a) noble metals (b) alkali metals
(c) inert gas solids [✓ (d) transition metals]

9. A metal with body centered cubic (bcc) structure shows the first (i.e. smallest angle) diffraction peak at Bragg angle of $\theta = 30°$. The wavelength of X-ray used is 2.1 Å. The volume of the PRIMITIVE unit cell of the metal is:
(a) 26.2 $(\text{Å})^3$ [✓ (b) 13.1 $(\text{Å})^3$] (c) 9.3 $(\text{Å})^3$ (d) 4.6 $(\text{Å})^3$.

10. The potential in a divalent solid at a particular temperature is represented by a one-dimensional periodic model. The solid should behave electrically as:
(a) semiconductor (b) a conductor
[✓ (c) an insulator] (d) a superconductor

11. In a cube system with cell edge a, two phonos with wave vectors $\vec{q}_1$ and $\vec{q}_2$ collide and produce a third phonon with a wave vector $\vec{q}_3$ such that:

$$\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R},$$

Where $\vec{R}$ is a lattice vector. Such a collision process will lead to:
[✓ (a)] finite thermal resistance
(b) zero thermal resistance
(c) an infinite thermal resistance
(d) a finite thermal resistance for certain $|\vec{R}|$ only

12. For an NaCl crystal, the cell-edge a = 0.542 nm. The smallest angle at which Bragg reflection can occur corresponds to a set of planes whose indices are
(a) 100 (b) 110 (c) 111 (d) 200

13. Consider the Fermi-Dirac distribution function f(E) at room temprature (300 K) where E refers to energy. If $E_F$ is the Fermi energy, which of the following is true?
(a) f(E) is a step function
[✓ (b)] f(EF) has a value of 1/2
(c) States with $E<E_F$ are filled completely
(d) f(E) is large and tends to infinity as E decreases much below $E_F$

14. The effective mass of an electron in a semiconductor can be:
(a) negative near the bottom of the band
(b) a scalar quantity with a small magnitude
(c) zero at the center of the band
[✓ (d)] negative near the top of the band

15. Which one of the following is TRUE for a semiconductor p–n junction with no external bias?
(a) The total charge in the junction is not conserved
(b) The p side of the junction is positively charged
[✓ (c)] The p side of the junction is negatively charged
(d) No charge develops anywhere in the junction

16. Consider the energy E in the first Brillouin zone as a function of the magnitude of the wave vector k for a crystal of lattice constant a. Then
[✓ (a)] the slope of E versus k is proportional to the group velocity
[✓ (b)] the slope of E versus k has its maximum value at $|k| = \pi/a$
(c) the plot of E versus k will be parabolic in the interval $(-\pi/a) < k < (\pi/a)$
(d) the slope of E versus k is non-zero for all k the interval $(-\pi/a) < k\ (\pi/a)$

17. The specific heat of an ideal Fermi gas in 3-dimension at very low temperature (T) varies as:
[✓ (a)] T (b) $T^{3/2}$ (c) $T^2$ (d) $T^3$

18. Which one of the following is the first order phase transition?
[✓ (a)] Vaporization of a liquid at its boiling point
(b) Ferromagnetic to paramagnetic }
(c) Normal liquid He to superfluid He } 2nd order
(d) Superconducting to normal state }

19. The C/a ratio for an ideal hexagonal closed packed structure is:
(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{8}$ (c) $\sqrt{5}$ [✓ (d)] $\sqrt{\frac{8}{3}}$

20. The effective mass of an electron in a semiconductor
(a) can never be positive (b) can never be negative
(c) can be positive or negative (d) depends on its spin

21. For a closed packed BCC structure of hard spheres, the lattice constant a is related to the sphere radius R as:
(a) $a = 4R/\sqrt{3}$ (b) $a = 4R\sqrt{3}$ (c) $a = 4R\sqrt{2}$ (d) $a = 2R\sqrt{2}$

22. An n-type semiconductor has an electron concentration of $3\times10^{20}\,m^{-3}$. If the electron drift velocity is 100 ms–1 in an electric field of 200 Vm–1, the conductivity (in $\Omega^{-1}m^{-1}$) of this material is:
(a) 24 (b) 36 (c) 48 (d) 96

23. For a conventional superconductor, which of the following statements is NOT true?
(a) Specific heat is discontinuous at transition temperature $T_c$
(b) The resistivity falls sharply at $T_c$
(c) It is diamagnetic below $T_c$
(d) It is paramagnetic below $T_c$

24. The target of an X-ray tube is subjected to an excitation voltage V. The wavelength of the emitted X-rays is proportional to:
(a) $\frac{1}{\sqrt{V}}$ for e (b) $\sqrt{V}$ (c) $\frac{1}{V}$ (d) V

25. Which one of the following is NOT a correct statement about semiconductors?
(a) The electrons and holes have different mobilities in a semiconductor
(b) In an n-type semiconductor, the Fermi level lies closer to the conduction band edge
(c) Silicon is a direct band gap semiconductor ( it is indirect)
(d) Silicon has diamond structure

26. Which one of the following axes of rotational symmetry is NOT permissible in single crystals?
(a) two fold axis (b) three-fold axis
(d) four-fold axis (d) five-fold axis

27. Which one of the following statements about superconductors is NOT true?
(a) A type I superconductor is completely diamagnetic
(b) All type II superconductor exhibits Meissner effect upto second critical magnetic field $(H_{C_2})$.
(c) A type II superconductor exhibits zero reistance upto the second critical magnetic field
(d) Both type I and type II superconductors exhibit sharp fall in resistance at the superconducting transition temperature. ( ∵ Tc is same for both)

28. A solid superconductor is placed in an external magnetic field and then cooled below its criticalk temperature. The superconductor.
(a) retains its magnetic flux becuase the surface current supports it.
(b) expels out its magnetic flux because it behaves like a paramagnetic material.
(c) expels out its magnetic flux because it behaves like an anti-ferromagnetic material
(d) expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field.

behaves as (diamagnetic)

29. Consider the atomic packing factor (APF) of the following crystal structures:

**P.** Simple Cubic　　**Q.** Body Centred Cubic

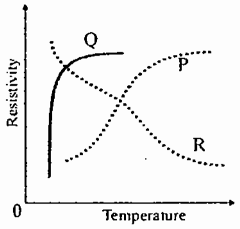
**R.** Face Centred Cubic　　**S.** Diamond

**T.** Hexagonal Close Packed

Which two of the above structures have equal APF?

(a) P and Q　(b) S and T　(c) R and S　[✓ (d) R and T]

30. Variation of electrical resistivity ρ with temperature T of three solids is sketched (on different scales) in the figure, as curves P, Q and R.



Which of the following statements describes the variations most appropriately?

(a) P is for a superconductor and R for a semiconductor

[✓ (b) Q is for a superconductor and P for a conductor]

(c) Q is for a superconductor and R for a conductor

(d) R is for a superconductor, and P for a conductor

**Statement for Linked Answer Q.31 and Q.32.**

The atomic density of a solid is $5.85\times10^{28}\ \text{m}^{-3}$. Its electrical resistivity is $1.6\times10^{-8}\,\Omega\,\text{m}$. Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

31. The drift mobility (in $m^2V^{-1}s^{-1}$) of the conduction electrons is:

[✓ (a) $6.67\times10^{-3}$]　(b) $6.67\times10^{-6}$　(c) $7.63\times10^{-3}$　(d) $7.63\times10^{-6}$

32. The relaxation time (mean free time), in seconds, of the conduction electrons is:

(a) $3.98\times10^{-15}$　[✓ (b) $3.79\times10^{-14}$]　(c) $2.84\times10^{-12}$　(d) $2.64\times10^{-11}$

33. Metallic monovalent sodium crystallizes in body centered cubic structure. If the length of the unit cell is $4\times10^{-8}$ cm, the concentration of conduction electrons in metallic sodium is:

(a) $6.022\times10^{23}\ \text{cm}^{-3}$　[✓ (b) $3.125\times10^{22}\ \text{cm}^{-3}$]

(c) $2.562\times10^{21}\ \text{cm}^{-3}$　(d) $1.250\times10^{20}\ \text{cm}^{-3}$

34. The primitive translation vectors of the body centered cubic lattice are $\vec{a}=\frac{a}{2}(\hat{x}+\hat{y}-\hat{z})$, $\vec{b}=\frac{a}{2}(\hat{x}+\hat{y}+\hat{z})$, and $\vec{c}=\frac{a}{2}(\hat{x}-\hat{y}+\hat{z})$. The primitive translation vectors $\vec{A},\vec{B}\,and\,\vec{C}$ of the reciprocal lattice are:

(a) $\vec{A}=\frac{2\pi}{a}(\hat{x}-\hat{y});\ \vec{B}=\frac{2\pi}{a}(\hat{y}+\hat{z});\vec{C}=\frac{2\pi}{a}(\hat{x}+\hat{z})$

(b) $\vec{A}=\frac{2\pi}{a}(\hat{x}+\hat{y});\ \vec{B}=\frac{2\pi}{a}(\hat{y}-\hat{z});\vec{C}=\frac{2\pi}{a}(\hat{x}+\hat{z})$

(c) $\vec{A}=\frac{2\pi}{a}(\hat{x}+\hat{y});\ \vec{B}=\frac{2\pi}{a}(\hat{y}+\hat{z});\vec{C}=\frac{2\pi}{a}(\hat{x}-\hat{z})$

[✓ (d) $\vec{A}=\frac{2\pi}{a}(\hat{x}+\hat{y});\ \vec{B}=\frac{2\pi}{a}(\hat{y}+\hat{z});\vec{C}=\frac{2\pi}{a}(\hat{x}+\hat{z})$]

35. The structure factor of a single cell of identical atoms of form factor f is given by

$Shkl = f\sum_{1}\exp\left(-i2\pi\left(x_jh+y_jk+z_jl\right)\right)$ where $(x_j, y_j, z_jl)$ is the coordinate of an atom, and hkl are the Miller indices. Which one of the following statement is correct for the diffraction peaks of body centered cubic (BCC) and face centered cubic (FCC) lattices?

[✓ (a)] BCC : (200; (110); (222) FCC : (111); (311); (400)
(b) BCC : (210); (110); (222) FCC : (111); (311); (400)
(c) BCC : (200); (110); (222) FCC : (111); (211); (400)
(d) BCC : (200); (210); (222) FCC : (111); (211); (400)

36. The kinetic energy of a free electron at a corner of the first Brillouin zone of a two dimensional square lattice is larger than that of an electron at the mid-point of a side of the zone by a factor b. The value of b is:

(a) $b=\sqrt{2}$ [✓ (b)] b = 2 (c) b = 4 (d) b = 8

37. Which of the following bonds has more directional:
(a) Ionic bonds [✓ (b)] Covalent (c) Metalic (d) Vander Waals

38. If there are n-atoms in the primitive cell, the phonon dispersion relation had optical branches:-
[✓ (a)] 3n – 3 (b) 3n – 2 (c) 3n – 1 (d) none

39. Silicon has diamond structure with unit-cell-edge a = 0.542 nm. The interatomic separation is :
(a) 0.122 nm [✓ (b)] 0.234 nm (c) 0.383 nm (d) 0.542 nm.

40. A metal has free-electron density n = $10^{29}$ $m^{-3}$. Which of the following wavelengths will excite plasma oscillations?
[✓ (a)] 0.033 μm (b) 0.330 μm (c) 3.300 μm (d) 33.000 μm

41. The critical magnetic field for a solid in superconducting state
(a) Does not depend upon temperature
(b) Increases if the temperature
[✓ (c)] Increases if the temperature decreases
(d) Does not depend on the transition temperature.

42. Which one of the following statements is NOT TRUE?
(a) Entropy decreases markedly on cooling a superconductor below the critical temperature, $T_C$.
(b) The electronic contribution to the heat capacity in the superconducting state has an exponential form with an argument proportional to $T^{-1}$, suggestive of an energy gap
(c) A type I superconductor is a perfect diamagnet.
[✓ (d)] Critical temperature of superconductors does not vary with the isotopic mass.

43. The valence electrons do not directly determine the following property of a metal.
(a) Electrical conductivity (b) Thermal conductivity
[✓ (c)] Shear modulus (mechanical property) (d) metallic lustre.

44. Consider X-ray diffraction from a crystal with a face-centered-cubic (fcc) lattice. The lattice. The lattice plane for which there is NO diffraction peak is:
[✓ (a)] (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

45. The Hall coefficient, $R_H$, of sodium depends on
(a) The effective charge carrier mass and carrier density.
(b) The charge density and relaxation time.
[✓ (c)] The charge carrier density only.
(d) The effective charge carrier mass.

46. For a two-dimensional free electron gas, the electronic density n, and the Fermi energy $E_F$, are related by

(a) $n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ [✓ (b)] $n = \frac{mE_F}{\pi\hbar^2}$ (c) $n = \frac{mE_F}{2\pi\hbar^2}$ (d) $n = \frac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

47. Identify which one is a first order phase transition?
[✓ (a)] A liquid to gas transition at its critical temperature.
(b) A liquid to gas transition close to its triple point.
(c) A paramagnetic to ferromagnetic transition in the absence of a magnetic field.
(d) A metal to superconductor transition in the absence of a magnetic field.

48. A superconducting ring is cooled in the presence of a magnetic field below its critical temperature ($T_C$). The total magnetic flux that passes through the ring is:

(a) zero [✓ (b)] $n\frac{h}{2e}$ (c) $\frac{nh}{4\pi e}$ (d) $\frac{ne^2}{hc}$

49. In a cubic crystal, atoms of mass $M_1$ lie on one set of planes and atoms of mass $M_2$ lie on planes interleaved between those of the first set. If C is the force constant between nearest neighbour planes, the frequency of lattice vibrations for the optical phonon branch with wavevector k = 0 is:

[✓ (a)] $\sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (b) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (c) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{2M_2}\right)}$ (d) 0

50. For a Fermi gas of N particles in three dimensions at T = 0 K, the Fermi energy, $E_F$

[✓ (a)] $N^{2/3}$ (b) $N^{3/2}$ (c) $N^3$ (d) $N^2$

51. Compared to a p-n junction with $N_A = N_D = 10^{14}/cm^3$, which one of the following statement is TRUE for a p-n junction with $N_A = N_D = 10^{20}/cm^3$?
(a) Reverse breakdown voltage is lower and depletion capacitance is lower.
(b) Reverse breakdown voltage is higher and depletion capacitance is lower.
[✓ (c)] Reverse breakdown voltage is lower and depletion capacitance is higher.
(d) Reverse breakdown voltage is higher and depletion capacitance is higher.

52. The ratio of the mobility to the diffusion coefficient in a semiconductor has the units:

[✓ (a)] $V^{-1}$ (b) $cm.V^{-1}$ (c) $V.cm^{-1}$ (d) $V.s$

53. Silicon is doped with boron to a concentration of $4\times10^{17}$ atoms/cm$^3$. Assume the intrinsic carrier concentration of silicon to be $1.5\times10^{10}$/cm$^3$ and the value of kT/q to be 25 mV at 300 K. Compared to un-doped silicon, the Fermi level of doped silicon:
(a) Goes down by 0.13 eVs (b) Goes up by 0.13 eV
[✓ (c)] Goes down by 0.427 eV (d) Goes up by 0.427 eV

54. The electron and hole concentrations in an intrinsic semiconductor are $n_i$ per cm$^3$ at 300K. Now, if acceptor impurities are introduced with a concentration of $N_A$ per cm$^3$ (where $N_A >> n_i$), the electron concentration per cm$^3$ at 300 K will be:

(a) $n_i$ (b) $n_i + N_A$ (c) $N_A - n_i$ [✓ (d)] $\frac{n_i^2}{N_A}$

55. In a $p^+$ n junction diode under reverse bias, the magnitude of electric field is maximum at:
(a) The edge of the depletion region on the p-side (b) The edge of the depletion region on the n-side
[✓ (c)] The $p^+$ n junction (d) The centre of the depletion region on the n-side.

56. A $P^+$–N junction has a built-in potential of 0.8 V. The depletion layer width at a reverse bias of 1.2 V is 2 $\mu m$. For a reverse bias of 7.2 V, the depletion layer width will be:
[✓ (a)] 4 $\mu m$ (b) 4.9 $\mu m$ (c) 8 $\mu m$ (d) 12 $\mu m$

57. The majority carriers in n-type semiconductor have an average drift velocity V in a direction perpendicular to a uniform magnetic field B. The electric field E induced due to Hall effects acts in the direction:
(a) V×B [✓ (b) B×V] (c) along V (d) opposite to V

58. The bandgap of silicon at room temperature is:
(a) 1.3 eV (b) 0.7 eV [✓ (c) 1.1 eV] (d) 1.4 eV

59. A silicon PN junction diode under reverse bias has depletion region of width 10 $\mu m$. The relative permitivity of Silicon, $\varepsilon_r = 11.7$ and the permitivity of free space $\varepsilon_o = 8.85\times10^{-12}\ F/m$. The depletion capacitance of the diode per square metter is:
(a) 100 $\mu F$ [✓ (b) 10 $\mu F$] (c) 1 $\mu F$ (d) 20 $\mu F$

60. In an abrupt p-n junction, the doping concentrations on the p-side and n-side are $N_A = 9\times10^{16}/cm^3$ and $N_D = 1\times10^{16}\ /cm^3$ respectively. The p-n junction is reverse biased and the total depletion width is 3 $\mu m$. The depletion width on the p-side is:
(a) 2.7 $\mu m$ [✓ (b) 0.3 $\mu m$] (c) 2.25 $\mu m$ (d) 0.75 $\mu m$

61. The longest wavelength that can be absorbed by silicon, which has the band-gap of 1.12 eV, is 1.1$\mu m$. If the longest wavelength that can be absorbed by another material is 0.87 $\mu m$, then the bandgap of this material is:
[✓ (a) 1.416 eV] (b) 0.886 eV (c) 0.854 eV (d) 0.706 eV

62. N-type silicon bar 0.1 cm long and 100 $\mu m^2$ in cross-sectional area has a majority carrier concentration of $5\times10^{20}/cm^3$ and the carrier mobility is 0.13 $m^2/V$-s at 300 K. If the charge of an electron is $1.6\times10^{-19}$ coulomb, then the resistance of the bar is:
[✓ (a) $10^6$ ohm] (b) $10^4$ ohm (c) $10^{-1}$ ohm (d) $10^{-4}$ ohm

63. A particular green LED emits light of wavelength 5490Å. The energy bandgap of the semiconductor material used there is (Planck's constant = $6.626\times10^{-34}$ J-s):
[✓ (a) 2.26 eV] (b) 1.98 eV (c) 1.17 eV (d) 0.74 eV

64. A long specimen of p-type semiconductor material:
(a) Is positively charged [✓ (b) Is electrically neutral]
(c) Has an electric field directed along its length (d) Acts as a dipole

65. The probability that an electron in a metal occupies the Fermi-level at any temperature (> 0 K):
(a) 0 (b) 1 [✓ (c) 0.5] (d) 1.0

66. In a P-type silicon sample, the hole concentration is $2.25\times10^{15}/cm^3$. If the intrinsic carrier concentration is $1.5\times10^{10}/cm^3$, the electron concentartion is:
(a) Zero (b) $10^{10}/cm^3$ [✓ (c) $10^5/cm^3$] (d) $1.5\times10^{25}/cm^3$

67. An infra-red LED is usually fabricated from:
(a) Ge (b) Si [✓ (c) Ga As] (d) Ga As P

68. Which of the following statements is/are true for a good conductor of electricity?
1. Its conductivity decreases with increasing temperature.
2. Number of free electrons is around $10^{28} m^{-3}$.
3. Its conductivity decreases with addition of impurities.
4. It is a good conductor of heat also.
[✓ (a) 1, 2, 3 and 4] (b) 1 only (c) 2 and 3 only (d) 3 and 4 only

69. Consider the following statements:
The conductivity of a metal has negative temperature coefficient since:
1. The electron concentration increases with temperature. (Remain constant)
2. The electron mobility decreases with temperature. $\longrightarrow \mu \propto T \propto T^{-1}$
3. The electron-lattice scattering rate increases with temperature.

Which of the above statements is/are correct?
(a) 1 only (b) 1 and 2 [✓ (c) 2 and 3] (d) 3 only

70. Which one of the following is a trivalent material?
(a) Antimony (b) Phosphorus (c) Arsenic [✓ (d) Boron]

71. Consider the following:
1. Si 2. Ge 3. GaAs 4. In P
Which of the above semiconductors should be used for making highly efficient photodiodes?
(a) 1 and 4 only [✓ (b) 3 and 4 only] (c) 1, 3 and 4 (d) 2, 3 and 4.

72. The materials not having negative temperature coefficient of resistivity are
[✓ (a) Metals] (b) Semiconductors (c) Insulators (d) None of the above.

73. In a materials, the Fermi level is located between the centre of the forbidden band and the conduction band. Then what is that material?
(a) A p-type semiconductor (b) An n-type semiconductor
[✓ (c) An intrinsic semicondutor] (d) An insulator.

74. An intrinsic semiconductor with energy gap 1 eV has a carrier cocentration N at temperature 200 K. Another intrinsic semiconductor has the same value of carrier concentration N at temperature 600K. What is the energy gap value for the second semiconductor?
(a) (1/3) eV (b) (3/2) eV [✓ (c) 3 eV] (d) 9 eV

75. Consider the following statements:
n-type of silicon can be
1. Formed by adding impurity of phosphorus
2. Formed by adding impurity of arsenic
3. Formed by adding impurity of boron.
4. Formed by adding impurity of aluminium.
Which of the statements given above are correct?
(a) 1 and 3 only (b) 3 and 4 only [✓ (c) 1 and 2 only] (d) 1, 2, 3 and 4.

76. Consider the following statements for a photo conducting material:
1. Its dark conductivity is small.
2. With the absorption of radiation, equal numbers of electrons and of holes are produced.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only [✓ (c) Both 1 and 2] (d) Neither 1 nor 2.

77. In a step-graded p-n junction diode, what is the ratio of depletion-region penetration depths into p and n regions (if the ratio of acceptor to donor impurity atoms densities is 1:2)?
[✓ (a) 2:1] (b) 4:1 (c) 1:2 (d) 1:4

78. Match **List-I** with **List-II** and select the corret answer using the code given below the lists:

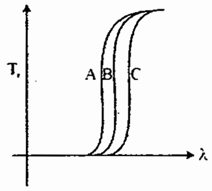
| **List-I** **(Item)** | **List-II** **(Position)** |
|---|---|
| A. Donor energy band | 1. At the middle of the forbidden energy gap. |
| B. Fermi level of p-type semiconductor at room temperature | 2. Close to the conduction band. |
| C. Acceptor energy band | 3. Very close to the valance band. |
| D. Fermi level in intrinsic semiconductor | 4. Close to the valence band. |

**Code:**

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 4 | 3 | 2 | 1 |
| (b) | 2 | 1 | 4 | 3 |
| (c) | 4 | 1 | 2 | 3 |
| [✓ (d)] | 2 | 3 | 4 | 1 |

79. The depletion layer in a p-n junction is made of which of the following?
(a) Ionized donors in p-side and ionized acceptors in n-side.
(b) Ionized acceptors in p-side and ionized donors in n-side.
(c) Accumulated holes in p-side and accumulated electrons in n-side.
(d) Accumulated electrons in p-side and accumulated holes in n-side.

80. Photovoltaic devices may be made from various semiconductors as optical materials for which the transmission coefficient ($T_r$) is shown in the figure given below as a function of photon wavelength.



Which one of the following is the correct sequence of A, B and C pertaining to Ge, Ga, As and Si, respectively?
(a) A–B–C (b) B–A–C (c) C–A–B (d) C–B–A

81. Match the **List-I**(Current) with **List-II** (Variation) and select the correct answer using the code given below the lists:

| **List-I (Current)** | **List-II (Variation)** |
|---|---|
| A. Hole deffusion current | 1. n.E B |
| B. Electron drift current | 2. p.E C |
| C. Hole drift current | 3. –dp/dx A |
| D. Electron diffusion current | 4. dn/dx D |

(a) A-2, B-1, C-3, D-4 (b) A-3, B-4, C-2, D-1
(c) A-2, B-4, C-3, D-1 (d) A-3, B-1, C-2, D-4

82. Photons of energy $1.53\times10^{-19}$ Joule are incident on a photodiode which has a responsivity of 0.65 A/W. If the optical power level is 10 $\mu$W. What is the photo current generated?
(a) 64 nA (b) 1.5 $\mu$A (c) 2.1 $\mu$A (d) 6.5 $\mu$A

83. A thin resistive film having length (l), width (w) and thickness (t) is used as a resistor ($R_1$) for conducting current in the direction of l. If the film is connected to serve as a resistor ($R_2$) for conducting current in the direction of w, then what is the value of $R_1/R_2$.
(a) l/w (b) w/l (c) $l^2/w^2$ (d) $w^2/l^2$.

84. **Match List-I (Type of Material)** with **List-II (Type of Bonding)** and the select the correct answer using the codes given below the lists:

| **List-I (Typed of Material)** | **List-II (Type of Bonding)** |
|---|---|
| A. Elemental semiconductor | 1. Ionic D |
| B. Hydrogen molecule | 2. Covalent A |
| C. Copper | 3. Van der Waals' B |
| D. AgI | 4. Metallic. C |

**Codes:**
(a) A-3 B-2 C-1 D-4 (b) A-2 B-3 C-1 D-4
(c) A-2 B-3 C-4 D-1 (d) A-3 B-4 C-2 D-1

85. An LED made usig GaAs emits radiation in:
(a) Visile region (b) Ultraviolet region
(c) Infra red region (d) Microwave frequency region.

86. The light emitting diode (LED) emits light of a particular colour because
(a) It is fabricated from a fluorescent material.
(b) Transition between energy levels of the carriers takes place while crossing the p-n junction.
(c) Heat generated in the diode is converted into light.
(d) The band gap of the semi-conductor material used in fabrication of the diode is equal to the energy $h_\nu$ of the light photon.

87. Depletion capacitance in a diode depends on:
1. Applied junction voltage.
2. Junction built-in potential
3. Current through junction
4. Doping profile across the junction
Select the correct answer using the codes given below:
(a) 1 and 2 (b) 1 and 3 [✓ (c) 1, 2 and 4] (d) 2, 3 and 4

88. The depletion layer across a $p^+$-n junction lies
(a) mostly in the p region
[✓ (b) mostly in the n region]
(c) equally in both $p^+$ and n regions
(d) Entirely in the $p^+$ region.

89. The Ohm's law for conduction in metals is:
[✓ (a) $J = \sigma E$] (b) $J \propto \sigma E$ (c) $J = E/\sigma$ (d) $J \propto E/\sigma$

90. Match **List-I (Crystal type)** with **List-II (Name of the solid)** and select the correct answer using the codes given below the Lists:

| **List-I** | **List-II** |
|---|---|
| A. Ionic | 1. Solid argon |
| B. Covalent | 2. Copper |
| C. Metallic | 3. Silicon |
| D. Van der Waal's | 4. Sodium chloride |

**Codes:**
(a) A-3 B-4 C-2 D-1
(b) A-4 B-3 C-1 D-2
(c) A-3 B-4 C-1 D-2
[✓ (d) A-4 B-3 C-2 D-1]

91. The carrier mobility in a semiconductor is 0.4 $m^2/Vs$. Its diffusion constant at 300 K will be (in $m^2/s$)
(a) 0.43 (b) 0.16 (c) 0.04 [✓ (d) 0.01]

92. GaAs has an energy gap 1.43 eV. The optical cut-off wavelength of GaAs would lie in the
(a) Visible region of the spectrum
[✓ (b) Infrared region of the spectrum]
(c) Ultraviolet region of the spectrum
(d) Far ultraviolet region of the spectrum

93. At very high temperatures, the extrinsic semiconductors become intrinsic because
(a) Of drive-in diffusion of dopants and carries
[✓ (b) Band to band transition dominates over impurity ionization.]
(c) Impurity ionization dominates over band to band transition.
(d) Band to band transition is balanced by impurity ionization

94. Given that the band gap of cadmium sulphide is 2.5 eV, the maximum photon wavelength, for electron-hole pair generation will be
(a) 5400 μm (b) 540 μm [✓ (c) 5400Å] (d) 540 Å

95. The increasing temperature, the electrical conductivity would:
(a) Increase in metals as well as in intrinsic semiconductor
(b) Increase in metals but decrease in intrinsic semiconductor
[✓ (c) Decrease in metals but increase in intrinsic semiconductor]
(d) Decrease in metals as well as in intrinsic semiconductors.

96. Which one of the following statements with reference to effective mass is incorrect?
(a) It is a function of wave vector K
(b) It can be positive or negative
(c) It is different from free mass because of lattice interaction
[✓ (d) Its concept is applicable only to electrons and not to holes.]

97. Hall effect is observed in a specimen when it (metal or a semiconductor) is carrying current and is placed in a magnetic field. The resultant electric field inside the specimen will be in
[✓ (a) A direction normal to both current and magnetic field.]
(b) The direction of current
(c) A direction antiparallel to magnetic field
(d) An arbitrary direction depending upon the conductivity of the specimen.

98. The minority carrier life-time and diffusion constant in a semiconducting material are respectively 100 microsecond and 100 cm$^2$/s. The diffusion length of the carriers is:

[✓ (a) 0.1 cm] (b) 0.01 cm (c) 0.0141 cm (d) 1 cm

99. The wavelength of light emitted by a GaAs laser s $8670\times10^{-10}$ m. Given Planck's constant = $6.626\times10^{-34}$ Js, velocity of light = $2.998\times10^{8}$ m/s and 1 eV = $1.602\times10^{-19}$ J, the energy gap in GaAs is

(a) 0.18 eV (b) 0.7 eV [✓ (c) 1.43 eV] (d) 2.39 eV

100. The difference between the electron and hole of a semiconductor laser is 1.5 eV and the band gap of the semiconductor is 1.43 eV. The upper and lower frequency limits of the laser will be respectively

(a) $3.3\times10^{14}$ and $9.9\times10^{13}$ Hz [✓ (b) $3.7\times10^{14}$ and $3.5\times10^{14}$ Hz]

(c) $6.28\times10^{14}$ and $3.1\times10^{14}$ Hz (d) $1.16\times10^{15}$ and $8.97\times10^{14}$ Hz

101. The order of magnitude of the energy gap of a typical superconductor is:

(a) 1 MeV (b) 1 KeV (c) 1 eV [✓ (d) 1 meV]

102. For a three-dimensional crystal having N primitive unit cells with a basis of p atoms, the number optical branches is:

(a) 3 (b) 3p [✓ (c) 3p–3] (d) 3N–3p

103. For a intrinsic semiconductor, $m_e^*$ and $m_h^*$ are respectively the effective masses of electrons and holes near the corresponding band edges. At a finite temperature, the position of hte Fermi level

(a) Depends on $m_e^*$ but not on $m_h^*$ (b) Depends on $m_h^*$ but not on $m_e^*$

[✓ (c) Depends on $m_e^*$ and $m_h^*$] (d) depends neither on $m_e^*$ nor on $m_h^*$

# Assignment-II: Solid State Physics

1. Ionic crystals are very good insulators at room temperature conducting at higher temperatures. This is due to:
(a) Thermal expansion of the ionic solid
(b) Generation of more free electrons
(c) Transition of large number of electrons from valence band
(d) Movement of larger number of ions in vacancies

2. At low temperature the specific heat of insulating crystals varies as:
(a) $AT^3$ (b) $BT + CT^3$
(c) $D \exp (E/T)$ (d) will not change with temperature

3. The band gap at 300°K of diamond cubic crystals of C, Si, Ge and Sn is 5.5, 1.2, 0.67 and 0.1 eV respectively. Identify the crystal which will have the highest electrical condictivity at 300 K
(a) Ge (b) Sn (c) C (d) Si

4. The $\theta - 2\theta$ X-ray powder diffraction pattern of ionic crystals KCl and KBr both of which form fcc structure will have:
(a) same number of diffraction lines with equal intensities
(b) pattern of KBr will have more lines
(c) Pattern of KCl will have more lines
(d) number of lines will be same but their intensities will be different

5. A lattice is characterized by the following primitive vectors (in angstroms) :
$\vec{a} = 2(\hat{i} + \hat{j}), \vec{b} = 2(\hat{j} + \hat{k}), \vec{c} = 2(\hat{k} + \hat{i})$. The reciprocal lattice corresponding to the above is:
(a) body centered cubic lattice with cube edge $\pi$ Å$^{-1}$
(b) body centered cubic lattice with cube edge $2\pi$ Å$^{-1}$
(c) face centered cubic lattice with cube edge $\pi$ Å$^{-1}$
(d) face centered cubic lattice with cube edge $2\pi$ Å$^{-1}$

6. Experimental study shows that there exists an energy gap in a superconductor. Which one of the following statements is incorrect?
(a) The gap separates the lowest excited state in a superconductor from the ground state.
(b) The temperature dependence of electronic specific heat indicates the existence of an energy gap.
(c) The existence of an energy gap is accounted for in the BCS theory.
(d) The energy gap in supercondocturs is similar to the energy gap which occurs in insulators.

7. The resistivity of metals: ( below debey Temp) ($T < \theta_D$)
(a) Always varies as $T^5$
(b) is independent of temperature
(c) increases liniearly with temperature at room temperature and above ✓
(d) attains a constant value as temperature tends to absolute zero

8. Magnons in ferromagnets: (To decrease the free energy (To stabilize))
(a) decrease the magnetization
(b) increase the magnetization
(c) stabilize the magnetization
(d) cause critical magnetic fluctuations

9. X-rays with a wave vector $\vec{K}$ are scattered from a simple cubic lattice with lattice spacing a = 2π. The scattered X-rays have wave vector $\vec{K}$. The possible values of $\Delta K_1 = K_x - K'_x$ for which there are peaks in the scattered intensity are:
(a) $0 \leq \Delta K_1 \leq p/4$
[✓ (b)] $\Delta K_x$ = integer
(c) $\Delta K_x$ = integer multiple of 2π
(d) $\Delta K_x$ = integer/2π

10. For an energy state E of a photon gas, the density of states is proportional to:
(a) $\sqrt{E}$
(b) E
(c) $E^{3/2}$
[✓ (d)] $E^2$

11. In a one-dimensional Kroning Penny model, the total number of possible wave functions is equal to:
(a) twice the number of unit cells
[✓ (b)] number of unit cells
(c) half the number of unit cells
(d) independent of the number of unit cells

12. For a perfect free-electron gas in a metal, the magnitudes of phase velocity ($v_p$) and group velocity ($v_g$) are such that:
(a) $v_p = v_g$
[✓ (b)] $v_p = \frac{1}{2} v_g$
(c) $v_p = \sqrt{2} v_g$
(d) $v_p = 2 v_g$

13. A second order phase transition is one in which:
(a) the plot of entropy as a function of temeprature shows a discontinuity
[✓ (b)] the plot of specific heat as a function of temperature shows a discontinuity
(c) the plot of volume as a function of pressure shows a discontinuity
(d) the plot of comprehensibility as a function of temperature is continuous

14. A cubic cell consists of two atoms of masses $m_1$ and $m_2$ ($m_1 > m_2$) with $m_1$ and $m_2$ atoms situated on alternate planes. Assuming only nearest neighbour interactions, the center of mass of the two atoms:
(a) moves with the atoms in the optical mode and remains fixed in the acoustic mode
[✓ (b)] remains fixed in the optical mode and moves with the atoms in the acoustic mode
(c) remains fixed in both optical and acoustic modes
(d) moves with the atoms in both optical and acoustic modes

15. In simple metals the phonon contribution to the electrical resistivity at temperature T is:
[✓ (a)] directly proportional to T above Debye temperature and to $T^3$ below it.
(b) inversely proportional to T for all temperatures
(c) independent of T for all temperatures
(d) directly proportional to T above Debye temperature and to $T^5$ below it

16. An external magnetic field of magnitude H is applied to a Type-I superconductor at a temperature below the transition ponit. Then which one of the following statements is NOT true for H less than the ciritical field $H_C$?
(a) the sample is diamagnetic
(b) it magnetization varies linearly with H
(c) the lines of magnetic induction are pushed out from the sample
[✓ (d)] the sample exhibits mixed states of magnetization near $H_C$ (Type II)

17. A ferromagnetic material has a Curie temperature 100 K. Then:
(a) its succeptibility is double when it is cooled from 300 K to 200 K
(para) → (b) all the atomic magnets in it get oriented in the same direction above 100 K
(c) the plot of inverse susceptibility versus temperature is linear with a slope $T_C$ (intercept $T_C$)
(d) the plot if its succeptibility versus temperature is linear with an intercept $T_C$

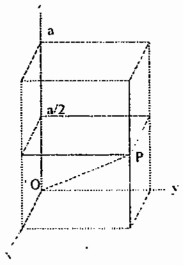
**Common Data for Q. 18 and Q. 19.**

Acrystal belongs to a face centered cubic lattice with four atoms in the unit cell. The size of the crystal is 1 cm and its unit cell dimension is 1 nm. f is the scattering factor of the atom.

18. The number of atoms in the crystal is:
(a) $2 \times 10^{21}$ [✓ (b) $4 \times 10^{21}$] (c) $2 \times 10^{23}$ (d) $4 \times 10^{24}$

19. The structure factors for (0 1 0) and (2 0 0) reflections respectively are:
(a) 2 f and zero [✓ (b) zero and 4 f] (c) 2 f and 2 f (d) zero and zero

20. In crystallographic notation the vector $\overline{OP}$ in the cubic cell shown in the figure is:



[✓ (a) [2 2 1]] (b) [1 2 2] (c) [1 2 1] (d) [1 1 2]

21. The de Broglie wavelength λ for an electron of energy 150 eV is:
(a) $10^{-8}$ m [✓ (b) $10^{-10}$ m] (c) $10^{-12}$ m (d) $10^{-14}$ m

22. The energy of a ferromagnet as a function of magnetization M is given by:

$$F(M) = F_0 + 2(T - T_C)\,M_2 + M_4,\ F_0 > 0$$

The number of minima in the function F(M) for $T > T_C$ is:
(a) 0 [✓ (b) 1] (c) 3 (d) 4

23. Density of states of free electrons in a solid moving with an energy 0.1 eV is given by $2.15 \times 10^{21}$ $eV^{-1}$ $cm^{-3}$. The density of states in $eV^{-1}$ $cm^{-3}$) for electrons moving with an energy of 0.4 eV will be:
(a) $1.07 \times 10^{21}$ (b) $1.52 \times 10^{21}$ (c) $3.04 \times 10^{21}$ [✓ (d) $4.30 \times 10^{21}$]

24. The effective density of states at the conduction band edge of Ge is $1.04 \times 10^{19}$ $cm^{-3}$ at room temeprature 300 K). Ge has an optical bandgap of 0.66 eV. The intrinsic carrier concentration (in $cm^{-3}$) in Ge at room temperature (300 K) is approximately:
(a) $3 \times 10^{10}$ [✓ (b) $3 \times 10^{13}$] (c) $3 \times 10^{16}$ (d) $6 \times 10^{16}$

**Statement for Linked Answer Q. 25 and Q. 26.**

Lead has atomic weight of 207.2 amu and density of 11.35 gm $cm^{-3}$.

25. Number of atoms per $cm^3$ for lead is:
(a) $1.1 \times 10^{25}$ [✓ (b) $3.3 \times 10^{22}$] (c) $1.1 \times 10^{22}$ (d) $3.3 \times 10^{25}$

26. If the energy of vacancy formation in lead is 0.55 eV/atom, the number of vacancies $cm^3$ at 500K is:
(a) $3.2 \times 10^{16}$ (b) $3.2 \times 10^{19}$ (c) $9.5 \times 10^{19}$ (d) $9.5 \times 10^{16}$

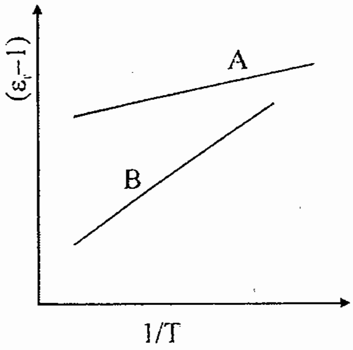
27. Which one of the following statements is NOT correct about the Brillouin zones (BZ) of a square lattices with constant a?
(a) The first BZ is a square of size $2\pi/a$ in $k_x$–$k_y$ plane
(b) The areas of the first BZ and third BZ are the same
(c) The k-points are equidistant in $k_x$ as well as in $k_y$ directions
[✓ (d) The area of the second BZ is twice that of the first BZ]

28. In a crystal of N primitive cells, each cell contains two monovalent atoms. The highest occupied energy band of the crystal is:

(a) one-fourth filled (b) one-third filled

(c) half filled [✓ (d) completely filled]

29. Two dielectric materials A and B exhibit both ionic and orientational polarizabilities. The variation of the susceptibilities c(=$\varepsilon_r$−1) with temperature T is shown in the figure, where ε, is the relative dielectric constant. It can be inferred from the figure that:



(a) A is more polar and it has a smaller value of ionic polarizability than that of B

(b) A is more polar and it has a higher value of ionic polarizability than that of B

(c) B is more polar and it has a higher value of ionic polarizability than that of A

[✓ (d) B is more polar and it has a smaller value of ionic polarizability than that of A]

**Statement for Linked Answer Q.30 and Q.31.**

The powder diffraction pattern of a body centred cubic crystal is recorded by using $\mathrm{Cu}k_\alpha$ X-rays of wavelength 1.54Å.

30. If the (002) planes diffract at 60°, then lattice parameter is:

(a) 2.67 Å [✓ (b) 3.08 Å] (c) 3.56 Å (d) 5.34 Å

31. Assuming the atomic mass of the constituent atoms to be 50.94 amu, then density of the crystal in units of kg m$^{-3}$ is:

(a) 3.75 × $10^3$ (b) 4.45 × $10^3$ [✓ (c) 5.79 × $10^3$] (d) 8.89 × $10^3$

32. When liquid oxygen is poured down close to a strong bar magnet, the oxygen stream is:

(a) repliecd towards the lower field because it is diamagnetic

(b) attracted towards the higher field because it is diamagnetic

(c) repelled towards the lower field becaue it is paramagnetic

[✓ (d) attracted towards the higher field because it is paramagnetic]

33. In a powder differaction pattern recorded from a face-centred cubic sample using x-rays, the first peak appears at 30°. The second peak will appear at:

(a) 32.8° (b) 33.7° [✓ (c) 34.8°] (d) 35.3°

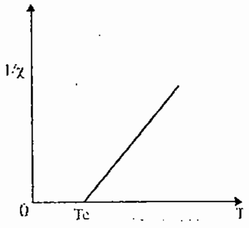
34. An extrinsic semiconductor sample of cross-section A and length L is doped in such a way that the doping concentration varies as $N_D(x) = N_0 \exp\left(-\frac{x}{L}\right)$, where $N_0$ is a constant. Assume that the mobility μ of the majority carriers remains constant. The resistance R of the sample is given by:

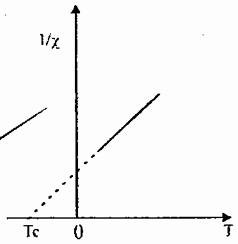
[✓ (a) $R = \frac{L}{A\mu e N_0}\left[\exp(1.0)-1\right]$] (b) $R = \frac{L}{\mu e N_0}\left[\exp(1.0)-1\right]$

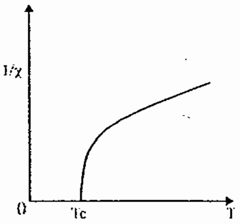
(c) $R = \frac{L}{A\mu e N_0}\left[\exp(-1.0)-1\right]$ (d) $R = \frac{L}{A\mu e N_0}$

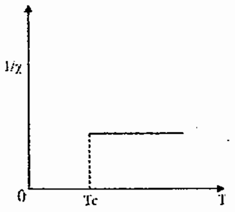
35. A ferromagnetic mixture of iron and copper having 75% atoms of Fe exhibits a saturation magnetization of $1.3 \times 10^6\,\mathrm{A\,m^{-1}}$. Assume that the total number of atoms per unit volume is $8 \times 10^{28}\,\mathrm{m^{-3}}$. The magnetic moment of an iron atom, in terms of the Bohr Magneton, is:
(a) 1.7 (b) 2.3 (c) 2.9 (d) 3.8

36. The plot of inverse magnetic susceptibility $\frac{1}{\chi}$ versus temperature T of an antiferromagnetic sample corresponds to:

(a)


[✓ (b)]


(c)


(d)


37. The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model. Einstein modle and Debye model. At low tesmperature $\hbar\omega >> k_BT$, which one of the following statements is true (a and A are constants)

(a) Dulong Petit : $C \propto \exp\left(\frac{-a}{T}\right)$; Einstein: $C = \text{constant}$; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(b) Dulong Petit : $C = \text{constant}$; Einstein: $C \propto \left(\frac{T}{A}\right)^3$ Debye : $C \propto \left(\frac{-a}{T}\right)$

[✓ (c)] Dulong Petit : $C = \text{constant}$; Einstein: $C \propto \frac{e^{-a/T}}{T^2}$ Debye : $C \propto \left(\frac{T}{A}\right)^3$

(d) Dulong Petit : $C \propto \left(\frac{T}{A}\right)^3$ ; Einstein: $C \propto \frac{e^{-a/T}}{T^2}$ Debye : $C = \text{constant}$

38. A linear diatomic lattice of lattice constant a with masses M and m (M > m) are coupled by a force constant C. The dispersion relation is given by:

$$\omega_{\pm}^2 = C\left(\frac{M+m}{Mm}\right) \pm \left[C^2\left(\frac{M+m}{M}\right)^2 - \frac{4C^2}{Mm}\sin^2\frac{ka}{2}\right]^{1/2}$$

Which one of the following statements is incorrect?
(a) The atoms vibrating in transverse modecorrespond to the optical branch
[✓ (b)] The maximum frequency of the acoustic branch depends on the mass of the lighter atom m.
Imp (c) The dispersion of frequency in the optical branch is smaller than that in the acoustic branch.
(d) No normal modes exist in the acoustic branch for any frequency greater than the maximum frequency at $k = \frac{\pi}{a}$.

39. Which is loose packed structure?
[✓ (a)] Body centred cubic structure (b) Hexagonal close-packed structure
(c) Face-centred cubic structure (d) None of these

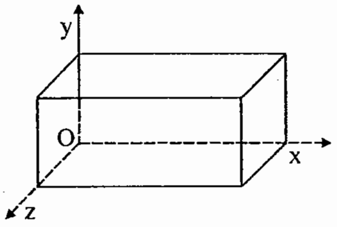
40. A doped Germanium crystal of length 2 cm, breadth 1 cm and width 1 cm, carries a current of 1 mA along its length parallel to +x axis. A magnetic field of 0.5 T is applied along +z axis. Hall voltage of 6 mV is measured with negative polarity at y = 0 plane. The sign and concentration of the majority charge carrier are, respectively [Given : $e = 1.6\times10^{-19}C$]

(a) Positive and $5.2\times10^{19}\ m^{-3}$

(b) Negative and $5.2\times10^{19}\ m^{-3}$

(c) Positive and $10.4\times10^{19} m^{-3}$

(d) Negative and $10.4\times10^{19} m^{-3}$



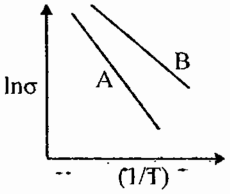
41. The temperature dependence of the electrical conductivity $\sigma$ of two intrinsic semi-conductors A and B is shown in the figure. If $E_A$ and $E_B$ are the band gaps of A and B respectively, which one of the following is TRUE?

(a) $E_A > E_B$

(b) $E_A < E_B$

(c) $E_A = E_B$

(d) $E_A$ and $E_B$ both depend on temperature.



42. If the static dielectric constant of NaCl crystal is 5.6 and its optical refractive index is 1.5, the ratio of its electric polarizability to its total polarizability is:

(a) 0.5 (b) 0.7 (c) 0.8 (d) 0.9

43. Which one of the following statements is TRUE?

(a) Magnetic tapes are made of Iron.

(b) Permanent magnets are made from ferrites.

(c) Ultrasonic transducers are made from quartz crystals.

(d) Optoelectronic devices are made from soft ferrites

44. The Bloch theore states that within a crystal, the wavefunction, $\psi(\vec{r})$, of an electron has the form

(a) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}.\vec{r}}$ where $u(\vec{r})$ is an arbitrary function and $\vec{k}$ is an arbitrary vector.

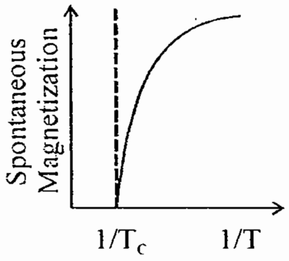
(b) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}.\vec{r}}$ where $u(\vec{r})$ is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector.

(c) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}.\vec{r}}$ where $u(\vec{r}) = u(\vec{r}+\vec{\Lambda}), \vec{\Lambda}$ is a lattice vector and $\vec{G}$ is a reciprocal lattice vector.

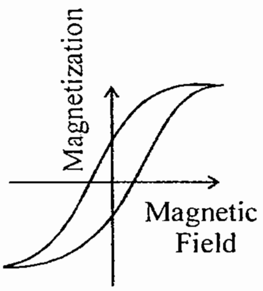
(d) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}.\vec{r}}$ where $u(\vec{r}) = u(\vec{r}+\vec{\Lambda}), \vec{\Lambda}$ is a lattice vector and $\vec{k}$ is an arbitrary vector.

45. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represented the property of the medium? ($T_C$ is the Curie temperature)

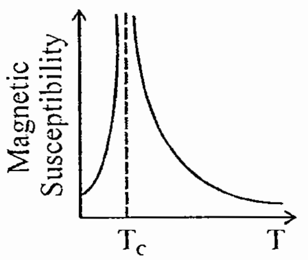
(a)



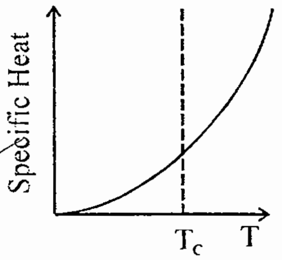
(b)



(c)



[✓ (d)]

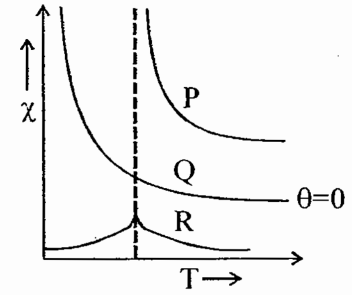


46. The thermal conductivity of a given material reduces when it undergoes a transition from its normal state to the superconducting state. The reason is:
[✓ (a) ~~The~~ cooper pairs cannot transfer energy to the lattice.]
(b) Upon the formation of cooper pairs, the lattice becomes less efficient in heat transfer.
(c) The electrons in the normal state lose their ability to transfer heat because of their coupling to the cooper pairs.
(d) The heat capacity increases on transition to the superconducting state leading to a reduction in thermal conductivity.

47. Far away from any of the resonance frequencies of a medium, the real part of the dielectric permitivity is:
(a) Always independent of the frequency (b) Monotonically decreasing with frequency
[✓ (c) Monotonically increasing with frequency] (d) A non-monotonic function of frequency.

48. In an insulating solid which one of the following physical phenomena is a coonsequence of Pauli's exclusion principle?
(free $e^-$ / conduction $e^-$)
(a) Ionic conductivity (b) Ferromagnetism [✓ (c) Paramagnetism] (d) Ferroelectricity.
(insulator

49. The dependence of the magnetic susceptibility $(\chi)$ of a material with temperature (T) can be represented by $\chi \propto \frac{1}{T-\theta}$, where $\theta$ is the Curie-Weiss temperature. The plot of magnetic susceptibility versus temperature is sketched in the figure, as curves P, Q and R with curve Q having $\theta = 0$. Which one of the following statements is correct?



(a) Curve R represents a paramagnet and Q a ferromagnet.
(b) Curve Q represents a ferromagnet and P an anterromagnet.
[✓ (c) Curve R represents an antiferromagnet and Q a paramagnet.]
(d) Curve R represents an antiferromagnet and Q a ferromanget.

50. The dielectric constant of a material at optical frequencies is mainly due to
(a) Ionic polarizability. [✓ (b) Electronic polarizability]
(c) Dipolar polarizability (d) Ionic and dipolar polarizability.

51. An electron of wavevector $\vec{k}_e$, velocity $\vec{\upsilon}_e$ and effective mass $m_c$ is removed from a filled energy band. The resulting hole has wavevevtor $\vec{k}_h$, velocity $\vec{\upsilon}_h$, and effective mass $m_h$. Which one of the following statements is correct?

[✓ (a)] $\vec{k}_h = -\vec{k}_e; \vec{\upsilon}_h = \vec{\upsilon}_e; m_h = -m_e$
(b) $\vec{k}_h = \vec{k}_e; \vec{\upsilon}_h = \vec{\upsilon}_e; m_h = m_e$
(c) $\vec{k}_h = \vec{k}_e; \vec{\upsilon}_h = -\vec{\upsilon}_e; m_h = -m_e$
[✓ (d)] $\vec{k}_h = -\vec{k}_e; \vec{\upsilon}_h = \vec{\upsilon}_e; m_h = -m_e$

**Linked Answer Questions:**
**Statement for Linked Answer Q.52 and Q. 53:**
The primitive translatin vectors of the face centered cubic (fcc) lattice are

$$\hat{a}_1 = \frac{a}{2}\left(\hat{j}+\hat{k}\right);\ \hat{a}_2 = \frac{a}{2}\left(\hat{i}+\hat{k}\right); a_3 = \frac{a}{2}\left(\hat{i}+\hat{j}\right)$$

52. The primitive translation vectos of the fcc reciprocal lattice are

[✓ (a)] $\hat{b}_1 = \left(\frac{2\pi}{a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{2\pi}{a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{2\pi}{a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

(b) $\hat{b}_1 = \left(\frac{\pi}{a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{\pi}{a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{\pi}{a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

(c) $\hat{b}_1 = \left(\frac{\pi}{2a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{\pi}{2a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{\pi}{2a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

(d) $\hat{b}_1 = \left(\frac{3\pi}{a}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right); \hat{b}_2 = \left(\frac{3\pi}{a}\right)\left(\hat{i}-\hat{j}+\hat{k}\right); \hat{b}_3 = \left(\frac{3\pi}{a}\right)\left(\hat{i}+\hat{j}-\hat{k}\right)$

53. The volume of the primitive cell of the fcc reciprocal lattice is:

[✓ (a)] $4\left(\frac{2\pi}{a}\right)^3$
(b) $4\left(\frac{\pi}{a}\right)^3$
(c) $4\left(\frac{\pi}{2a}\right)^3$
(d) $4\left(\frac{3\pi}{a}\right)^3$

54. An intrinsic semiconductor with mass of a hole $m_h$ and mass of an electron $m_C$ is at a finite temperature T. If the top of the valence band enegy is $E_V$ and the bottom of the conduction band energy is $E_C$, the Fermi energy of the semiconductor is:

(a) $E_F = \left(\frac{E_V+E_C}{2}\right) - \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$
(b) $E_F = \left(\frac{k_B T}{2}\right) + \frac{3}{4}\left(E_V+E_C\right)\ln\left(\frac{m_h}{m_e}\right)$
[✓ (c)] $E_F = \left(\frac{E_V+E_C}{2}\right) + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$
(d) $E_F = \left(\frac{k_B T}{2}\right) - \frac{3}{4}\left(E_V+E_C\right)\ln\left(\frac{m_h}{m_e}\right)$

55. A silicon sample A is doped with $10^{18}$ atoms/cm$^3$ of Boron. Another sample B of identical dimensions is doped with $10^{18}$ atoms/cm$^3$ of Phosphorus. The ratio of electron to hole mobility is 3. The ratio of conductivity of the sample A to B is:
(a) 3
[✓ (b) 1/3]
(c) 2/3
(d) 3/2

56. N-type silicon is obtained by doping silicon with:
(a) Germanium
(b) Aluminium
(c) Boron
[✓ (d) Phosphorus]

57. The band-gap of silicon at 300 K is:
(a) 1.36 eV
[✓ (b) 1.10 eV]
(c) 0.80 eV
(d) 6.67 eV

58. The intrinsic carrier density at 300 K is $1.5\times10^{10}\ /cm^3$, in silicon. For n-type silicon doped to $2.25\times10^{15}$ atoms / $cm^3$, the equilibrium electron and hole densities are:

(a) $n = 1.5\times10^{15}\ /cm^3,\ p = 1.5\times10^{10}\ /cm^3$ (b) $n = 1.5\times10^{10}\ /cm^3,\ p = 2.25\times10^{15}\ /cm^3$

[✓ (c) $n = 2.25\times10^{15}\ /cm^3,\ p = 1.0\times10^5\ /cm^3$] (d) $n = 1.5\times10^{10}\ /cm^3,\ p = 1.5\times10^{10}\ /cm^3$

59. The drift velocity of electrons in silicon:

(a) Is proportional to the electric field for all values of electric field.

(b) Is independent of the electric field.

(c) Increases at low values of electric field and decreases at high values of electric field exhibiting negative differential resistance.

[✓ (d) Increases linearly with electric field at low values of electric field and gradually saturates at higher values of electric field.]

60. A silicon sample is uniformly doped with $10^{16}$ phosphorous atoms/cm$^3$ and $2\times10^{16}$ boron atoms/cm$^3$. If all the dopants are fully ionized, the material is:

(a) n-type with carrier concentration of $10^{16}\ /cm^3$

[✓ (b) p-type with carrier concentration of $10^{16}\ /cm^3$]

(c) p-type with carrier concentration of $2\times10^{16}\ /cm^3$

(d) $T_2$-will get damaged and $T_1$ will be safe.

61. In a uniformly doped abrupt p-n junction, the doping level of n-side is four (4) times the doping level of the p-side. The ratio of the depletion layer widths is:

[✓ (a) 0.25] (b) 0.5 (c) 1.0 (d) 2.0

62. LED is a

[✓ (a) p-n diode] (b) Thermistor (c) Gate (d) Transistor

63. As per Hall effect, if any specimen carrying a current I is placed in a transverse magnetic field B, then an electric field E is induced in the specimen in the direction.

(a) Parallel to I [✓ (b) Perpendicular to B and parallel to I]

(c) Parallel to I and B (d) Perpendicular to both I and B.

64. The doping concentration on the n-side of a p-n junction diode is enhanced. Which one of the following will get affected?

[✓ (a) Width of the depletion region on n-side.]

(b) Width of the depletion region on p-side.

(c) Width of the depletion region on both sides.

(d) No change in width of depletion regions

65. Which one of the following expressions may be used to correctly describe the temperature(T) variation of the intrinsic carrier density ($n_i$) of a semiconductor?

(a) $n_i(T) = (A/T)\exp(-E_g/kT^2)$ (b) $n_i(T) = A(E_g/kT)^{10}$

(c) $n_i(T) = A\exp(-E_g/2kT)$ [✓ (d) $n_i(T) = AT^{3/2}\exp(-E_g/2kT)$]

66. An intrinsic semiconductor (intrinsic electron density $= 10^{16}\ m^{-3}$) is doped with donors to a level of $10^{22}\ m^{-3}$. What is the hole density assuming all donors to the ionized?

(a) $10^7\ m^{-3}$ (b) $10^8\ m^{-3}$ [✓ (c) $10^{10}\ m^{-3}$] (d) $10^6\ m^{-3}$.

67. Consider the following statements in respect of graphite:

1. It has the three coplanar covalent bonds.
2. It has good electrical and thermal conductivity. (Poor thermal conductivity)
3. Sheets of graphite are held together by van der waals interaction.

Which of the statements given above are correct?

(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 [✓ (d) 1 and 3]

68. The free electron density in a conductor is $(1/1.6)\times 10^{22}$ cm$^{-3}$. The electron mobility is 10 cm$^2$/V s. What is the value of its resistivity?

(a) $10^{-4}$ Wm  (b) $1.6\times 10^{-2}$ Wm  [✓ (c) $10^{-4}$ $\Omega$cm]  (d) $10^{-4}$ $\Omega\,\text{cm}^{-1}$

69. Which one of the following is the correct relationship between the band gap of a material used in a photo detector and the energy of the incident photon?

(a) $E_g \geq hc/\lambda$  (b) $h\upsilon > E_g$  [✓ (c) $h\upsilon \geq E_g$]  (d) $h\upsilon \leq E_g$

70. Two pure specimen of a semiconductor material are taken. One is doped with $10^{18}$ cm$^{-3}$ number of donors and the other is doped with $10^{16}$ cm$^{-3}$ number of acceptors. The minority carrier density in the first specimen is $10^7$ cm$^{-3}$. What is the minority carrier density in the other specimen?

(a) $10^{16}$ cm$^{-3}$  (b) $10^{27}$ cm$^{-3}$  (c) $10^{18}$ cm$^{-3}$  [✓ (d) $10^{9}$ cm$^{-3}$]

71. A copper wire is 1 metre long and has a uniform cross-section of 0.1 mm$^2$. The resistance of the wire at room temperature is 0.171 ohm. What is the resistivity of the material?

(a) $1.71\times 10^{-6}$ $\Omega$.m  (b) $1.71\times 10^{-7}$ $\Omega$.m  [✓ (c) $1.71\times 10^{-8}$ $\Omega$.m]  (d) $1.71\times 10^{-9}$ $\Omega$.m

72. A conductor carries a current of 4A and if magnitude of charge of an electron $e = 1.6\times 10^{-19}$ coulomb, then th number of electrons which flow past the cross section per second is:

[✓ (a) $2.5\times 10^{19}$]  (b) $1.6\times 10^{19}$  (c) $6.04\times 10^{19}$  (d) $0.4\times 10^{19}$

73. In an extrinsic semiconductor the Hall coefficient $R_H$.

(a) Increases with increase of temperature
(b) Decreaes with increase of temperature
[✓ (c) Is independent of the change of temperature]
(d) Changes with the change of magnetic field.

74. As the Fermi energy of silver is $8.8\times 10^{-19}$ joule, the velocity of the fastest electron in the silver at 0 K. (Given: Rest mass of electron = $9.1\times 10^{-31}$ kg) is:

(a) $3.33\times 10^{5}$ m/s  (b) $4.40\times 10^{7}$ m/s  [✓ (c) $1.39\times 10^{6}$ m/s]  (d) $3\times 10^{8}$ m/s

75. Which of the following statements related to the Hall effect?

1. A potential difference is developed across a current-carrying metal strip when the strip is placed in a transverse magnetic field.
2. The Hall effect is very weak in metals but is large in semiconductors
3. The Hall effect is very weak in semiconductors but is large in metals.
4. It is applied in the measurement of the magnetic field intensity.

Select the correct answer using the code given below:

(a) 1, 2 and 3  (b) 2 and 4  [✓ (c) 1, 3 and 4]  (d) 1, 2 and 4

76. Which of the following elements act as donor impurities?

1. Gold  2. Phosphorus  3. Boron  4. Antiomony
5. Arsenic  6. Indium.

Select the corrent answer using the codes given below:

(a) 1, 2 and 3  (b) 1, , 4 and 5  (c) 3, 4, 5 and 6  [✓ (d) 2, 4 and 5]

77. Germanium and silicon photo-sensors have their maximum spectral response in the

[✓ (a) Infrared region]  (b) Ultraviolet region
(c) Visible region  (d) X-ray region

78. The conductivity of an intrinsic semicoductor is (symbols have the usual meanings) where e is electron charge

(a) Generally less than that of a doped semiconductor.

(b) Given by $\sigma_i = en_i\left(\mu_n - \mu_p\right)$

[✓ (c) Given by $\sigma_i = en_i\left(\mu_n + \mu_p\right)$]

(d) Given by $\sigma_i = n_i\left(\mu_n + \mu_p\right)$

79. Measurement of Hall effect coefficient in a semiconductor provides information on the

(a) Sign and mass of charge carriers
(b) Mass and concentration of charge carriers
(c) Sign of charge carriers alone
[✓ (d) Sign and concentration of charge carriers.]

80. The temperature (T) dependence of magnetic susceptibility $(\chi)$ of a ferromagnetic substance with a Curie temperature ($T_c$) is given by

(a) $\frac{C}{T-T_c}$, for $T < T_c$
[✓ (b) $\frac{C}{T-T_c}$, for $T > T_c$]
(c) $\frac{C}{T+T_c}$, for $T > T_c$
(d) $\frac{C}{T+T_c}$, for all temperature.

**Common data Q.81 and Q.82:**

The tight binding energy dispersion (E–k) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is:

$$E = E_0 - \beta - 2\gamma\cos(ka)$$

Where $E_0$, $\beta$ and $\gamma$ are constants and k is the wave-vector.

81. The density of states of electrons (including spin degeneracy) in the band is given by

(a) $\frac{L}{\pi\gamma a\sin(ka)}$
[✓ (b) $\frac{L}{2\pi\gamma a\sin(ka)}$]
(c) $\frac{L}{2\pi\gamma a\cos(ka)}$
(d) $\frac{L}{\pi\gamma a\cos(ka)}$

82. The effective mass of electrons in the band is given by

(a) $\frac{\hbar^2}{\gamma a^2\cos(ka)}$
[✓ (b) $\frac{\hbar^2}{2\gamma a^2\cos(ka)}$]
(c) $\frac{\hbar^2}{\gamma a^2\sin(ka)}$
(d) $\frac{\hbar^2}{2\gamma a^2\cos(ka)}$

83. Light of wavelength 660 nm and power of 1 mW is incident on a semiconductor photodiode with an absorbing layer of thickness of (ln 4) μm .

**A.** If the absorption coefficient at this wavelength is $10^4$ cm$^{-1}$ and if 1% power is lost on reflection at the surface, the power absorbed will be

[✓ (a) 750 μW]
(b) 675 μW
(c) 250 μW
(d) 225 μW

**B.** The generated photo-current for a quantum efficiency of unity will be

(a) 360 μA
[✓ (b) 400 μA]
(c) 133 μA
(d) 120 μA

84. A narrow beam of X-rays with wavelength 1.5Å is reflected from an ionic crystal with an fcc lattice structure with a density of 3.32 g cm$^{-3}$. The molecular weight is 108 AMU (1 AMU= $1.66\times10^{-24}$ g).

**A.** The lattice constant is:

[✓ (a) 6.00Å]
(b) 4.56Å
(c) 4.00Å
(d) 2.56Å

**B.** The sine of the angle corresponding to (111) reflection is:

(a) $\sqrt{3}/4$
[✓ (b) $\sqrt{3}/8$]
(c) 1/4
(d) 1/8

85. A flux quantum (fluxoid) is approximately equal to $2\times10^{-7}$ gauss-cm$^2$. A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is $\frac{2}{\pi}\times10^5$ gauss

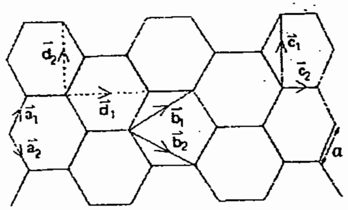
**A.** The penetration depth of this superconductor is

[✓ (a) 100Å]
(b) 10Å
(c) 1000Å
(d) 314 Å

**B.** The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi}\times10^5$ gauss. The correlation length of the superconductor is: (Coherence length)

(a) 20Å (b) 200Å (c) 628 Å (d) 2000Å

86. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing a, as shown below. The carbon atoms occupy the vertices.



**A.** The Wigner-Seitz cell has an area of ( smallest area-cell)

(a) $2a^2$ (b) $\frac{\sqrt{3}}{2}a^2$ (c) $6\sqrt{3}\,a^2$ (d) $\frac{3\sqrt{3}}{2}a^2$

**B.** The Bravais lattice for this array is a

(a) Rectangular lattice with basis vectors $\bar{d}_1$ and $\bar{d}_2$

(b) Rectangular lattice with basis vectors $\bar{c}_1$ and $\bar{c}_2$

(c) Hexagonal lattice with basis vectors $\bar{a}_1$ and $\bar{a}_2$ (lattice vecto)

(d) Hexagonal lattice with basis vectors $\bar{b}_1$ and $\bar{b}_2$.

87. The solid phase of an element follows van der Waals bonding with iner-atomic potential $V(r)=-\frac{P}{r^6}+\frac{Q}{r^{12}}$, where P and Q are constants. The bond length can be expressed as

(a) $\left(\frac{2Q}{P}\right)^{-6}$ (b) $\left(\frac{Q}{P}\right)^{-6}$ (c) $\left(\frac{P}{2Q}\right)^{-6}$ (d) $\left(\frac{P}{Q}\right)^{-6}$

88. The total number of $Cl^-$ ions in a unit cell of NaCl crystal is

(a) 4 (b) 2 (c) 8 (d) 10

89. If the distance between a Na ion and a chlorine ion in NaCl crystal is 0.28 nm, then the lattice parameter of the crystal is

(a) 0.14 nm (b) 0.8 nm (c) 56 Ao (d) 0.56 nm

90. At the equilibrium spacing of a diatomic molecule, the resultant force is

(a) zero (b) minimum (c) maximum (d) unity

91. The potential energy in the above said spacing is

(a) zero (b) minimum (c) maximum (d) unity

92. Which of the interatomic bonds are directional?

(a) covalent (b) metallic (c) ionic (d) van der Waals

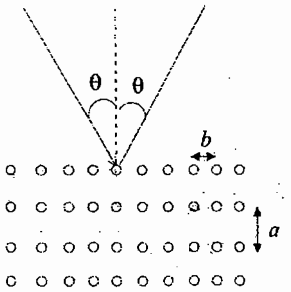
93. The co-ordination number of $Na^+$ and $Cl^-$ in the rock salt structure is respectively

(a) 8 and 6 (b) 6 and 8 (c) 6 and 6 (d) 4 and 4

94. In a scanning tunnelling microscope, a fine Platinum needle is held close to a metallic surface in vacuum and electrons are allowed to tunnel across the tiny gap $\delta$ between the surface and the needle. The tunnelling current I is related to the gap $\delta$, through positive constants a and b, as

(a) $I = a - b\,\delta$ (b) $I = a + b\,\delta$ [✓ (c) $\log I = a - b\,\delta$] (d) $\log I = a + b\,\delta$

95. A monochromatic beam of X-ray with wavelength $\lambda$ is incident at an angle $\theta$ on a crystal with lattice spacing a and b as sketched in the figure below.

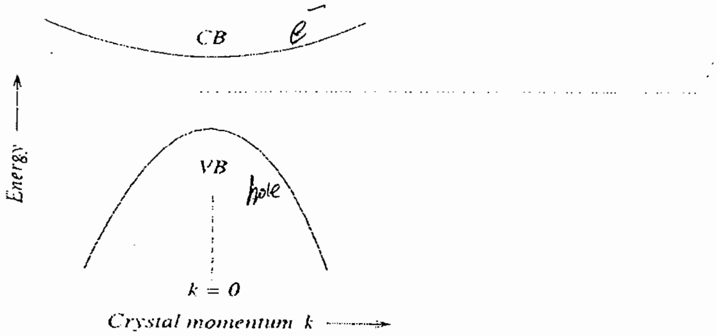


A condition for there to a maximum in the diffracted X-ray intensity is

(a) $2\sqrt{a^2 + b^2}\sin\theta = \lambda$ (b) $2b\cos\theta = \lambda$

[✓ (c) $2a\cos\theta = \lambda$] (d) $(a+b)\sin\theta = \lambda$

96. Suppose the energy band of a certain pure crystalline solid is as shown in the figure below, where the energy (E) varies with crystal momentum (k) as $E \propto k^2$



At finte temperatures the bottom of the conduction band (CB) is partially filled with electrons (e) and the top of the valence band (VB) is partially filled holes (h). If an electric fields is applied to this solid, both e and h will start moving. If the time between collisions is the same for both e and h, then

(a) e and h will move with the same speed in opposite directions
[✓ (b) h will on an average achieve higher speed than e]
(c) e will on an average achieve higher speed than h
(d) e and h will recombine and after a while there will be no flow of charges.

97. Metallic Copper is known to from cubic crystals and the lattice constant is measured from X-ray diffraction studies to be about 0.36nm. If the specific gravity of Copper is 8.96 and its atomic weight is 63.5, one can conclude that

(a) the crystals are of simple cubic type (b) the crystals are of b.c.c. type

[✓ (c) the crystals are of f.c.c. type] (d) the crystals are a mixture of f.c.c. and b.c.c. types.

98. The potencial of a diatomic molecule as a function of the distance r between the atoms is given by $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. The value of the potential at equilibrium separation between the atoms is:

(a) $-4a^2/b$ (b) $-2a^2/b$ (c) $-a^2/2b$ [✓ (d) $-a^2/4b$]

99. Assume that the free energy of a magnetic system has an expansion in the order parameter M of the form $F(M,T) = a(T-T_C)M^2 + bM^4 + cM^6$, with a, b and c > 0. As the temperature is lowered below $T_C$, the system undergoes a phase transition. The behaviour of the order parameter just below the transition, where $(T-T_C)$ is very small, is best described by

(a) $M \propto (T_C - T)^{-1/2}$ [✓ (b) $M \propto (T_C - T)^{1/2}$] (c) $M \propto (T_C - T)$ (d) $M \propto (T_C - T)^3$

100. The energy levels of electrons of mass 'm' and charge 'e' confined in an area A in the xy-plane with a uniform magnetic field B applied in the z-direction are given by $E_n = \left(n + \frac{1}{2}\right)\frac{eB\hbar}{mc}, n = 0,1,2....$ The degeneracy of each level is $\frac{eBA}{\hbar c}$. The lowest level is completely filled the other are empty. The fermi energy $\frac{\hbar^2 N}{2\pi m A}$, where N is the total number of electrons, is:

(a) coincident with the n = 0 level (b) coincident with the n = 1 level

(c) midway between the n = 0 and the n = 1 levels

(d) midway between the n = 1 and the n = 2 levels.

101. The excitation of a three-dimensional solid are bosonic in nature with their frequency $\omega$ and wave-number k are related by $\omega \propto k^2$ in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to

(a) $T^{1/2}$ (b) T [✓ (c) $T^{3/2}$] (d) $T^3$

102. Monochromatic light of wavelength 660 nm and intensity 100 $\text{mW}/\text{cm}^2$ falls on a solar cell of area 30 $\text{cm}^2$. The conversion efficiency of the solar cell is 10%. If each coverted photon results in an electron-hole pair, what is the maximum circuit current supplied by the solar cell? (Take $h = 6.6\times10^{-34}\,\text{J-s}$, $c = 3\times10^8\,\text{m/s}$ and $e = 1.6\times10^{-19}\,\text{C}$).

[✓ (a) 160 mA] (b) 320 mA [✓ (c) 1600 mA] (d) 3200 mA

103. The magnetization M of a system in equilibrium at temperature T is given by the equation $M = \tanh\left(M/T\right)$, where all quantities are in dimensionless units. This system will show

(a) a first order phase transition at T = 1 [✓ (b) a second order phase transition at T=1]

(c) no phase transition at any T (d) a second order phase transition at T = $\frac{1}{2}$.

104. The basis vectors of a lattice are $2\hat{i}, \hat{i}+2\hat{j}$ and $\hat{k}$. The basis vectors of the reciprocal lattice are

(a) $\frac{2}{\pi}(\hat{i}+\hat{j}), \frac{2}{\pi}\hat{j}$ and $\frac{2}{\pi}\hat{k}$ (b) $\pi(2\hat{i}-\hat{j}), \pi\hat{j}$ and $4\pi\hat{k}$

[✓ (c) $\frac{\pi}{2}(2\hat{i}-\hat{j}), \pi\hat{j}$ and $2\pi\hat{k}$] (d) $2\pi\hat{i}, \pi(\hat{i}+2\hat{j})$, and $\pi\hat{k}$

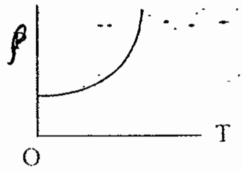
105. The number of phonon branch in graphite, with four atoms in the primitive cell, is
(a) 8, out of which two are acoustic, the rest optical
(b) 9, out of which three are acoustic, the rest optical
(c) 12, out of which three are acoustic, the rest optical
(d) 12, out of which two are acoustic, the rest optical

106. The nature of the interaction that gives rise to the van der Waals force in a molecular crystal is
(a) dipolar (b) nuclear (c) magnetic (d) quadrupolar

107. The experimental method used most frequently for detecting magnetic structure of solids is
(a) electron diffraction (b) neutron diffraction
(c) x-ray diffraction (d) photo-emission spectroscopy

108. We know that the allowed wave vectors for phonons in a crystal have discrete values. This is a consequence of
(a) quantization of phonons (b) periodic boundary conditions
(c) uncertainty principle (d) periodic structure of the lattice

109. The temperature dependence of the electrical resistivity $\rho$ of a material is shown in the plot below. This material is a



(a) insulator (b) semiconductor (c) normal metal (d) superconductor

110. The specific heat of a normal metal at low temperatures is dominated by the contribution of
(a) conduction electrons (b) acoustic phonons
(c) core electrons (d) optical phonons

111. The Fermi level in an intrinsic semiconductor at zero temperature lies
(a) in the conduction band (b) at the top of the valence band
(c) at the bottom of the conduction band
(d) in the energy gap between the valence and the conduction bands

112. Let $\xi$ and $\lambda$ denote the coherence length and the penetration depth of a superconductor. For a type-I, superconductor
(a) $\xi < \lambda$ (b) $\xi > \lambda$ (c) $\xi = \lambda$ (d) $\lambda = 0$

113. A KCl crystal grown in an environment containing some $CaCl_2$ will have
(a) excess positive charge (b) higher density
(c) lower ionic conductivity (d) more lattice vacancies.

114. According to the Debye model, the internal energy of a solid a at temperature T close to absolute zero varies as
(a) $T^2$ (b) $T^4$ (c) $T^3$ (d) $T$

115. In a period potential given by $V(\vec{r}) = V(\vec{r}+\vec{R})$, where $\vec{R}$ is the lattice vector, the eigenstates of the Hamiltonian $H = \frac{-\hbar^2}{2m}\nabla^2 + V(\vec{r})$ are $\psi_k(\vec{r}) = e^{i\vec{k}.\vec{r}}u_{\vec{k}}(\vec{r})$, where

(a) $u_{\vec{k}}(\vec{r}) = e^{i\vec{k}.\vec{R}}(\vec{k} \neq \vec{k}')$ (b) $u_{\vec{k}}(\vec{r}) = e^{i\vec{k}.\vec{R}}\, u_{\vec{k}}\left(\vec{r}+\frac{\vec{R}}{3}\right)$

(c) $u_{\vec{k}}(\vec{r}) = u_{\vec{k}}\left(\vec{r}+\frac{\vec{R}}{2}\right)$ [✓ (d)] $u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}+\vec{R})$

116. The magnetic susceptibility of a type I superconductor at a temperature T below the transition temperature $T_c$

(a) increases linearly from a negative value to zero as temperature increases from 0 to $T_c$

(b) decreases linearly from a positive value to zero as temperature increases from 0 to $T_c$

[✓ (c)] is negative and constant at all T (d) is positive and constant at all T

117. In an intrinsic semiconductor, as the temperature increases the conductivity $\sigma$, electron concentration n, and mobility $\mu$ vary as follows:

(a) $\sigma$ decreases, n increases and $\mu$ decreases (b) $\sigma$ decreases, n is unaffected and $\mu$ decreases

[✓ (c)] $\sigma$ increases, n increases and $\mu$ decreases (d) $\sigma$ increases, n increases and $\mu$ is unaffected

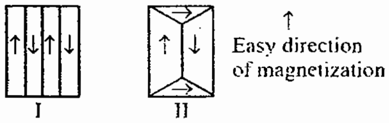
118. The value of a primitive unit cell defined by axes $(\hat{k}+\hat{i})\text{Å}, (\hat{i}+\hat{j})\text{Å}, (\hat{j}+\hat{k})\text{Å}$, is

(a) 1Å$^3$ (b) 3Å$^3$ [✓ (c)] 2Å$^3$ (d) 4Å$^3$

119. In the Hall effect the sign of the Hall voltage depends upon the carrier concentration (n,p) and mobility $(\mu_n\ \mu_p)$ of electrons and holes, respectively. The Hall voltage in intrinsic and n-type semiconductors is negative because

[✓ (a)] $n\mu_n > p\mu_p$ (b) $n > p, \mu_n < \mu_p$ (c) $n > p, \mu_n = \mu_p$ (d) $n = p, \mu_n > \mu_p$

120. Two possible domain structures (I and II) with an easy direction of magnetization (along the arrow) for a ferromagnetic material are shown. $E_{ms}$ and $E_{ca}$ are magnetostatic and crystalline anisotropy energy of the domain structure. Which of the following statements is correct?



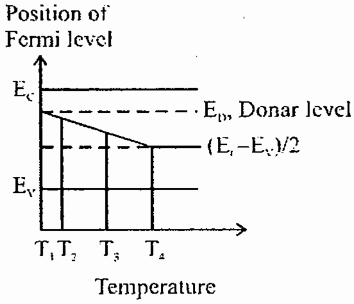
(a) $E_{ms}(I) < E_{ms}(II)$ (b) $E_{ca}(I) > E_{ca}(II)$

(c) $E_{ms}(I) < E_{ms}(II),\ E_{ca}(I) < E_{ca}(II)$ (d) $E_{ms}(I) > E_{ms}(II),\ E_{ca}(I) < E_{ca}(II)$

121. The Miller indices of a plane passing through the points $3\hat{a}$, $4\hat{b}$ and $5\hat{c}$, where $\hat{a}$, $\hat{b}$ and $\hat{c}$, are lattice vectors are

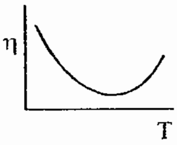
(a) (12, 15, 20) [✓ (b)] (20, 15, 12) (c) (5, 4, 3) (d) (3, 4, 5)

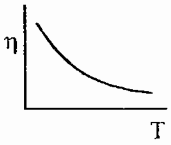
122. In the following figure, the position of the Fermi level is shown as a function of temperature along with the band diagram of an n-type semiconductor. The temperature at which 50% of the donor atoms are ionized is

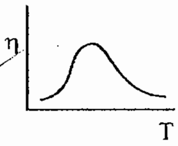


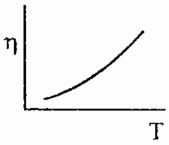
(a) $T_3$ (b) $T_1$ (c) $T_4$ [✓ (d) $T_2$]

123. Which of the following figures represents the temperature dependence of the thermal conductivity $\eta$ of a solid according to the Drude model.

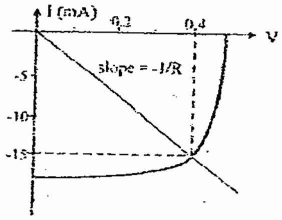
(a) 

(b) 

[✓ (c)] 

(d) 

124. The I-V characteristics of a solar cell of dimension 1 cm × 1 cm illuminated with light of intensity 600 $Wm^{-2}$ are shown in the figure. If the cell is used to drive a load R (see the load line in the figure) the efficiency of the solar cell is:



(a) 1% (b) 5% (c) 20% [✓ (d) 10%]

125. In an experiment to measure the Hall co-efficient of a particular sample, the generated Hall voltage was found to be too small to measure. To improve the measurement, you would
(a) Increase the magnetic field (b) Increase the temperature of the sample
[✓ (c) Use a longer and thinner sample] (d) Use a shorter and thicker sample.

126. When the sample is heated the X-ray diffraction pattern of a powder changes as follows:
(a) The line spacing decreases and line intensity increases
(b) The line spacing and line intensity both increases
(c) The line spacing increases and line intensity decreases
[✓ (d) The line spacing and line intensity both decrease]

127. Let $\Phi(\vec{q}) = \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}} V(\vec{r})\; q \neq 0$, where $V(\vec{r})$ is the potential seen by an electron in a crystal lattice. If $\vec{G}$ is a reciprocal lattice vector, then $\Phi(\vec{q})$ is nonzero,

[✓ (a) only if $\vec{q} = \vec{G}$] (b) for $\vec{q} = \frac{1}{2}\vec{G}$ (c) for all $\vec{q}$ (d) for all $\vec{q}$ within the Brillouin zone.

128. In the continuum model of an elastic solid, the frequency of sound waves
(a) is constant in some directions and proportional to the wave-vector in other directions
(b) is proportional to the square of the wave-vector
(c) is a constant independent of the wave-vector
[✓ (d) is proportional to the wave-vector in all directions.]

129. The number of acoustic phonon branches in a three-dimensional crystal with 3 atoms per primitive cell is
(a) 6 (b) 9 [✓ (c) 3] (d) 12

130. The most suitable experimental method for studying the dispersion relation of phonons is
(a) X-ray diffraction (b) elastic scattering of neutorns
[✓ (c) inelastic scattering of thermal neutrons] (d) photo-electron emission.

131. For Li metal, the resistivity at 77K is 1.01 $\mu\Omega-\text{cm}$ and that at 273K is 8.57 $\mu\Omega-\text{cm}$. The ratio of the mean collision times, $\frac{\tau_{77}}{\tau_{273}}$ is approximately,
(a) 3 (b) 1/9 [✓ (c) 9] (d) 1/3

132. The real part of the ac conductivity of a metal at a large frequency $\omega(\omega\tau >> 1, \tau = \text{mean collision time})$.
(a) increases as $\omega$ [✓ (b) decreases $\frac{1}{\omega^2}$]
(c) becomes independent of $\omega$ (d) varies as $\sqrt{\omega}$

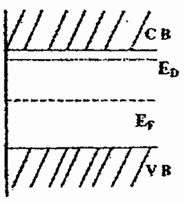
133. The (Pauli) paramagnetic susceptibility of a metal at a low temperature T is
(a) proportional to $\frac{1}{T}$ [✓ (b) independent of T]
(c) proportional $T^2$ (d) proportional to T

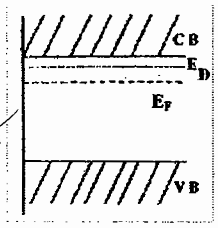
134. The isotope effect in superconductors says that the critical temperature
(a) increases as the isotopic mass increases [✓ (b) decreases as the isotopic mass increases]
(c) increases linearly with the isotopic mass (d) decreases linearly with the isotopic mass.

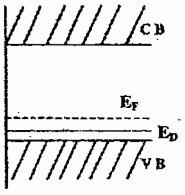
135. For an ideal Fermi gas in three dimensions, the electron velocity $V_F$ at he Fermi surface is related to electron concentration n as,
(a) $V_F \propto n^{2/3}$ (b) $V_F \propto n$ (c) $V_F \propto n^{1/2}$ [✓ (d) $V_F \propto n^{1/3}$]

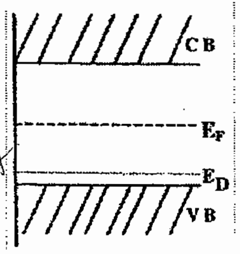
136. A Ge semiconductor is doped with acceptor impurity concentration of $10^{15}$ atoms/cm$^3$. For the given hole mobility of $1800\ \text{cm}^2/\text{V}-\text{s}$, the resistivity of this material is:
(a) 0.288 Ω cm (b) 0.694 Ω cm [✓ (c) 3.472 Ω cm] (d) 6.944 Ω cm

137. Identify the CORRECT energy band diagram for silicon doped with Arsenic. Here CB, VB, $E_D$ and $E_F$ conduction band, valence band, impurity level and Fermi level, respectively.

(a)


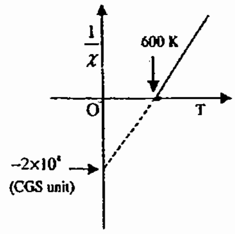
[✓ (b)]


(c)


[✓ (d)]


138. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids ?
(a) Debye's $T^3$ law (b) Dulong petit's law
(c) Optical baranches in lattices [✓ (d) Thermal expansion]

139. Inverse susceptibility $(1/\chi)$ as a function of temperature, T for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. the values of the Curie constant, C and the Weiss molecular field constant, $\lambda$, in CGS units, are



(a) $C = 5\times10^{-5}, \lambda = 3\times10^{-2}$ (b) $C = 3\times10^{-2}, \lambda = 5\times10^{-5}$

[✓ (c) $C = 3\times10^{-2}, \lambda = 2\times10^{4}$] (d) $C = 2\times10^{4}, \lambda = 3\times10^{-2}$

140. A simple cubic crystal with lattice parameter $a_c$ undergoes transition into a tetragonal structure with lattice parameter $a_t = b_t = \sqrt{2}\, a_c$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacings of (101) planes for the cubic and the tetragonal structure is

(a) $\sqrt{\frac{1}{6}}$ (b) $\frac{1}{6}$ [✓ (c) $\sqrt{\frac{3}{8}}$] (d) $\frac{3}{8}$

**Common data for Q. 141 and Q. 142**

The dispersion relation for a one dimensional monatomic crystal with lattice spacing a, which interacts via nearest neighbour harmonic potential is given by $\omega = A\left|\sin\frac{Ka}{2}\right|$, where A is a constant of a appropriate unit.

141. The group velocity at the boundary of the first Brillouin zone is

[✓ (a) 0] (b) 1 (c) $\sqrt{\frac{Aa^2}{2}}$ (d) $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

142. The force constant between the nearest neighbour of the lattice is ( M is the mass of the atom)

[✓ (a) $\frac{MA^2}{4}$] (b) $\frac{MA^2}{2}$ (c) $MA^2$ (d) $2MA^2$

143. A beam of x-rays is incident on a BCC crystal. If the difference between the incident and the scattered wave vectors is $\vec{K} = h\hat{x} + k\hat{y} + \ell\hat{z}$ where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of the associate cubic lattice, the necessary condition for the scattered beam to given a Laue maximum is

[✓ (a) h + k + l = even] (b) h = k = 1 (c) h, k, l are all distinct (d) h + k + l = odd

144. The Dulong Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye temperature is

[✓ (a) $\gg 300$ K] (b) $\sim 300$ K (c) $\ll 300$ K (d) 0 K

145. the second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees with respect to the crystal plane. The distance between the lattice planes is

(a) 1 Angstrom (b) 2 Angstroms [✓ (c) 4 Angstroms] (d) 8 Angstroms

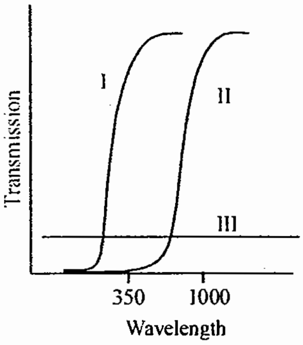
146. The net charge of an n-type semiconductor is

(a) positive [✓ (b) zero] (c) negative (d) dependent on the dopant density

147. An LED operates at 1.5 V and 5 mA in forward bias. Assuming an 80% external efficiency of the LED, how many photons are emitted per second?

(a) $5.0\times10^{16}$ (b) $1.5\times10^{16}$ (c) $0.8\times10^{16}$ [✓ (d) $2.5\times10^{16}$]

148. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond respectively, to



[✓ (a) insulator, semiconductor and metal] (b) semiconductor, metal and insulator
(c) metal, semiconductor and insulator (d) insulator, metal and semiconductor

149. The magnetization M of a ferromagnet, as a function of the temperature T and the magnetic field H, is described by the equaltion $M = \tanh\left(\frac{T_C}{T}M + \frac{H}{T}\right)$. In these units, the zero-field magnetic susceptibility in terms of $M(0) = M(H=0)$ is given by

(a) $\frac{1-M^2(0)}{T-T_c(1-M^2(0))}$ (b) $\frac{1-M^2(0)}{T-T_c}$ [✓ (c) $\frac{1-M^2(0)}{T+T_c}$] (d) $\frac{1-M^2(0)}{T}$

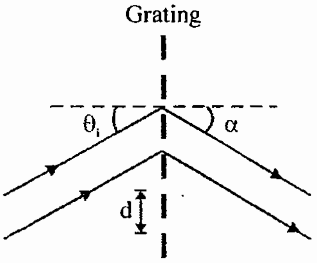
150. The energy required to create a lattice vacancy in a crystal is equal to 1 eV. The ratio of the number densities of vacancies $n(1200K)/n(300K)$, when the crystal is at equilibrium at 1200K and 300K, respectively is approximately

(a) exp(–30) (b) exp(–15) (c) exp(15) [✓ (d) exp(30)]

151. The dispersion relation of phonons in a solid is given by $\omega^2(k) = \omega_0^2(3 - \cos k_x a - \cos k_y a - \cos k_z a)$ The velocity of the phonons at large wavelenght is

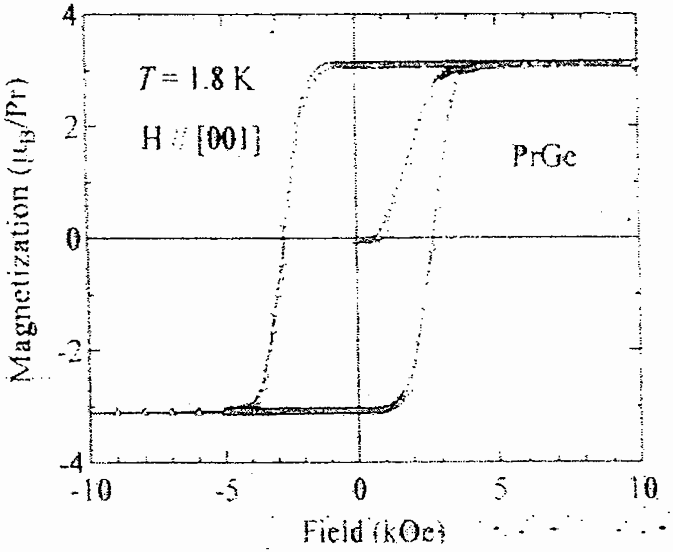
(a) $\omega_0 a/\sqrt{3}$ (b) $\omega_0 a$ (c) $\sqrt{3}\omega_0 a$ (d) $\omega_0 a/\sqrt{2}$

152. A parallel beam of light of wavelength $\lambda$ is incident on a transmission grating with groove spacing d, at an angle $\theta_i$, as shown in the figure on the left. The plane of incidence is normal to the grooves. After diffraction, the transmitted beam is seen to be at an angle $\alpha$ relative to the normal. Which of the following conditions must be satisfied for this to happen?



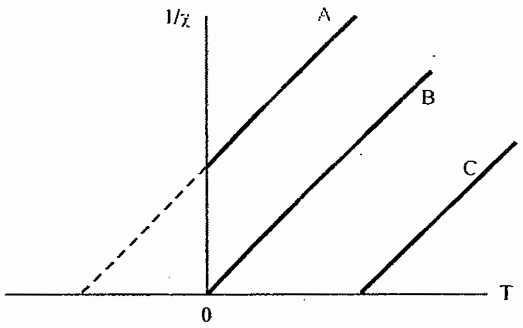
(a) $d(\sin\theta_i - \sin\alpha) = n\lambda$ (b) $2d\sin(\theta_i - \alpha) = n\lambda$
[✓ (c) $d(\sin\theta_i + \sin\alpha) = n\lambda$] (d) $2d(\alpha + \theta_i) = n\lambda$

153. The Curie temperature of a single crystal of PrGe is known to be 41 K. The magnetization data of this sample is measured at 1.8K for the magnetic field applied parallel to the [001] direction is shown in the figure on the left. At a temperature of 38 K, the hysteresis loop in the figure will



(a) Have the same width
[✓ (b) decrease in width]
(c) increase in width
(d) shrink to a line.

154. The magnetic susceptibility $\chi$ of three samples A, B and C, is measured as a function of their absolute temperature T, leading to teh graphs shown below.



From these graphs, the magnetic nature of the samples can be inferred to be

[✓ (a) A:anti-ferromagnet B: paramagnet C: ferromagnet]
(b) A:diamagnet B: paramagnet C: anti-ferromagnet
(c) A:paramagnet B: anti-ferromagnet C: ferromagnet
(d) A:anti-ferromagnet B: diamagnet C: paramagnet.

155. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is $a^3$, is

[✓ (a) $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$]
(b) $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$
(c) $\left(\frac{\pi^2}{a^3}\right)^{1/3}$
(d) $\frac{1}{a}$

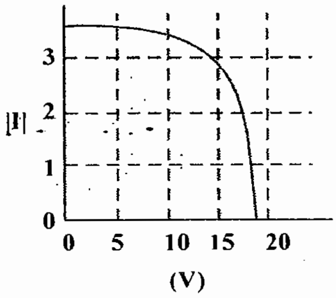
156. A magnetic field sensor based on the Hall effect is to be fabricated by implanting. As into a Si film of thickness 1 μm. The specifications require a magnetic field sensitivity of 500 mV/Tesla at an excitation current of 1 mA. The implanation dose is to be adjusted such that the average carrier density, after activation, is

(a) $1.25\times10^{26}\ m^{-3}$ [✓ (b) $1.25\times10^{22}\ m^{-3}$] (c) $4.1\times10^{21}\ m^{-3}$ (d) $4.1\times10^{20}\ m^{-3}$

157. The free energy difference between the superconducting and the normal states of a material is given by $\Delta F = F_S - F_N = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$, where $\psi$ is an order parameter and $\alpha$ and $\beta$ are constants such that $\alpha > 0$ in the normal and $\alpha < 0$ in the superconducting state, while $\beta > 0$ always. The minimum value of $\Delta F$ in the superconducting state is

(a) $-\alpha^2/\beta$ [✓ (b) $-\alpha^2/2\beta$] (c) $-3\alpha^2/2\beta$ (d) $-5\alpha^2/2\beta$

158. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of 5 Ω, the power drawn by the load is:

(a) 97W (b) 73 W (c) 50 W [✓ (d) 45 W]

159. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta\left(\cos k_x a + \cos k_y a + \cos k_z a\right)$$

where $\beta$ is a constant and a is the lattice constant. The effective mass at the boundary of the first Brilliouin zene is

(a) $\frac{2\hbar^2}{5\beta a^2}$ (b) $\frac{4\hbar^2}{5\beta a^2}$ [✓ (c) $\frac{\hbar^2}{2\beta a^2}$] (d) $\frac{\hbar^2}{3\beta a^2}$

160. A phosphorous doped silicon semiconductor (doping density: $10^{17}/cm^3$) is heated from 100°C to 200°C. Which one of the following statements is CORRECT?

(a) Position of Fermi level moves towards conduction band
(b) Position of dopant level moves towards conduction band
[✓ (c) Position of Fermi level moves towards middle of energy gap]
(d) Position of dopant level moves towards middle of energy gap

161. Considering the BCS theory of superconductors, which one of the following statements is NOT CORRECT? (*h* is the Planck's constant and *e* is the electronic charge)

(a) Presence of energy gap at temperatures below the critical temperature
(b) Different critical temperatures for isotopes
[✓ (c) Quantization of magnetic flux in superconducting ring in the unit of $\left(\frac{h}{e}\right)$]
(d) Presence of Meissner effect

162. **Group-I** contains elementary excitations in solids. **Group-II** gives the associated fields with these excitations. MATCH the excitations with their associated field and select your answer as per codes given below.

| **Group-I** | **Group-II** |
|---|---|
| (P) phonon | S (i) photon + lattice vibration |
| (Q) plasmon | R (ii) electron + elastic deformation |
| (R) polaron | Q (iii) collective electron oscillations |
| (S) polariton | P (iv) elastic wave |

**Codes:**

(a) (P–iv), (Q-iii), (R-i), (S-ii)

(b) (P–iv), (Q-iii), (R-ii), (S-i)

(c) (P–i), (Q-iii), (R-ii), (S-iv)

(d) (P–iii), (Q-iv), (R-ii), (S-i)

163. A lattice has the following primitive vectors (in A): = $\vec{a} = 2(\hat{j}+\hat{k}), \vec{b} = 2(\hat{k}+\hat{i}), \vec{c} = 2(\hat{i}+\hat{j})$. The reciprocal lattice corresponding to the above lattice is

(a) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$

(b) BCC lattice with cube edge of $(2\pi)A^{-1}$

(c) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$

(d) FCC lattice with cube edge of $(2\pi)A^{-1}$

164. The total energy of an ionic solid is given by an expression $E = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where $\alpha$ is Madelung constant, $r$ is the distance between the nearest neighbours in the crystal and B is a constant. If $r_0$ is the equilibrium separation between the nearest neighbours then the value of B is

(a) $\frac{\alpha e^2 r_0^8}{36\pi \epsilon_0}$ (b) $\frac{\alpha e^2 r_0^8}{4\pi \epsilon_0}$ (c) $\frac{2\alpha e^2 r_0^{10}}{9\pi \epsilon_0}$ (d) $\frac{\alpha e^2 r_0^{10}}{36\pi \epsilon_0}$

165. Using the frequency-dependent Drude formula, what is the effective kinetic inductance of a metallic wire that is to be used as a transmission line? [In the following, the electron mass is m, density of electrons is n, and the length and cross-sectional area of the wire are $\ell$ and A respectively]

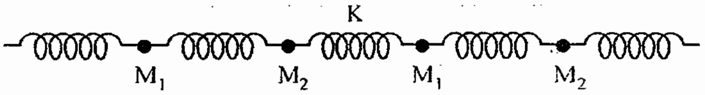
(1) $mA/(ne^2\ell)$

(2) zero

(3) $m\ell/(ne^2A)$

(4) $m\sqrt{A}/(ne^2\ell^2)$

166. The phonon dispersion for the following one-dimensional diatomic lattice with masses $M_1$ and $M_2$ (as shown in the figure)



is given by

$$\omega^2(q) = K\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

where $a$ is the lattice parameter and K is spring constant. The velocity of sound is

(1) $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}}a$

(2) $\sqrt{\frac{K}{2(M_1+M_2)}}a$

(3) $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}}a$

(4) $\sqrt{\frac{KM_1M_1}{2(M_1+M_2)^3}}a$

167. The electron dispersion relation for a one-dimensional metal is given by

$$\varepsilon_k = 2\varepsilon_0\left[\sin^2\frac{ka}{2} - \frac{1}{6}\sin^2 ka\right]$$

where k is the momentum, a is the lattice constant, $\varepsilon_0$ is a constant having dimensions of energy and $|ka| \leq \pi$. If the average number of electrons per atom in the conduction band is 1/3, then the Fermi energy is

(1) $\varepsilon_0/4$ (2) $\varepsilon_0$ (3) $2\varepsilon_0/3$ (4) $5\varepsilon_0/3$

168. Which one of the following diffraction techniques has most surface sensitivity?

(a) X-ray [✓ (b) Electron] (b) Neutron (d) Optical

169. The inter-planar spacing for (110) plane of a monoatomic FCC crystal of lattice constant 2Å is

(a) 1/2Å (b) 2Å [✓ (c) $\sqrt{2}$ Å] (d) $\sqrt{3}$ Å

170. Which one of the following diffraction peaks will be absent in a powder diffraction pattern for BCC-crystalline powder such as Fe

(a) (110) (b) (111) [✓ (c) (200)] (d) (211)

171. The position of an X-ray diffraction peak may be shifted with respect to the peak of a perfect crystal due to

(Broadening)

[✓ (a) homogenous strain] (b) inhomogeneous strain (c) small unstrained crystallites

(d) inhomogeneously strained nano-sized crystallites

172. The Burgers vector of a screw dislocation in an FCC crystal is $\frac{1}{2}[110]$. The possible line direction for the dislocation is

[✓ (a) [110]] (b) [011] (c) [101] (d) [111]

173. A Schottky defect in $Al_2O_3$ involves

(a) an $Al^{3+}$ vacancy and an $Al^{3+}$ interstitial (b) an $Al^{3+}$ vacancy and an $O^{2-}$ interstitial

(c) an $Al^{3+}$ vacancy and an $O^{2-}$ vacancy (d) two $Al^{3+}$ vacancies and three $O^{2-}$ vacancies

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174. Which one of the following type of dislocations can cross-slip?

(a) Edge dislocations (b) Screw dislocations

(c) Mixed dislocations (d) Sessile dislocations

175. If the gap between ground state and excited state energy levels of electrons in an atom is 1eV. Estimate the wavelength of the light the atom will absorb? The value of the Planck's constant is $6.6\times10^{-34}$ Js.

(a) 1 mm [✓ (b) 1.24 μm] (c) 1.24 angstrom (d) 1cm

176. Which one of the statements about density of states is not true?

[✓ (a) Variation in the energy dependence of density of states is dimension independent.]

(b) Variation in the energy dependence of density of states in 3D follows $E^{1/2}$

(c) Variation in the energy dependence of density of states in 2D follows $E^0$

(d) Variation in the energy dependence of density of states in 1D follows $E^{-1/2}$

177. By using X-ray diffraction one cannot obtain

(a) crystal structure [✓ (b) type of bonding] (c) residual stress (d) grain size

178. Band gap of a semiconductor nanocrystal (keeping the same lattice structure)

[✓ (a) increases with decreasing size] (b) decreases with decreasing size

(c) is same as that of bulk (d) is always materials dependent

179. The voltage required to produce 483.6 MHz through a Josephson junction is

(a) 1nV [✓ (b) 1μV] (c) 1mV (d) 1V

180. Structure of $C_{60}$ fullerite is
(a) face center cubic (b) body center cubic (c) primitive cubic (d) base center cubic

181. The fraction of electrons excited across the energy gap of 0.7 eV in Ge, at room temperature is
(a) 0 [✓ (b)] $1.4\times10^{-6}$ (c) $1.6\times10^{-8}$ (d) $1.03\times10^{-12}$

182. A typical semiconductor such as Ge is formed by what kind of bonds?
[✓ (a)] Covalent (b) Ionic (c) van der Waals (d) Metallic

183. When a pentavalent impurity is added to Si, it becomes
(a) an insulator (b) a conductor
(c) an intrinsic semiconductor [✓ (d)] an *n*-type semiconductor

184. An *n*-type semiconductor is
(a) positively charged (b) negatively charged
[✓ (c)] electrically neutral (d) positively or negatively charged

185. When a pure semiconductor is heated, its resistance
(a) increases (b) decreases linearly
[✓ (c)] decreases exponentially (d) remains the same

186. The transition temperature of a BCS superconductor with an average atomic mass of 199.5 is 4.185K. What is the transition temperature for an average atomic mass of 203.4?
(a) 4.185 [✓ (b)] 4.145 (c) 4.226 (d) 4.105

187. A contribution to the heat capacity is large in magnetic materials at the temperature at which magnetic moments
[✓ (a)] are ordered (b) are disordered
(c) are antiparallel to each other (d) become zero

188. A flat surface is covered with non-overlapping disks of same size. What is the largest fraction of the area that can be covered?
(a) $\frac{3}{\pi}$ (b) $\frac{5\pi}{6}$ (c) $\frac{6}{7}$ [✓ (d)] $\frac{\pi}{2\sqrt{3}}$

189. A metal suffers a structural phase transition from face-centred cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbour distances $d_{fc}$ and $d_{sc}$ for the FCC and the SC structures respectively are in the ratio $(d_{fc}/d_{sc})$ [Given $2^{1/3} = 1.26$]
(a) 1.029 [✓ (b)] 1.122 (c) 1.374 (d) 1.130

190. The energy of an electron in a band as a function of its wave vector **k** is given by
$E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$, where $E_0, B$ and $a$ are constants. The effective mass of the electron near the bottom of the band is
(a) $\frac{2\hbar^2}{3Ba^2}$ (b) $\frac{\hbar^2}{3Ba^2}$ (c) $\frac{\hbar^2}{2Ba^2}$ [✓ (d)] $\frac{\hbar^2}{Ba^2}$

191. A DC voltage V is applied across a Josephson junction between two superconductors with a phase difference $\phi_0$. If $I_0$ and $k$ are constants that depend on the properties of the junction, the current flowing through it has the form
[✓ (a)] $I_0 \sin\left(\frac{2eVt}{\hbar} + \phi_0\right)$ (b) $kV \sin\left(\frac{2eVt}{\hbar} + \phi_0\right)$ (c) $kV \sin\phi_0$ (d) $I_0 \sin\phi_0 + kV$

192. A sample of Si has electron and hole mobilities of 0.13 and 0.05 $m^2/V\text{-}s$ respectively at 300K. It is doped with P and Al with doping densities of $1.5 \times 10^{21}/m^3$ and $2.5 \times 10^{21}/m^3$ respectively. The conductivity of the doped Si sample at 300K is

[✓ (a)] $8\Omega^{-1}m^{-1}$ (b) $32\Omega^{-1}m^{-1}$ (c) $20.8\Omega^{-1}m^{-1}$ (d) $83.2\Omega^{-1}m^{-1}$

193. A uniform linear monoatomic chain is modeled by a spring-mass system of masses $m$ separated by nearest neighbor distance $a$ and spring constant $m\omega_0^2$. The dispersion relation for this system is

(a) $\omega(k) = 2\omega_0\left(1-\cos\left(\frac{ka}{2}\right)\right)$ (b) $\omega(k) = 2\omega_0 \sin^2\left(\frac{ka}{2}\right)$

[✓ (c)] $\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$ (c) $\omega(k) = 2\omega_0 \tan\left(\frac{ka}{2}\right)$

194. The physical phenomenon that cannot be used for memory storage applications is

(a) large variation in magnetoresistance as a function of applied magnetic field

(b) variation in magnetization of a ferromagnet as a function of applied magnetic field

(c) variation in polarization of a ferroelectric as a function of applied electric field

[✓ (d)] variation in resistance of a metal as a function of applied electric field

195. A solid sample has the property that, when cooled below a certain temperature, it expels any small applied magnetic field from within the material. Which of the following best describes this sample in the cooled state?

(a) Paramagnet [✓ (b) Diamagnet] (c) Ferromagnet [✓ (d) Anti-ferromagnet]

196. In a low temperature experiment, the resistance of a sensor is used as a thermometer. In order to have better sensitivity in the range 100 mK to 1.0 K, which material would make the best sensor?

[✓ (a) insulator] (b) p-n junction (c) pure semiconductor [✓ (d) metal]

197. For a pure germanium semiconductor cooled in liquid nitrogen, the average density of conduction electrons is about $n = 10^{12}$ per $cm^3$. At this temperature the electron and hole mobilities are equal and have the common value $\mu = 5.0\times10\ cm^2V^{-1}s^{-1}$. If a potential of 100V is applied across a 1 cm cube of this cooled germanium sample, the current observed can be estimated as

[✓ (a)] 80 μA (b) 160 mA (c) 16 mA (d) 16 A

198. Melting temperature of strained nanoparticles having the same structure as that of bulk is

(a) higher than that of bulk [✓ (b)] lower than that of bulk

(c) does not change (d) always 2/3 times that of bulk

199. What is the value of potential energy V, in a free electron gas theory of metals?

(a) A constant (b) Infinite

(c) It varies with the distance [✓ (d)] zero

200. The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model, Einstein model and Debye model. At low temperature $\hbar\omega >> K_BT$, which one of the following statements is true (a and A are constants)

(a) Dulong Petit: $C \propto \exp\left(\frac{-a}{T}\right)$; Einstein:C=.constant, Debye:$C \propto \left(\frac{T}{A}\right)^3$

(b) Dulong Petit:C = .constant; Einstein:$C \propto \left(\frac{T}{A}\right)^3$; Debye:$C \propto \exp\left(\frac{-a}{T}\right)$

[✓ (c)] Dulong Petit:$C = .\text{constant}$; Einstein:$C \propto \dfrac{e^{-a/T}}{T^2}$;Debye:$C \propto \left(\dfrac{T}{A}\right)^3$

(d) Dulong Petit:$C \propto \left(\dfrac{T}{A}\right)^3$ ; Einstein:$C \propto \dfrac{e^{-a/T}}{T^2}$;Debye:$C$=.constant

201. At low temperature the specific heat of insulating crystals varies as

[✓ (a)] $AT^3$ (b) $BT + CT^3$

(c) D exp (E/T) (d) will not change with temperature.

202. Which of the following statement best explains why the specific heat of electrons in metals is much smaller than that expected in a non-interacting (free) electron gas model?

(a) The mass of electron is much smaller than that of the ions in the crystal.

[✓ (b)] The Pauli exclusion principle restricts the number of electrons which can absorb thermal energy

(c) Electron spin can take only two different values.

(d) Electrons in a metal cannot be modelled as non-interacting.

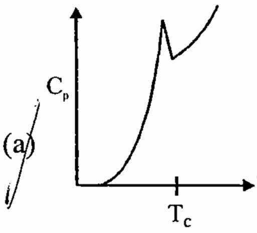
203. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

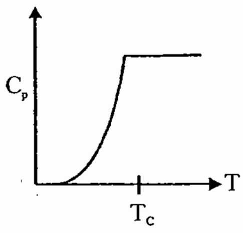
(a) 0° (b) 45° [✓ (c)] 90° (d) 120°

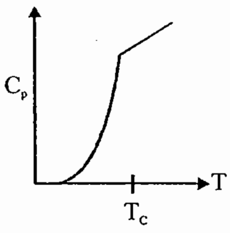
204. The Miller indices of a plane passing through the three points having coordinates (0, 0, 1), (1, 0, 0), $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}\right)$ are

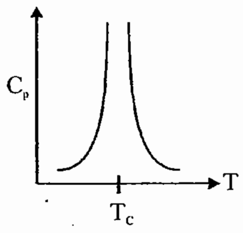
(a) (2 1 2) (b) (1 1 1) [✓ (c)] (1 2 1) (d) (2 1 1)

205. The plot of specific heat versus temperature across the superconducting transition temperature $(T_C)$ is most appropriately represented by

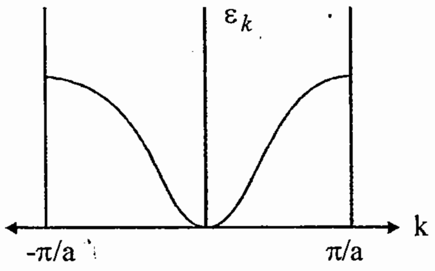
[✓ (a)]


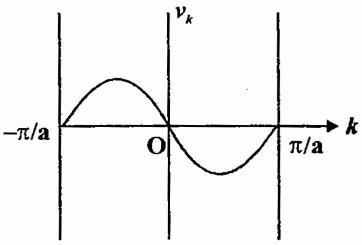
(b)


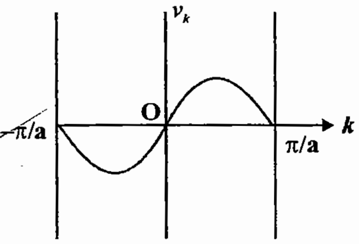
(c)


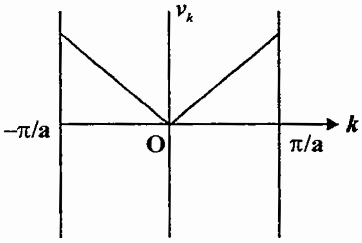
(d)


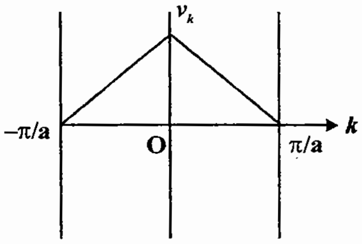
206. The energy, $\varepsilon_k$ for band electrons as a function of the wave vector, k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as (a is latticec constant)



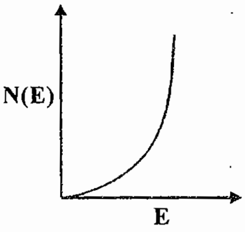
(a)


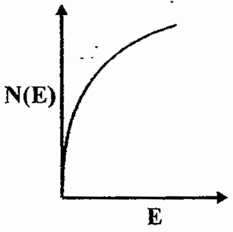
~~(b)~~


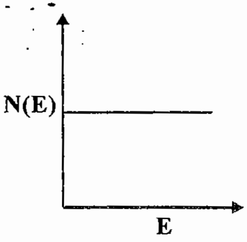
(c)


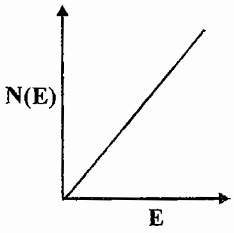
(d)


207. For a free electron gas in two dimensions, the variation of the density of states, N(E) as a function of energy E, is the best represented by

(a)


(b)


(c)


(d)


208. For Nickel, the number density is $8\times10^{23}$ atoms/cm$^3$ and electronic configuration is $1s^2\,2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic states is ________ $\times\,10^9$ A/m.

(Given the value of Bohr magneton $\mu_B = 9.21\times10^{-21}\ Am^2$)

209. The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is ________ meV.

$(k_B T = 25 meV$ at 300K$)$

210. Neutrons moving with speed $10^3$ m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30°, the interplanarspacing of the crystal is ________ Å.

(Given: $m_n = 1.675\times10^{-27}\,kg,\ h = 6.626\times10^{-34}\ J.s$)

211. Consider an electron in a b.c.c. lattice with lattice constant a. A single particle wavefunction that satisfies the Bloch theorem will have the form $f(\vec{r})\exp(i\vec{k}.\vec{r})$, with $f(\vec{r})$ being

(a) $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$ —> fcc

(b) $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

(c) $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$ —> BCC

(d) $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

212. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation by

$$\varepsilon(k)=-4\varepsilon_0\left[\cos\frac{k_x a}{2}\cos\frac{k_y a}{2}+\cos\frac{k_y a}{2}\cos\frac{k_z a}{2}+\cos\frac{k_z a}{2}\cos\frac{k_x a}{2}\right]$$

where 'a' is the lattice constant and $\varepsilon_0$ is a constant with the dimension of energy. The $x$-component of the velocity of the electrons at $\left(\frac{\pi}{a},0,0\right)$ is

(a) $-\frac{2\varepsilon_0 a}{\hbar}$ (b) $\frac{2\varepsilon_0 a}{\hbar}$ (c) $-\frac{4\varepsilon_0 a}{\hbar}$ [✓ (d) $\frac{4\varepsilon_0 a}{\hbar}$]

200

## Assignment-I: Solid State Physics [Answer Key]

1. (c) 2. (d) 3. (a) 4. (d) 5. (c) 6. (b,d) 7. (c)
8. (d) 9. (b) 10. (c) 11. (a) 12. (c) 13. (b) 14. (d)
15. (c) 16. (a) 17. (a) 18. (a) 19. (d) 20. (c) 21. (a)
22. (a) 23. (d) 24. (c) 25. (c) 26. (d) 27. (b) 28. (d)
29. (d) 30. (b) 31. (a) 32. (b) 33. (b) 34. (d) 35. (a)
36. (b) 37. (b) 38. (a) 39. (b) 40. (b) 41. (c) 42. (d)
43. (c) 44. (a) 45. (c) 46. (b) 47. (a) 48. (b) 49. (a)
50. (a) 51. (a) 52. (a) 53. (a) 54. (d) 55. (d) 56. (a)
57. (a) 58. (c) 59. (a) 60. (b) 61. (a) 62. (a) 63. (a)
64. (b) 65. (c) 66. (c) 67. (c) 68. (a) 69. (c) 70. (d)
71. (b) 72. (a) 73. (c) 74. (c) 75. (c) 76. (c) 77. (a)
78. (b) 79. (b) 80. (c) 81. (d) 82. (d) 83. (c) 84. (c)
85. (c) 86. (d) 87. (c) 88. (b) 89. (a) 90. (d) 91. (d)
92. (b) 93. (b) 94. (c) 95. (c) 96. (d) 97. (a) 98. (b)
99. (c) 100. (b) 101. (d) 102. (c) 103. (c)

## Assignment-II: Solid State Physics (Answer Key)

1. (d) 2. (a) 3. (b) 4. (b) 5. (a) 6. (d) 7. (c)
8. (c) 9. (b) 10. (d) 11. (b) 12. (b) 13. (b) 14. (b)
15. (a) 16. (d) 17. (a) 18. (b) 19. (b) 20. (a) 21. (b)
22. (b) 23. (d) 24. (b) 25. (b) 26. (a) 27. (d) 28. (d)
29. (d) 30. (b) 31. (c) 32. (d) 33. (c) 34. (a) 35. (b)
36. (b) 37. (c) 38. (b) 39. (a) 40. (b) 41. (a) 42. (a)
43. (c) 44. (d) 45. (d) 46. (c) 47. (c) 48. (c) 49. (c)
50. (b) 51. (a) 52. (a) 53. (a) 54. (c) 55. (d) 56. (b)
57. (c) 58. (d) 59. (b) 60. (a) 61. (a) 62. (d) 63. (a)
64. (d) 65. (c) 66. (c) 67. (c) 68. (b) 69. (d) 70. (b)
71. (a) 72. (c) 73. (c) 74. (c) 75. (d) 76. (a) 77. (c)
78. (d) 79. (d) 80. (b) 81. (b) 82. (b) 83_A(a) 83_B(b)
84_A(a) 84_B(b) 85_A(a) 85_B(a) 86_A(b) 86_B(c) 87. (c)
88. (a ) 89. (d) 90. (a) 91. (b) 92. (a) 93. (c) 94. (c)
95. (c) 96. (b) 97. (c) 98. (d) 99. (b) 100. (c) 101. (c)
102. (a) 103. (b) 104. (c) 105. (a) 106. (a) 107. (b) 108. (a)
109. (c) 110. (a) 111. (d) 112. (b) 113. (d) 114. (b) 115. (d)
116. (c) 117. (c) 118. (c) 119. (a) 120. (d) 121. (b) 122. (b)
123. (c) 124. (d) 125. (c) 126. (d) 127. (a) 128. (d) 129. (c)
130. (c) 131. (c) 132. (b) 133. (b) 134. (b) 135. (d) 136. (c)
137. (b) 138. (d) 139. (c) 140. (c) 141. (a) 142. (a) 143. (a)
144. (a) 145. (c) 146. (b) 147. (d) 148. (a) 149. (a) 150. (d)
151. (d) 152. (c) 153. (b) 154. (a) 155. (a) 156. (b) 157. (b)
158. (d) 159. (d) 160. (c) 161. (c) 162. (b) 163. (a) 164. (a)
165. (c) 166. (b) 167. (a) 168. (b) 169. (a) 170. (b) 171. (a)
172. () 173. () 174. () 175. (b) 176. (a) 177. (b) 178. (a)
179. (b) 180. (a) 181. () 182. (a) 183. (d) 184. (c) 185. (c)
186. () 187. (a) 188. (d) 189. () 190. (d) 191. (a) 192. (a)
193. (c) 194. (d) 195. (b) 196. (c) 197. (b) 198. () 199. ()
200. (c) 201. (b) 202. (b) 203. (c) 204. (c) 205. (a) 206. (b)
207. (c) 208. (40 to 43) 209. (114 to 117) 210. (3.91 to 4.15)
211. (b) 212. (d)

## CSIR-UGC-NET/JRF | GATE - PHYSICS

### Assignment - I Solid State

1. Sodium transfoms from bcc to hcp crystal structure at about 23 kelvin without change of density in the cubic phase a= 4.23 Å and the hcp phase $^c/_a$ has ideal value of 1.63. Find the lattice constant.
2. The lattice parameter and the atomic mass of a diamond crystal are 3.57 Å and 12, respectively. Calculate the density of the same.
3. Calculate the lattice constant for a rock salt crystal (density = 2180 kg/m$^3$), assuming that it has fcc lattice. Mol. wt. of NaCl = 58.5.
4. Calculate the number of atoms per unit cell for an fcc lattice of copper crystal. It is given that a = 3.60Å, at wt. of Cu = 63.6, density of Cu = 8960 kg/m$^3$ and the Avogadro's number, N = $6.023\times10^{26}$.
5. Aluminium has an fcc structure. Its density is $2.7\times10^3$ kg/m$^3$. Calculate the unit cell dimensions and the atomic diameter.
6. Determine the Miller indices of a plane that makes an intercept of 3Å, 4Å, and 5Å on the coordinate axes of an orthorhombic crystal with a: b: c = 1:2:5.
7. Find the Miller indices of a plane that makes intercepts on a, b and c axes equal to 4Å, 6Å and 4Å in a tetragonal crystal with the cla ratio as 2.
8. Determine the value of "i" in the following Miller-Bravais indices: (10.3), (11.3), (01.4), $(1\bar{1}4),(10.4),(2\bar{2}.4),(2\bar{2}.3)(11.0),(32.5)$ and $(3\bar{4}.2)$
9. Calculate the number of atoms per unit cell for rock - salt crystal. Given a = 5.63Å, mol.wt. of NaCl = 58.5 and the density is 2180 kg/m$^3$.
10. In a tetragonal crystal, the lattice parameters a = b = 2.42Å, and c = 1.74Å. Deduce the interplanar spacing between consecutive (101) planes.
11. The distance between consecutive (1 1 1) planes in a cubic crystal is 2Å. Determine the lattice parameter.
12. Calculate the atomic density in (100), (110) and (111) planes of copper (fcc) with the lattice parameter of 3.61Å. Can you pack atoms more closely than in (111) plane?
13. Calculate the number of atoms per unit area in (100), (110) and (111) plane of a bcc crystal with the lattice parameter of 2.5Å.
14. $\alpha-$Co has an hcp structure with lattice spacings of a = 2.51 Å and c = 4.07Å, $\beta-$Co is fcc, with a cubic lattice spacing of 3.35 Å. What is the difference in density between the two forms?
15. Compute the packing fraction of the 2-D hexagonal lattice structure.
16. The density of copper is 8.96 g/cm$^3$. It forms an FCC lattice, and has an atomic weight of 63.4 g/mole. Find the length, a, of a side of the cubic unit cell (conventional cell) of copper. Give your result in nanometers.
17. Define lattice constant of a material. Given the silicon lattice constant a = 0.543 nm, calculate the Si-Si bond length. Also calculate the density of silicon, in g cm$^{-3}$. [Atomic mass of silicon = 28.09, Avogadro's number = $6.022\times10^{23}$ mol$^{-1}$].
18. Assuming that the phase transition of metallic Fe, from a body centred cubic structure to a face-centred cubic structure does not affect the atomic radius, find the ratio of the density of Fe in the two phases.
19. The distance between (111) planes in a face centered cubic crystal is 2 Å. Determine the lattice parameter and the atomic diameter.

20. (a) Calcualte the diffraction angle $(2\theta)$ for the first three peaks in aluminium powder diffraction pattern. Note that the crystal structure of the Aluminium is fcc and lattice constant is 0.404 nm.
(b) If the lattice constant of Al was larger than 0.404 nm. Describe how pattern would change?

21. In a powder X-ray diffractoin pattern using X-rays of wave length 1.54Å, three consecutive first order peaks are observed at '$2\theta$' values of 60°, 90° and 112°. If system is cubic and peaks at 90°. Corresponds to (220) planes.
(a) Calculate the value of lattice constant (a)
(b) Obtain the miller indices and identify the system.

22. A body-centred cubic (bcc) cell has a lattice constant a
(a) How many atoms are there in a cubic unit cell?
(b) What is the spacing between the (1 1 1) planes?
(c) What will be the Bragg's diffraction angle for scattering from the (100) planes of the crystal?

23. A transmission electron microscope is use to produce a diffraction ring for the polycrystalline sample of copper the ring (1 1 1) is 12 nm from centre. How far would (200) ring be from the centre.

24. Calculate the packing fraction for
(a) (110) and (111) planes of sc.
(b) (100), (110) and (111) of fcc.
(c) (100), (110) and (111) of fcc
(d) Triangular lattice in hcp crystal having each side (a). Where a is lattice parameter.

25. Assume the potential energy of two particles in the field of each other is given by

$$U(R)=\frac{-A}{R}+\frac{B}{R^9}$$

Where A and B are constants.
(a) Calculate equilibrium distance.
(b) Show that stable configuration, the energy of attracton is nine times the energy of repulsion.
(c) Show that potential energy of the system under stable configuration is $\frac{8A}{9R}$

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## CSIR-UGC-NET/JRF | GATE - PHYSICS

## Assignment - 2 Solid State

1. As we know for diffraction studies of crystal, the wavelength of incident wave should be around 1Å. X-rays, neutrons and electrons used to explore the crystal structure, then compare the energies of X-rays, neutrons and electrons for 1Å wavelength also find out which one better for the studies of
(i) Crystal lattice structure (ii) Surface structure (iii) Magnetic structure

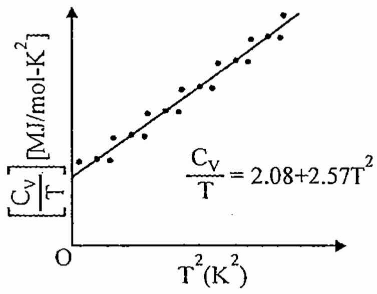
2. (a) Show that as diffraction angle increases accuracy in calculation of lattice parameter increases
(b) In an X-ray diffraction experiment using $\lambda_{cu} = 1.542\text{Å}$, the (311) planeof a cubic lattice gives a dffraction peak at $\theta = 45^\circ$ with an uncertaininty of $\pm 0.2^\circ$. Calculate the value of lattice parameter and estimated error.

3. Consider the monoatomic lattice in which second nearest neighbour interaction is also considered. The dispersion relation for phonon is given by

$$\omega^2 = \left(\frac{4C}{m}\right)\sin^2\left(\frac{Ka}{2}\right)\left[1+4\cos^2\left(\frac{Ka}{2}\right)\right]$$

Calculate the velocity of sound in the solid.

4. The experimental heat capacity of potassium (K) is plotted in the graph below:



from this graph calculate
(a) Fermi temperature ($T_F$) (b) Fermi energy ($E_F$)
(c) Debye temperature ($\theta_D$) (d) Debye frequency ($\nu_D$)
(e) If potassium is monovalent bcc, calculate its lattice parameter (a)
(f) Speed of sound in potassium (K)

5. (a) Consider two branches of a phonon spectrum of a one dimensional lattice $\omega_1(\bar{k}) = A|\sin(k)|$ and $\omega_2(\bar{k}) = B(2+\cos(k))$
(a) in the Debye phonon dispersion approximations give the phonon dispersion reation.
(b) Band width of optical branch, Band width of acoustical branch and Band gap

* Dispersion relation
* Band theory
Imp-

6. What is the temperature at which there is a 1% probability that an energy state lying 0.2 eV below the fermi level in gold is empty. $E_F$ of gold is 5.4 eV.

7. Calculate the thermal conductivity of sodium (Na) at 300K. The $e^-$ concentration and mean free time ($\tau$) for sodium is $2.65 \times 10^{28}$ / $m^3$ and $3 \times 10^{-14}$ sec respectively.

8. Take the Fermi energy of silver to be 5.52 eV.

(a) Find the corresponding velocity of conduction electron.

(b) If the resistivity of silver at room temperature is $1.62 \times 10^{-8}$ $\Omega$m estimate the average time between collisions.

(c) Determine the mean free path. Assume the number of conduction electrons as $5.86 \times 10^{28}$ $m^{-3}$.

9. Aluminium is trivalent with atomic weight 27 and density 2.7 g/cm$^3$, while the mean collision time between electrons is $4 \times 10^{-14}$s. Calculate the current flowing through an aluminium wire 20m long and 2mm$^2$ cross-sectional area when a potential of 3V is applied to its ends.

10. Given that the inter-collision time in copper is $2.3 \times 10^{-14}$ S, calculate its thermal conductivity at 300K. Assume the Wiedemann-Franz constant is $C_{WF} = 2.31 \times 10^{-8} W \Omega K^{-2}$.

11. The Fermi energy in gold is 5.54 eV (a) calculate the average energy of the free electrons in gold at 0°K. (b) Find the corresponding speed of free electrons (c) Which temperature is necessary for the average kinetic energy of gas molecules to posses this value?

12. The density of copper is 8.94g/cm$^3$ and its atomic weight is 63.5 per mole, the effective mass of electron being 1.01. Calculate the Fermi energy in copper assuming that each atom gives one electron.

13. Find the probability of occupancy of a state of energy (a) 0.05eV above the Fermi energy (b) 0.05eV below the Fermi energy (c) equal to the Fermi energy. Assume a temperature of 300K.

14. What is the probability at 400K that a state at the bottom of the conduction band is occupied in silicon. Given that $E_g = 1.1$eV.

15. The Debye temperature $\theta$ for iron is known to be 360K. Calculate $\nu_m$, the maximum frequency.

16. (a) A fcc crystal of cube side a. Calculate the fermi energy for (111) plane and compare this fermi energy to the fermi energy of (100) plane. Here we are assuming that fcc crystal giving two conduction electrons per atom.

(b) Calculate the Fermi energy and wave vector for simple cubic crystal in one dimension, cube side is a.

17. Sodium metal with bcc structure has radius of sodium atom to be 1.85 Å. Calculate

(a) Fermi energy, fermi velocity and diameter of fermi sphere.

(b) Electrical resistavity from drude model. (Given: $\tau = 3 \times 10^{-14}$ second)

18. Sodium has bcc structure with atomic radius 1.82 Å and one valence electron per atom. Resistivity of Na at 20°C is $\rho = 1.72 \times 10^{-8}\ \Omega - m$. Calculate:

(i) Characteristic time $(\tau)$ from Drude model. (ii) Fermi energy

(iii) Diameter of Fermi sphere $= 2K_F$ (iv) Mean free path $(\lambda)$. $\lambda = V_F \tau$

(v) Fermi temperature $(T_F)$. $E_F = KT_F$

19. Lattice vectors of rectangular primitive lattice is given as $\vec{a} = 2\hat{i},\ \vec{b} = \hat{j}$ (a and b are in Å).

(i) Calculate the area of all the Brillouin zones.

(ii) Radius of fermi circle.

(iii) Fermi energy.

(iv) Average velocity of electrons at which they are moving in crystal.

20. Calculate the fermi energy of electron gas in copper. The density of copper at 300 K is 8.96 g/cm and atomic weight = 63.5. Take one conduction electron per atom.

21. Calculate drift velocity and mobility of electron in a copper wire of length 2.4 m when a potential differene of 32v is maintained across its ends. Assume that each copper atom contributes one conduction electron. Avogadro number $N = 6.02 \times 10^{26}$ / kg mole.

    Resistivity $\rho = 0.172 \times 10^{-7}$ ohm – m

    Density $= 8.9 \times 10^3$ kg / m$^3$

    Atomic weight = 63.5.

22. A body centred cubic metal having electrons as charge carries density of electrons $2.64 \times 10^{28}$ /m$^3$. Using free electron theory calculate:
    (i) Fermi energy of metal in eV.
    (ii) Lattice constant of solid in Ang.
    (iii) Fermi momentum.

23. Copper has monovalent fcc structure. Calculate the diameter of fermi sphere and shortest distance in first Brillouin zone.

24. $E(\bar{k}) = E_0 - 24\left[\cos(k_x a) + \cos(k_y a)\right]$

    where $k^2 = k_x^2 + k_y^2$

    Calculate:
    (i) Band width of lowest band.
    (ii) Plot first Brillouin zone.
    (iii) E–k diagram along [10] and [11] direction.

25. A hypothetical semi conductor has a conduction band that can be described by $E_{cb} = E_1 - E_2\cos(ka)$ and a valence band that can be described by $E_{vb} = E_3 - E_4\sin^2\left(\frac{ka}{2}\right)$

    where $E_3 < (E_1 - E_2)$ and $\frac{-\pi}{a} \le k \le \frac{\pi}{a}$

    (i) Sketch the variation of the energy of two bands as function of k for $-\frac{\pi}{a} \le k \le \frac{\pi}{a}$
    (ii) Band width of the conduction band and that of the valence band?
    (iii) Band gap of the material?
    (iv) What is the effective mass of the electron at the bottom of the conduction band?

26. For a solid energy is given by $E(k) = E_0 - c\cos\left(\frac{k_x a}{\cancel{2}}\right)\cos\left(\frac{k_y a}{\cancel{2}}\right)$ ← 2-D version of bcc & fcc & its lattice parameter is (2

    (a) Expand $E(\bar{k})$. (b) Band width of the band (c) Effective mass at $(0, \pi/a)$ and at $(0,0)$.

27. The energy band relation in a linear chain with interatomic distance a is given by $E_k = E_0 - T\cos(ka)$. When width of this band is increased by 20%. Calculate the percentage change in the effective mass of electron at the bottom of the band.

## CSIR-UGC-NET/JRF | GATE - PHYSICS

### Assignment - 3 Solid State

1. The band structure of a simple cubic lattice is given by $E = E_0 - A(\cos ka + \cos ka + \cos ka)$ where $a$ is the lattice constant and A is a positive constant.

   (a) Find the effective mass of the electron at values of k corresponding to the top and bottom the band.

   (b) For a particular simple cubic solid with a lattice constant a = 2.5Å, the effective mass of the electron at the bottom of the band is equal to the free electron mass. Find the value of the constant A and the width of the band in eV.

2. A certain metal has a body centered cubic space lattice with cube side $a$. The electrons are assume to be free.

   (a) If the metal is monovalent, what is the radius of fermi sphere?

   (b) For the electrons density of part (a) the fermi sphere does not reach the Brillouin zone boundary. What is the minimum value of the free electron density so that the fermi sphere just touches the zone boundary?

3. Imp An organic conductor has recently been found by optical studies to have $\omega_p = 1.80 \times 10^{15}\ \text{sec}^{-1}$ for plasma frequency and $\tau = 2.83 \times 10^{-15}$ sec for the electron relaxation time at room temperature.

   (a) Calculate electrical conductivity

   (b) From crystal and chemical structure; the conduction electron concentration is $4.7\times10^{21}$ /cm$^3$. Calculate effective mass m*.

4. Calculate the hall coefficient of sodium (Na) on based on free electron theory. Sodium has bcc structure and side of cube is a = 4.28Å.

5. A sample of silicon of thickness 200μm is doped with $10^{23}$ phosphorus atoms/m$^3$. If the sample is kept in a magnetic field of 0.2 wb/m$^2$ and a current of 1 mA is passed through the sample, calculate the hall voltage produced.

6. Hall coefficient of a body-centred cubic metal having electrons as charge carriers is $R_H = -2.36\times10^{10}$ m$^3$/C. Using free electron theory. Calculate the lattice constant of metal.

7. A silicon bar of length $l$ = 10.0 mm and cross-section a$^2$ = 0.5×0.5 mm$^2$ carries current I = 5mA along its length in $x$-direction. It is placed between pole pieces of an electron magnet of field $B_z$ = 1.6 tesla along $z$-direction. A pairs of voltage probles, plaed across the cross-section of the sample and perpendicular to the current and field, measures a voltage, $V_H$ = –5mV. find dopent density.

8. The donor concentration of an $n$-type semiconductor ($E_g$ = 2.303 eV) is $N_D = 2\times10^{16}$ /cm$^3$. Find the temperature (in terms of Boltzman constant) at which the semiconductor starts behaving like an intrinsive semiconductor. The density of states in the conduction and the valence bands are $4\times10^{9}$ /cm$^3$ and $1\times10^{19}$/cm$^3$ respectively.

9. (a) Show that minimum conductivity of a semi conductor sample occurs. When $n_0 = n_i\sqrt{\dfrac{\mu_p}{\mu_n}}$

   (b) What is the expression for the minimum conductivity $\sigma_{\min}$ ?

   (c) Calculate $\sigma_{\min}$ for Si at 300K and compare with the intrinsic conductivity.

10. A Si solar cell has a short circulated current of 100 mA and an open circuit voltage of 0.8 volt under full solar illumination. The fill factor is 0.7. What is the maximum power delivered to a load by this cell?

11. A Si solar cell with a dark current Ith of 5 nA is illuminated such that the short circuit current is 200 mA. Plot I-V characteristics for the cell? [Temperature is T = 300K].

12. A Si solar cell 2cm × 2cm with Ith = 32nA has an optical generation rate of $10^8$ EHA cm$^3$–sec within $L_p = L_n = 2\mu m$ of the junction. If the depletion width is 1 μm, calculate the short-circuit current and open circuit voltage for this cell?

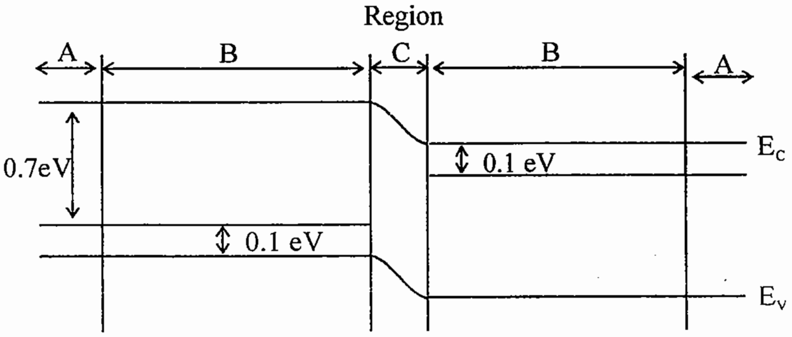
13. If one LED made from a semiconductor with a band gap of 2.5 eV. What wavelength of light will it emit? Can you use it to efficiently detect photons of wavelength 1900 nm and 100 nm?

14. A solar cell has a short circuit current of 50 mA and an open circuit voltage of 0.7V under full illumination. What is the maximum power delivered by this cell if the fill factor is 0.8?

15. A photo conductive cell, 1×1×1 ~~mm~~ mm$^3$/Cds crystal is illuminated with light of wavelength 450 nm and intensity 1 mW/cm$^2$. Assuming quantum efficiency of unity. Calculate the number of electron hole pairs (EHP/second) generated per second. Given the energy band gap of Cds, $E_g$ = 2.6 eeV.

16. What wavelength of light can be observed by a silicon photodetector band gap silicon = 1.1 eV.

17. Consider a band diagram of a simple light-emitting diode. Assume essentially all recombination is direct and result in light emission. The forward biased current consists of holes injected from a conctact on the left and electrons injected from a contact on the right.



(a) In which region would you expect the optical recombination rate to be greatest?

(b) What is the approximation energy of emitted photons is eV?

(c) For steady state current I = 10 mA, assuming all photons escape, what is the optical output power consists with your answer to part (b)

(d) If the voltage drop across depletion region is 0.4 volt, what is the separation of quasi fermi level in region C. How can this be less than the total forward bias of 1.4 volt?

18. A 0.46μm thick sample of GaAs is illuminated with monochromatic light of hν = 2eV. The absorption coefficient $\alpha$ is $5\times10^4$ cm$^{-1}$. The power incident on the sample is 10 mW. (Band gap $E_g$ = 1.43 eV)

(a) Find the total energy absorbed by the sample per second (J/sec).

(b) Find the rate of excess thermal energy given up by the electrons to the lattice before recombination.

(c) Find the number of photons per second given off from recombination events assuming perfect quantum efficiency.

19. A Si pin photodiode has an active light receiving area of dimensions (0.4mm × 0.4 mm). When radiation of wavelength 700 nm and intensity 0.1 mW/cm$^2$ is incident, it generated a photon current of 20 nA. Calculate the

(a) Responsivity in units of A/W, and the

(b) Qunatum efficiency of hte photodiode at 700 nm (within an accuracy of 1%).

20. (a) Calculate the static dielectric constant $(\varepsilon_r)$ of solid argon using classius-Mossotti equation and the following data?

$\alpha_e = 1.7\times10^{-40}\ F.m^2$, atomic mass of Avogadro number
Ar = 39.95 g/mole,
N = $6.02\times10^{23}$ per mole, density of solid Ar
= 1.8 gm/cm$^3$.

(b) Will result be affected if $(\varepsilon_r)$ is calculated from $P=\varepsilon_0(\varepsilon_r-1)E=N\alpha_e E$. Give reason for your answer.

21. Calculate the dielectric constant for saturated water vapour at 20°C given the density of saparated water vapour is 17.3 g/m$^3$.

22. A radiation of 1 kW/m$^2$ incident on the solar panel of area $1\times1.6$ m$^2$. The light has wavelength $\lambda = 732\ nm$.
(i) Determine the number of photons/second incident on the panel.
(ii) Calculate the current generated in (A) if panel has 100% efficiency.

23. Iron has bcc structure with lattice constant a = 2.86Å. The saturation magnetisation of iron is $1.74\times10^6$ amp/m. Calcualte magnetic moment (μ) of Fe atom.

24. Calculate saturation magnetisation for Nickel and cobalt having densities equal to 8.67 and 9.04 gm/cm$^3$, respectively. Number of Bohr magnetons per atom are 1.72 for Co and 0.54 for Ni. Atomic masses are $M_{CO}$ = 58.93 and $M_{Ni}$ = 58.69.

25. The diamagnetic susceptibility due to the ion cores in metallic copper is $-0.20\times10^{-6}$ knowing the densities of Cu is 8.93 g/cm$^3$ and that its atomic weight is 63.5. Calculate the average radius of Cu ion.

26. The binding energy of a copper pair at absolute zero according to BCS theory is a about 3.5 $KT_c$. Where 'K' is Boltzmann constant and $T_c$ is critical temperature. The critical temperature of type I super conductor is 10K.
(a) What is the energy gap in electron volt.
(b) Calculate the wavelength of photons whose energy is the just sufficient to break the copper pair in this super conductivity
(c) In which region of electromagnetic spectrum do this photon lie?

27. In lead (Pb) the binding energy of Cooper pair is 10 meV. Calculate
(i) Critical temperature of Pb.
(ii) Maximum wavelength of photons require to break the cooper pair.

28. For BCS super conductor $T_C$ = 5K, $E_F$ = 5 eV, and *n* density of conduction electrons = $10^{29}$/m$^3$. Evaluate
(a) Approximate magnitude of energy gap in (eV)
(b) Density of super electrons close to T = 0K.

29. The maximum wavelength of photons require to break the cooper pair in Sn(tin) is $1.08\times10^{-3}$m. Calculate the critical temperature of Sn.

30. Calculate the frequency of AC current flows across Josephson junction. If 1.5μV potential applied on the junction. Also calculate switching time of the junction.

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## CSIR-UGC-NET/JRF | GATE- PHYSICS

## Assignment - Experimental Methods

1. In which counter, for detection of thermal neutron baron is introduced in the form of $BF_3$?
(a) Ionisation Chamber (b) Proportional counter
(c) G.M. counter (d) All of the above

2. The pulse rise time is defined as the time taken by the pulse :
(a) To go from 10% to 90% of its amplitude
(b)To go from 0% to 100% of its amplitude
(c) To go from 0% to 90% of its amplitude
(d) To go from 10% to 100% of its amplitude

3. In ionisation types of vaccume gauges, the pressure of P is :
(a) $P\ \alpha\ I_G/I_P$ (b) $P\ \alpha\ I_P/I_G$
(c) $P\ \alpha\ I_K/I_G$ (d) $P\ \alpha\ I_P/I_K$
Where $I_G$ = Grid current, $I_K$ = Cathode current
$I_P$ = Plate current

4. Radiation Pyrometers are usded in the temperature range of :
(a) 0 – 500°C (b) 500 – 1000°C (c) –250 – 500°C (d) 1200 – 2500°C

5. The mean deviation $\bar{D}$ in terms of deviation from the mean value of n readings is :
(a) $\frac{\Sigma|d|}{n}$ (b) $\frac{\Sigma d}{n}$ (c) $\frac{\sqrt{\Sigma d^2}}{n}$ (d) $\sqrt{\frac{\Sigma d^2}{n}}$

6. The Gaussian distribution can be mathematically expressed as :
(a) $y=\frac{h}{\sqrt{\pi}}\exp\left(h^2x^2\right)$ (b) $y=\frac{h}{\pi}\exp\left(-h^2x^2\right)$ (c) $y=-\frac{h}{\sqrt{\pi}}\exp\left(h^2x^2\right)$ (d) $y=\frac{h}{\sqrt{\pi}}\exp\left(-h^2x^2\right)$

7. One of the organic crystal used in scintillator is :
(a) Anthracene (b) Urea
(c) Xylene (d) Benzene

8. In a Hall effect experiment donor density $=10^{21}m^{-3}$, $B=0.5\ W/m^2$, $J=500\ \text{amps}/m^2$, thickness of the sample = 3 mm. calculate the Hall voltage :
(a) 4.7 mV (b) 7.4 mV (c) 8.7 mV (d) 3.9 MV

9. The quenching agents used in GM counter is :
(a) Ethyl alcohol or Halogens (b) Ar
(c) He (d) $CH_4$

10. In order to obtain good gain stability in a negative voltage feedback amplifier ($A_o$ = voltage gain without feedback; β= feedback fraction) the required condition is :
(a) $A_o\beta=1$ (b) $A_o\beta>>1$ (c) $A_o\beta<1$ (d) None of the above

11. Which of the following statement is false :
(a) CR differentiator is a high pass filter
(b) RC integrator is a low pass filter
(c) (RC) – $(CR)^n$ shaping produce Gaussian shape
(d) Pole zero cancellation removes the undershoat

12. At the input on amplifier has a signal voltage level of 3μV and a noise voltage level of 1 μV, the signal to noise ratio at the input is :
(a) 1 (b) 3 (c) 9 (d) 27

13. Which of the following statement is wrong?
(a) In photoemission, absorption of photon frees an electron from material
(b) In photo voltaic effect, absorption of photon leads to the generation to a potential defference across a junction between two material
(c) In photoconductivity, absorption of photon increases the number of charge carrier in a material
(d) In thermopile, absorption of photons causes change in resistance

14. A 0 – 300V voltmeter has an error of ± 2% of full scale deflection. What would be the range of reading if true voltage is 30V.
(a) 24V – 36 V (b) 29.4V – 30.6 V (c) 20 V – 40 V (d) None of these

15. Johnson noise is :
(a) caused by thermal agitation of free electrons carrying current there by modulating the current
(b) Noise carried into a circuit through conductors
(c) Noise of an electromagnetic origin that is radiated into a circuit
(d) Shat noise which results from random emission of electrons across a p-n junction.

16. In solid state detectors, contact is made with the n-type silicon layer by a thin evaporated film of :
(a) Gold (b) Aluminium (c) Silver (d) Alloy

17. The impedance matching between detector and Pri-amplifier is required as it :
(a) Reduce the noise level is signal (b) Increases the band width of the Pre-amplifier
(c) Reduce the reflection of signal (d) All of the above

18. The variance of a mean in which of the following is wrongly matched
(a) $\frac{\sigma_u}{u} = \pm m\sigma_x$ for $u = x^{\pm m}$ (b) $\frac{\sigma_u}{u} = \pm(b\,\ell n\,a)\sigma_x$ for $u = a^{\pm bx}$
(c) $\frac{\sigma_u}{u} = \sqrt{\frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2}}$ for $u = \pm axy$ (d) $\sigma(\bar{x}) = \sqrt{N}$ for $\bar{x} = \frac{1}{N}\Sigma x$

19. The flow process for which the mean free path of the molecules is large compared with the dimensions of the apparatus is :
(a) Viscous flow (b) Molecular flow (c) Knudson flow (d) None of these

20. The sensitive region of a p-n junction particle detector is the :
(a) P-region (b) Depletion region
(c) n-region (d) P, n and Depletion region

21. The length and radius of a perfect cylinder are each measured with an RMS error of 1%. The RMs error on the inferred volume of the cylinder is roughly
(a) 1.7% (b) 3.3% (c) 0.5% (d) 1%

22. A resistance strain gauge is fastened to a steel fixture and subjected to a stress of 1000 kg/$m^2$. IF the gauge factor is 3 and the modulus of elasticity of steel is $2\times10^{10}$ kg / $m^2$, then the fractional change in resistance of the strain gauge due to the applied stress is
(Note: The gauge factor is defined as the ratio of the fractional change in resistance to the fractional change in lenght.)
(a) $1.5\times10^{-7}$ (b) $3.0\times10^{-7}$ (c) $0.16\times10^{-10}$ (d) $0.5\times10^{-7}$

23. Consider a sinusoidal waveform of amplitude 1V and frequency $f_0$. Starting from an arbitrary intial time, the waveform is sampled at intervals of $1/(2f_0)$. IF the corresponding Fourier spectrum peaks at a frequency $\bar{f}$ and an amplitude $\bar{A}$, then

(a) $\bar{f} = 2f_0$ and $\bar{A} = 1V$ (b) $\bar{f} = f_0$ and $0 \le \bar{A} \le 1V$

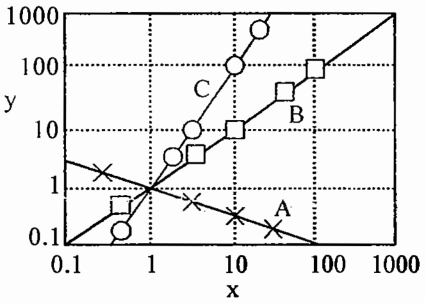
(c) $\bar{f} = 0$ and $\bar{A} = 1V$ (d) $\bar{f} = \frac{f_0}{2}$ and $\bar{A} = \frac{1}{\sqrt{2}}V$

24. The acceleration due to gravity g is determined by measuring the time period T and the length L of a simple pendulum. If the uncertainties in the measurements of T and L are $\Delta T$ and $\Delta L$ respectively, the fractional error $\Delta g/g$ in measuring g is best approximated by

(1) $\frac{|\Delta L|}{L} + \frac{|\Delta T|}{T}$ (2) $\frac{|\Delta L|}{L} + \frac{|2\Delta T|}{T}$

(3) $\sqrt{\frac{|\Delta L|^2}{L} + \frac{|\Delta T|^2}{T}}$ [✓ (4) $\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$]

25.. Three sets of data $A$, $B$ and $C$ from an experiment, represented by $\times$, $\square$ and $\bigcirc$, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure.



The functional dependence $y(x)$ for the sets A, B and C are, respectively

(a) $\sqrt{x}, x$ and $x^2$ (b) $-\frac{x}{2}, x$ and $2x$

(c) $\frac{1}{x^2}, x$ and $x^2$ (d) $\frac{1}{\sqrt{x}}, x$ and $x^2$

26. One gram of salt is dissolved in water that is filled to a height of 5 cm in a beaker of diameter 10 cm. The accuracy of length measurement is 0.01. The accuracy of length measurement is 0.01 cm while that of mass measurement is 0.01 mg. When measuring the concentration C, the fractional error $\Delta C/C$ is

(a) 0.8% (b) 0.14% [✓ (c) 0.5%] (c) 0.28%

27. A resistance is measured by passing current through it and measuring the resulting voltage drop. If the voltmeter and the ammeter have uncertainties of 3% and 4%, respectively, then

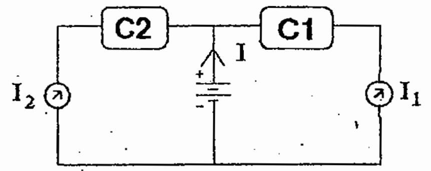
**A.** The uncertaintty in the value of resistance is:

(a) 7.0 % (b) 3.5 % [✓ (c) 5.0 %] (d) 12.0 %

**B.** The uncertainty in the computed value of the power dissipated in resistance is

(a) 7% [✓ (b) 5%] (c) 11% (d) 9%

28. A battery powers two circuits $C_1$ and $C_2$ as shown in t the figure



The total current I drawn from the battery is estimated by measuring the currents $I_1$ and $I_2$ through the individual circuits. If $I_1$ and $I_2$ are both 200 mA and if the errors in their measurement are 3 mA and 4mA respectively, the error in the estimate of I is:

(a) 7.0 mA
(b) 7.5 mA
(c) 5.0 mA
(d) 10.5 mA

## Answer Key: Experimental Methods

1. (b)
2. (a)
3. (b)
4. (d)
5. (a)
6. (d)
7. (a)
8. (a)
9. (a)
10. (c)
11. (c)
12. (c)
13. (d)
14. (a)
15. (a)
16. (a)
17. (c)
18. (d)
19. (b)
20. (b)
21. ()
22. ()
23. ()
24. (d)
25. (d)
26. (d)
27._A(c_
27_B(b)
28. (c)

# CSIR-UGC-NET/JRF | GATE

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# Assignment

# Electromagnetic Theory

## PHYSICAL SCIENCES

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**South Delhi Centre:**
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**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T: 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## CSIR-UGC-NET/JRF | GATE Physics

### Assignment –I: Electromagnetic Theory

1. In a region of electrostatic field which of the following statements is true for a charged particle:
(a) There exists only one stable equilibrium point
(b) Many stable equilibrium points may exist.
(c) No stable equilirium point exists.
(d) Number of stable equilibrium points depends on how the field is produced

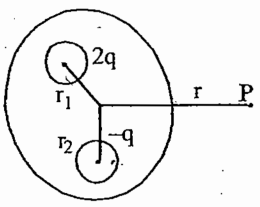
2. A shpere of radius R carrries a volume charge density $\rho = \rho_0 r$, where $\rho_0$ is a constant and r is the radial distance from the centre of the charged sphere. The intensity of the electric field is proportional to:

(a) $r^2$ for $r \le R$ and $1/r^2$ for $r \ge R$
(b) $r$ for $r \le R$ and $1/r^2$ for $r \ge R$
(c) $1/r^2$ for both $r \le R$ and $r \ge R$
(d) $1/r$ for $r \le R$ and $1/r^2$ for $r \ge R$

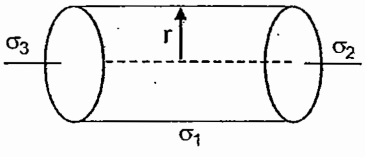
3. A uniformly charged thin spherical shell of radius R carries a total charge q. At a point inside (r < R) the spherical shell, if E and V are intensity of the electric field and electric potential respectively then:
(a) Both E and V are zero.
(b) E is zero but V is a nonzero constant.
(c) E is proportional to $1/r^2$ and V is proportional to $1/r$.
(d) Both E and V are nonzero constants.

4. A spherical conductor of radius R has two cavities centered at $r_1$ and $r_2$ respectively from the centre of the conductor. The cavities contain point charges +2q and –q at their respective centers as shown in the figure. The magnitude of the electric field at a point P is:

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
(b) $\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{2q}{(r-r_1)^2}\right)^2+\left(\frac{-q}{(r-r_2)^2}\right)^2\right]^{1/2}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-r_1-r_2)^2}$
(d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-r_1)^2+(r-r_2)^2}$



5. A cylinder of radius R and length L carries a polarization $P = P_0\bar{r}$, where $P_0$ is a constant and $\bar{r}$ is a radial vector from the axis. The surface bound charges on the cylindrical surface $(\sigma_1)$ and at the circular surfaces $(\sigma_2$ and $\sigma_3)$ as shown in the firgure are:



(a) $\sigma_1 = 0, \sigma_2 = \sigma_3 = P_0R$ (b)
(b) $\sigma_1 = P_0R, \sigma_2 = \sigma_3 = 0$
(c) $\sigma_1 = P_0R^2/L, \sigma_2 = \sigma_3 = P_0R$
(d) $\sigma_1 = P_0R, \sigma_2 = \sigma_3 = P_0R^2/L$

6. **STATEMENT I :**
The work done by the magnetic force on a moving charged particle is zero.
**STATEMENT II :**
The magnetic force tends to change the direction of a moving charged particle in a direction perpendicular to both the magnetic field and the instantaneous velocity of the particle.
[✓ (a) Statements I and II both are ture. Statement II is the correct explanation for statement I.]
(b) Statements I and II both are true. Statement II is not the correct explanation for statement I.
(c) Statement I is true but statement II is false.
(d) Statement I is false but statement II is true

7. A dipole of magnetic moment $\vec{m}$ is placed in an inhormogeneous magnetic field $\vec{B}$. The force experienced by the dipole is :
(a) $\vec{F}=\vec{m}\times\left(\vec{\nabla}\times\vec{B}\right)$ [✓ (b) $\vec{F}=\left(\vec{m}.\vec{\nabla}\right)\vec{B}$] (c) $\vec{F}=\left(\vec{\nabla}.\vec{m}\right)\vec{B}$ (d) $\vec{F}=\vec{\nabla}\times\left(\vec{m}\times\vec{B}\right)$

8. An electro-magnetic field is defined by a scalar potential $\phi(\vec{r},t)=0$ and vector potential $\vec{A}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$. The corresponding electric field is :
(a) $\vec{E}=0$ (b) $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ [✓ (c) $\vec{E}=-\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$] (d) $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\theta}$

9. Which one of the following current densities, $\vec{J}$, can generate the magnetic vector potential $\vec{A}=\left(y^2\hat{i}+x^2\hat{j}\right)$?
(a) $\frac{2}{\mu_0}\left(x\hat{i}+y\hat{j}\right)$ [✓ (b) $-\frac{2}{\mu_0}\left(\hat{i}+\hat{j}\right)$] (c) $\frac{2}{\mu_0}\left(\hat{i}-\hat{j}\right)$ (d) $\frac{2}{\mu_0}\left(x\hat{i}-y\hat{j}\right)$

10. In a non-conducting medium characterized by $\varepsilon=\varepsilon_0, \mu=\mu_0$ and conductivity $\sigma=0$, the electric field $\left(\text{in Vm}^{-1}\right)$ is given by $\vec{E}=20\sin\left[10^8t-kz\right]\hat{j}$. The magnetic field, $\vec{H}\left(\text{in A m}^{-1}\right)$, is given by:
(a) $20k\cos\left[10^8t-kz\right]\hat{i}$ (b) $\frac{20k}{10^8\mu_0}\sin\left[10^8t-kz\right]\hat{j}$
[✓ (c) $-\frac{20k}{10^8\mu_0}\sin\left[10^8t-kz\right]\hat{i}$] (d) $-20k\cos\left[10^8t-kz\right]\hat{j}$

**Statement for Linked Answer Q.11 and Q.12 :**

A dielectric sphere of radius R carries polarization $\vec{P}=kr^2\,\hat{r}$, wherer r is the distance from the centre and k is a constant. In the spherical polar coordinate system, $\hat{r}, \hat{\theta}$ and $\hat{\varphi}$ are the unit vectors.

11. The bound volume charge density inside the sphere at a distance r from the centre is :
(a) –4kR [✓ (b) –4kr] (c) $-4kr^2$ (d) $-4kr^3$

12. The electric field inside the sphere at a distance d from the centre is :
[✓ (a) $\frac{-kd^2}{\varepsilon_0}\hat{r}$] (b) $\frac{-kR^2}{\varepsilon_0}\hat{r}$ (c) $\frac{-kd^2}{\varepsilon_0}\hat{\theta}$ (d) $\frac{-kR^2}{\varepsilon_0}\hat{\theta}$

13. Which one of the following sets of Maxwell's equations for time - independent charge density $\rho$ and current density $\hat{J}$ is correct?

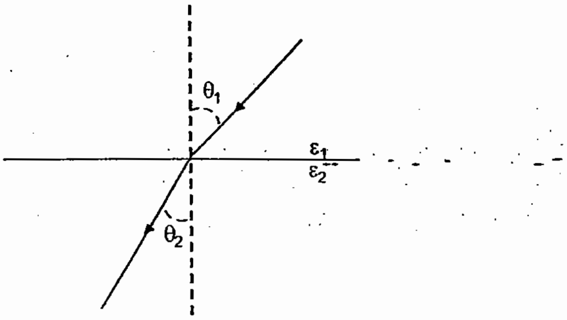
(a) $\vec{\nabla}.\vec{E}=\rho/\varepsilon_0$, $\vec{\nabla}.\vec{B}=0$, $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

[✓ (b)] $\vec{\nabla}.\vec{E}=\rho/\varepsilon_0$, $\vec{\nabla}.\vec{B}=0$, $\vec{\nabla}\times\vec{E}=0$, $\vec{\nabla}\times\vec{B}=\mu_0\hat{J}$

(c) $\vec{\nabla}.\vec{E}=0$, $\vec{\nabla}.\vec{B}=0$, $\vec{\nabla}\times\vec{E}=0$, $\vec{\nabla}\times\vec{B}=\mu_0\hat{J}$

(d) $\vec{\nabla}.\vec{E}=\rho/\varepsilon_0$, $\vec{\nabla}.\vec{B}=\mu_0\hat{J}$, $\vec{\nabla}\times\vec{E}=0$, $\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

14. At the interface between two linear dielectrics (with dielectric constants ($\varepsilon_1$ and $\varepsilon_2$), the electric field lines bend, as shown in the figure. Assume that there are no free charges at the interface. The ratio $\varepsilon_1$ and $\varepsilon_2$ is :



[✓ (a)] $\frac{\tan\theta_1}{\tan\theta_2}$ (b) $\frac{\cos\theta_1}{\cos\theta_2}$ (c) $\frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{\cot\theta_1}{\cot\theta_2}$

15. The vector potential in a region is given as $\vec{A}(x,y,z)=-y\hat{i}+2x\hat{j}$. The associated magnetic induction is $\vec{B}$ is :

(a) $\hat{i}+\hat{k}$ [✓ (b)] $3\hat{k}$ (c) $-\hat{i}+2\hat{j}$ (d) $-\hat{i}+\hat{j}+\hat{k}$

16. Three point charges q, q and $-2q$ are located at $(0,-a,a),(0,a,a)$ and $(0,0,-a)$ and $(0,0,-a)$, respectively. The net dipole moment of this charge distribution is :

[✓ (a)] $4qa\hat{k}$ (b) $2qa\hat{k}$ (c) $-4qa\hat{i}$ (d) $-2qa\hat{j}$

17. A shere of radius R has uniform volume charge density. The electric potential at a point r (r<R) is
(a) Due to the charge inside a sphere of radius r only
[✓ (b)] Due to the entire charge of the sphere
(c) Due to charge in the spherical shell of inner and outer radii r and R, only
(d) Independent of r

18. In an electromagnetic field, which one of the following remains invariant under Lorentz transformation?

(a) $\vec{E}\times\vec{B}$ [✓ (b)] $E^2-c^2B^2$ (c) $B^2$ (d) $E^2$

19. The value of $\oint_c \vec{A}.d\vec{\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is :

[✓ (a)] 0 (b) 2LA (c) 4LA (d) $L^2A$

20. For a vector potential $\vec{A}$, the divergence of $\vec{A}$ is $\vec{\nabla}.\vec{A} = -\frac{\mu_0}{4\pi}\frac{Q}{r^2}$, where Q is a constant of appropriate dimension. The corresponding scalar potential $\varphi(\vec{r},t)$ that makes $\vec{A}$ and $\varphi$ Lorentz gauge invariant is :

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$  (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r}$  (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$  [✓ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r^2}$]

21. The work done in bringing a charge +q from infinity in free space, to a position at a distance d in front of a semi-infinite grounded metal surface is :

(a) $-\frac{q^2}{4\pi\varepsilon_0(d)}$  (b) $-\frac{q^2}{4\pi\varepsilon_0(2d)}$  [✓ (c) $-\frac{q^2}{4\pi\varepsilon_0(4d)}$]  (d) $-\frac{q^2}{4\pi\varepsilon_0(6d)}$

22. If the electrostatic potential were given by $\phi = \phi_0\left(x^2+y^2+z^2\right)$, where $\phi_0$ is constant, then the charge density giving rise to the above potential would be :

(a) 0  [✓ (b) $-6\,\phi_0\,\varepsilon_0$]  (c) $-2\,\phi_0\,\varepsilon_0$  (d) $-\frac{6\phi_0}{\varepsilon_0}$

23. A solid sphere of radius R carries a uniform volume charge density $\rho$. The magnitude of electric field inside the sphere at a distance r from the centre is :

(a) $\frac{r\rho}{3\varepsilon_0}$  (b) $\frac{R\rho}{3\varepsilon_0}$  (c) $\frac{R^2\rho}{r\varepsilon_0}$  (d) $\frac{R^3\rho}{r^2\varepsilon_0}$

24. A Plane electromagnetic wave of frequency $\omega$ is incident normally on an air-dielectric interface. He dielectric is linear, isotropic, non-magnetic and its refractive index is n. The reflectance (R) and transmittance (T) from the interface are :

[✓ (a) $R = \left(\frac{n-1}{n+1}\right)^2, T = \frac{4n}{(n+1)^2}$]  (b) $R = -\left(\frac{n-1}{n+1}\right), T = \frac{2}{(n+1)^2}$

(c) $R = \left(\frac{n-1}{n+1}\right)^3, T = \frac{4n^3}{(n+1)^3}$  (d) $R = \left(\frac{(n-1)^2}{n+1}\right), T = \frac{4n^2}{(n+1)^2}$

25. A conducting sphere of radius R has charge + Q on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will :

(a) Increase four times  [✓ (b) Increase eight times]

(c) Remain the same  (d) Decrease four times

26. A charge + q is kept at a distance of 2R from the center of a grounded conducting sphere of radius R. The image charge and its distance from the centre are, respectively :

[✓ (a) $-\frac{q}{2}$ and $\frac{R}{2}$]  (b) $-\frac{q}{2}$ and $\frac{R}{4}$  (c) $-q$ and $\frac{R}{2}$  (d) $+\frac{q}{2}$ and $\frac{R}{2}$

27. An infinite conducting sheet in the x-y plane carries a surface current density K along the y-axis. The magnetic field B for z > 0 is :

(a) B = 0  (b) $B = \mu_0 K\,\hat{k}/z$  [✓ (c) $B = \mu_0 K\,\hat{i}/2$]  (d) $B = \mu_0\,K\,\hat{j}/\left(x^2+z^2\right)^{0.5}$

28. The dielectric constant of water is 80. However its refractive index is 1.75 invalidating the expression $n=\varepsilon^{1/2}$. This is because

[✓ (a) The water molecule has a permanent dipole moment]
(b) The boiling point of water is 100°C
(c) The two quantities are measured in different experiments
(d) Water is transparent to visible light

29. An electromagnetic wave is propagating in free space in the z-direction. If the electric field is given by $E=\cos(\omega t-kz)i$, where $\omega t=ck$, then the magnetic field is given by :

[✓ (a) $B=(1/c)\cos(\omega t-kz)j$]
(b) $B=(1/c)\sin(\omega t-kz)j$
(c) $B=(1/c)\cos(\omega t-kz)i$
(d) $B=(1/c)\cos(\omega t-kz)ji$

30. Which one of the following Maxwell's equations implies the absence of magnetic monopoles?

(a) $\nabla.E=\pi/\varepsilon_0$
[✓ (b) $\nabla.B=0$]
(c) $\nabla\times E=-\partial B/\partial t$
(d) $\nabla\times B=(1/c^2)\partial B/\partial t+\mu_0 J$

31. In a two beam interference pattern, the maximum and minimum intensity values are found to be $25\,I_0$ and $9\,I_0$ respectively, where $I_0$ is a constant. The intensities of the two interfereing beams are :

[✓ (a) $16\,I_0$ and $I_0$]
(b) $5\,I_0$ and $3\,I_0$
(c) $17\,I_0$ and $8\,I_0$
(d) $8\,I_0$ and $2\,I_0$

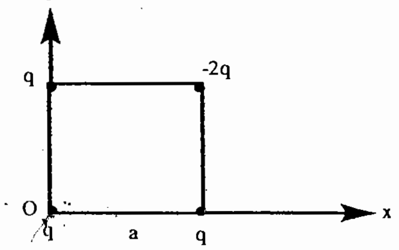
32. A left circularly polarized light beam of wavelength 600 nmis incident on a crystal of thickness d and propagated perpendicular to its optic axis. The ordinary and extraordinary refractive indices of the crystal are $n_0=1.54$ and $n_e=1.55$ respectively. The emergent light will be right circularly polarized if d is :

(a) 120 μm
(b) 60 μm
[✓ (c) 30μm]
(d) 15μm

33. An infinitely long closely would solenoid carries a sinusoidally varying current. The induced electric field is :

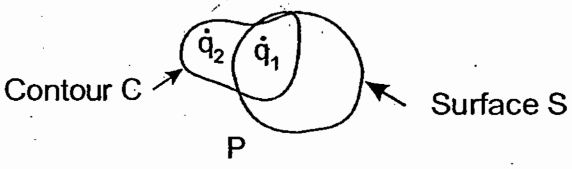
(a) Zero everywhere
(b) Non-zero inside and zero outside the solenoid
[✓ (c) Non-zero inside as well as outside the solenoid]
(d) Zero inside and non-zero outside the solenoid

34. Four charges are placed at the four corners of a square of side a as shown in the figure. The electric dipole moment of this configuration is :



(a) $\vec{p}=qa\hat{i}+qa\hat{j}$
(b) $\vec{p}=-qa\hat{i}+qa\hat{j}$
[✓ (c) $\vec{p}=-qa\hat{i}-qa\hat{j}$]
(d) $\vec{p}=qa\hat{i}-qa\hat{j}$

35. Consider a set of two stationary point charges $q_1$ and $q_2$ as shown in the figure. Which of the following statements is correct?



(a) The electric field at P is independent of $q_2$

[✓ (b)] The electric flux crossing the closed surface S is independent of $q_2$

(c) The line integral of the electric field $\vec{E}$ over the closed contour C depends on $q_1$ and $q_2$.

(d) $\vec{\nabla}.\vec{E} = 0$ everywhere

36. Consider an infinitely long straight cylindrical conductor of radius R with its axis along the z-direction, which carries a current of 1 A uniformly distributed over its cross section. Which of the following statements is correct?

(a) $\vec{\nabla} \times \vec{B} = 0$ everywhere

(b) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ everywhere

[✓ (c)] $\vec{\nabla} \times \vec{B} = 0$ for $r > R$

(d) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ for $r > R$

where r is the radial distance from the axis of the cylinder.

37. Consider an electric field $\vec{E}$ existing in the interface between a conductor and free space. Then the electric field $\vec{E}$ is :

[✓ (a)] External to the conductor and normal to the conductor's surface

(b) Internal to the conductor and normal to the conductor's surface

(c) External to the conductor and tangential to the conductor's surface

(d) Both external and internal to the conductor and normal to the conductor's surface

38. Two point charges $Q_1$ = 1 nC and $Q_2$ = 2 nC are kept in free space such that the distance between them is 0.1 m.

(a) The force on $Q_2$ is along the direction from $Q_2$ to $Q_1$

[✓ (b)] The force on $Q_2$ is the same in magnitude as that on $Q_1$

(c) The force on $Q_1$ is attractive

(d) A point charge $Q_3$ = –3 nC, placed at the midpoint between $Q_1$ and $Q_2$, experiences no net force.

39. A very long solenoid with n turns per unit length carries a current I. The magnetic field at a point, which is on its axis and its end face, is :

(a) $\mu_0 nI$ (b) $(2/3)\mu_0 nI$ (c) $(1/3)\mu_0 nI$ [✓ (d)] $(1/2)\mu_0 nI$

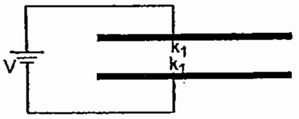
40. The space between the plates of a parallel plate capacitor is filled with two dielectric slabs of dielectric constants $k_1$ and $k_2$ as shown in the figure. If the capacitor is charged to a potential V, then at the interface between the two dielectrics.

[✓ (a)] $\vec{E}$ is discontinuous and $\vec{D}$ is continuous

(b) $\vec{E}$ is discontinuous and $\vec{D}$ is discontinuous

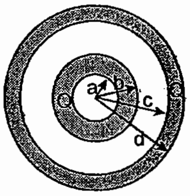
(c) $\vec{E}$ is continuous and $\vec{D}$ is continuous

(d) $\vec{E}$ is continuous and $\vec{D}$ is discontinous



**Common data for Q. 41 and Q.42.**

Consider two concentric conducting spherical shells with inner and outer radii a, b and c, d as shown in the figure. Both the shells are given Q amount of positive charges.



41. The electric field in different regions are :

(a) $\vec{E}=0$ for $r<a$; $\vec{E}=\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a<r<b$; $\vec{E}=0$ for $b<r<c$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r>d$

(b) $\vec{E}=\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r<a$; $\vec{E}=0$ for $a<r<b$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b<r<c$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r<d$

(c) $\vec{E}=\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r<a$; $\vec{E}=0$ for $a<r<b$; $\vec{E}=0$ $b<r<c$; $\vec{E}=\frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r>d$

(d) $\vec{E}=0$ for $r>a$; $\vec{E}=0$ for $a<r<b$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b<r<c$; $\vec{E}=\frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r>d$

42. In order to have equal surface charge densities on the outer surfaces of both the shells, the following conditions should be satisfied

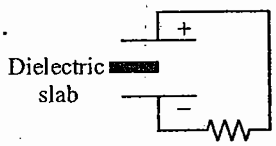
(a) $d=4b$ and $c=2a$
(b) $d=2b$ and $c=\sqrt{2}a$
(c) ~~$d \neq \sqrt{2}b$~~ and $c>a$ $d=\sqrt{2}\,b$
(d) $d>b$ and $c=\sqrt{2}a$

43. Consider an infinite line charge with linear charge density $\lambda$. At a distance r from the line, the electrostatic potential has the form

(a) $\frac{\lambda a}{4\pi\varepsilon_0 r}$
(b) $\frac{\lambda}{4\pi\varepsilon_0}\exp(-r/a)$
(c) $\frac{\lambda}{4\pi\varepsilon_0}\ln(r/a)$
(d) $\frac{\lambda}{4\pi\varepsilon_0}\frac{r}{a}$

44. A thin dielectric slab is slowly introduced between the plates of a charged parallel plate capacitor, as shown in the figure.

Which of the following is true?



(a) The slab is pushed out of the region between the capacitor plates.
(b) The slab is sucked into the region between the capacitor plates.
(c) The slab moves towards the positively charged plate.
(d) The slab moves towards the negatively charged plate.

45. If in interior of a unit sphere we have $\vec{\nabla}.\vec{J}=a$ positive constant, where $\vec{J}$ is the current density then we may conclude that;

(a) According to Gauss' theorem, the charge contained in the unit sphere is constant in time.
(b) According to Gauss' theorem, charge is flowing into the unit sphere.
(c) According to the continuity equation, the charge density within the unit sphere must necessarily be uniform.
(d) According to the continuity equation, the change density inside the unit sphere diminishes with time.

46. Choose the correct statement from the following:

(a) The magnetic field $\vec{B}$ is a vector and not a pseudovector.

(b) $\vec{E}.\vec{B}$ is a scalar and not a pseudoscalar.

(c) $\vec{E}\times\vec{B}$ is a pseudovector.

(d) The magnetic vector potential $\vec{A}$ is a pseudovector.

47. Consider the operations $P:\vec{r}\to-\vec{r}$ (parity) and $T:t\to-t$ (time-reversal). For the electric and magnetic field $\vec{E}$ and $\vec{B}$, which of the following set of transformations is correct?

(a) $P:\vec{E}\to-\vec{E},\vec{B}\to\vec{B};\quad T:\vec{E}\to\vec{E},\vec{B}\to-\vec{B}$

(b) $P:\vec{E}\to\vec{E},\vec{B}\to\vec{B};\quad T:\vec{E}\to\vec{E},\vec{B}\to\vec{B}$

(c) $P:\vec{E}\to\vec{E},\vec{B}\to\vec{B};\quad T:\vec{E}\to-\vec{E},\vec{B}\to-\vec{B}$

(d) $P:\vec{E}\to\vec{E},\vec{B}\to-\vec{B};\quad T:\vec{E}\to-\vec{E},\vec{B}\to\vec{B}$

48. A charged particle moving with a constant velocity enters a magnetic field perpendicular to its velocity. In which direction y an electric field should be applied to compensate the magnetic force?

(a) Along the initial velocity

(b) Perpendicular to the initial velocity and parallel to the magnetic field.

(c) Perpendicular to both, the initial velocity and the magnetic field.

(d) Along the magnetic field.

49. The neutral pion $\pi^0$ at rest decays into two photons. One of the photons is right circularly polarized. The other photon is:

(a) Also right circularly polarized. (b) Left circularly polarized.

(c) Unpolarized. (d) Plane polarized.

50. Consider a parallel plate capacitor connected to an AC voltage source (as shown in fig.1). A conducting slab is introduced in the space between the plates from above (as shown in fig.2)

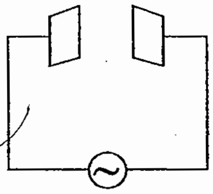


**Fig. 1**

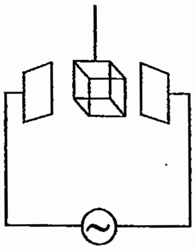


**Fig. 2**

$C = \frac{Q}{V}$

$Q = \frac{1}{2}CV^2$

The capacitance of the parallel plate capacitor.

(a) Goes to zero. (b) Increases to a finite value.

(c) Decreases to a non-zero value. (d) Becomes infinite.

51. When a particle of charge q is at rest, it produces

(a) Only electric field (b) Only magnetic field

(c) Both electric and magnetic fields. (d) A magnetic field depending upon the sign of the charge.

52. A charge q is revolving with angular velocity ω in a plane perpendicular to a uniform magnetic field B. If the field is reduced to B/2.

(a) The linear velocity is doubled (b) The linear velocity is halved

(c) The radius is halved (d) The radius is doubled.

53. An electromagnetic wave is normally incident upon a perfectly conducting surface. In the reflected wave,

(a) $\vec{H}$ is shifted 180° in phase. ✓(b) $\vec{E}$ is shifted 180° in phase

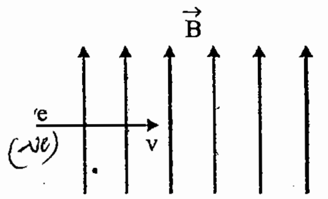
(c) $\vec{H}$ is shifted 90° in phase (d) $\vec{E}$ is shifted 90° in phase.

54. The condition for a vector potential $\vec{A}$ to satisfy the wave equation is:

(a) $\vec{\nabla}.\vec{A} = 0$ ✓(b) $\vec{\nabla}.\vec{A} + \mu_0 \in_0 \frac{\partial \phi}{\partial t} = 0$

(c) $\frac{\partial \vec{A}}{\partial t} + \mu_0 \in_0 \vec{\nabla}\phi = 0$ (d) $\vec{A} + \vec{\nabla}\phi = 0$

55. A stream of electrons eneters, from left to right, a uniform magnetic field (B) which is pointing upwards as shown in the figure. The field deflects the stream of electrons.



(a) Downward (in the plane of the paper) (b) Upward (in the plane of the paper)

✓(c) Into the paper (d) Out of the Paper.

uniform distrubted over its crossection

56. A wire of radius R carries a current I. The magnetic induction B, inside the wire, at a distance r(<R)

(a) Is independent of R (b) Is proportional to ln R

✓(c) Is proportional to r (d) Is proportional to 1/r.

($B_{in} \propto R$)

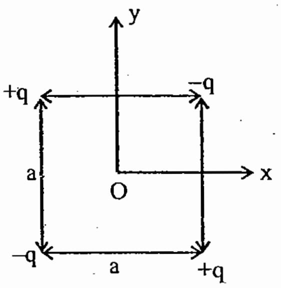
57. A charged capacitor is made of two concentric cylinders of radii $r_1$ and $r_2$. The region between the cylindrical plates is filled with a medium of dielectric constant, k. The displacement vector $(\vec{D})$ at a distance $r\,(r_1 < r < r_2)$.

$\propto$

(a) Is independent of r. (b) Varies as ln r.

(c) Is proportional to r. ✓(d) Varies as 1/r.

58. The dipole moment of the following configuration about the centre:



(a) $4\,qa\left(\hat{i}+\hat{j}\right)$ (b) $2\,qa\left(\hat{i}-\hat{j}\right)$ ✓(c) 0 (d) $8\,qa^2\,\hat{k}$

59. In which region of the electromagnetic spectrum do metals have the highest reflectivity?

✓(a) Infrared (b) Visible

(c) Ultraviolet (d) In the entire region of the spectrum

60. At the interface between two dielectric media,

(a) Normal components of $\vec{E}$ and $\vec{B}$ must be continuous.

(b) Tangential components of $\vec{E}$ and $\vec{B}$ must be continuous.

(c) Normal components of $\vec{D}$ and $\vec{H}$ must be continuous

✓(d) Tangential component of $\vec{E}$ and normal component of $\vec{B}$ must be continuous.

61. The refractive indices of water and glass are $\frac{4}{3}$ and $\frac{3}{2}$ respectively. The ratio of their dielectric constants at $10^{14}$ Hz is:

(a) $\frac{8}{9}$ [✓ (b) $\frac{64}{81}$] (c) $\frac{81}{64}$ (d) $\frac{9}{8}$

62. A certain charge distribution consists of two equivalent dipoles which are oppositely oriented. The leading term in the expansion of the potential is that due to a

(a) Monopole (b) Dipole [✓ (c) Quadrupole] (d) Octupole

63. An electron is moving with an acceleration 10 m/s$^2$ whereas a positron is moving with an acceleration 20 m/s$^2$. The ratio of the power radiated by the positron to that of electron is:

(a) 2 (b) 1/2 (c) 1/4 [✓ (d) 4]

64. The magnetic field $\vec{B}$ can be expressed in terms of the vector potential as $\vec{B}=\vec{\nabla}\times\vec{A}$. If $\vec{B}=B\hat{k}$ then $\vec{A}$ can be expressed as

[✓ (a) $\left(-\frac{1}{2}By, \frac{1}{2}Bx, 0\right)$] (b) $\left(\frac{1}{2}By, -\frac{1}{2}Bx, 0\right)$ (c) $(0,0,B)$ (d) $\left(\frac{Bx}{3}, \frac{By}{3}, \frac{Bz}{3}\right)$

65. Current flowing in a circuit containing a capacitor. To satisfy the continuity equation, $\vec{\nabla}.\vec{J}=-\frac{\partial\rho}{\partial t}$, for charge and current, the current density $\vec{J}$ must be replaced by

(a) $\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}$ (b) $\vec{J}-\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}$ (c) $\vec{D}+\frac{1}{4\pi}\frac{\partial\vec{J}}{\partial t}$ [✓ (d) $\vec{J}+\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}$]

66. The electric field due to a charge q is given by $\vec{E}=\frac{q}{r^2}\hat{r}$. The value of the surface integral $\oint_s \vec{E}.\vec{ds}$ depends only

(a) On the area of the surface (b) On the radial distance r.
[✓ (c) On the charge] (d) On the shape of the surface

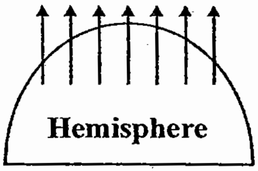
67. The relation between $\vec{H}.\vec{B}$ and $\vec{M}$ representing the magnetic intensity, magnetic field and magnetic dipole moment vectors, respectively, is

(a) $\vec{H}=-\left(\mu_0\vec{M}+\vec{B}\right)$ (b) $\vec{H}=\mu_0\vec{M}-\vec{B}$
[✓ (c) $\vec{H}=\vec{B}-\mu_0\vec{M}$] (d) $\vec{H}=\vec{B}+\mu_0\vec{M}$

68. Electrical charge is stored in

(a) Battery [✓ (b) Capacitor] (c) Voltmeter (d) Wire

69. What is the resultant Electric flux



(a) $2\pi R^2E$ (b) $4\pi R^2E$ (c) 0 [✓ (d) $\pi R^2E$]

70. Magnetic field associated with a wire depends as

(a) $B\propto\frac{1}{r^2}$ (b) $B\propto\frac{1}{r}$ (c) $B\propto\frac{1}{r^3}$ (d) None of these

71. For circular wire which of the following is correct

(a) $E\propto\frac{1}{r}, B\propto\frac{1}{r}$ (b) $E\propto\frac{1}{r^2}, B\propto\frac{1}{r^2}$ (c) $E\propto\frac{1}{r}, B\propto r$ (d) $E\propto\frac{1}{r^2}, B\propto\frac{1}{r}$

72. A point charge is at a distance d from a metal plate. The force acting on the charge is:

(a) Proportional to $1/d^2$ and repulsive. (b) zero

(c) Proportional to $1/d^2$ and attractive. (d) Proportional to 1/d and attractive.

73. One of electrostatic boundary conditions for the potential v for a surface carrying a surface charge density $\sigma$ can be written as

(a) $V_{above}-V_{below}=-\frac{\sigma}{\epsilon_0}$ (b) $V_{above}-V_{below}=\frac{\sigma}{\epsilon_0}$

(c) $\frac{\partial V_{above}}{\partial n}-\frac{\partial V_{below}}{\partial n}=\frac{\sigma}{\epsilon_0}$ (d) $\frac{\partial V_{above}}{\partial n}-\frac{\partial V_{below}}{\partial n}=-\frac{\sigma}{\epsilon_0}$

74. The magnitude of the electrostatic field of an infinite straight wire carrying a line charge density $\lambda$ at a distance Z from it is given by

(a) $E=\frac{1}{4\pi\varepsilon_0}.\frac{\lambda}{Z^2}$ (b) $E=\frac{1}{4\pi\varepsilon_0}.\frac{2\lambda}{Z^2}$ (c) $E=\frac{1}{4\pi\varepsilon_0}.\frac{2\lambda}{Z}$ (d) $E=\frac{1}{4\pi\varepsilon_0}.\frac{\lambda}{Z}$

75. The relation $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$ where

$\vec{D}\rightarrow$ dielectric constant; $\vec{E}\rightarrow$ electric field intensity; $\vec{P}\rightarrow$ polarization vector is applicable

(a) For all medium (b) For dielectric medium

(c) Free space (d) Magnetic medium

76. For a medium of electric and magnetic field the relation between E and B with propagation vector k

(a) $\bar{E}\times\bar{k}=\omega\bar{B}$ (b) $\bar{E}\times\bar{R}=\bar{\omega}\times\bar{B}$ (c) $\bar{E}=\frac{k}{\omega}\times\bar{B}$ (d) $\bar{B}=\frac{\bar{k}\times\bar{E}}{\omega}$

77. The condition that the magnetic field $\vec{B}$ is a curl of a vector potential $\vec{A}$ should be

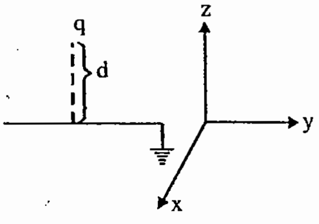
(a) $\vec{\nabla}.B=0$ (b) $\vec{\nabla}\times\vec{A}=0$ (c) $\vec{\nabla}\vec{A}=0$ (d) $\vec{\nabla}\times\vec{A}-\nabla^2\vec{A}=0$

78. A hollow, closed conductor is placed in an external electric field $\vec{E}$. Then, at equilibrium, the electric field within the cavity is:

(a) $\vec{E}$ (b) 0

(c) Can be less than $\vec{E}$ but non-zero (d) Greater than $\vec{E}$

79. Let a point charge +q be at a distance d from an infinite conducting plate which is grounded.



The force experienced by the charge is given by

(a) $\vec{F}=+\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\hat{k}$ (b) $\vec{F}=-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\hat{k}$

(c) $\vec{F}=-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}\hat{k}$ (d) $\vec{F}=+\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}\hat{k}$

80. In an electrostatic boundary value problem, the surface charge density $\sigma$ is specified at $x = 0$. The corresponding boundary condition for potential V can be written as

(a) $\left(\frac{\partial^2 V}{\partial x^2}\right)_{x=0} = \frac{1}{\varepsilon_0}\sigma(y.z)$ (b) $\left(\frac{\partial^2 V}{\partial x^2}\right)_{x=0} = -\frac{1}{\varepsilon_0}\sigma(y.z)$

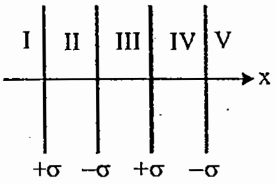
(c) $\left(\frac{\partial V}{\partial x}\right)_{x=0} = +\frac{1}{\varepsilon_0}\sigma(y.z)$ (d) $\left(\frac{\partial V}{\partial x}\right)_{x=0} = -\frac{1}{\varepsilon_0}\sigma(y.z)$

81. Let a right circularly polarized light be incident normally on an ordinary mirro. The reflected light is:

(a) Elliptically polarized (b) Right circularly polarized

(c) Left circularly polarized (d) Linearly polarized

82. Four infinite conducting plates carry equal but opposite uniform charge densities $+\sigma$ as shown. The electrostatic fields in the regions I, II, III, IV and V are given respectively by



(a) $0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0$ (b) $\frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}$

(c) $0, \frac{\sigma}{\varepsilon_0}\hat{x}, -\frac{\sigma}{\varepsilon_0}\hat{x}, \frac{\sigma}{\varepsilon_0}\hat{x}, 0$ (d) $0, +\frac{\sigma}{\varepsilon_0}\hat{x}, \frac{\sigma}{2\varepsilon_0}\hat{x}, +\frac{\sigma}{\varepsilon_0}\hat{x}, 0$

83. A charge q moves in a combined electric field $\vec{E}$ and magnetic induction $\vec{B}$. Then, as the charge moves

(a) Both the electric field and magnetic induction do work on it.

(b) The electric field does work but magnetic induction does no work on it.

(c) Magnetic induction does work but electric field does no work on it.

(d) Both electric field and magnetic induction do no work on it.

84. In charge free and current free space, the following is true:

(a) $\vec{\nabla}\times\vec{H} = \frac{\partial\vec{B}}{\partial t}$ (b) $\vec{\nabla}\times\vec{H} = 0$ (c) $\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$ (d) $\vec{\nabla}\times\vec{H} = -\frac{\partial\vec{D}}{\partial t}$

85. Let $\vec{A}$ and $\varphi$ denote the vector and scalar potentials, respectively, which are Lorentz gauge invariant. $\vec{A}$ and $\varphi$ satisfy.

(a) $\text{div}\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0$ (b) $\text{div}\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$ (c) $\nabla^2\vec{A} + \frac{1}{c}\frac{\partial\varphi}{dt} = 0$ (d) $\nabla^2\vec{A} + \frac{1}{c}\frac{\partial A}{dt} = 0$

86. Two point charges q and 4q are located at $r_1$ and $r_2$, respectively, on the xy-plane. The magnitude Q and location R of a third charge to be placed on the plane such that total force on the each of the three charges vanishes, are

(a) $Q = -4q/9$ and $R = (2r_1 + r_2)/3$ (b) $Q = -4q/5$ and $R = (r_1 + 2r_2)/3$

(c) $Q = -4q/25$ and $R = (16r_1 + r_2)/\sqrt{17}$ (d) $Q = -9q/4$ and $R = (r_1 + 2r_2)/3$

87. Consider a spherical cavity in an infinite, homogeneous and isotropic dielectric material of permittivity $\varepsilon$. When placed in an external electric field E, the electric field inside the cavity is:

(a) $3\varepsilon E/(2\varepsilon + \varepsilon_0)$ (b) $(2\varepsilon + \varepsilon_0)E/3\varepsilon$ (c) $-\varepsilon E$ (d) $E/\varepsilon$

88. A circular coil of radius 120 mm with 10 turns is placed with its plane parallel to the earth's magnetic field. When a current of 0.45 A flows through the coil, a compass needle placed at the center of the coil is seen to be deflected by an angle of 45° to the plane of the coil. The intensity of the earth's magnetic field is calculated to be

(a) 0.3 $\mu T$ [✓ (b) 24 $\mu T$] (c) 42 $\mu T$ (d) 100 $\mu T$

89. Electric current of uniform current density is flowing through an infinitely long wire with a circular cross-section of constant radius R. Which of the following relations gives the magnitude B of the magnetic field measured inside the wire at a distance r from the central axis of the wire (so that r < R)?

[✓ (a) $B \propto r$] (b) $B \propto \ln r$ (c) $B \propto 1/r$ (d) $B \propto 1/r^2$

90. The amount of energy required to charge a metallic sphere of radius 2.0 m to a potential of 3000 V in air is:

[✓ (a) $1\times10^{-3}$ Joule] (b) $2\times10^{-3}$ Joule (c) $3\times10^{-3}$ Joule (d) $4\times10^{-3}$ Joule

91. In a region of space that does not contain any electric charge, the electrostatic potential satisfies Laplace's equation. Suppose that the potential in such a situation is given by the formula

$$V(x,y,z) = A\left(bx^2 + \frac{1}{2}y^2 - z^2\right)$$

Where A and b are constants. Then the value of b

(a) Can be arbitrary [✓ (b) Must be zero] [✓ (c) Must be 1/2] (d) Must be 1

92. Suppose that the electric field on the xy-plane is given by

$$\vec{E} = P\left[xy\hat{i} + \left(\frac{1}{2}x^2 + y^2\right)\hat{j}\right]$$

$$\boxed{V = -\int_a^b \vec{E}.d\vec{l}}$$

The magnitude of the difference in potential between the origin (0, 0) and the point (1, 2) is:

(a) $\frac{5}{4}P$ (b) $\frac{11}{3}P$ (c) $\frac{7}{2}P$

[✓ (d) Not unique, it depends on the path joining the two points.]

93. Four electric charges of strength Q, Q, –Q and –Q are placed respectively at the points (–b, 0), (b, 0), (0, –a) and (0, a) on the xy plane. At a distance d very far away from these charges (d >> a, b), the electric field will appear to be approximately that of

(a) A point charge (b) A dipole

[✓ (c) A quadrupole] (d) A mixture of a dipole and a quadrupole

94. The potential due to an electric dipole P placed at the origin is known to be of the form $\phi(r) = P.r/r^3$. The total flux of the electric field through a spherical surface of radius R with the dipole at the centre is given by

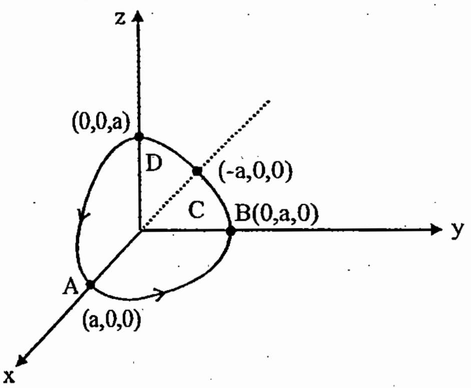
(a) Zero (b) $-\frac{2\pi}{3}\frac{|p|}{R}$ (c) $+\frac{2\pi}{3}\frac{|p|}{R}$ (d) $+\frac{4\pi}{3}\frac{|p|}{R}$

$$V_{dip}(r,\theta) = \frac{\hat{r}.P}{4\pi\epsilon_0 r^2} = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\Phi_E = \int_S \vec{E}.d\vec{a}$$

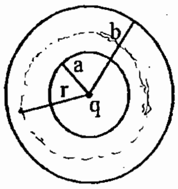
$$\boxed{\vec{E} = -\nabla V}$$

95. In the following figure, ABC and CDA are two semicircular conducting wires of radius a with the origin as their common centre. The arc ABC lies on the xy-plane whereas CDA lies on the xz-plane. They are joined as shown. This combined loop carries a current in the direction indicated in the figure below. Which of the following unit vectors denotes the direction of the resulting magnetic field vector at the origin?



(a) $(1,0,0)$ [✓ (b) $\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$] (c) $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ (d) $(-1,0,0)$

96. A conducting spherical shell of inner radius a and outer radius b has a point charge q located at the centre of the shell. The potential at a distance r from the centre $(a < r < b)$ is:



(a) 0 (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ [✓ (c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$] (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

97. Consider the electric and magnetic fields of an accelerating charge. How should the fields vary with r (the retarded distance) for the poynting vector to remain finite at arbitrary large distances?

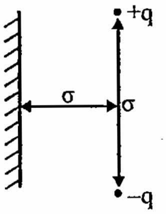
[✓ (a) $1/r$] (b) $1/r^2$ (c) r d) $r^2$

98. According to Kirchoff's laws for the circuits, the sum of the currents at any junction is equal to zero. Which of the following equations for the current density $\vec{j}$ describes this situation?

(a) $\vec{\nabla}\times\vec{j}=0$ [✓ (b) $\vec{\nabla}.\vec{j}=0$]

(c) $\nabla^2\vec{j}=0$ (d) $\partial\vec{j}/\partial t=0$

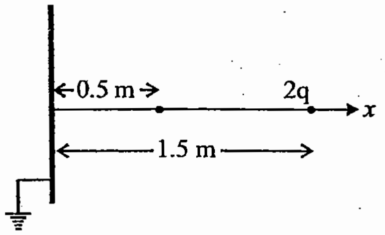
99. Two equal and opposite charges are placed at a distance r from each other. A large metallic sheet is placed at a distance d from them as shown in figure. Due to presence of the sheet, the attractive force between the charges along the direction joining them



[✓ (a) decreases] (b) Increases

(c) Remains unchanged. (d) Decreases if $r > d/2$ and increases if $r < d/2$.

100. Two charges q and 2q are placed along the x-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is:



(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$ (c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

101. A uniform surface current is flowing in the positive y-direction over an infinite sheet lying in x-y plane. The direction of the magnetic field is:

[✓ (a) Along $\hat{i}$ for $z > 0$ and along $-\hat{i}$ for $z < 0$.] (b) Along $\hat{k}$ for $z > 0$ and along $-\hat{k}$ for $z < 0$

(c) Along $-\hat{i}$ for $z > 0$ and along $\hat{i}$ for $z < 0$ (d) Along $-\hat{k}$ for $z > 0$ and along $\hat{k}$ for $z < 0$

102. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is:

(a) $\vec{r}\times(\vec{m}\times\vec{B})$ (b) $\vec{r}\times\vec{\nabla}(\vec{m}.\vec{B})$ (c) $\vec{m}\times\vec{B}$ [✓ (d) $\vec{m}\times\vec{B}+\vec{r}\times\vec{\nabla}(\vec{m}.\vec{B})$]

103. Which of the following expressions for a vector potential $\vec{A}$ **DOES NOT** represent a uniform magnetic field of magnitude $B_0$ along the z-direction?

(a) $\vec{A}=(0, B_0x, 0)$ (b) $\vec{A}=(-B_0y, 0, 0)$ [✓ (c) $\vec{A}=\left(\frac{B_0x}{2},\frac{B_0y}{2},0\right)$] (d) $\vec{A}=\left(-\frac{B_0y}{2},\frac{B_0x}{2},0\right)$

104. A conducting sheet has infinite dimensions in the Y and Z directions and semi-infinite in the X direction. The Z component of the current density $\vec{J}$ is:

(a) Exponentially increasing (b) Exponentially decreasing

(c) Constant (d) Oscillating.

105. The ratio of cyclotron frequencies of an α-particle and a proton moving in the same magnetic field $\vec{B}$ is: qmB

[✓ (a) 1/2] (b) 2 (c) 4 (d) 1/4

106. The vector potential of a moving charge is dependent upon

(a) Its velocity only (b) Its charge only (c) Its momentum [✓ (d) Its velocity and charge] $\boxed{J = \sigma V}$

107 In vacuum the electricfield of individual particles are $\vec{E}_1$ and $\vec{E}_2$ then $\vec{E}=\vec{E}_1+\vec{E}_2$ the total energy will be

(a) The sum of individual energy (b) The sum of the square of individual energy.

(c) The square of the sum of individual energy [✓ (d) None of these.]

108. For constant uniform electric and magnetic fields $\vec{E}=\vec{E}_0$ and $\vec{B}=\vec{B}_0$, it is possible to choose a gauge such that the scalar potential $\phi$ and vector potential $\vec{A}$ are given by

d

(a) $\phi = 0$ and $\vec{A}=\frac{1}{2}(\vec{B}_0\times\vec{r})$ [✓ (b) $\phi=-\vec{E}_0.\vec{r}$ and $\vec{A}=\frac{1}{2}(\vec{B}_0\times\vec{r})$]

(c) $\phi=-\vec{E}_0.\vec{r}$ and $\vec{A}=0$ (d) $\phi = 0$ and $\vec{A}=-\vec{E}_0t$

109. The electrostatic potential V(x,y) in free space in a region where the charge density $\rho$ is zero is given by $V(x,y) = 4e^{2x} + f(x) - 3y^2$. Given that the x-component of the electric field, $E_x$, and V are zero at the origin, f(x) is:

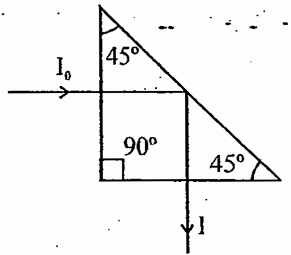
(a) $3x^2 - 4e^{2x} + 8x$ (b) $3x^2 - 4e^{2x} + 16x$ (c) $4e^{2x} - 8$ (d) $3x^2 - 4e^{2x}$

110. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by $E(x,y,z,t) = E_0(\hat{x} + A\hat{z})\exp\left[ik_0\left\{-ct + \left(x + \sqrt{3}z\right)\right\}\right]$, where c is the speed of light in vacuum, $E_0$, A and $k_0$ are constants and $\hat{x}$ and $\hat{z}$ are unit vectors along the x- and z-axes. The relative dielectric constant of the medium, $\varepsilon_r$ and the constant A are

(a) $\varepsilon_r = 4$ and $A = -\frac{1}{\sqrt{3}}$ (b) $\varepsilon_r = 4$ and $A = +\frac{1}{\sqrt{3}}$

(c) $\varepsilon_r = 4$ and $A = \sqrt{3}$ (d) $\varepsilon_r = 4$ and $A = -\sqrt{3}$

111. Circularly polarized light with intensity $I_0$ is incident normally on a glass prism as shown in the figure. The index of refraction of glass is 1.5. The intensity I of light emerging from the prism is:



(a) $I_0$ (b) $0.96\,I_0$ (c) $0.92\,I_0$ (d) $0.88\,I_0$.

# Assignment-II: Electromagnetic Theory

1. A wire of circular cross section of radius R carries a volume current density $\vec{J} = kr^2\hat{z}$ along the length of the wire. The magnetic field at a position r < R is :

(a) $\frac{\mu_0}{2}kr^3\hat{z}$ (b) $\frac{\mu_0}{4}k\,r^3\hat{\phi}$ (c) $\frac{\mu_0}{4\pi}k\,r^3\hat{r}$ (d) $\frac{\mu_0}{4\pi}k\,r^2\hat{\phi}$

2. Three infinitely long parallel wires separated by a distance d from each other, carrying current I each are kept in the same plane. The magnetic field is found to be zero at a distance x on both sides of the central wire. The values of x are :

(a) $\pm d/2$ (b) $\pm d/\sqrt{2}$ (c) $\pm d/\sqrt{3}$ (d) $\pm d/3$

3. Consider an infinite plane at z = 0 with uniform surface current density $\vec{K} = K\hat{i}$. The direction of the magnetic field $\vec{B}$ and the vector potential $\vec{A}$ of this current distribution at a distance d along the positive z-axis are :

(a) $\vec{B}$ along $\hat{j}$ and $\vec{A}$ along $\hat{k}$ (b) $\vec{B}$ along $\hat{k}$ and $\vec{A}$ along $\hat{j}$

(c) $\vec{B}$ along $-\hat{j}$ and $\vec{A}$ along $\hat{i}$ (d) $\vec{B}$ along $\hat{i}$ and $\vec{A}$ along $-\hat{j}$

4. The electric field of two linearly polarized waves are given by $\vec{E}_1 = \hat{i}E_x\cos(\omega t - \theta_1)$ and $\vec{E}_2 = \hat{j}E_y\cos(\omega t - \theta_2)$. The nature of polarization of the superimposed wave is :

(a) Unpolarized (b) Linearly polarized

(c) Circularly polarized (d) Elliptically polarized (is general type of polarization)

5. In a free space, which is the correct set of Maxwell's equations in absence of any charges and current densities :

(a) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0, \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}.\vec{B} = 0, \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(b) $\vec{\nabla}.\vec{E} = 0, \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}.\vec{B} = 0, \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

(c) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0, \vec{\nabla}\times\vec{E} = 0$ (d)

$\vec{\nabla}.\vec{B} = 0, \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(d) $\vec{\nabla}.\vec{E} = 0, \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

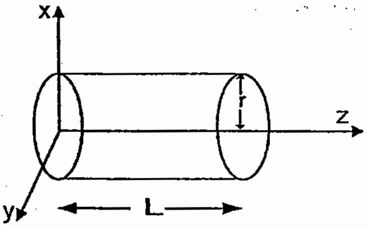
$\vec{\nabla}.\vec{B} = 0. \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

6. A conducting loop L of surface area S is moving with a velocity $\vec{\upsilon}$ in a magnetic field $\vec{B}(\vec{r},t) = \vec{B}_0t^2, B_0$ is a positive constant of suitable dimensions. The emf induced, $V_{emf}$, in the loop is given by :

(a) $-\int_S\frac{\partial\vec{B}}{\partial t}.d\vec{S}$ (b) $\oint_L(\vec{\upsilon}\times\vec{B}).d\vec{L}$

(c) $-\int_S\frac{\partial\vec{B}}{dt}.d\vec{S} - \oint_L(\vec{\upsilon}\times\vec{B}).d\vec{L}$ (d) $-\int_S\frac{\partial\vec{B}}{dt}.d\vec{S} + \oint_L(\vec{\upsilon}\times\vec{B}).d\vec{L}$

7. A cylindrical rod of length L and radius r, made of an inhomogeneous dielectric, is placed with its axis along the z direction with one end at the origin as shown below :



If the rod carries a polarization, $\vec{P} = (5z^2 + 7)\hat{k}$, the volume bound charge inside the dielectric is :

(a) Zero (b) $10\pi r^2 L$ (c) $-5\pi r^2 L$ (d) $-5\pi r^2 L^2$

8. A long cylindrical kept along z-axis carries a current density $\hat{J} = J_0 r\hat{k}$, where $J_0$ is a constant and r is the radial distance form the axis of the cylinder. The magnetic induction $\hat{B}$ inside the conductor at a distance d from the axis of the cylinder is :

(a) $\mu_0 J_0 \hat{\phi}$ (b) $\frac{\mu_0 J_0 d}{2}\hat{\phi}$ [✓ (c) $\frac{\mu_0 J_0 d^2}{3}\hat{\phi}$] (d) $\frac{\mu_0 J_0 d^3}{4}\hat{\phi}$

9. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is :

(a) Circular (b) Elliptical (c) Parabolic (d) Hyperbolic

**Statement for linked Answer Q.10 and Q.11:**

An infinitely long hollow cylinder of radius R carrying a surface charge density σ is rotated about its cylinderical axis with a constant angular speed ω

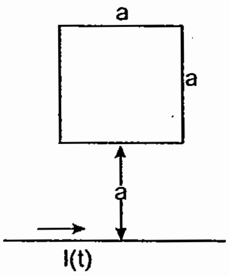
10. The magnitude of the surface current is :

[✓ (a) σRω] (b) 2σRω (c) πσRω (d) 2πσRω

11. The magnitude of vector potential inside the cylinder at a distance from its axis is :

(a) $2\mu_0\sigma R\omega r$ (b) $\mu_0\sigma R\omega r$ [✓ (c) $\frac{1}{2}\mu_0\sigma R\omega r$] (d) $\frac{1}{4}\mu_0\sigma R\omega r$

12. An infinitely long wire carrying a current $I(t) = I_0 \cos(\omega t)$ is placed at a distance a from a square loop of side a as shown in the figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is :



[✓ (a) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}\text{In } 2$] (b) $\frac{\mu_0}{\pi}\frac{aI_0\omega}{R}\text{In } 2$

(c) $\frac{2\mu_0}{\pi}\frac{aI_0\omega}{R}\text{In } 2$ (d) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}$

13. A plane electromagnetic wave travelling in vaccum is incident normally on a non magnetic, non-absorbing medium of refractive index n. The incident ($E_i$), reflected ($E_r$) and transmitted ($E_t$) electric fields are given as,

$$E_i = E\exp\left[i(kz-\omega t)\right], E_r = E_{or}\exp\left[i(k_r z-\omega t)\right], E_t = E_{0t}\exp\left[i(k_t z-\omega t)\right]$$

If E = 2 V/m and n = 1.5 then the application of appropriate boundary conditions leads to :

(a) $E_{0r} = -\frac{3}{5}\text{V/m}, E_{0t} = \frac{7}{5}\text{V/m}$ (b) $E_{0r} = -\frac{1}{5}\text{V/m}, E_{0t} = \frac{8}{5}\text{V/m}$

[✓ (c)] $E_{0r} = -\frac{2}{5}\text{V/m}, E_{0t} = \frac{8}{5}\text{V/m}$ (d) $E_{0r} = \frac{4}{5}\text{V/m}, E_{0t} = \frac{6}{5}\text{V/m}$

14. Consider the following three independent cases :

(i) Particle A of charge +q moves in free space with a constant velocity $\vec{\upsilon}(\upsilon << \text{speed of light})$

(ii) Particle B of charge +q moves in free space in a circle of radius R with same speed $\upsilon$ as in case (i)

(iii) Particle C having charge −q moves as in case (ii)

If the powers radiated by A, B and C are $P_A$, $P_B$ and $P_C$ respectively, then :

(a) $P_A = 0, P_B > P_C$ [✓ (b)] $P_A = 0, P_B = P_C$

(c) $P_A > P_B > P_C$ (d) $P_A = P_B = P_C$

15. The electric (E) and magnetic (B) field amplitudes associated with an electromagnetic radiation from a point source behave at a distance r from the source as :

(a) E = constant, B = constant [✓ (b)] $E \propto \frac{1}{r}, B \propto \frac{1}{r}$

(c) $E \propto \frac{1}{r^2}, B \propto \frac{1}{r^2}$ (d) $E \propto \frac{1}{r^3}, B \propto \frac{1}{r^3}$

16. The electric field $\vec{E}(\vec{r},t)$ for a circularly polarized electromagnetic wave propagating along the positive z direction is :

(a) $E_0(\hat{x}+\hat{y})\exp\left[i(kz-\omega t)\right]$ [✓ (b)] $E_0(\hat{x}+i\hat{y})\exp\left[i(kz-\omega t)\right]$

(c) $E_0(\hat{x}+i\hat{y})\exp\left[i(kz+\omega t)\right]$ (d) $E_0(\hat{x}+\hat{y})\exp\left[i(kz+\omega t)\right]$

17. Three infinitely long wires are placed equally apart on the circumference of a circle of radius a, perpendicular to its plane. Two of the wires carry current I each, in the same direction, while the third carries current 2I along the direction opposite to the other two. The magnitude of the magnetic induction $\vec{B}$ at a distance r from the centre of the circle, for r > a, is :

[✓ (a)] 0 (b) $\frac{2\mu_0}{\pi}\frac{I}{r}$ (c) $-\frac{2\mu_0}{\pi}\frac{I}{r}$ (d) $\frac{2\mu_0}{\pi}\frac{Ia}{r^2}$

18. Which one of the following statements is TRUE?

(a) The magnitude of the electric field is attenuated as the wave propagates
(b) The energy of the e.m. wave flows along the x-direction
(c) The magnitude of the electric field of the wave is a constant
(d) The speed of the wave is the same as c(speed of light in free space)

19. The magnetic field $\vec{B}$ of the wave is :

(a) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp[i(zk\cos\phi-\omega t)]$

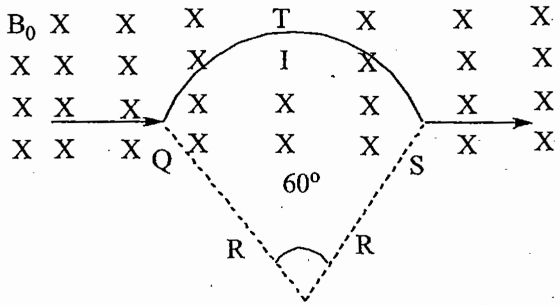
(b) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp[i(zk\cos\phi-\omega t+\phi)]$

(c) $\hat{y}\frac{k}{\omega}E_0 \exp[i(zk\cos\phi-\omega t+\phi)]$

(d) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\cos\phi)\exp[i(zk\sin\phi-\omega t)]$

20. Which one of the following statements is TRUE?
(a) The magnitude of the electric field is attenuated as the wave propagates
(b) The energy of the e.m. wave flows along the x-direction
(c) The magnitude of the electric field of the wave is a constant
(d) The speed of the wave is the same as c(speed of light in free space)

21. A circular arc QTS is kept in an external magnetic field $\vec{B}_0$ as shown in figure. The arc carries a current I. The magnetic field is directed normal and into the page. The force acting on the arc is :



(a) $2IB_0\hat{k}R$ (b) $IB_0R\hat{k}$ (c) $-2IB_0R\hat{k}$ (d) $-IB_0R\hat{k}$

22. A conducting sphere of radius R is placed in a uniform electric field $\vec{E}_0$ directed along + z axis. The electric potential for outside points is given as :

$V_{out} = -E_0\left(1-\frac{R^3}{r^3}\right)r\cos\theta$, where r is the distance from the centre and θ is the polar angle. The charge density on the surfae of the sphere is :

(a) $3\varepsilon_0E_0\cos\theta$ (b) $\varepsilon_0E_0\cos\theta$ (c) $3\varepsilon_0E_0\cos\theta$ (d) $\frac{\varepsilon_0}{3}E_0\cos\theta$

23. The state of polarization of light with the electric field vector
$\vec{E} = \hat{i}E_0\cos(kz-\omega t)-\hat{j}E_0\cos(kz-\omega t)$ is :
(a) Linearly polarized along z-direction
(b) Linearly polarized at –45° to x-axis
(c) Circularly polarized
(d) Elliptically polarized with the major axis along x- axis

**Data for Q. 24 and Q.25.**

Consider two conducting plates of infinite extent, one plate at z = 0 and the other at z = L, both parallel to the xy plane. The vector and scalar potential in the region between the plates is given by:

$$A(r,t) = A_0\, i \cos(kz + \alpha)\cos(kct);\ \phi(r,t) = 0$$

24. For this to represent a standing wave in the empty region between the plates :

(a) $k = \pi / L$ and $\alpha = 0$ (b) $k = 2\pi / L$ and $\alpha = \pi / 2$

[✓ (c) $k = \pi / (2L)$ and $\alpha = \pi / 2$] (d) $k = \pi / 2L$ and $\alpha = 0$

25. The energy density at z = 0 and t = 0 is :

(a) 0 (b) $\varepsilon_0 c^2 k^2 A_0^2$

(c) $(1/2)\mu_0 A_0^2 k^2$ (d) $(1/2)\mu_0 A_0^2 k^2 + (1/2)\varepsilon_0 c^2 k^2 A_0^2$

26. Consider the given statements about E(r, t) and B(r, t), the electric and magnetic vectors respectively in a region of free space

**P.** Both E and B are conservative vector fields
**Q.** Both E and B are central force fields
**R.** E and B are mutually perpendicular in the region
**S.** Work done by B on a moving charge in the region is zero

Choose the right combination of correct statements from the following :

(a) P and R [✓ (b) R and S] (c) S only (d) P and Q

27. Electromagnetic waves are propagating along a hollow, metallic waveguide whose cross-section is a square of side W. The minimum frequency of the electromagnetic waves is :

(a) c/W (b) 2c/W [✓ (c) $\pi c / W$] (d) $\sqrt{2}\, \pi c / W$

28. An electric field applied along the length of a long cylinder produces a polarization P. The depolarization field produced in this configuration is :

(a) $4\pi P / 3$ (b) $-4\pi P / 3$ (c) $2\pi P$ [✓ (d) 0]

29. An electric charge, +Q is placed on the surface of a solid, conduction sphere of radius a. The distance measured from the centre of the sphere is denoted as r. Then :

(a) The charge gets distributed uniformly through the volume of the sphere
[✓ (b) The electrostatic potential has the same value for r < a]
(c) An equal and opposite charge gets induced in the bottom half of the sphere
(d) The electric field is given by $1/(4\pi\varepsilon_0 r^2)$ for $r < a$

30. A large circular coil of N turns and radius R carries a time varying current $i = i_0 \sin(\omega t)$. A small circular coil of n turns and radius r ( r <<R) is placed at the center of the large coil such that the coils are concentric and coplanar. The induced emf in the small coil

(a) Leads the current in the large coil by $\pi / 2$
(b) Lags the current in the large coil by $\pi$
(c) Is in phase with the current in the large coil
[✓ (d) lags the current in the large coil by $\pi / 2$]

31. The electric field E(r, t) at a point r at time t in a metal due to the passage of electrons can be described by the equation

$$\nabla^2\vec{E}(\vec{r},t)=\frac{1}{c^2}\left[\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2}+\omega'^2\vec{E}(\vec{r},t)\right]$$

Where $\omega'$ is a characteristic associated with the metal and c is the speed of light in vacuum. The dispersion relation corresponding to the plane wave solutions of the form $\exp\left[i\left(\vec{k}.\vec{r}-\omega t\right)\right]$ is given by :

(a) $\omega^2=c^2k^2-\omega'^2$ [✓ (b) $\omega^2=c^2k^2+\omega'^2$] (c) $\omega^2=ck-\omega'$ (d) $\omega=ck+\omega'$

32. The x and y plane carries a uniform surface current of density $\vec{K}=50\,\hat{e}_z\ A/m$. The magnetic field at the point z = –0.5 m is :

(a) $10\times10^{-6}$ Wb (b) $1\times10^{-6}$ Wb (c) $\pi\times10^{-6}$ Wb [✓ (d) $10\pi\times10^{-6}$ Wb]

33. A coaxial cable of uniform cross-section contains an insulating material of dielectric constant 3.5. The radius of the central wire is 0.01 m and that of the sheath is 0.02 m. The capacitance per kilometer of a cable is :

[✓ (a) 280.5 nF] (b) 28.05 nF (c) 56.10nF (d) 2.805 nF

34. A thin conducting wire is bent into a circular loop of radius r and placed in a time dependent magnetic field of magnetic induction.

$\vec{B}(r)=B_0e^{-\alpha t}\hat{e}_z,\ (B_0>0 \text{ and } \alpha>0),$

such that, the plane of the loop is perpendicular to $\vec{B}(t)$. Then the induced emf in the loop is :

[✓ (a) $\pi r^2\alpha B_0\,e^{-\alpha t}$] (b) $\pi r^2B_0\,e^{-\alpha t}$ (c) $-\pi r^2\alpha B_0\,e^{-\alpha t}$ (d) $-\pi r^2\,B_0\,e^{-\alpha t}$

35. A current I flows in the anticlockwise direction through a square loop of side a lying in the xoy plane with its center at the origin. The magnetic induction at the center of the square loop is :

(a) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_x$ [✓ (b) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_z$] (c) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_z$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_x$

36. Which of the following equations is relativistically invariant ? ($\alpha,\beta,\gamma$ and $\delta$ are constants of suitable dimensions).

(a) $\frac{\partial\phi(x,t)}{\partial t}=\alpha\frac{\partial^2\phi(x,t)}{dx^2}$ [✓ (b) $\frac{\partial^2\phi(x,t)}{\partial t^2}=\beta\frac{\partial^2\phi(x,t)}{\partial x^2}$]

(c) $\frac{\partial^2\phi(x,t)}{\partial t^2}=\gamma\frac{\partial\phi(x,t)}{\partial x}$ (d) $\frac{\partial^2\phi(x,t)}{\partial t}=\delta\frac{\partial^3\phi(x,t)}{\partial x^3}$

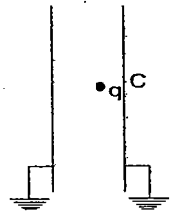
37. Three plane waves are given as $\bar{y}_1=A_1\,\hat{y}.\cos(kx-\omega t+\delta_1),\ \bar{y}_2=A_2\,\hat{z}\cos(kx-\omega t+\delta_2)$

$\bar{y}_3=A_3\,\hat{y}\cos(kx-\omega t+\delta_3)$, where $\delta_1,\delta_2,\delta_3$ and $A_1,A_2,A_3$ are constants. If these waves are superposed pairwise, which superposition will lead to interference?

(a) $\bar{y}_1$ and $\bar{y}_2$ (b) $\bar{y}_2$ and $\bar{y}_3$

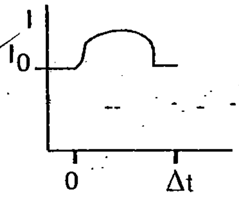
[✓ (c) $\bar{y}_1$ and $\bar{y}_3$] (d) No interference in any pair

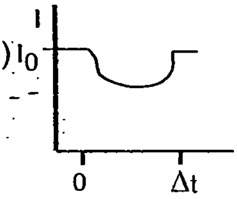
38. A point charge q is kept at the mid-point between two large parallel grounded conducting plates. Assume no gravity. The charge is displaced a little towards the right plate. The charge will now,

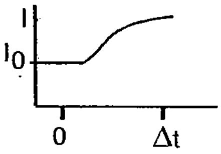


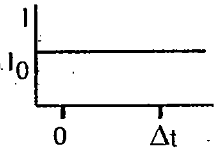
(a) Stay where it is
(b) Move towards the right plate
(c) Move towards the left plate
(d) Oscillate between the plates

39. A solenoid with an iron core is connected in series with a battery of emf V and it is found that a constant current $I_0$ passes through the solenoid. If at t = 0, the iron core is pulled out from the solenoid quickly in a time $\Delta t$, which one of the following could be a correct description of the current passing through the solenoid?

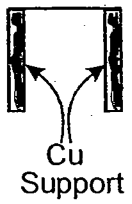
(a)


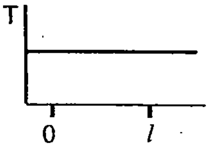
(b)


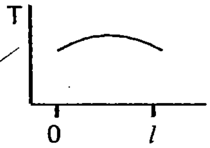
(c)


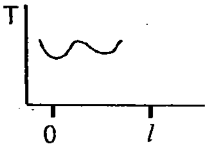
(d)


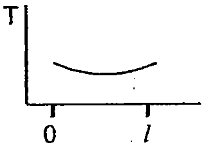
40. A tungsten wire of uniform cross section and high resistance is supported by two copper supports of low resistance in vacuum. The length of the wire is $l$. A constant current I is sent through the wire. The temperature profile (T vs. x) of the wire will look like.



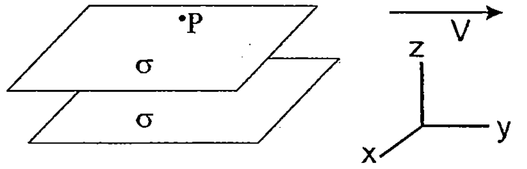
(a)


(b)


(c)


(d)


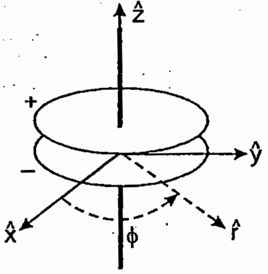
41. Two large parallel plates move with a constant speed v in the positive y-direction as shown in the figure. If both the plates have a surface charge density $\sigma > 0$, the magnetic field at the point P just above the top plate will have :



$B=0$
$B=0$

(a) Larger magnitude than the field at the mid-point between the plates and point towards $-\hat{x}$
(b) Smaller magnitude than the field at the mid-point between the plates and point towards $+\hat{x}$.
(c) Larger magnitude than the field at the mid-point between the plates and point towards $+\hat{x}$
(d) Smaller magnitude than the field at the mid-point between the plates and point towards $-\hat{x}$

42. A parallel plate capacitor is being discharged. What is the direction of the energy flow in terms of the poynting vector in the space betwen the plates?



(a) Along the wire in the positive z axis (b) Radially inward $(-\hat{r})$

[✓ (c) Radially outward $(\hat{r})$] (d) Circumferential $(\phi)$

43. Unpolarized light falls from air to a planar air-glass interface (refractive index of glass is 1.5) and the reflected light is observed to be plane polarized. The polarization vector and the angle of incidence $\theta_i$ are :

(a) Perpendicular to the plane of incidence and $\theta_i = 42^\circ$

(b) Parallel to the plane of incidence and $\theta_i = 56^\circ$

[✓ (c) Perpendicular to the plane of incidence and $\theta_i = 56^\circ$]

(d) Parallel to the plane of incidence and $\theta_i = 42^\circ$

44. A finite wave train, of an unspecified nature, propagates along the positive x axis with a constant speed v and without any change of shape. The differential equation among the four listed below, whose solution it must be, is :

[✓ (a) $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = 0$] [✓ (b) $\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = 0$]

(c) $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial t^2} - i\hbar\frac{\partial}{\partial t}\right)\psi(x,t) = 0$ (d) $\left(\nabla^2 + a\frac{\partial}{\partial t}\right)\psi(\vec{r},t) = 0$

45. A plane electromagnetic wave is given by $E_0\left(\hat{x} + e^{i\delta}\hat{y}\right)\exp\{i(kz - \omega t)\}$. At a given location, the number of times $\vec{E}$ vanishes in one second is :

[✓ (a) An integer near $\frac{\omega}{\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer]

(b) An integer near $\frac{\omega}{\pi}$ and is independent of $\delta$

(d) An integer near $\frac{\omega}{2\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer

(d) An integer near $\frac{\omega}{2\pi}$ and is independent of $\delta$

46. Consider an infinite horizontal surface with fixed surface charge density $\sigma$, where $\hat{n}$ is the upward normal to the surface. If an electric field $\vec{E}_b = E\hat{n}$ is applied from below, the electric field $\vec{E}_a$ in the region above the surface is:

(a) $\vec{E}_a = \vec{E}_b - \frac{\sigma}{2\varepsilon_0}\hat{n}$
[✓ (b)] $\vec{E}_a = \vec{E}_b + \frac{\sigma}{2\varepsilon_0}\hat{n}$
(c) $\vec{E}_a = \vec{E}_b - \frac{\sigma}{\varepsilon_0}\hat{n}$
(d) $\vec{E}_a = \vec{E}_b + \frac{\sigma}{\varepsilon_0}\hat{n}$

47. 2 A charged particle is travelling in the positive x-direction with a constant velocity. An observer located at a point P on the y-axis observes the electric field due to this charge. Let $\hat{e}(t)$ be the unit vector along the electric field at any time t, and $\hat{n}(t)$ the unit vector in the direction of the position vector of the charge with respect to the observer. Then

(a) $\hat{e}(t)$ lags behind $\hat{n}(t)$

(b) $\hat{e}(t)$ leads $\hat{n}(t)$

(c) $\hat{e}(t)$ is always in the same direction as $\hat{n}(t)$

(d) $\hat{e}(t)$ is always perpendicular to $\hat{n}(t)$

48. A current carrying loop lying in the plane of the paper is in the shape of an equilateral triangle of side a. It carries a current I in the clockwise sense. If $\hat{k}$ denotes the outward normal to the plane of the paper, the magnetic moment $\vec{m}$ due to the current loop is:

(a) $\vec{m} = a^2 I\,\hat{k}$
(b) $\vec{m} = -\frac{1}{2}a^2 I\,\hat{k}$
(c) $\vec{m} = \frac{\sqrt{3}}{2}a^2 I\,\hat{k}$
[✓ (d)] $\vec{m} = -\frac{\sqrt{3}}{2}a^2 I\,\hat{k}$

49. An electric point charge +q is placed at the point (1, 1) of the xy-plane in which two grounded semi-infinite conducting plates along the positive x and y-axes meet see figure). The electric potential in the positive quadrant at a large distance r goes as

(a) $V(r) \sim r^{-1}$
(b) $V(r) \sim r^{-2}$
[✓ (c)] $V(r) \sim r^{-3}$
(d) $V(r) \sim r^{-4}$

50. In a given frame of refrence, it is found that the electric field $\vec{E}(\vec{r},t)$ and the magnetic field $\vec{B}(\vec{r},t)$ are perpendicular to each other at all points, i.e., $\vec{E}(\vec{r},t).\vec{B}(\vec{r},t) = 0$. If the fields observed in any other inertial frame are $\vec{E}'$ and $\vec{B}'$, then

(a) $\vec{E}'//\vec{B}'$ at all points
(b) $\vec{E}'.\vec{B}' < 0$ at all points
(c) $\vec{E}'.\vec{B}' > 0$
[✓ (d)] $\vec{E}'.\vec{B}' = 0$

51. Given any arbitrary electric and magnetic field $\vec{E}(\vec{r},t)$ and $\vec{E}(\vec{r},t)$, it is always possible to choose the scalar potential $\phi(\vec{r},t)$ and vector potential $\vec{A}(\vec{r},t)$ such that

(a) $\vec{A}(\vec{r},t)$ is identically zero.

(b) $\phi(\vec{r},t)$ is identically zero.

(c) $\left|\vec{A}(\vec{r},t)\right|$ is any given non-zero constant.

[✓ (d) The conditions $\nabla.\vec{A}=0$ and $\frac{1}{c}\frac{\partial\phi}{\partial t}+\nabla.\vec{A}=0$ are simultaneously satisfied.]

52. In a certain region R, Maxwell's equations for the electric and magnetic fields are given by

$$\nabla.\vec{E}=0, \nabla.\vec{B}=0, \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

We may conclude that

(a) Both the scalar and the vector potential are necessarily constant in the region R.

(b) The electric field $\vec{E}$ and the magnetic field $\vec{B}$ must necessarily be uniform in R.

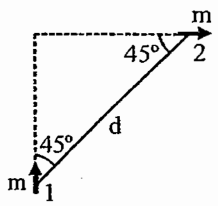
[✓ (c) There are no sources for electric charges and currents in R.]

both [✓ (d) The electric field $\vec{E}$ is necessarily perpendicular to the magnetic field $\vec{B}$ at every point in R.]

53. An insulating sphere of radius a carries a charge density $\rho(\bar{r})=\rho_0(a^2-r^2)\cos\theta; r<a$. The leading order term for the electric field at a distance d, far away from the charge distribution, is proportional to

(a) $d^{-1}$ (b) $d^{-2}$ [✓ (c) $d^{-3}$] (d) $d^{-4}$

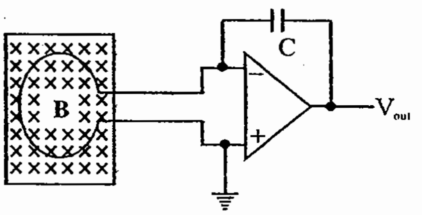
54. Two magnetic dipoles of magnitude m each are placed in a plane as shown



The energy of interaction is given by

(a) Zero (b) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$ (c) $\frac{3\mu_0}{2\pi}\frac{m^2}{d^3}$ [✓ (d) $-\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$]

55. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux $\phi_0$. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged. Vo o/p independent of B



(o/p independent of B)

[✓ (a) $\phi_0$] (b) u (c) R (d) C

**Linked Answer Questions:**

**Statement for Linked Answer Q.56 and Q.57.**

Consider the propagation of electromagnetic waves in a linear, homogenous and isotropic material medium with electric permitivity $\varepsilon$, and magnetic permeability $\mu$.

56. For a plane wave of angular frequency $\omega$ and propagation vector $\vec{k}$ propagating in the medium Maxwell's equations reduce to

(a) $\vec{k}.\vec{E}=0; \vec{k}.\vec{H}=0; \vec{k}\times\vec{E}=\omega\varepsilon\vec{H}; \vec{k}\times\vec{H}=-\omega\mu\vec{E}$

(b) $\vec{k}.\vec{E}=0; \vec{k}.\vec{H}=0; \vec{k}\times\vec{E}=-\omega\varepsilon\vec{H}; \vec{k}\times\vec{H}=\omega\mu\vec{E}$

(c) $\vec{k}.\vec{E}=0; \vec{k}.\vec{H}=0; \vec{k}\times\vec{E}=-\omega\mu\vec{H}; \vec{k}\times\vec{H}=\omega\varepsilon\vec{E}$

[✓ (d)] $\vec{k}.\vec{E}=0; \vec{k}.\vec{H}=0; \vec{k}\times\vec{E}=\omega\mu\vec{H}; \vec{k}\times\vec{H}=-\omega\varepsilon\vec{E}$

57. If $\varepsilon$ and $\mu$ assume negative values in a certain frequency range, then the directions of the propagation vector $\vec{k}$ and the Poynting vector $\vec{S}$ in that frequency range are related as

(a) $\vec{k}$ and $\vec{S}$ are parallel

[✓ (b)] $\vec{k}$ and $\vec{S}$ are anti-parallel.

(c) $\vec{k}$ and $\vec{S}$ are perpendicular to each other

(d) $\vec{k}$ and $\vec{S}$ make an angle that depends on the magnitude of $|\varepsilon|$ and $|\mu|$.

58. Consider an inifinite horizontal surface with fixed surface charge density $\sigma$, where $\hat{n}$ is the upward normal to the surface. If an electric field $\vec{E}_b = E\hat{n}$ is applied from below, the electric field $\vec{E}_a$ in the region above the surface is:

(a) $\vec{E}_a = \vec{E}_b - \frac{\sigma}{2\varepsilon_0}\hat{n}$ (b) $\vec{E}_a = \vec{E}_b + \frac{\sigma}{2\varepsilon_0}\hat{n}$ (c) $\vec{E}_a = \vec{E}_b - \frac{\sigma}{\varepsilon_0}\hat{n}$ (d) $\vec{E}_a = \vec{E}_b + \frac{\sigma}{\varepsilon_0}\hat{n}$

59. A charged particle is travelling in the positive x-direction with a constant velocity. An observer located at a point P on the y-axis observes the electric field due to this charge. Let $\hat{e}(t)$ be the unit vector along the electric field at any time t, and $\hat{n}(t)$ the unit vector in the direction of the position vector of the charge with respect to the observer. Then

(a) $\hat{e}(t)$ lags behind $\hat{n}(t)$ (b) $\hat{e}(t)$ leads $\hat{n}(t)$

(c) $\hat{e}(t)$ is always in the same direction as $\hat{n}(t)$

(d) $\hat{e}(t)$ is always perpendicular to $\hat{n}(t)$

60. A current carrying loop lying in the plane of the paper is in the shape of equilateral triangle of side a. It carries a current I in the clockwise sense. If $\hat{k}$ denotes the outward normal to the plane of the paper, the magnetic moment $\vec{m}$ due to the current loop is:

(a) $\vec{m} = a^2 I\,\hat{k}$ (b) $\vec{m} = -\frac{1}{2}a^2 I\,\hat{k}$ (c) $\vec{m} = \frac{\sqrt{3}}{2}a^2 I\,\hat{k}$ (d) $\vec{m} = -\frac{\sqrt{3}}{4}a^2 I\,\hat{k}$

61. An electric point charge +q is placed at the point (1, 1) of the xy-plane in which two grounded semi-infinite conducting plates along the positive x and y-axes meet. The electric potential in the positive quadrant at a large distance r goes as

(a) $V(r)\sim r^{-1}$ (b) $V(r)\sim r^{-2}$ (c) $V(r)\sim r^{-3}$ (d) $V(r)\sim r^{-4}$

62. In a given frame of reference, it is found that the electric field $\vec{E}(\vec{r},t)$ and the magnetic field $\vec{B}(\vec{r},t)$ are perpendicular to each other at all points, i.e. $\vec{E}(\vec{r},t).\vec{B}(\vec{r},t)=0$. If the fields observed in any other inertial frame are $\vec{E}'$ and $\vec{B}'$, then

(a) $\vec{E}' \parallel \vec{B}'$ at all points (b) $\vec{E}'.\vec{B}'<0$ at all points.
(c) $\vec{E}'.\vec{B}'>0$ at all points (d) $\vec{E}'.\vec{B}'=0$ at all points

63. Given any arbitrary electric and magnetic fields $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$, it is always possible to choose the scalar potential $\phi(\vec{r},t)$ and vector potential $\vec{A}(\vec{r},t)$ such that

(a) $\vec{A}(\vec{r},t)$ is identically zero.

(b) $\phi(\vec{r},t)$ is identically zero.

(c) $|\vec{A}(\vec{r},t)|$ is any given non-zero constant,

(d) The conditions $\nabla.\vec{A}=0$ and $\frac{1}{c}\frac{\partial\phi}{\partial t}+\nabla.\vec{A}=0$ are simultaneously satisfied.

64. In a certain region, R, Maxwell's equations for the electric and magnetic fields are given by

$$\nabla.\vec{E}=0,\ \nabla.\vec{B}=0,\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},\ \nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

We may conclude that:
(a) Both the scalar and the vector potential are necessarily constant in the region R.
(b) The electric field $\vec{E}$ and the magnetic field $\vec{B}$ must necessarily be uniform in R.
(c) There are no sources for electric charges and currents in R.
(d) The electric field $\vec{E}$ is necessarily perpendicular to the magnetic field $\vec{B}$ at every point in R.

65. The magnitude of vector potential at a point $r(r<R)$ in an infinite solenoid with N-turns per unit length, radius R and carrying a current I, is given by

(a) $A=\mu_0 N I r^2$ (b) $A=\frac{\mu_0 N I r^2}{2}$ (c) $A=\mu_0 N I r$ [✓ (d)] $A=\frac{\mu_0 N I r}{2}$

66. The gauge transformations of the scalar and vector potentials that leave the electric and magnetic field invariant are

(a) $\vec{A}\to\vec{A}'=A-\frac{\partial\Lambda}{\partial t}$, $\phi\to\phi'=\phi+\nabla\Lambda$
[✓ (b)] $\vec{A}\to\vec{A}'=A+\nabla\Lambda$, $\phi\to\phi'=\phi-\frac{\partial\Lambda}{\partial t}$
(c) $\vec{A}\to\vec{A}'=A-\nabla\Lambda$, $\phi\to\phi'=\phi-\frac{\partial\Lambda}{\partial t}$
(d) $\vec{A}\to\vec{A}'=\vec{A}+\nabla\Lambda$, $\phi\to\phi'=\phi+\frac{\partial\Lambda}{\partial t}$

67. For plane electromagnetic waves varying in time as $e^{-iwt}$, the time averaged Poynting vector is:

both [✓ (a)] $\frac{1}{2}R_e(\vec{E}\times\vec{H})$ [✓ (b)] $R_e(\vec{E}\times\vec{H})$ (c) $\frac{1}{2}R_e(\vec{E}\times\vec{H})$ (d) $\frac{1}{2}(R_e\vec{E})\times(R_e\vec{H})$

68. A dipole is formed by charges q and –q. The energy radiated by the dipole is maximum.
(a) Along the line joining the charges
[✓ (b)] Along the line perpendicular to the dipole and passing midway between the dipole.
(c) Along the line inclined to the dipole axis at $+45°$.
(d) At the centre of the dipole.

69. The electric field of an electric dipole $\vec{p}$ at a distance $\vec{r} = r\,\hat{r}$ where $\hat{r}$ is a unit vector along $\vec{r}$ is proportional to

(a) $[3\hat{r}.\vec{p}]/r^2$ (b) $[3\vec{r}.\vec{p}]/r^3$ (c) $3[(\hat{r},\vec{p})\hat{r}-\vec{p}]/r^3$ (d) $[3(\hat{r}.\vec{p})\hat{r}-\vec{p}]/r^3$

70. A large charged copper sheet is surrounded by a dielectric medium of dielectric constant $\in$. At any point on the flat surface of the sheet.

both

(a) The normal component of $\vec{E}$ is zero. (b) The normal component of $\vec{E}$ is not zero.

(c) The tangential component of $\vec{E}$ is zero. (d) The tangential component of $\vec{E}$ is not zero.

71. Magnetic induction $|B|$ due to current I in a coil of radius R placed in y-z plane at a distance X along the axis of the coil is given by $\frac{\mu_0 I}{2}\frac{R^2}{\left(R^2+X^2\right)^{3/2}}$. Another similar coil of radius R, carrying equal current I is placed in the y-z plane at a distance 2R from the first coil along the same axis. The value of $|B|$ at a point midway between the two coils and on the x-axis is:

(a) $\frac{\mu_0 I}{2\sqrt{2}\,R}$ (b) $\frac{\mu_0 I R}{2\sqrt{2}}$ (c) 0 (d) $\frac{\mu_0 I}{4\sqrt{2}\,R}$

72. A metal rod AB rotates clockwise in the plane of the paper about the end B with an angular velocity $\omega$ in a uniform magnetic field emerging out of the plane of the paper normally. The end B about which the rod rotates will be

(a) Positive with respect to the end A (b) Negative with respect to the end A.

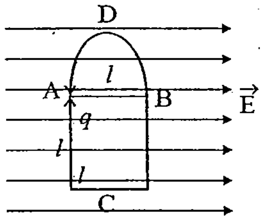
(c) Having same potential as the remaining part of the rod.

(d) Having same potential as the end B, but the centre of the rod will be at higher potential.

73. An electromagnetic wave travelling with speed $V_i$ is incident on an interface of two media parallel to x-z plane. The magnetic field vector of the incident wave given by $\vec{B}_i(x,t) = \hat{k}\frac{E_{OT}}{V_i}e^{i(k_1x-wt)}$. The magnetic field vector $\vec{B}(x,t)$ of the transmitted wave is:

(a) $\hat{k}\frac{E_{OT}}{V_2}e^{i(k_2x-wt)}$ (b) $-\hat{k}\frac{B_{OT}}{V_2}e^{i(k_2x-wt)}$ (c) $\hat{j}V_2B_{OT}e^{i(k_2x+wt)}$ (d) $\hat{j}V_2B_{OT}e^{i(k_2x-wt)}$

74. A charge q is taken from point A to point B along two different paths ACB and ADB in an uniform electric field.



The difference in the work done between these two paths is:

(a) $3qlE-(\pi/2)lqE$ (b) 0 (c) $\frac{1}{2}\pi lqE$ (d) 3qlE

75. An electric dipole $\vec{p}$ located in a uniform electric field with its direction along the electric field $\vec{E}$. The work required to rotate the dipole through 180° is:

(a) $2(\vec{p}.\vec{E})$ (b) $\vec{p}.\vec{E}$ (c) $\vec{p}\times\vec{E}$ (d) $2(\vec{p}\times\vec{E})$

76. The angular distribution of the average power radiated from a short dipole antenna of length d, is dependent on d and polar angle $\theta$. This dependence is proportional to

(a) $\sin^2\theta$ (b) $d\sin^2\theta$ [✓ (c) $d^2\sin^2\theta$] (d) $d\sin\theta$

77. Maxwell's equations can be written in terms of two coupled equations satisfied by the vector potential $\vec{A}$ and scalar potential $\phi$. The Lorentz gauge which decouples these equations is given by

(a) $\vec{\nabla}.\vec{A}=0$ (b) $\nabla^2\phi=\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}$ [✓ (c) $\vec{\nabla}.\vec{A}=-\frac{1}{c^2}\frac{\partial\phi}{\partial t}$] (d) $\vec{\nabla}.\vec{A}=\frac{1}{c}\frac{\partial\phi}{\partial t}$

78. An electromagnetic field is described by the scalar potential $\phi=ay$ and vector potential $\vec{A}=\hat{k}\times t$ where a is a constant. The electric field $\vec{E}$ is:

[✓ (a) Linearly polarized] (b) Circularly polarized
(c) Elliptically polarized (d) Unpolarized.

79. An electron moves with velocity $\hat{v}=\hat{j}.v_0$ under the combined influence of $\vec{E}=\hat{i}E_0$ and $\vec{B}=-\hat{k}B_0$ fields. The force on the electron is in the

(a) Y direction (b) Z direction (c) YZ plane [✓ (d) X direction]

80. An electromagnetic wave is elliptically polarized in the X-Y plane. The direction of the Poynting vector is in the

[✓ (a) Z-direction] (b) X-direction (c) Y-direction (d) XY-plane

81. The electrostatic potential is given by,

$$\Phi=0 \text{ at } x=0 \text{ and } x=a\,;\ \Phi=V\ \ 0\le x\le a.$$

Then $\Phi$ is given by $\Phi=\sum_{n=1}^{\infty}A_n\sin(\alpha x)$ where $\alpha$ is:

(a) $\frac{n\pi}{2}$ [✓ (b) $\frac{n\pi}{a}$] (c) $\frac{na}{\pi}$ (d) $\frac{na}{2\pi}$

82. A certain charge distribution is symmetric about the Z-axis which is the principal axis. The off-diagonal quadrupole tensor elements $Q_{ij}$ are all zero. It is given that $Q_{11}=1$, then

d

(a) $Q_{33}=-2$ (b) $Q_{22}=-1$ (c) $Q_{33}=1$ (d) $Q_{22}=-2$

83. Which of the following is Lorentz Invarient

[✓ (a) $E^2-B^2C^2$] (b) $\vec{E}\times\vec{B}$ (c) $E^2-BC^2$ (d) $\vec{E}\times\vec{C}$

84. For a co-axial TE mode

(a) $E_z\neq0, H_z=0$ [✓ (b) $E_z=0, H_z\neq0$]

(c) $E_z=0, H_z=0$ (d) $E_z\neq0, H_z=1$

85. Polarisation can be achieved by

[✓ (a) Beam splitter] (b) Grating (c) Plane Mirror (d) None of these

86. The poynting vector in the radial direction, at the surface of a long straight wire, of radius b and resistance per unit length R, carrying a current I is:

[✓ (a) $\frac{-RI^2}{2\pi b}\hat{r}(\text{inward})$] (b) $\frac{RI^2}{2\pi b}\hat{r}(\text{outward})$

(c) $\frac{bI}{2\pi R}\hat{r}(\text{inward})$ (d) $\frac{2\pi I}{Rb}\hat{r}(\text{inward})$

87. Of A and V are vector and scalar potential $v(r,t)=0$, $A(r,t)=\frac{-q}{4\pi\epsilon_0}\frac{t\,\bar{r}}{r^2}$. Scalar function $\lambda=\frac{-q}{4\pi\epsilon_0}\frac{t}{r^2}$

Due to gauge transformation transformed potential is

(a) $v'(r,t)=0 \quad A(r,t)=\frac{q}{4\pi\epsilon_0}t\frac{1}{r^2}$

(b) $v'(r,t)=\frac{q}{4\pi\epsilon_0 r^2} \quad A(r,t)=\frac{q}{4\pi\epsilon_0}t\frac{1}{r^2}$

(c) $v'(r,t)=\frac{-q}{4\pi\epsilon_0}\frac{1}{r^2} \quad A'(r,t)=\frac{-qt}{4\pi\epsilon_0}\frac{1}{r^2}$

(d) $v'(r,t)=\frac{q}{4\pi\epsilon_0}\frac{1}{r^2} \quad A'(r,t)=\frac{qt}{4\pi\epsilon_0 r^2}$

88. A medium with refractive index $n(\omega)$, where $\omega$ is the frequency, is said to posses anomalous dispersion when

(a) $\frac{\partial n}{\partial\omega}>0$ (b) $\frac{\partial n}{\partial\omega}<0$ (c) $\frac{\partial^2 n}{\partial\omega^2}>0$ (d) $\frac{\partial^2 n}{\partial\omega^2}<0$

89. The field $\vec{F}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ may represent

(a) Only magnetostatic field (b) Only an electrostatic field

(c) Both electrostatic and magnetostatic field (d) Neither electrostatic nor magnetostatic field.

90. Electromagnetic wave in vacuum is given by $\vec{E}(\vec{r},t)=\hat{i}E_0e^{-i(\omega t-\vec{k}.\vec{x})}$. Magnetic field $\vec{B}$ is:

(a) $\hat{j}E_0\hat{k}\exp\left[-i\left(\omega t-\vec{k}.\vec{x}\right)\right]$ (b) $\hat{j}E_0\exp\left[-i\left(\omega t-k_z z\right)\right]$

(c) $\hat{k}\,E_0\exp\left[-i\left(\omega t-\vec{k}.\vec{x}\right)\right]$ (d) $\hat{j}E_0\exp\left[-i\left(\omega t-\vec{k}.\vec{x}\right)\right]$

91. In a non-magnetic dielectric medium with dielectric constant $\varepsilon_r=4$, the electric field of a propagating plane wave with $\omega=10^8$ rad/s is given by, $\vec{E}=\left(-\hat{i}+\sqrt{3}\hat{j}\right)\exp\left[j\left(\omega t-\vec{k}.\vec{r}\right)\right]$. The propagation vector $\vec{k}$ (in units of m$^{-1}$) is given by

(a) $\vec{k}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{3}\hat{j}$ (b) $\vec{k}=\frac{1}{3}\hat{z}$

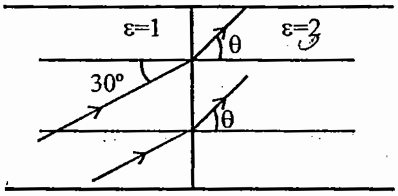
(c) $\vec{k}=\frac{1}{2\sqrt{3}}\hat{i}+\frac{1}{6}\hat{j}$ (d) $\vec{k}=-\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{3}\hat{j}$

92. Two small circular loops, each of area 1 cm² carry currents 1A and 2A respectively. They are placed in a plane at a distance of 5 meters from each other. If one of the loops is lifted 5 meters in the vertical direction while maintaining it flat, the electrical work done in the process is ($\mu_0$ is the magnetic permeability of vacuum)

(a) $2.75\times10^{-9}\,\mu_0^2\,J$ (b) $1.46\times10^{-10}\,\mu_0^2\,J$

(c) $1.88\times10^{-10}\,\mu_0^2\,J$ (d) $4.74\times10^{-9}\,\mu_0^2\,J$

93. A large slab consists of two materials with dielectric constants 1 and 3 divided by a plane interface as shown in the figure. An electric field exists in this slab, which uniformly makes an angle of 30° with the normal to the interface on the left side of the $(\varepsilon = 1)$. The angle the field makes on the other side of the interface is:



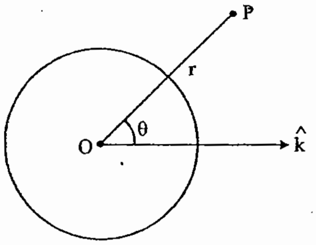
(a) $\tan^{-1}(3)$ (b) $\sin^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ [✓ (c) 60°] (d) $\tan^{-1}(1/3)$

94. A particle of mass m and charge q is constrained to move along a straight line joining two other equal charges q fixed at $x = \pm a$. The time period of small oscillation is:

[✓ (a) $T = \frac{2\pi a}{q}\sqrt{\varepsilon_0 am}$] (b) $T = \frac{4\pi a}{q}\sqrt{r\varepsilon_0 am}$ (c) $T = \frac{a}{q}\sqrt{r\varepsilon_0 am}$ (d) $T = \frac{2\pi a}{q}\sqrt{\varepsilon_0 am}$

95. A spherical conductor of radius a is placed in a uniform electric field $\vec{E} = E_0\hat{k}$. The potential at a point $P(r,\theta)$ for r > a, is given by, $\varphi(r,\theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$

Where 'r' is the distance of P from the centre O of the sphere and $\theta$ is the angle OP makes with the z-axis.



The charge density on the sphere at $\theta = 30°$ *is* :

[✓ (a) $3\sqrt{3}\,\varepsilon_0 E_0/2$] (b) $3\varepsilon_0 E_0/2$ (c) $\sqrt{3}\varepsilon_0 E_0/2$ (d) $\varepsilon_0 E_0/2$

**Statement for Linked Answer Questions 96 and 97:**

A plane electromagnetic wave has the magnetic field given by $\vec{B}(x,y,z,t) = B_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{k}$

Where k is the wave number and $\hat{i}, \hat{j}$ and $\hat{k}$ are the Cartesian unit vectors is x, y and z directions, respectively.

96. The electric field $\vec{E}(x,y,z,t)$ corresponding to the above wave is given by

[✓ (a) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$] (b) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(c) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{i}$ (d) $cB_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{j}$

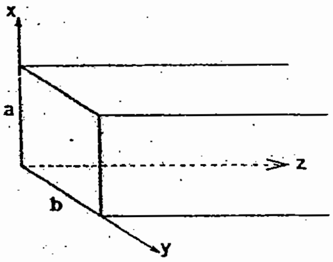
97. The average Poynting vector is given by

(a) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (c) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ [✓ (d) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$]

98. A static, spherically symmetric charge distribution is given by $\rho(r)=\frac{A}{r}e^{-kr}$ where A and K are positive constants. The electrostatic potential corresponding to this charge distribution varies with r as

(a) $r\,e^{-Kr}$ [✓ (b) $\frac{1}{r}e^{-Kr}$] (c) $\frac{1}{r^2}e^{-Kr}$ (d) $\frac{1}{r}\left(1-e^{-Kr}\right)$

99. The magnetic field of the $TE_{11}$ mode of a rectangular waveguide of dimensions a×b as shown in the figure is given by $H_z = H_0\cos(0.3\pi x)\cos(0.4\pi y)$, where x and y are in cm.



**A.** The dimensions of the waveguide are

[✓ (a) a = 3.33 cm, b = 2.50 cm] (b) a = 0.40 cm, b = 0.30 cm

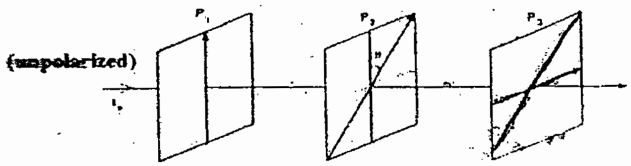
(c) a = 0.80 cm, b = 0.60 cm (d) a = 1.66 cm, b = 1.25 cm

**B.** The entire range of frequencies f for which the $TE_{11}$ mode will propagate is:

(a) 6.0 GHz < f < 7.5 GHz (b) 7.5 GHz < f < 9.0 GHz

(c) 7.5 GHz < f < 12.0 GHz [✓ (d) 7.5 GHz < f]

100. Consider three polarizers $P_1, P_2$ and $P_3$ placed along an axis as shown in the Figure.



The pass axis of $P_1$ and $P_3$ are at right angles to each other while the pass axis of $P_2$ makes an angle $\theta$ with that of $P_1$. A beam of unpolarized light of intensity $I_0$ is incident on $P_1$ as shown. The intensity of light emerging from $P_3$ is

(a) 0 (b) $\frac{I_0}{2}$ [✓ (c) $\frac{I_0}{8}\sin^2 2\theta$] (d) $\frac{I_0}{4}\sin^2 2\theta$

101. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e., points where the electric field vanishes will be found inside the square?

(a) 3 (b) 4 [✓ (c) 5] (d) 7

102. In a Young's double slit interference expriment, the slits are at a distance 2L from each other and the screen is at a distance D from the slits. If a glass slab of refractive index $\mu$ and thickness d is placed in the path of one of the beams, the minimum value of d for the central fringe to be dark is

(a) $\frac{\lambda D}{(\mu-1)\sqrt{D^2+L^2}}$ (b) $\frac{\lambda D}{(\mu-1)L}$ (c) $\frac{\lambda}{(\mu-1)}$ [✓ (d) $\frac{\lambda}{2(\mu-1)}$]

103. Consider a solenoid of radius R with n turns per unit length, in which a time dependent current $I = I_0 \sin\omega t$ (where $\omega R/c \ll 1$) flows. The magnitude of the electric field at a perpendicular distance $r < R$ from the axis of symmetry of the solenoid, is:

(a) 0

(b) $\frac{1}{2r}\omega\mu_0 n I_0 R^2\cos\omega t$

(c) $\frac{1}{2}\omega\mu_0 n I_0 r\sin\omega t$

[✓ (d) $\frac{1}{2}\omega\mu_0 n I_0 r\cos\omega t$]

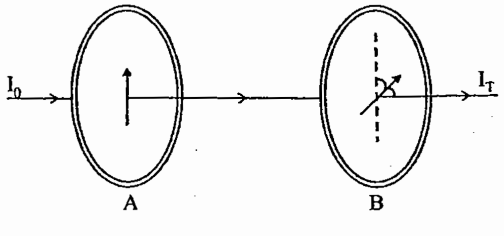
104. A constant electric current I in an infinitely long straight wire is suddenly switched on at $t = 0$. The vector potential at a perpendicular distance r from the wire is given by $\vec{A} = \frac{\hat{k}\mu_0 I}{2\pi}\ell n\left[\frac{1}{r}\left(ct+\sqrt{c^2t^2-r^2}\right)\right]$. The electric field at a distance $r(<ct)$ is:

(a) 0 (b) $\frac{\mu_0 I}{2\pi t}\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$ (c) $\frac{c\mu_0 I}{2\pi\sqrt{c^2t^2-r^2}}\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ [✓ (d) $-\frac{c\mu_0 I}{2\pi\sqrt{c^2t^2-r^2}}\hat{k}$]

105. A static charge distribution gives rise to an electric field of the form $\vec{E} = \alpha\left(1-e^{-r/R}\right)\frac{\hat{r}}{r^2}$, where $\alpha$ and R are positive constants. The charge contained within a sphere of radius R, centred at the origin is:

(a) $\pi\alpha\varepsilon_0\frac{e}{R^2}$ (b) $\pi\alpha\varepsilon_0\frac{e^2}{R^2}$ (c) $4\pi\alpha\varepsilon_0\frac{R}{e}$ (d) $\pi\alpha\varepsilon_0\frac{R^2}{e}$

106. Unpolarised light of intensity $I_0$ Passes successively through two identical linear polarisers A and B, placed such that their polarisation axes are at an angle of 45° (see figure) with respect to one another.



Assuming A and B to be perfect polarisers (i.e. no absorption losses), the intensity of the transmitted light will be $I_T =$

[✓ (a) $I_0/4$] (b) $I_0/2\sqrt{2}$ (c) $I_0/2$ (d) $I_0/\sqrt{2}$

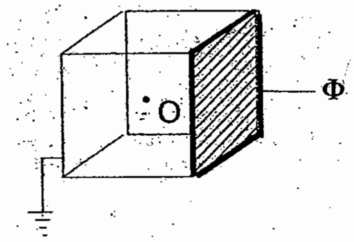
107. Three equal chrages Q are successively brought from infinity and each is placed at one of the three vertices of an equilateral tringle. Assuming the rest of the Universe as a whole to be neutral, the energy $E_0$ of the electrostatic field will increase, successively, to

$$E_0 + \Delta_1, \qquad E_0 + \Delta_1 + \Delta_2, \qquad E_0 + \Delta_1 + \Delta_2 + \Delta_3$$

Where $\Delta_1 : \Delta_2 : \Delta_3 =$

(a) 1 : 2 : 3 : (b) 1 : 1 : 1 (c) 0 : 1 : 1 [✓ (d) 0 : 1 : 2]

108. Five sides of a hollow metallic cube are grounded and the sixth side is insulated from the rest and is held at a potential $\Phi$ (see figure).



The potential at the centre O of the cube is

(a) 0 [✓ (b) $\Phi/6$] (c) $\Phi/5$ (d) $2\Phi/3$

109. If $\vec{D} = (2y^2 + z)\hat{i} + 4xy\hat{j} + x\hat{k}$, units of $C/m^2$, the total charge enclosed by the cube $0 \le x < 1, 0 \le y < 1,\ 0 \le z \le 1$

[✓ (a) 2C] (b) 4C (c) 0 C (d) 1C

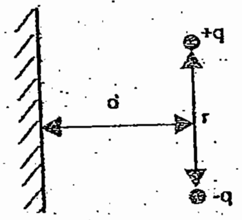
110. The electric fields of a long straight wire varies with distance r (perpendicular to the wire) as

[✓ (a) $\frac{1}{r}$] (b) In r (c) $1/r^2$ (d) $1/r^3$

111. If $\vec{V} = \hat{i} + \hat{j} + \hat{k}$, where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in three-dimensions, the value of $\vec{\nabla}.\vec{V}$ is :

(a) -3 (b) 2 [✓ (c) 0] (d) 3

112. Two equal and opposite charges are placed at a distance r from each other. A large metallic sheet is placed at a distance d from them as shown in the figure. Due to the presence of the sheet, the attractive force between the charges along the direction joining them



[✓ (a) decreases] (b) increases (c) remains unchanged

(d) decreases or increases depending on the conductivity of the sheet.

113. The magnetic induction $\vec{B}$ and the field $\vec{H}$ inside a bar magnet are

[✓ (a) parallel to the bar, and opposite to each other with $\vec{B}$ directed from south to north pole]

(b) parallel to each other along the bar, but their magnitude differ by a factor of permeability

(c) parallel to the bar, and opposite to each other with $\vec{B}$ directed from north to south pole

(d) parallel to each other along the bar with equal magnitude

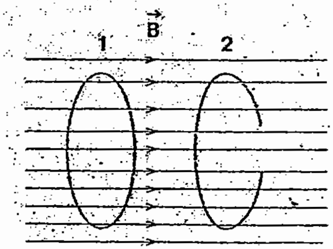
114. Two beams of light with electric field amplitudes $\hat{i}E$ and $(\hat{i}-2\hat{j})E$ interfere at a point with path difference $\lambda/2$. The magnitude of the electric field at the point of interference is

(a) $2\sqrt{2}\,E$ [✓ (b) 0] (c) $2\,E$ (d) $(1+\sqrt{5})E$

115. A propagating electromagnetic wave is given by $\vec{E}=E_0(\hat{j}+i\hat{k})e^{i(kx-\omega t)}$. The wave is

[✓ (a) left circularly polarized] (b) right circularly polarized
(c) linearly polarized (d) elliptically polarized

116. Two conducting coils 1 and 2 (identical except that 2 is split) are placed in a uniform magnetic field as shown in the figure. The magnetic field decreases at constant rate. If the plane of the coils is perpendicular to the field lines, which of the following statements is true.



(a) an emf is induced in the split coil 2
(b) an emf is induced in both coils
(c) equal Joule heating occurs in both coils
[✓ (d) Joule heating does not occur in either coil]

117. If the scalar and vector potentials $\phi(\vec{r},t), \vec{A}(\vec{r},t)$ are transformed to $\phi'(\vec{r},t)=\phi(\vec{r},t)+\phi_0 e^{-\lambda t}$ and $\vec{A}'(\vec{r},t)=\vec{A}(\vec{r},t)+\phi_0\vec{r}$ respectively, the corresponding electric and magnetic fields transform as

(a) $\vec{E}'=\vec{E}, \vec{B}'=B_0+\phi_0\vec{r}\frac{e^{-\lambda t}}{r}$ (b) $\vec{E}'=\vec{E}+\phi_0\vec{r}, \vec{B}'=\vec{B}_0+\phi_0\vec{r}\;e^{-\lambda t}$

(c) $\vec{E}'=\vec{E}-\phi_0\vec{r}\;e^{-\lambda t}, \vec{B}'=\vec{B}$ [✓ (d) $\vec{E}'=\vec{E},\quad \vec{B}'=\vec{B}$]

118. For a wave propagating in an air filled metallic rectangular waveguide, the frequency and the wavevector are related as (c is the speed of light and $\omega_c$ is the cut off frequency)

[✓ (a) $c^2k^2=\omega^2+\omega_c^2$] (b) $c^2k^2=\omega^2$ (c) $c^2k^2=\omega^2-\omega_c^2$ (d) $c^2k^2=\sqrt{\omega^4-\omega_c^4}$

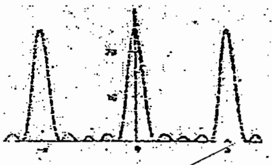
119. In a distortionless transmission line the attenuation constant $\alpha$ is frequency independent while the propagation constant depends linearly on frequency. If R, L, C and G denote the transmission line parameters, for the distortion less line

(a) $RG=\frac{1}{\sqrt{LC}}$ (b) $\frac{R}{L}=\frac{C}{G}$ (c) $R=\sqrt{\frac{L}{C}}$ [✓ (d) $\frac{R}{L}=\frac{G}{C}$]

120. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields respectively, identify the Lorentz invariant quantity from the following.

(a) $\vec{E}^2\,\vec{B}$ (b) $|\vec{E}\times\vec{B}|^2$ [✓ (c) $\vec{E}^2-c^2\vec{B}^2$] (d) $\vec{E}^2+c^2\vec{B}^2$

121. The diffraction pattern from a grating with N slits is shown in the figure. The value of N is

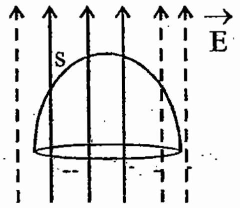


(a) 3 (b) 5 [✓ (c) 4] (d) 2

122. The one dimensional potential $V(x)$ for an electrostatic fields in a region of space which does not contain any charge has the form (in the following a and b are constants)

(a) $a\sin bx$ (b) $ae^{-x^2}$ (c) $\frac{1}{2}ax^2$ [✓ (d) $ax+b$]

123. A hemispherical surface of radius R is in a constant electric field $\vec{E}$ directed perpendicular to the base, as shown. The flux through the curved surface S of the hemisphere is



(a) 0 [✓ (b) $\pi R^2E$] (c) $2\pi R^2E$ (d) $4\pi R^2E$

124. When a plane electromagnetic wave is incident angle i = 60°, the reflected light is completely polarized. The dielectric constant is

(a) 4/3 (b) $\sqrt{3}$ (c) 2 [✓ (d) 3]

125. When a charged particle with kinetic energy E enters a region of uniform magnetic field $\vec{B}$ directed perpendicular to its velocity, it is found to move in circle of radius R. If the particle had kinetic energy 2E, the radius of the path would have been

(a) $R/\sqrt{2}$ [✓ (b) $\sqrt{2}R$] (c) $R/2$ (d) $2R$

126. A dielectric sphere of radius R carries a polarization $\vec{P} = k\vec{r}$ throughout its volume. The volume charge density pb of the bound charges is

[✓ (a) $-3k$] (b) $3k$ (c) $-k$ (d) $k$

127. The expression $\vec{A} = \frac{B}{2}\left(-\hat{i}y + \hat{j}x\right)$ for the vector potential represents

[✓ (a) a uniform magnetic field in the z-direction] (b) a non- uniform magnetic field in the z-direction
[✓ (c) a uniform magnetic field in xy-plane] (d) a non-uniform magnetic field in xy-plane

128. In a region of space it is found that $\vec{\nabla}.\vec{E} = 0$ but $\vec{\nabla}\times\vec{E} \neq 0$. Which of the following statements is not true?

(a) The charge density must be zero [✓ (b) A displacement current must exist]
(c) The divergence of magnetic field must be zero
(d) A time-varying magnetic field must exist

129. An electromagnetic wave propagating along the z-direction in a rectangular waveguide in the TE mode is characterized by

[✓ (a) $E_z = 0, H_z \neq 0$] (b) $H_z = 0, E_z \neq 0$ (c) $E_z = 0, H_z = 0$ (d) $E_z \neq 0, H_z \neq 0$

130. To determine the electrostatic potential inside an enclosure with insulating walls, one needs to know the

[✓ (a) value of the potential at all points on the surface of the enclosure]

(b) value of the potential at just two points on the surface of the enclosure

(c) value of the potential as well as its gradient at all points on the surface of the enclosure

(d) value of the potential and its gradient at one point on the surface

131. Consider the plane defined by $(x, y, z = 10\text{ m})$. If it carries a charge of $20\text{ nC/m}^2$, what is the electric field intensity at the origin?

(a) $-720\pi\hat{k}\frac{V}{m}$ [✓ (b) $-360\pi\hat{k}\frac{V}{m}$] (c) $-72\pi\hat{k}\frac{V}{m}$ (d) $-18\pi\hat{k}\frac{V}{m}$

132. The electrostatic potential in a given region of space is $\phi(\bar{r}) = \frac{1}{\epsilon_0}\left[x^2+y^2-2z^2\right]$. The charge density $\rho(\bar{r})$ in the region is

(a) $\frac{-x-y+4z}{r^3}$ (b) $\frac{x+y-2z}{r}$ [✓ (c) zero] (d) $\frac{4}{r^2}$

133. A square loop of a thin wire with side a carries a current I. Choosing the centre of the loop to be the coordinate origin and the z-axis to be normal to the loop, the magnetostatic potential $\phi(\bar{r})$ is given as

(a) $\frac{\mu_0 I}{r^3}$ (b) $\frac{\mu_0 I a^2}{r^2}$ (c) $\frac{\mu_0 I a^2}{4\pi}\frac{z}{r^3}$ (d) $\mu_0 I a^2\left(\frac{3xy-z^2}{r^3}\right)$

134. Which of the following groups of boundary conditions is satisfied at an interface between two nonmagnetic dielectric media ? [$\vec{D}_1, \vec{E}_1, \vec{B}_1$ and $\vec{D}_2, \vec{E}_2, \vec{B}_2$ represent the displacement vector, electric field and magnetic field in region 1 and 2 respectively and $\hat{n}$ defines the unit vector normal to the interface]

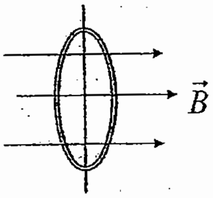
(a) $\vec{D}_1 \times \hat{n} = \vec{D}_2 \times \hat{n}$ & $\vec{B}_1 . \hat{n} = \vec{B}_2 . \hat{n}$ (b) $\vec{E}_1 . \hat{n} = \vec{E}_2 . \hat{n}$ & $\vec{B}_1 \times \hat{n} = \vec{B}_2 \times \hat{n}$

(c) $\vec{D}_1 \times \hat{n} = \vec{D}_2 \times \hat{n}$ & $\vec{B}_1 \times \hat{n} = \vec{B}_2 \times \hat{n}$ [✓ (d) $\vec{D}_1 . \hat{n} = \vec{D}_2 . \hat{n}$ & $\vec{B}_1 \times \hat{n} = \vec{B}_2 \times \hat{n}$]

135. There is a uniform magnetic field B in the z-direction in a certain region of space. The corresponding vector potential is given by.

[✓ (a) $\vec{A} = Bx\,\vec{j}$] (b) $\vec{A} = \frac{B\vec{r}}{r^2}$ (c) $\vec{A} = B\vec{r}$ (d) $\vec{A} = B\left(x\vec{i} + y\vec{k}\right)$

136. A loop of wire of radius a and carrying a current I is suspended vertically in a uniform horizontal magnetic field B as shown. Calculate the work done in rotating the loop around the vertical axis through an angle $\pi$



(a) $\frac{\pi^2 I^2}{2a}$ [✓ (b) $2\pi a^2 I$] (c) $\frac{\pi a^2 I}{2}$ (d) 0

137. The electric field of an electromagnetic wave is given by $\vec{E} = E_0\left(3\hat{i}+4\hat{j}\right)\cos k\left(4x-3y-ct\right)$. The directions of the magnetic field $\vec{B}$ and the poynting vector $\vec{p}$ are

(a) $-\hat{k}, \frac{4\hat{i}+3\hat{j}}{5}$ [✓ (b) $\hat{k}, \frac{4\hat{i}-3\hat{j}}{5}$] (c) $\frac{4\hat{i}-3\hat{j}}{5}, \hat{k}$ (d) $\frac{4\hat{i}-3\hat{j}}{5}, \frac{4\hat{i}+3\hat{j}}{5}$

138. A right circularly polarized beam is incident on a calcite half wave plate. The emergent wave will be

(a) linearly polarized [✓ (b) left circularly polarized]

(c) elliptically polarized (d) unpolarized

139. A circular loop of wire of radius a carries an oscillating current I of frequency $\omega$. The electric and magnetic fields at distance r much larger than $a\left(r >> a\right)$ very as

(a) $E \propto \frac{1}{r},\ B \propto \frac{1}{r^2}$ (b) $E \propto \frac{1}{r^2},\ B \propto \frac{1}{r^2}$ [✓ (c) $E \propto \frac{1}{r},\ B \propto \frac{1}{r}$] (d) $E \propto \frac{1}{r^2},\ B \propto \frac{1}{r}$

140. A microwave source of wave length $\lambda = 3\text{ cm}$ is used in a double slit interference experiment. The distance D from the slits to the detector is 1m. If the distance d between the slits increases from 1mm to 5 cm, then

(a) the fringes being to overlap

(b) the central maximum becomes more intense

(c) the secondary sources at the slits lose coherence

[✓ (d) the separation between the maxima decreases.]

141. The space-time dependence of the electric field of a linearly polarized light in free space is given by $\hat{x}E_0\cos\left(\omega t - kz\right)$ where $E_0$ $\omega$ and k are the amplitude, the angular frequency and the wavevector, respectively. The time averaged energy density associated with the electric field is

[✓ (a) $\frac{1}{4}\varepsilon_0 E_0^2$] (b) $\frac{1}{2}\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2$ (d) $2\varepsilon_0 E_0^2$

142. A plane electromagnetic wave traveling in free space is incident normally on a glass plate of refractive index 3/2. If there is no absorption by the glass, Its reflectivity is

[✓ (a) 4%] (b) 16 % (c) 20% (d) 50%

143. The electric and the magnetic fields $\vec{E}(z,t)$ and $\vec{B}(z,t)$, respectively corresponding to the scalar potential $\phi(z,t) = 0$ and vector potential $\vec{A}(z,t) = \hat{i}tz$ are

(a) $\vec{E} = \hat{i}z$ and $\vec{B} = -\hat{j}t$ (b) $\vec{E} = \hat{i}z$ and $\vec{B} = \hat{j}t$ (c) $\vec{E} = -\hat{i}z$ and $\vec{B} = -\hat{j}t$ [✓ (d) $\vec{E} = -\hat{i}z$ and $\vec{B} = \hat{j}t$]

144. A plane polarized electromagnetic wave in free space at time t = 0 is given by $\vec{E}(x,z) = 10\hat{j}\exp\left[i\left(6x+8z\right)\right]$. The magnetic field $\vec{B}(x,z,t)$ is given by

[✓ (a) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i\left(6x+8z-10ct\right)\right]$.]

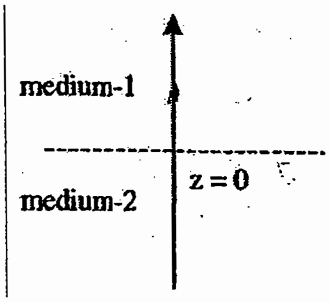
(b) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\exp\left[i\left(6x+8z-10ct\right)\right]$.

(c) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i\left(6x+8z-ct\right)\right]$.

(d) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\exp\left[i\left(6x+8z+ct\right)\right]$.

145. Two infinitely extended homogeneous isotropic dielectric media ( medium -1 and medium −2 with dielectric constants $\frac{\varepsilon_1}{\varepsilon_0}=2$ and $\frac{\varepsilon_2}{\varepsilon_0}=5$, respectively) meet at the $z=0$ plane as shown in the figure. A uniform electric field exists everywhere. For $z \geq 0$, the electric field is given by $\vec{E}_1 = 2\hat{i}-3\hat{j}+5\hat{k}$. The interface separating the two media is charge free.

The electric displacement vector in the medium-2 is given by



(a) $\overrightarrow{D_2}=\varepsilon_0\left[10\hat{i}+15\hat{j}+10\hat{k}\right]$ [✓ (b) $\overrightarrow{D_2}=\varepsilon_0\left[10\hat{i}-15\hat{j}+10\hat{k}\right]$]

(c) $\overrightarrow{D_2}=\varepsilon_0\left[4\hat{i}-6\hat{j}+10\hat{k}\right]$ (d) $\overrightarrow{D_2}=\varepsilon_0\left[4\hat{i}+6\hat{j}+10\hat{k}\right]$

**Statement for Linked Answer Q.146 and Q. 147**

In a hydrogen atom, consider that the electronic charge is uniformly distributed in a spherical volume of radius $a\left(=0.5\times10^{-10}\text{m}\right)$ around the proton. The atom is placed in a uniform electric field $E=30\times10^5$ V/m. Assume that the spherical distribution of the negative charge remains undistorted under the electric field.

146. In the equilibrium condition, the separation between the positive and the negative charge centers is

(a) $8.66\times10^{-16}$m (b) $2.60\times10^{-15}$m [✓ (c) $2.60\times10^{-16}$m] (d) $8.66\times10^{-15}$m

147. The polarizability of the hydrogen atom in unit of $\left(\text{C}^2\text{m}/\text{N}\right)$ is

(a) $2.0\times10^{-40}$ [✓ (b) $1.4\times10^{-41}$] (c) $1.4\times10^{-40}$ (d) $2.0\times10^{-39}$

148. An observer in an inertial frame that a point P the electric field vanishes but the magnetic field does not. This implies that in any other initial frame the electric field $\vec{E}$ and the magnetic field $\vec{B}$ satisfy

(a) $|\vec{E}|^2=|B|^2$ [✓ (b) $\vec{E}.\vec{B}=0$] (c) $\vec{E}\times\vec{B}=0$ (d) $\vec{E}=0$

149. A magnetic field $\vec{B}=B_0\left(\hat{i}+2\hat{j}-4\hat{j}\right)$ exists at a point. If a test charge moving with a velocity $\vec{\upsilon}=\upsilon_0\left(3\hat{i}-\hat{j}+2\hat{k}\right)$ experiences no force at a certain point. The electric field at that point in SI units is

(a) $\vec{E}=-\upsilon_0B_0\left(3\hat{i}-2\hat{j}-4\hat{k}\right)$ (b) $\vec{E}=-\upsilon_0B_0\left(\hat{i}+\hat{j}+7\hat{k}\right)$

(c) $\vec{E}=\upsilon_0B_0\left(14\hat{j}+7\hat{k}\right)$ [✓ (d) $\vec{E}=-\upsilon_0B_0\left(14\hat{j}+7\hat{k}\right)$]

150. A circular conducting ring of radius 'R' rotates with constant angular velocity $\omega$ about its diameter placed along the x-axis. A uniform magnetic field B is applied along the y-axis. If at time t =0 the ring is entirely in the xy- plane, the emf induced in the ring at time $t>0$ is

(a) $B\omega^2\pi R^2 t$ (b) $B\omega\pi R^2\tan(\omega t)$ (c) $B\omega\pi R^2\sin(\omega t)$ [✓ (d) $B\omega\pi R^2\cos(\omega t)$]

151. An electric field in a region is given by $\vec{E}(x,y,z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$. For which of a, b, c does this represent an electrostatic field ?

(a) 13, 1, 12 (b) 17, 6, 1 (c) 13, 1, 6 (d) 45, 6, 1

$\nabla \times E = 0$

152. A cube has a constant electric potential V on its surface. If there are no charges inside the cube. the potential at the center of the cube is

(a) V (b) V/8 (c) 0 (d) V/6

153. Consider a particle of electric charge e and mass moving under the influence of constant horizontal electric field E and constant vertical gravitational field described by acceleration due to gravity 'g'. If the particle starts from rest, what will be its trajectory ?

(a) Parabolic (b) elliptic (c) straight line (d) circular

154. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet

(a) Attains a constant velocity (b) moves with a constant acceleration
(c) moves with a constant deceleration (d) executes simple harmonic motion.

155. A point charge +q placed at ( 0, 0, d) above a grounded infinite conducting plane defined by z = 0. There are no charges present anywhere else. What is the magnitude of electric field at (0, 0, –d) ?

(a) $q/\left(8\pi\varepsilon_0 d^2\right)$ (b) $-\infty$ (c) 0 (d) $q/\left(16\pi\varepsilon_0 d^2\right)$

156. When unpolarised light is incident on a glass plate at a particular angle, it is observed that the reflected beam is linearly polarised. What is the angle of the refracted beam with respect to the surface normal ? Refractive index of glass is 1.52

(a) 56.7° (b) 33.4° (c) 23.3°
(d) The light is completely reflected and there is no refracted beam.

157. A time-dependent magnetic field $\vec{B}(t)$ is produced in a circular region of space, infinitely long and of radius R. The magnetic field is given as $\vec{B} = B_0 t\hat{z}$ for $0 \le r < R$ and is zero for $r > R$. Where $B_0$ is a positive constant. The electric field for $r > R$ is

(a) $\frac{B_0R^2}{r}\hat{r}$ (b) $\frac{B_0R^2}{2r}\hat{\theta}$ (c) $-\frac{B_0R^2}{r}\hat{r}$ (d) $-\frac{B_0R^2}{2r}\hat{\theta}$

158. The magnetic field corresponding to the vector potential, $\bar{A} = \frac{1}{2}\bar{F} \times \bar{r} + \frac{10}{r^3}\bar{r}$ where $\bar{F}$ is a constant vector, is

(a) $\bar{F}$ (b) $-\bar{F}$ (c) $\bar{F} + \frac{30}{r^4}\bar{r}$ (d) $\bar{F} - \frac{30}{r^4}\bar{r}$

159. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field, $E_\perp$, of the reflected wave into the water is found to remain the same for ail angles of incidence. The phase of the magnetic field H.

(a) does not change (b) changes by $3\pi/2$ (c) changes by $\pi/2$ (d) changes by $\pi$

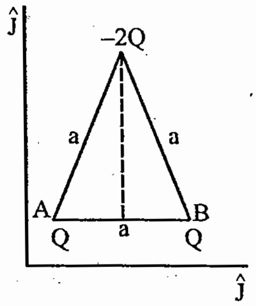
160. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to

(a) IR (b) $I/R^2$ (c) $I^2/R^2$ (d) I/R

161. Which of the following quantities is Lorentz invariant?

(a) $|E \times B|^2$ (b) $|E|^2 - |B|^2$ (c) $|E|^2 + |B|^2$ (d) $|E|^2 |B|^2$

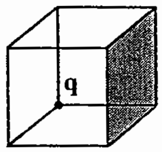
162. Charges Q, Q and -2Q are placed on the vertices of an equilateral triangle ABC of sides of length a, as shown in the figure



The dipole moment of this configuration of charges, irrespective of the choice of origin, is

(a) $+2aQ\hat{i}$ (b) $+\sqrt{3}aQ\hat{j}$ [✓ (c) $-\sqrt{3}aQ\hat{j}$] (d) 0

163. A point charge 'q' sits at a corner of a cube of side 'a', as shown in the figure on the right. The flux of the electric field vector through the shaded side is

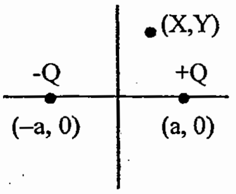


(a) $\frac{q}{8\varepsilon_0}$ (b) $\frac{q}{16\varepsilon_0}$ [✓ (c) $\frac{q}{24\varepsilon_0}$] (d) $\frac{q}{6\varepsilon_0}$

164. A parallel plate capacitor of circular cross section with radius r >>d, where 'd' is the spacing between the plates, is charged to a potential V and then disconnected from the charging circuit. If, now, the plates are slowly pulled apart (keeping then parallel) so that their separation is increased from d to d', the work done will be

(a) $\frac{\pi\varepsilon_0 r^2 V^2}{2d}\left(1-\frac{d}{d'}\right)$ (b) $\frac{\pi\varepsilon_0 r^2 V^2}{2d}\frac{d'}{d}$ [✓ (c) $\frac{\pi\varepsilon_0 r^2 V^2}{2d}\left(\frac{d'}{d}-1\right)$] (d) $\frac{\pi\varepsilon_0 r^2 V^2}{2d}\frac{d}{d'}$

165. Consider two charges +Q and –Q placed at the points (a, 0) and (–a, 0) in a plane, as shown in the figure on the right. If the origin is moved to the point (X, Y), the magnitude of the dipole moment of the given charge distribution with respect to this origin will be



(a) $Q\sqrt{(a-X)^2+y^2}-Q\sqrt{(a+X)^2+y^2}$ (b) $Q(a-X)-Q(-a+X)$

[✓ (c) 2Qa] (d) $2Q\sqrt{X^2+Y^2}$

166. A plane electromagnetic wave travelling in a vacuum is characterised by the electric and magnetic fields

$$\vec{E}=\hat{i}\left(30\pi\ \text{Vm}^{-1}\right)\exp i\left(\omega t+kz\right)$$

$$\vec{H}=\hat{j}\left(H_0\ \text{Am}^{-1}\right)\exp i\left(\omega t+kz\right)$$

If $\omega, k>0$, the value of $H_0$ must be

(a) $2\pi$ (b) 0.67 [✓ (c) 0.25] (d) 0.94

167. The magnetic vector potential $\vec{A}(\vec{r})$ corresponding to a uniform magnetic field $\vec{B}$ is taken in the form

$$\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$$

where '$\vec{r}$' is the position vector. If the electric field has the time dependent form $\vec{E} = \vec{E}_0(\vec{r})e^{i\omega t}$, where '$\omega$' is a constant, the gauge choice corresponding to this potential is a

[✓ (a) Lorenz gauge] [✓ (b) Coulomb gauge] (c) Non-linear gauge (d) Time-varying gauge.

168. Consider a hollow charged shell of inner radius 'a' and outer radius 'b'. The volume charge density is $\rho(r) = \frac{k}{r^2}$ (where k is a constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is:

(a) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > a$

(b) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\varepsilon_0 r^2}$ for $r > b$

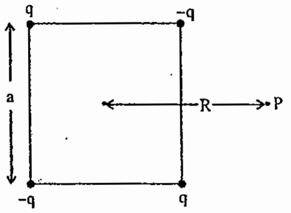
[✓ (c) $\frac{k(r-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$]

(d) $\frac{k(r-a)}{\varepsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 a^2}$ for $r > b$

169. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by $\vec{E}_1 = \hat{i}E_0\cos\omega t$ and $\vec{E}_2 = \hat{j}E_0\cos(\omega t + \varphi)$ where $\varphi$ is the phase difference. The intensity of the resulting wave is given by $\frac{\varepsilon_0}{2}\langle E^2\rangle$, where $\langle E^2\rangle$ is the time average of $E^2$. The total intensity is

(a) 0 [✓ (b) $\varepsilon_0 E_0^2$] (c) $\varepsilon_0 E_0^2\sin^2\varphi$ (d) $\varepsilon_0 E_0^2\cos^2\varphi$

170. Four charges (two +q and two –q) are kept fixed at the four vertices of a square of side 'a' as shown



At the point P which is at a distance R from the centre (R >> a), the potential is proportional to

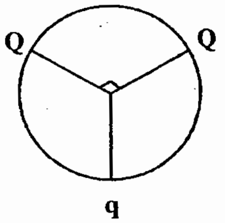
(a) $\frac{1}{R}$ (b) $\frac{1}{R^2}$ [✓ (c) $\frac{1}{R^3}$] (d) $\frac{1}{R^4}$

171. A point charge 'q' of mass 'm' is kept at a distance 'd' below a grounded infinite conducting sheet which lies in the xy-plane. What is the value of 'd' for which the charge remains stationary?

[✓ (a) $q/4\sqrt{mg\pi\varepsilon_0}$] (b) $q/\sqrt{mg\pi\varepsilon_0}$

(c) There is no finite value of 'd' (d) $\sqrt{mg\pi\varepsilon_0}/q$

172. Three charges are located on the circumference of a circle of radius 'R' as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge 'q' is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?



[✓ (a) $q/\sqrt{2}$]  (b) $\sqrt{2}q$  (c) 2q  (d) 4q

173. Consider an infinite conducting sheet in the xy-plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at (x, y, z) is given by

$$\vec{A} = \frac{\mu_0 K}{4c}(ct-z)^2\,\hat{i}$$

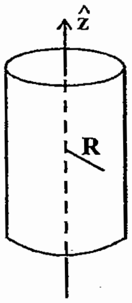
The magnetic field $\vec{B}$ is:

(a) $\frac{\mu_0 Kt}{2}\hat{j}$  (b) $-\frac{\mu_0 Kz}{2c}\hat{j}$  (c) $-\frac{\mu_0 K}{2c}(ct-z)\hat{i}$  [✓ (d) $-\frac{\mu_0 K}{2c}(ct-z)\hat{j}$]

174. When a charged particle emits electromagnetic radiation, the electric field $\vec{E}$ and the Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ at a large distance 'r' from the emitter vary as $\frac{1}{r^n}$ and $\frac{1}{r^m}$ respectively. Which of the following choices for 'n' and 'm' are correct?

(a) n = 1 and m = 1  (b) n = 2 and m = 2  [✓ (c) n = 1 and m = 2]  (d) n = 2 and m = 4

175. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I.



The vector potential $\vec{A}$ at a distance R from the axis.

(a) Is constant inside and varies as R outside the solenoid.
(b) Varies as R inside and is constant outside the solenoid.
(c) Varies as 1/R inside and as R outside the solenoid.
[✓ (d) Varies as R inside and as 1/R outside the solenoid.]

176. At a surface current, which one of the magnetostatic boundary condition is NOT CORRECT?

(a) Normal component of the magnetic field is continuous.
(b) Normal component of the magnetic vector potential is continuous.
(c) Tangential component of the magnetic vector potential is continuous.
[✓ (d) Tangential component of the magnetic vector potential is not continuous.]

177. Interference fringes are seen at an observation plane $z = 0$, by the superposition of two plane waves $A_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$ and $A_2 \exp[i(\vec{k}_2 \cdot \vec{r} - \omega t)]$, where $A_1$ and $A_2$ are real amplitudes. The condition for interference maximum is

(a) $(\vec{k}_1 - \vec{k}_2)\cdot\vec{r} = (2m+1)\pi$ (b) $(\vec{k}_1 - \vec{k}_2)\cdot\vec{r} = 2m\pi$ ~~(c)~~

(c) $(\vec{k}_1 + \vec{k}_2)\cdot\vec{r} = (2m+1)\pi$ (d) $(\vec{k}_1 + \vec{k}_2)\cdot\vec{r} = 2m\pi$

178. For a scalar function φ satisfying the Laplace equation, $\nabla\varphi$ has

(a) zero curl and non-zero divergence (b) non-zero curl and zero divergence

(c) zero curl and zero divergence (d) non-zero curl and non-zero divergence

179. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is CORRECT?

(a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.

(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.

(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.

(d) There will be no reflected light and the transmitted light is circularly polarized.

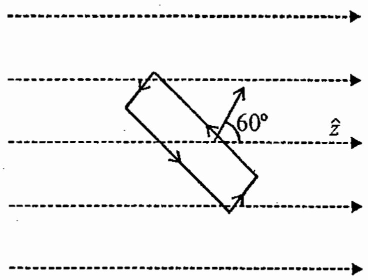
180. A charge distribution has the charge density given by $\rho = Q\{\delta(x - x_0) - \delta(x + x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

(a) $\dfrac{2Q\hat{x}}{9\pi \in_0 x_0^2}$ (b) $\dfrac{Q\hat{x}}{4\pi \in_0 x_0^3}$ (c) $\dfrac{Q\hat{x}}{4\pi \in_0 x_0^2}$ (d) $\dfrac{Q\hat{x}}{16\pi \in_0 x_0^2}$

181. A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If $\hat{k}_i$, $\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expression is correct?

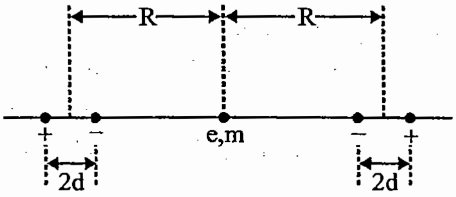
(a) $(\hat{k}_i - \hat{k}_r)\times\hat{n} \neq 0$ (b) $(\hat{k}_i - \hat{k}_r)\cdot\hat{n} = 0$ (c) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r = 0$ (d) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r \neq 0$

182. In a constant magnetic field of 0.6 Tesla along the $z$ direction, find the value of the path integral $\oint \vec{A}.\vec{dl}$ in the units of (Tesla m$^2$) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60° to the z-axis, as shown in he figure. The answer should be up to two decimal places. ____



0.654

183. A particle of charge e and mass m is located at the midpoint of the line joining two mixed collinear dipoles with unit charges as shown in the figure. (The particle is constrained to move only along the line joining the dipoles). Assuming that the length of the dipoles is much shorter than their separation, the natural frequency of oscillation of the particle is



(1) $\sqrt{\dfrac{6e^2R^2}{\pi\varepsilon_0 md^5}}$ (2) $\sqrt{\dfrac{6e^2R}{\pi\varepsilon_0 md^4}}$ [✓ (3)] $\sqrt{\dfrac{6e^2d^2}{\pi\varepsilon_0 mR^5}}$ (4) $\sqrt{\dfrac{6e^2d}{\pi\varepsilon_0 mR^4}}$

184. A current I is created by a narrow beam of protons moving in vacuum with constant velocity $\bar{u}$. The direction and magnitude, respectively, of the Poynting vector $\bar{S}$ outside the beam at a radial distance r (much larger than the width of the beam) from the axis, are

(1) $\bar{S}\perp\bar{u}$ and $|\bar{S}|=\dfrac{I^2}{4\pi^2\epsilon_0|\bar{u}|r^2}$ (2) $\bar{S}\parallel(-\bar{u})$ and $|\bar{S}|=\dfrac{I^2}{4\pi^2\epsilon_0|\bar{u}|r^4}$

[✓ (3)] $\bar{S}\parallel\bar{u}$ and $|\bar{S}|=\dfrac{I^2}{4\pi^2\epsilon_0|\bar{u}|r^2}$ (4) $\bar{S}\parallel\bar{u}$ and $|\bar{S}|=\dfrac{I^2}{4\pi^2\epsilon_0|\bar{u}|r^4}$

185. If the electric and magnetic fields are unchanged when the vector potential $\bar{A}$ changes (in suitable units) according to $\bar{A}\to\bar{A}+\hat{r}$, where $\bar{r}=r(t)\hat{r}$, then the scalar potential $\Phi$ must simultaneously change to

(1) $\Phi - r$ (2) $\Phi + r$ [✓ (3)] $\Phi - \partial r/\partial t$ (4) $\Phi + \partial r/\partial t$

186. Consider an axially symmetric static charge distribution of the form,

$\rho=\rho_0\left(\dfrac{r_0}{r}\right)^2 e^{-r/r_0}\cos^2\varphi$. The radial component of the dipole moment due to this charge distribution is

[✓ (1)] $2\pi\rho_0 r_0^4$ (2) $\pi\rho_0 r_0^4$ (3) $\rho_0 r_0^4$ (4) $\pi\rho_0 r_0^4/2$

187. A uniform cylinder of radius r and length $\ell$, and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is:

∝ (cm)

(1) $V\sqrt{\dfrac{14r\ell}{15R^2}}$ (2) $4V\sqrt{\dfrac{r}{15R}}$ (3) $\dfrac{4V}{\sqrt{15}}$ (4) $V\sqrt{\dfrac{14}{15}}$

188. The components of a vector potential $A_\mu \equiv (A_0, A_1, A_2, A_3)$ are given by $A_\mu = k(-xyz, yzt, zxt, xyt)$ where k is a constant. The three components of the electric field are

(1) k(yz, zx, xy) (2) k(x, y, z) [✓ (3)] (0, 0, 0) (4) k(xt, yt, zt)

189. An oscillating current $I(t) = I_0 \exp(-i\omega t)$ flows in the direction of the y-axis through a thin metal sheet of area 1.0 cm$^2$ kept in the xy-plane. The rate of total energy radiated per unit area from the surfaces of the metal sheet at a distance of 100m is

(1) $I_0\omega/(12\pi\varepsilon_0 c^3)$ [✓ (2) $I_0^2\omega^2/(12\pi\varepsilon_0 c^3)$]

(3) $I_0^2\omega/(12\pi\varepsilon_0 c^3)$ (4) $I_0\omega^2/(24\pi\varepsilon_0 c^3)$

190. The motion of an electron in an electromagnetic field is governed by
(a) Coulomb Force law (b) Gauss' Law (c) Lenz's law [✓ (d) Lorentz force law]

191. For the magnetic field measurement, Gauss-meters employ the principle of
(a) double exchange (b) super exchange (c) Jahn-Teller effect [✓ (d) Hall effect]

192. Which one of the following is not correct?

(a) $\nabla\cdot(\nabla\phi) = \nabla^2\phi$ (b) $\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$

(c) $\nabla\cdot(\nabla\times A) = 0$ [✓ (d) $\nabla\times(\nabla\phi) = \phi$]

193. For a transparent material the ratio of reflected intensity to the incident intensity depends on
(a) its absorption coefficient
[✓ (b) its refractive index only]
(c) its absorption coefficient and transmission coefficients
(d) its high transmission coefficient

194. The electric fields outside ($r > R$) and inside ($r < R$) a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$, $q$ being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is
(a) 1:3 (b) 9:16 (c) 3:8 [✓ (d) 5:6]

195. The vector field $xz\hat{i} + y\hat{j}$ in cylindrical polar coordinates is

[✓ (a) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1-z)\hat{e}_\phi$] (b) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1+z)\hat{e}_\phi$

(c) $\rho(z\sin^2\phi + \cos^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1+z)\hat{e}_\phi$ (d) $\rho(z\sin^2\phi + \cos^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1-z)\hat{e}_\phi$

196. At 'equilibrium' there can not be any free charge inside a metal. However, if you forcibly put charge in the interior then it takes some finite time to 'disappear', *i.e.* move to the surface. If the conductivity, $\sigma$, of a metal is $10^6(\Omega\, m)^{-1}$ and the dielectric constant $\epsilon_0 = 8.85\times10^{-12}$ Farad/m, this time will be approximately:
(a) $10^{-5}$ sec (b) $10^{-11}$ sec (c) $10^{-9}$ sec [✓ (d) $10^{-17}$ sec]

197. An electromagnetic wave of frequency $\omega$ travels in the $x$-direction through vacuum. It is polarized in the $y$-direction and the amplitude of the electric field is $E_0$. With $k = \omega/c$ where $c$ is the speed of light in vacuum, the electric and the magnetic fields are then conventionally given by

(a) $\vec{E} = E_0 \cos(ky - \omega t)\hat{x}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{z}$

[✓ (b) $\vec{E} = E_0 \cos(kx - \omega t)\hat{y}$ and $\vec{B} = \frac{E_0}{c}\cos(kx - \omega t)\hat{z}$]

(c) $\vec{E} = E_0 \cos(kx - \omega t)\hat{z}$ and $\vec{B} = \frac{E_0}{c}\cos(kx - \omega t)\hat{y}$

(d) $\vec{E} = E_0 \cos(kx - \omega t)\hat{x}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{y}$

198. If $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$ then

(a) Both are impossible electrostatic fields.
(b) Both are possible electrostatic fields.
(c) Only $\vec{E}_1$ is a possible electrostatic field.
[✓ (d) Only $\vec{E}_2$ is a possible electrostatic field.]

199. A charge $q$ is placed at the centre of an otherwise neutral dielectric sphere of radius $a$ and relative permittivity $\in_r$. We denote the expression $q/4\pi \in_0 r^2$ by $E(r)$. Which of the following statements is false?

(a) The electric field inside the sphere, $r < a$, is given by $E(r)/\in_r$.
(b) The field outside the sphere, $r > a$, is given by $E(r)$.
(c) The total charge inside a sphere of radius $r > a$ is given by $q$.
[✓ (d) The total charge inside a sphere of radius $r < a$ is given by $q$.]

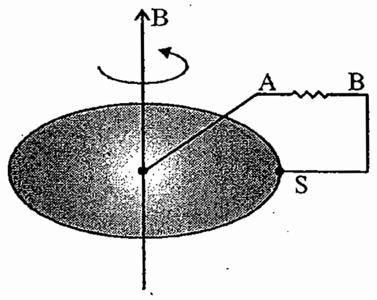
200. The electric and magnetic fields caused by an accelerated charged particle are found to scale as $E \propto r^{-n}$ and $B \propto r^{-m}$ at large distances. What are the values of $n$ and $m$?

(a) $n = 1, m = 2$
(b) $n = 2, m = 1$
[✓ (c) $n = 1, m = 1$]
(d) $n = 2, m = 2$

201. The conductors in a 0.75 km long two-wire transmission line are separated by a centre-to-centre distance of 0.2m. If each conductor has a diameter of 4cm, then the capacitance of the line is

(a) 8.85 μF
(b) 88.5 nF
(c) 8.85 pF
[✓ (d) 8.85 nF]

202. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field pointing up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contact S. The current that flows through the resistor is



(a) zero
(b) DC from A to B
[✓ (c) DC from B to A]
(d) AC

203. The force between two long and parallel wires carrying currents $I_1$ and $I_2$ and separated by a distance D is proportional to

[✓ (a) $I_1 I_2 / D$] (b) $(I_1 + I_2) / D$ (c) $(I_1 I_2 / D)^2$ (d) $I_1 I_2 / D^2$

204. Let $(V, A)$ and $(V', A')$ denote two sets of scalar and vector potentials, and $\psi$ a scalar function. Which of the following transformations leave the electric and magnetic fields (and hence Maxwell's equations) unchanged?

[✓ (a) $A' = A + \nabla\psi$ and $V = V - \frac{\partial\psi}{\partial t}$] (b) $A' = A - \nabla\psi$ and $V' = V + 2\frac{\partial\psi}{\partial t}$

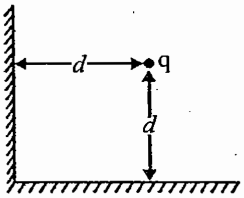
(c) $A' = A + \nabla\psi$ and $V' = V + \frac{\partial\psi}{\partial t}$ (d) $A' = A - 2\nabla\psi$ and $V' = V - \frac{\partial\psi}{\partial t}$

205. The electric field of an electromagnetic wave is given by $\vec{E} = E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$. The associated magnetic field $\vec{B}$ is

(a) $10^{-3} E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$ [✓ (b) $10^{-4} E_0 \cos[\pi(0.3x + 0.4y - 1000t)](4\hat{i} - 3\hat{j})$]

(c) $E_0 \cos[\pi(0.3x + 0.4y - 1000t)](0.3\hat{i} + 0.4\hat{j})$ (d) $10^2 E_0 \cos[\pi(0.3x + 0.4y - 1000t)](3\hat{i} + 4\hat{j})$

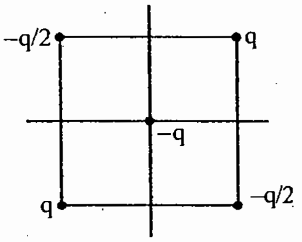
206. A point charge $q$ is placed symmetrically at a distance $d$ from two perpendicularly placed grounded conducting infinite plates as shown in the figure. The net force on the charge (in units of $1/4\pi\epsilon_0$) is



(a) $\frac{q^2}{8d^2}(2\sqrt{2} - 1)$ away from the corner [✓ (b) $\frac{q^2}{8d^2}(2\sqrt{2} - 1)$ towards the corner]

(c) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner (d) $\frac{3q^2}{8d^2}$ away from the corner

207. Let four point charges q, −q/2, q and −q/2 be placed at the vertices of a square of side a. Let another point charge −q be placed at the centre of the square (see the figure).



Let $V(r)$ be the electrostatic potential at a point P at a distance $r >> a$ from the centre of the square. Then $V(2r) / V(r)$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ [✓ (d) $\frac{1}{8}$]

208. If the electrostatic potential $V(r,\theta,\phi)$ in $a$ charge free region has the form $V(r,\theta,\phi)=f(r)\cos\theta$, then the functional form of $f(r)$ (in the following $a$ and $b$ are constants) is

(a) $ar^2+\frac{b}{r}$ [✓ (b)] $ar+\frac{b}{r^2}$ (c) $ar+\frac{b}{r}$ (d) $a\ln\left(\frac{r}{b}\right)$

209. A solid spherical conductor has a conical hole made at one end, ending in a point B, and a small conical projection of the same shape and size at the opposite side, ending in a point A. A cross-section through the centre of the conductor is shown in the figure on the right.

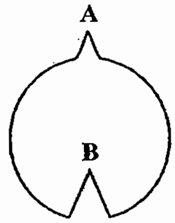
If now, a positive charge Q is transferred to the sphere, then

(a) The charge density at both A and B will be undefined.

(b) The charge density at A will be the same as the charge density at B.

[✓ (c)] The charge density at A will be more than the charge density at B.

(d) The charge density at B will be more than the charge density at A.



210. Solving Poisson's equation $\nabla^2\varphi=-\rho_0/\varepsilon_0$ for the electrostatic potential $\varphi(\vec{x})$ in a region with a constant charge density $\rho_0$, two students find different answers, viz,

$$\varphi_1(\vec{x})=-\frac{1}{2}\frac{\rho_0 x^2}{\varepsilon_0} \text{ and } \varphi_2(\vec{x})=-\frac{1}{2}\rho_0 y^2\varepsilon_0$$

The reason why these different solutions are both correct is because

(a) space is isotropic and hence $x$ and $y$ are physically equivalent.

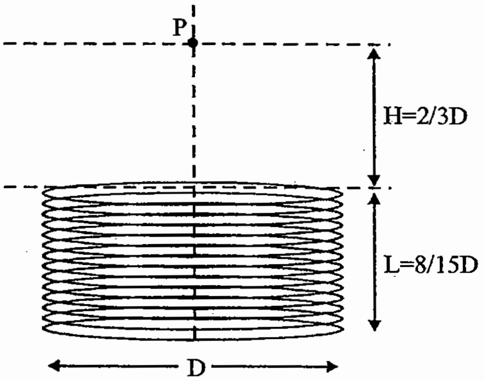
(b) We can add solutions of Laplace's equation to both $\varphi_1(\vec{x})$ and $\varphi_2(\vec{x})$

(c) The electrostatic energy is infinite for a constant charge density.

[✓ (d)] The boundary conditions are different in the two cases.

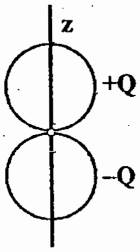
211. A short solenoid with n turns per unit length has diameter D and length L = 8D/15, as shown in the figure, and it carries a constant current I. The magnetic field B at a point P on the axis of the solenoid at a distance H = 2D/3 from its near end (see figure) is

[use $\int dx\left(1+x^2\right)^{-3/2}=x\left(1+x^2\right)^{-1/2}$]



[✓ (a)] $\frac{4}{65}\mu_0 nI$ (b) $\frac{4}{13}\mu_0 nI$ (c) $\frac{24}{15}\mu_0 nI$ (d) $\frac{112}{65}\mu_0 nI$

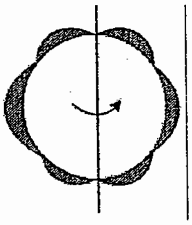
212. An electric dipole is constructed by fixing two circular charged rings, each of radius 'a', with an insulating contact (see figure). One of these rings has total charge +Q and the other has total charge −Q. If the charge is distributed uniformly along each ring, the dipole moment about the point of contact will be



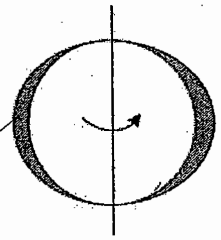
(a) $\frac{Qa}{\pi}\hat{z}$ (b) $4\pi Qa\hat{z}$ [✓ (c) $2Qa\,\hat{z}$] (d) Zero

213. A spherical conductor, carrying a total charge Q, spins uniformly and very rapidly about an axis coinciding with one of its diamters. In the diagrams given below, the equilibrium charge density on its surface is represented by the thickness of the shaded region. Which of these diagram is correct?

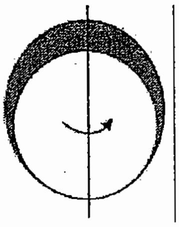
(a)



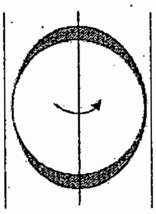
[✓ (b)]



(c)



(d)



214. The instantaneous electric and magnetic fields created at a distance 'r' by a point source at the origin are given by

$$\vec{E} = \frac{A\cos\omega t}{2\pi\varepsilon_0 r}\hat{\theta} \qquad \hat{H} = \frac{B\cos\omega t}{\mu_0 r}\hat{\varphi}$$

where $\omega, A, B$ are constants, and the unit vectors $(\hat{r}, \hat{\theta}, \hat{\varphi})$ form an orthonormal set. The time-averaged power radiated by the source is

(a) $\frac{\omega\varepsilon_0}{\mu_0}AB$ (b) $\frac{c^3}{2\pi}AB$ [✓ (c) $c^2AB$] (d) $\frac{2\pi\omega}{c}AB$

215. A thin uniform ring carrying charge $Q$ and mass $M$ rotates about its axis. What is the gyromagnetic ratio (defined as ratio of magnetic dipole moment to the angular momentum) of this ring?

(a) $Q/(2\pi M)$ (b) $Q/M$ [✓ (c) $Q/(2M)$] (d) $Q/(\pi M)$

216. A charged particle is at a distance $d$ from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance d/2 from the plane. At what distance from the plane will the particle reach the speed 2u?

(a) d/6 (b) d/3 (c) d/4 [✓ (d) d/5]

217. Consider an electromagnetic wave at the interface between two homogeneouos dielectric media of the dielectric constants $\varepsilon_1$ and $\varepsilon_2$. Assuming $\varepsilon_2 > \varepsilon_1$ and non charges on the surface, the electric field vector E and the displacement vector $\vec{D}$ in the two media satisfy the following inequalities

(a) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (b) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

[✓ (c)] $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (d) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

218. If the electrostatic potential in spherical polar coordinates is

$$\varphi(r) = \varphi_0 e^{-r/r_0}$$

where $\varphi_0$ and $r_0$ are constants, then the charge density at a distance $r = r_0$ will be

[✓ (a)] $\dfrac{\varepsilon_0\varphi_0}{er_0^2}$ (b) $\dfrac{\varepsilon\varepsilon_0\varphi_0}{2r_0^2}$ (c) $-\dfrac{\varepsilon_0\varphi_0}{er_0^2}$ (d) $-\dfrac{2\varepsilon\varepsilon_0\varphi_0}{r_0^2}$

219. A time-dependent current $\vec{I}(t) = Kt\hat{z}$ (where K is a constant) is switched on at t = 0 in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance 'a' from the wire is given (for time $t > a/c$) by

[✓ (a)] $\hat{z}\dfrac{\mu_0 K}{4\pi c}\displaystyle\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$ (b) $\hat{z}\dfrac{\mu_0 K}{4\pi}\displaystyle\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$

(c) $\hat{z}\dfrac{\mu_0 K}{4\pi c}\displaystyle\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$ (d) $\hat{z}\dfrac{\mu_0 K}{4\pi}\displaystyle\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{t}{\left(a^2+z^2\right)^{1/2}}$

optics

220. A doubel slit interference experiment uses a laser emitting light of two adjacent frequencies $\nu_1$ and $\nu_2$ $(\nu_1 < \nu_2)$. The minimum path difference between the interfering beams for which the interference pattern disappears is

(a) $\dfrac{c}{\nu_2+\nu_1}$ (b) $\dfrac{c}{\nu_2-\nu_1}$ (c) $\dfrac{c}{2(\nu_2-\nu_1)}$ (d) $\dfrac{c}{2(\nu_2+\nu_1)}$

221. A charge (–e) is placed in vacuum at the point (d, 0, 0), where d > 0. The region $x \le 0$ is filled uniformly with a metal. The electric field at the point $\left(\dfrac{d}{2}, 0, 0\right)$ is

(a) $-\dfrac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ [✓ (b)] $\dfrac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (c) $\dfrac{e}{\pi\varepsilon_0 d^2}(1,0,0)$ (d) $-\dfrac{e}{\pi\varepsilon_0 d^2}(1,0,0)$

222. A beam of light of frequency $\omega$ is reflected from a dielectric-metal interface at normal incience. the refractive index of the dielectric medium is n and that of the metal is $n_2 = n(1+i\rho)$. If the beam is polarised parallel to the interface, then the phase change experienced by the light upon reflection is

(a) $\tan\left(\dfrac{2}{\rho}\right)$ (b) $\tan^{-1}\left(\dfrac{1}{\rho}\right)$ (c) $\tan^{-1}\left(\dfrac{2}{\rho}\right)$ (d) $\tan^{-1}(2\rho)$

223. A thin infinitely long solenoid placed along the z-axis contains a magnetic flux $\phi$. Which of the following vector potentials corresponds to the magnetic field at an arbitrary point $(x,y,z)$?

(a) $(A_x, A_y, A_z)=\left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2}, \frac{\phi}{2\pi}, \frac{x}{x^2+y^2}, 0\right)$

(b) $(A_x, A_y, A_z)=\left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2+z^2}, \frac{\phi}{2\pi}, \frac{x}{x^2+y^2+z^2}, 0\right)$

(c) $(A_x, A_y, A_z)=\left(-\frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, \frac{\phi}{2\pi}, \frac{x}{x^2+y^2}, 0\right)$

(d) $(A_x, A_y, A_z)=\left(-\frac{\phi}{2\pi}\frac{x}{x^2+y^2}, \frac{\phi}{2\pi}, \frac{y}{x^2+y^2}, 0\right)$

224. An electromagnetically-shielded room is designed so that at a frequency $\omega = 10^7$ rad/s the intensity of the external radiation that penerates the room is 1% of the incident radiation. If $\sigma = \frac{1}{2\pi}\times 10^6 (\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that in 10 = 2.3)

(a) 4.60 mm (b) 2.30 mm (c) 0.23 mm (d) 0.46 mm

011 2615 2699

logic gate
Counters (FF)
ADC - DAC
Logic families

Transistor
BJT, Diode
Zener
Op-Amp

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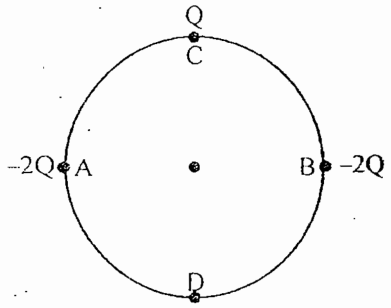
## PHYSICAL SCIENCES

## Assignment-01: Electromagnetic Theory

1. Three point charges are placed on the circumference of a circle as shown in the figure. What charge should be placed at D so that net force on C becomes zero?



2. Calculate total charge in the following cases.

(i) A infinitely long line charge lying along z axis has linear charge density $\lambda(z) = \frac{\lambda_o}{1+z^2}$

(ii) A circular disc of radius R has surface charge density $\sigma(r,\theta) = \sigma_o r^2 \sin^2\theta$

(iii) A solid sphere of radius R has volume charge density $\rho = \vec{a}\cdot\vec{r}$, where $\vec{a}$ is a constant vector and $\vec{r}$ is radial vector.

(iv) A spherical shell of radius R has surface charge density $\sigma(\theta) = \sigma_o \cos^2\theta$

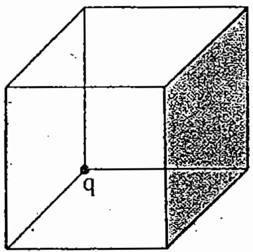
3. Find the electric field at (0, 3, 4)m due to a point charge $q = 1\mu C$ at (1, 1, 1).

4. Two point charges $q_1 = 10\mu C$ and $q_2 = 50\mu C$ are located at (–1, 1, –3)m and (3, 1, 0)m, respectively. Find the force on a point charge $q_3 = 1\mu C$ placed at the origin.

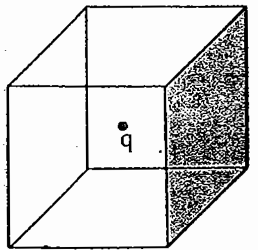
5. Find the force on a point charge of $50\mu C$ at (0, 0, 5)m due to the charge $500\pi\mu C$ uniformly distributed over a circular disk of radius $r = 5$m placed at $z = 0$.

6. Charge lies in the $z = -3m$ plane in the form of a square sheet defined by $-2 \le x \le 2m$, $-2 \le y \le 2m$ with surface charge density $\sigma = 2(x^2 + y^2 + 9)^{3/2} nC/m^2$. Find $\vec{E}$ field at the origin.

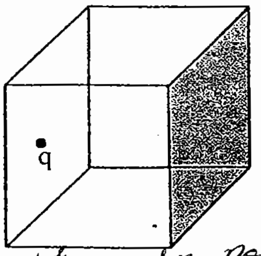
7. Calculate the flux of $\vec{E}$ through the shaded side?

(i)


corner position

(ii)


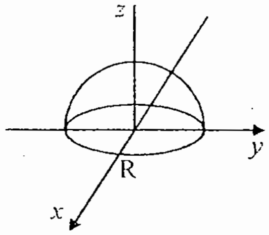
center position

(iii)


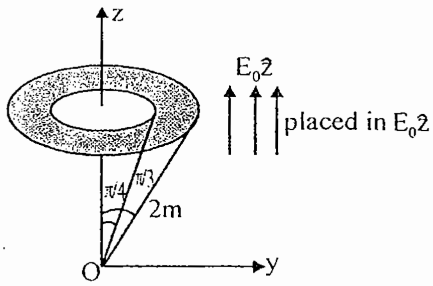
plane center position

12.

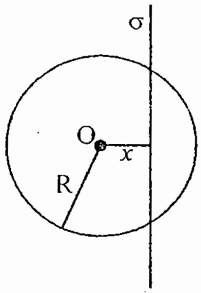
8. A closed hemispherical bowl as shown in the figure, is placed in an electric field given by $\vec{E} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$. Calculate the electric flux passing through the hemisphere.



9. (i) Calculate the electric flux through the following surface:

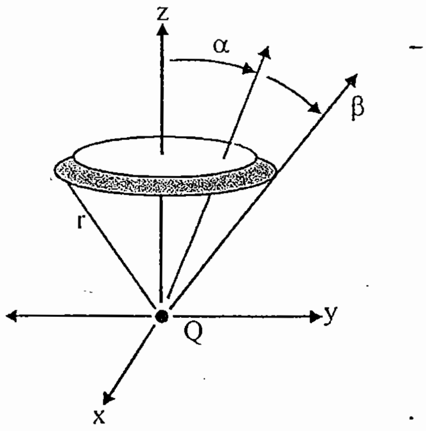


(ii) A plane sheet of surface charge density $\sigma$ is inserted into a sphere of radius R. The perpendicular distance of the plane from the centre is $x$. Calculate the flux passing through the sphere.



10. Electric field in space is given to be $\vec{E} = \alpha r\cos^2\theta\,\hat{r} + \alpha r\sin^2\theta\,\hat{\phi}$ where r, $\theta$ and $\phi$ are spherical polar coordinates. Calculate total charge enclosed by a sphere of radius $R$ centered at origin.

11. A point charge $Q$ is at the origin of a spherical coordinates system. Find the flux which crosses the portion of a spherical shell described by $\alpha \le \theta \le \beta$ (Fig.). What is the result if $\alpha = 0$ and $\beta = \pi/2$?



12. Given that $\vec{E} = 2r\cos\phi\hat{\phi} - \frac{\sin\phi}{3r}\hat{z}$ in cylindrical coordinates, find the flux crossing the portion of the $z = 0$ plane defined by $r \le a$, $0 \le \phi \le \pi/2$. Repeat for $3\pi/2 \le \phi \le 2\pi$. Assume flux positive in the $\hat{z}$ direction.

13. hw Suppose the electric field in some region is found to be $\vec{E} = kr^3\hat{r}$, in spherical coordinates ($k$ is some constant).

(i) Find the charge density $\rho$.

(ii) Find the total charge contained in a sphere of radius R, centered at the origin.

hw

14. Find the charge enclosed in the cylindrical volume $1 \le r \le 3m$, $0 \le \phi \le \pi/3$, $0 \le z \le 2m$ given the charge density $\rho = 2z\sin^2\phi (C/m^3)$.

$\int dr$

15. hw Electric field in a region is given to be $\vec{E} = \frac{1}{r}(\alpha\hat{r} + \beta\sin\theta\cos\phi\hat{\phi})$, calculate:

(i) The electric flux passing through a sphere of radius R centered at the origin.

(ii) Charge density $\rho$ corresponding to this electric field.

(iii) Total charge enclosed by the sphere.

16. hw Given that $\vec{E} = zs\cos^2\phi\hat{z}$ N/c. Calculate the charge density at $(1, \pi/4, 3)$ and the total charge enclosed by the cylinder of radius 1m with $-2 \le z \le 2$m.

17. hw If $\vec{E} = (2y^2 + z)\hat{x} + 4xy\hat{y} + x\hat{z}$ N/c, find

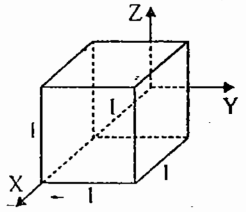
(a) The volume charge density at $(-1, 0, 3)$

(b) The flux through the cube defined by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$

(c) The total charge enclosed by the cube

18. If the electric flux density is $\vec{E} = \frac{10}{r}\hat{r}$ N/c, find the total charge within the sphere of radius 2m.

19. For the electric field $\vec{E} = (kx^{1/2})\hat{x}$ where k is a constant, compute the net electric flux through the cubic as shown in figure. What is the charge contained in the cube?



20. Given a charge density in spherical coordinates $\rho = \frac{\rho_0}{(r/r_0)^2}e^{-r/r_0}\cos^2\phi$ find the amounts of charge in the spherical volumes enclosed by $r = r_0$ and $r = \infty$.

21. Find the net charge enclosed in a cube of side 2m, parallel to the axes and centered at the origin, if the charge density is $\rho = 50x^2\cos\left(\frac{\pi}{2}y\right)(\mu C/m^3)$.

C

31.

22. A charge configuration in Spherical ~~cylindric~~al coordinates is given by $\rho = 5re^{-2r}\,(\text{C/m}^3)$. Find

(i) electric field $\left(\vec{E}\right)$ (ii) total charge in the whole space

23. The electric field in some region is given by $\vec{E}(r) = \frac{A}{r}\hat{r} + \frac{B\sin\theta\cos\phi}{r}\hat{\phi}$. Find

(i) charge density $\rho(r)$ in space

(ii) total charge in the sphere of radius R centered at the origin.

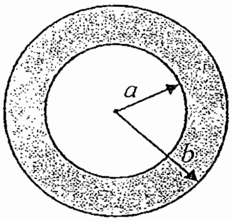
(iii) component of dipole moment in radial direction.

24. Find the electric field inside and outside a uniformly charged solid sphere of radius R and total charge $q$.

25. A long cylinder carries a charge density that is proportional to the distance from the axis: $\rho = ks$, for some constant $k$. Find the electric field inside this cylinder.

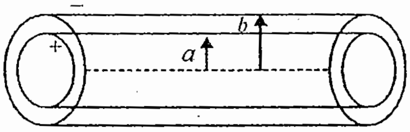
26. Find the electric field a distance $s$ from an infinity long straight wire, which carries a uniform line charge $\lambda$.

27. A hollow spherical shell carries charge density $\rho = \frac{k}{r^2}$ in the region $a \le r \le b$ (fig.). Find the electric field in the three regions (i) $r < a$ (ii) $a < r < b$, (iii) $r > b$. Also find total charge on the spherical shell.



28. An infinite plane carries a uniform surface charge $\sigma$. Find its electric field.

29. A long coaxial cable (Fig.) carries a uniform volume charge density $\rho$ on the inner cylinder (radius $a$), and a uniform surface charge density on the outer cylindrical shell (radius $b$). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically natural. Find the electric field in each of the three regions: (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$), (iii) outside the cable ($s>b$).



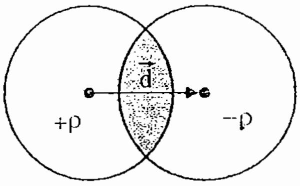
30. A spherical charge distribution is given by

$$\rho = \rho_0\left(1 - \frac{r^2}{a^2}\right) \qquad \text{for } r \le a,$$

$$\rho = 0 \qquad \text{for } r > a$$

where $a$ is the sphere. Find electric field intensity at a distance $r$ from the centre when $r > a$ and when $r < a$. Find the value of $r$ at which electric field is maximum.

31. Imagine overlapping of two solid spheres of same radii and carrying uniform volume charges of densities $\rho$ and $-\rho$. Calculate electric field in the overlapping region if distance between their centers is $d$.



32. A total charge Q is distributed in the volume of a solid sphere of radius $R$ such that density of the charge is $\rho = kd$ where $k$ is constant and $d$ is depth from surface. Find value of $k$ and also find value of electric field at depth $d$. At what value of $d$ field is maximum.

33. Charge density on the surface of a hemisphere is $\sigma = \sigma_0 \cos\theta$, if the radius of the hemisphere is R then find the electric field at this centre of the hemisphere.

34. A sphere of radius R contains space charge density $\rho(r) = A(R-r)$ for $0 < r < R$. The total electric charge inside the sphere is Q.
(a) Express A in terms of Q and R
(b) Find the electric field inside and outside the sphere

35. The electric field in a region is given to be $\vec{E} = ay\hat{i} + bx\hat{j} + cy\hat{k}$. Calculate the values of constants $a$, $b$ and $c$, so that $\vec{E}$ is electrostatic field.

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## PHYSICAL SCIENCES

## Assignment-02: Electromagnetic Theory

1. Find out which electric field is electrostatic?

   (a) $\vec{E} = k[xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}]$ (b) $\vec{E} = k[y^2\hat{x} + (2xy+z^2)\hat{y} + 2yz\hat{z}]$

2. Compute the rotor (curl) and divergence of the electric field given by $\vec{E}(x,y,z) = (x^2+z^2+5)\hat{x} + (y^2+x^2-93)\hat{y} + (z^2+y^2)\hat{z}$ . Is this field irrotational (conservative, electrostatic).

3. Determine which of the following are genuine electric fields (Electrostatic fields)

   (a) $\vec{E}_1 = 5x^3\,\hat{x} + 15x^2y\,\hat{y}$ (b) $\vec{E}_2 = 2s(z+1)\sin\phi\,\hat{s} + s(z+1)\cos\phi\,\hat{\phi} + \hat{s}\sin\phi\,\hat{z}$

   (c) $\vec{E}_3 = \dfrac{5}{r}\sin\theta\cos\phi\,\hat{r}$

4. Determine the electric field due to the following potentials:

   (a) $V = x^2 + 2y^2 + 4z^2$ (b) $V = \sin(x^2+y^2+z^2)^{1/2}$

   (c) $V = r^2(z+1)\sin\phi$ (d) $V = e^{-r}\sin\theta\cos 2\phi$

5. Find the potential inside and outside a spherical shell of radius R which carries a uniform surface charge $\sigma$.

6. Find the potential inside and outside a uniformly charged solid sphere whose radius is $R$ and whose total charge is $q$. Compute the gradient of $V$ in each region, and check that it yields the correct electric field.

7. Find the potential a distance $s$ from an infinitely long straight wire that carries a uniform line charge $\lambda$. Compute the gradient of your potential, and check that it yields the correct field.

8. For a spherical charge distribution

   $$\rho = \begin{cases} \rho_0(a^2 - r^2), & r < a \\ 0, & r > a \end{cases}.\ \text{Find}$$

   (a) E and V for $r \geq a$ (b) E and V for $r \leq a$

   (c) The total charge within the sphere (d) The value of r at which E is maximum.

9. The electric potential of some configuration is given by the expression $V(r) = A\dfrac{e^{-\lambda r}}{r}$, where $A$ and $\lambda$ are constants. Find the electric field $\vec{E}(r)$, the charge density $\rho(r)$, and the total charge $Q$.

10. Given the potential $V = \dfrac{10}{r^2}\sin\theta\cos\phi$ in spherical polar coordinate system.

    (a) Find the electric flux density $\vec{E}$ at $(2, \pi/2, 0)$.

    (b) Calculate the work done in moving a $10\,\mu C$ charge from point A(1, 30°, 120°) to B(4, 90°, 60°).

11. **The potential of the spherical charge distribution is given by**

$$V(r,\theta,\phi)=\left(\frac{V_0}{2}\right)\left(3-\frac{r^2}{R^2}\right) r<R$$

$$=\frac{V_0 R}{r} \qquad r>R$$

where $V_0$ and R are constants.
(a) Obtain the corresponding electric field distribution
(b) To what charge distribution would this correspond.
(c) Total charge on the sphere.

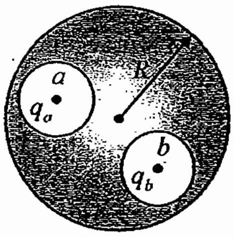
12. Determine the potential difference in volts between the points A(2, 0, –3) and B(1, $\frac{\pi}{2}$, 7) lying in a region having electric field intensity $\vec{E}=\frac{\cos\phi}{s^2}\hat{s}+\frac{\sin\phi}{s^2}\hat{\phi}$ V/m.

13. Given that $\vec{E} = (3x^2 + y)\hat{x} + x\hat{y}$ kV/m, find the work done in moving a –2μC charge from (0, 5, 0) to (2, –1, 0) by taking the straight line path.

14. Given that the electric field in a certain region is $\vec{E} = (z+1)\sin\phi\,\hat{s} + (z+1)s\cos\phi\,\hat{\phi} + s\sin\phi\,\hat{z}$ V/m determine the work done in moving a 4nC charge from
(a) A(1, 0, 0) to B(4, 0, 0) (b) B(4, 0, 0) to C(4, 30°, 0)

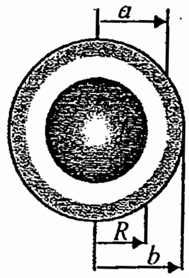
15. If $V = s^2 z\sin\phi$, calculate the energy stored within the region defined by $1<s<4, -2<z<2, 0<\phi<\pi/3$.

16. Two spherical cavities, of radii *a* and *b*, are hollowed out from the interior of a (neutral) conducting sphere of radius *R* shown in figure. At the centre of each cavity a point charge is placed - call these charges $q_a$ and $q_b$.



(a) Find the surface charges $\sigma_a, \sigma_b$ and $\sigma_R$
(b) What is the field outside the conductor?
(c) What is the field within each cavity?
(d) What is the force on $q_a$ and $q_b$?
(e) Which of these answers would change if a third charge, $q_c$, were brought near the conductor?

17. A metal sphere of radius *R*, carrying charge *q*, is surrounded by a thick concentric metal shell (inner radius *a*, outer radius *b*, as shown in figure). The shell carries no net charge.



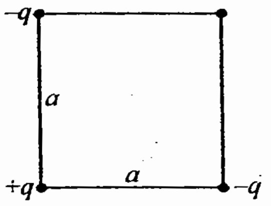
(a) Find the surface charge density σ at *R*, at *a*, and at *b*.
(b) Find the potential at the center, using infinity as the reference point.
(c) Now the outer surface is touched to a grounding wire,
which lowers its potential to zero (same as at infinity).
How do your answers to (a) and (b) change?

18. The force on a particle of charge $q = 2$ coulomb in an electrostatic field is given by $\vec{F} = A(yz\hat{i} + zx\hat{j} + xy\hat{k})$ where A = 10 Nm$^{-2}$. If the corresponding potential at the origin of coordinates is 10V, calculate the potential at the point $x=2, y=1, z=3$.

19. Two conducting spheres of charges $Q_1$ and $Q_2$ and radius $r_1$ and $r_2$ respectively, such that $(r_1 > r_2)$, now these two conductors are connected by a metallic wire. Calculate the potential and charge on two spheres after connection.

20. (a) Three charges are situated at the corners of a square (side $a$), as shown in figure. How much work does it take to bring in another charge, $+q$, from far away and place it in the fourth corner?

(b) How much work does it take to assemble the whole configuration of four charges?



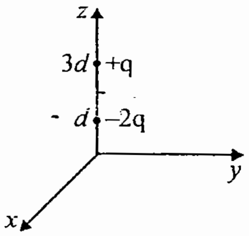
21. Calculate the workdone in assembling the charge configuration containing three points charges +q, –q and +q at the vertices of an equilateral triangle with side $a$.

22. Find the energy of a uniformly charged spherical shell of total charge $q$ and radius $R$.

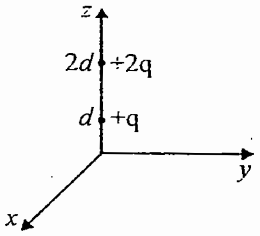
23. A sphere of radius R carries a charge density $\rho(r) = kr$ (where $k$ is a constant). Find the energy of this charge configuration.

24. Find the force on the charge $+q$ in figure. (The $xy$ plane is a grounded conductor).

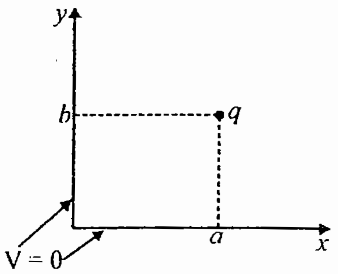
(a)



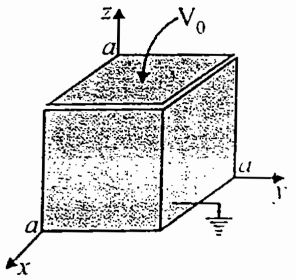
(b)



25. Two semi-infinite grounded conducting planes meet at right angles. In the region between them, there is a point charge $q$, situated as shown in figure. Set up the image configuration, and calculate the potential in this region. What is the force on $q$?



26. A cubical box (sides of length $a$) consists of five metal plates, which are welded together and grounded shown in figure. The top is made of a separate sheet of metal, insulated from the others, and held at a constant potential $V_0$. Find the potential inside the box.



## PHYSICAL SCIENCES

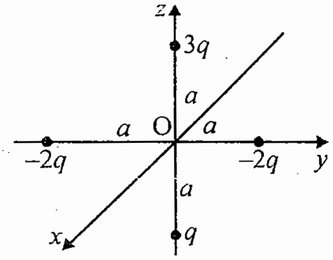
## Assignment-03: Electromagnetic Theory

1. Two dipoles with dipole moments $-5\,\hat{z}$ nCm and $9\,\hat{z}$ nCm are located at points (0, 0, –2) and (0, 0, 3), respectively. Find the potential at the origin.

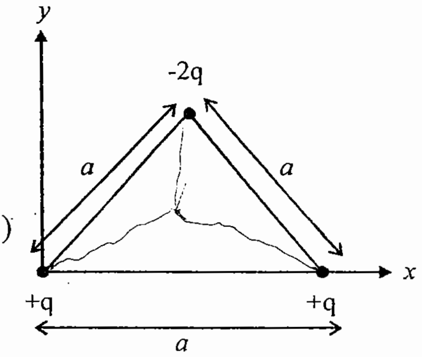
2. An electric dipole with $\vec{p} = p\hat{z}$ C.m is placed at (x, y, z) = (0, 0, 0). If the potential at (0, 0, 1m) is 9V, find the potential at (1m, 0, 1m).

3. A "pure" dipole $p$ is situated at the origin, pointing in the $z$-direction
(a) What is the force on a point charge $q$ at $(a, 0, 0)$ (Cartesian coordinates)?
(b) What is the force on $q$ at $(0, 0, a)$?
(c) How much work does it take to move $q$ from $(a, 0, 0)$ to $(0, 0, a)$?

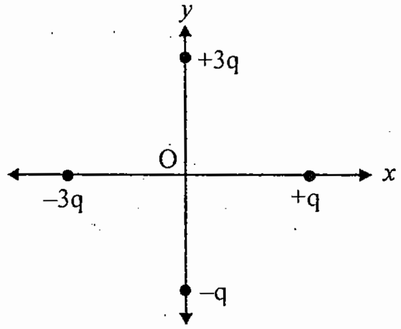
4. Four particles (one of charge $q$, one of charge $3q$, and two of charge $-2q$) are placed as shown in figure, each a distance $a$ from the origin. Find a simple approximate formula for the potential, valid at points far from the origin. (Express your answer in spherical coordinates).



5. (i)



(ii)



Calculate the dipole moment $(\vec{p})$ of the above charge configurations.

6. A static charge distribution has the form

$$\rho = \rho_0 \left(\frac{r_0}{r}\right)^2 e^{-r/r_0} \cos^2\phi$$

Calculate radial component of the dipole moment of the charge distribution.

7. A sphere of radius $R$, centered at the origin, carries charge density $\rho(r,\theta) = k\dfrac{R}{r^2}(R-2r)\sin\theta$, where $k$ is a constant, and $r$, $\theta$ are the usual spherical coordinates. Find the monopole moment and dipole moment for this charge distribution.

8. The electric potential of a grounded conducting sphere of radius R in a uniform electric field $E_0\hat{k}$ along z-direction is given by the following expression

$$\phi(r,\theta) = -E_0 r\cos\theta + \frac{E_0 R^3 \cos\theta}{r^2}$$

where 'r' is the distance from the centre of the sphere and $\theta$ is angle the radius vector makes with the z-axis.

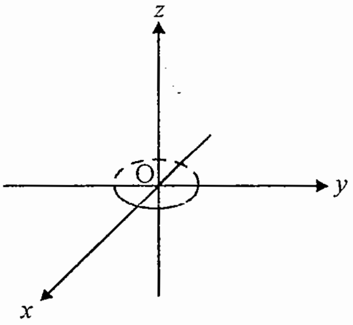
(a) What is dipole moment aquired by the sphere?
(b) Calculate the radial and tangential component of the electric field on the surface of the sphere
(c) What is the surface charge density on the sphere?

9. A spherical shell of radius R carries a surface charge density $\sigma = K\cos\theta$ where K is some constant. Calculate dipole moment of the spherical shell.

10. A circular disc of radius $a$ on the $xy$ plane has a surface charge density $\sigma = \frac{\sigma_0 r\cos\phi}{2}$, calculate the dipole moment of this charge distribution.

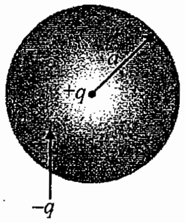
11. A thin insulating rod, running from $z = -a$ to $z = +a$, carries the indicated line charges. In each case, find the dipole moment and leading term in the multipole expansion of the potential:
(a) $\lambda = k\cos(\pi z/2a)$, (b) $\lambda = k\sin(\pi z/a)$, (c) $\lambda = k\cos(\pi z/a)$, where $k$ is a constant.

12. A thin circular ring of radius $R$ lies in the xy-plane with its centre at the origin of coordinates $O$ (see figure). the ring has a uniform linear change density $\lambda$.



Find the electric dipole moment $\vec{p}$, about the point O, of the charge distribution.

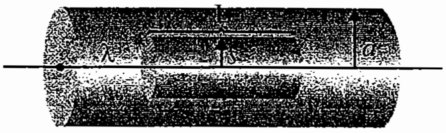
13. A primitive model for an atom consists of a point nucleus ($+q$) surrounded by a uniformly charged spherical could ($-q$) of radius $a$ shown in figure. Calculate the atomic polarizability of such an atom.



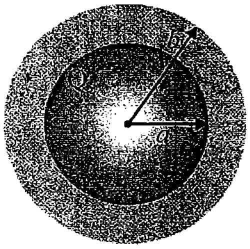
14. A sphere of radius $R$ carries a polarization $\vec{P}(r) = kr$, where $k$ is a constant and r is the vector from the centre.
(a) Calculate the bound charges $\sigma_b$ and $\rho_b$.(b) Find the field inside and outside the sphere.

15. A dielectric cube of side $a$, centered at the origin, carries a "frozen-in" polarization $\vec{P} = kr$, where k is a constant. Find all the bound charges, and check that they add up to zero.

16. A long straight wire, carrying uniform line charge $\lambda$, is surrounded by rubber insulation out to a radius $a$ shown in figure. Find the electric displacement.



17. A metal sphere of radius $a$ carries a charge Q shown in figure. It is surrounded, out to radius $b$, by linear dielectric material of permittivity $\in$. Find the potential at the center (relative to infinity).



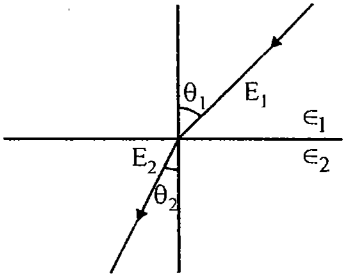
18. A dielectric sphere of radius R has polarization $\vec{P} = kr^2\vec{r}$. Calculate electric field inside and outside the sphere. Also calculate surface and volume bound charge densities and energy of the dielectric sphere.

19. A spherical conductor of radius a carries a charge Q. It is surrounded by a linear dielectric material of susceptibility $(\chi_e)$ upto radius b. Find the energy of this configuration.

20. A grounded spherical metal shell inner radius $R_1$ and outer radius $R_2$ is filled with uniform volume charge density ($\rho$) upto $R_1$. Find the electrostatic energy of the system.

21. A point charge $q$ is imbedded at the center of a sphere of linear dielectric material (with susceptibility $\chi_e$ and radius $R$). Find the electric field, the polarization, and the bound charge densities, $\rho_b$ and $\sigma_b$. What is the total bound charge on the surface? Where is the compensating negative bound charge located?

22. At the interface between one linear dielectric and another the electric field lines bend as shown in figure. Show that $\tan\theta_2 / \tan\theta_1 = \in_2 / \in_1$ assuming there is no free charge at the boundary.

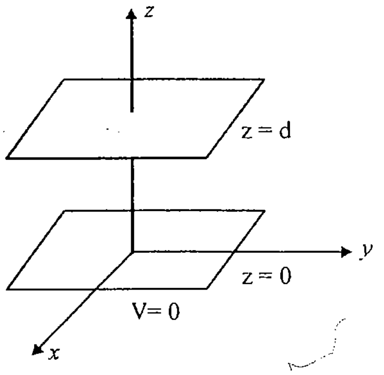


23. Two extensive homogeneous isotropic dielectric meet on plane $z = 0$, for $z > 0$, $\varepsilon_{r1} = 4$ and for $z < 0$, $\varepsilon_{r2} = 3$. A uniform electric field $\vec{E}_1 = 5\hat{i} - 2\hat{j} + 3\hat{k}$ KV/m exists for $z \geq 0$ find.

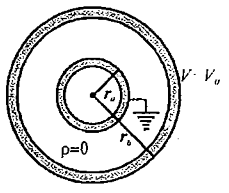
(a) $E_2$ for $z \leq 0$ (b) $\vec{D}_1$ and $\vec{D}_2$ (c) the angles $E_1$ and $E_2$ makes with the interface (d) the energy densities (in J/m$^3$) in both dielectrics.

24. The region $z > 0$ of a cartesian coordinate system contains a linear isotropic dielectric of dielectric constant 2.0. The region $z < 0$ is the free space. A free space charge density of $5nC/m^2$ is at the inteface $z = 0$. If the displacement vector in the dielectric is $\vec{D}_2 = 3\hat{x} + 4\hat{y} + 6\hat{z}\ nC/m^2$, find the corresponding displacement vector $\vec{D}_1$ is free space.

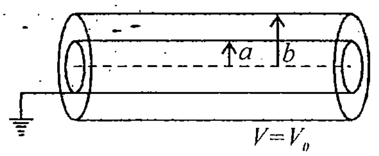
25. Consider the parallel conductors shown in the figure, where V = 0 at z = 0 and V = 100V at z = d. Assuming the region between the plates is charge free. Calculate the potential and electric field between the conductors.



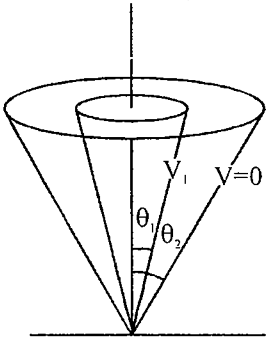
**26.** Consider two concentric spherical conducting shells at the origin. The outer radius of the inner shell is $r_a$ and inner radius of the outer shell is $r_b$. If charge density $\rho = 0$ in the region $r_a < r < r_b$. V = 0 at $r = r_a$ and $V = V_0$ at $r = r_b$. Find $V$ in the region $r_a < r < r_b$ by solving Laplace's equation.



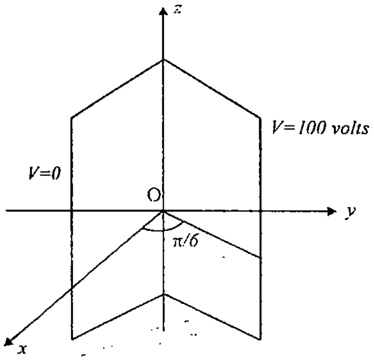
**27.** Two coaxial concentric cylinder is of radius $a$ and $b$, if the inner one grounded and outer is at potential $V_0$. Find potential and electric field between the cylinders by using the laplace equation. Also find capacitance per unit length of the cylinders.



**28.** Solve Laplace's equation for the region between coaxial cones as shown in figure. A potential $V_1$ is is assumed at $\theta_1$ and V = 0 at $\theta_2$. The cone verties are insulated at r = 0.

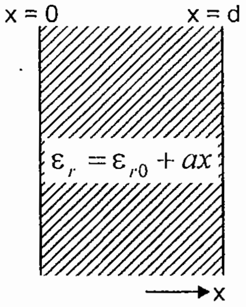


**29.** Two Semi infinite conducting planes $\phi = 0$ and $\phi = \frac{\pi}{6}$ are separated by an infintesimal insulated gap as shown in the figure. If $V(\phi = 0) = 0$ and $V\left(\phi = \frac{\pi}{6}\right) = 100\,volt$. Calculate $V$ and $\vec{E}$ in the region between the planes.

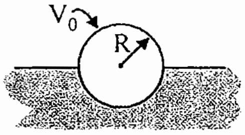


**30.** Consider a parallel plate capacitor filled with non-uniform dielectric of dielectric constant $\varepsilon_r = \varepsilon_{r0} + ax$. The distance between the plates is $d$. The free charge on the capacitor plates is Q and plate area is A.

$\frac{dt}{(t)^{3/2}}$



**31.** A conducting sphere at potential $V_0$ is half embedded in linear dielectric material of susceptibility $\chi_e$, which occupies the region $z < 0$ as shown in figure.

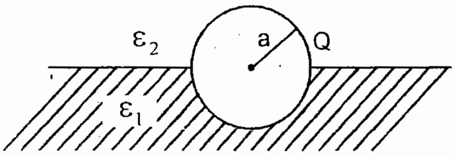


Pot constant

(a) Determine the potential, the electric field and polarization everywhere.
(b) The bound charge, and the free charge distribution on the sphere.

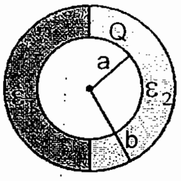
**32.** A perfectly conducting sphere of radius a and placed at the origin is half embedded in liquid dielectric medium of permittivity $\varepsilon_1$ as shown in the figure if the total free charge on the sphere is Q determine.

(i) Electric field intensity every where
(ii) Surface bound charge density $(\sigma_b)$.
(iii) Volume bound charge density $(\rho_b)$
(iv) Free surface charge density $(\sigma_f)$
(v) Potential on the surface of the sphere.



Free charge constant

**33.** One half of the region between the plates of a spherical capacitor of inner and outer radii a and b is filled with a linear isotropic dielectric of permittivity $\varepsilon_1$ and other half has permittivity $\varepsilon_2$ as shown in the figure if inner plate has total charge Q and outer plate has total charge –Q, find.



(a) The electric displacements $\vec{D}_1$ and $\vec{D}_2$ in the region of $\varepsilon_1$ and $\varepsilon_2$.
(b) The electric field in the region of $\varepsilon_1$ and $\varepsilon_2$.
(c) The total capacitance of the system.

**34.** The volume between two concentric conducting spherical surfaces of radii a and b (a < b) is filled with an inhomogeneous a permittivity $\varepsilon = \dfrac{\varepsilon_0 r}{R}$. Where $\varepsilon_0$ and R are constants. A charge Q is placed on the inner surface and outer one is grounded. Find

(a) $\vec{D}$ for a < r < b, r < a, r > b.
(b) $\vec{E}$ for a < r < b, r < a, r > b
(c) $\vec{P}$ for a < r < b, r < a, r > b.
(d) $\sigma_b|_{r=a}$ and $\sigma_b|_{r=b}$
(e) $\rho_b$ for a < r < b
(f) capacitance (C) of the capacitor.

**35.** Repeat the above problem for the relative permittivity $\varepsilon_r = \dfrac{\varepsilon_0}{(1+kr)}$. Where k is some constant.

### CSIR-UGC-NET/JRF | GATE - PHYSICS
### Unit Test : ELECTROMAGNETIC THEORY

**Time : 01:00 Hour** **Date : 04-10-2014**
**M.M. : 60**

**Instructions:**

1. **There are Two Parts. Part-A contains 10 objective type questions, each question carry 3.5 marks and Part-B contains 5 objective type questions, each question carry 5 marks.**
2. **There is negative marking, @ 25% will be deducted for each wrong answer.**
3. **Attempt all the questions, use of calculator is not allowed.**

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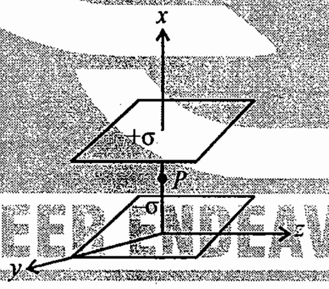
## PART-A

1. The magnetic field (in A/m) inside a long solid cylinderical conductor of radius a = 0.1m is: $\hat{H}=\frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r)-\frac{r}{\alpha}\cos(\alpha r)\right]\hat{\phi}$, where $\alpha=\frac{\pi}{2a}$. What is the total current (in A) in the conductor?

   (a) $5\pi$ (b) $\frac{800}{\pi}$ (c) $\frac{400}{\pi}$ (d) $\frac{300}{\pi}$

2. A large parallel plate capacitor with uniform surface charge '$\sigma$' on upper plate and $-\sigma$ on lower plate is moving with constant speed 'v' along z-direction as shown in the figure. The magnetic field at point 'P' between the plates is:

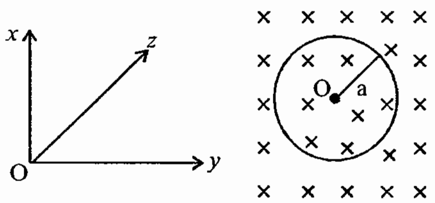


   (a) $\mu_0\sigma v\,\hat{z}$ [✓ (b)] $\mu_0\sigma v\,\hat{y}$ (c) $-\mu_0\sigma v\,\hat{y}$ (d) 0

3. A current distribution gives rise to the magnetic vector potential $\vec{A}=x^2y\hat{x}+y^2x\,\hat{y}-4x\,yz\,\hat{z}$ Wb/m. The magnetic flux passing through the surface defined by $z=1, 0\le x\le 1\ \&\ -1\le y\le 4$ is?

   [✓ (a)] 4 Wb (b) 10 Wb (c) 20 Wb (d) 40 Wb.

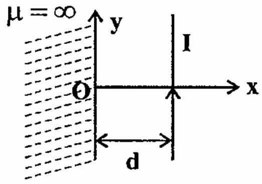
4. A circular conducting loop of radius 'a' placed in a time-dependent magnetic field $\boxed{B=B_0\cos\omega t\hat{z}}$ as shown in the figure



If resistance of the loop is R then average power drop in the loop is:

[✓ (a) $\frac{B_0^2\pi^2a^4\omega^2}{R}$] (b) $\frac{B_0^2\pi^2a^4\omega^4}{2R}$ (c) $\frac{B_0^2\pi^2a^4\omega^2}{2R}$ (d) 0

5. A long thin wire carrying a current 'I' lies parallel to and at a distance 'd' from a semi-infinite slab of iron of infinite permeability $\mu = \infty$ as shown in the figure below. The force per unit length on the wire is:



(a) $\frac{\mu_0 I^2}{2\pi d}\hat{y}$ (b) $\frac{\mu_0 I^2}{2\pi d}\hat{x}$ [✓ (c) $\frac{\mu_0 I^2}{4\pi d}\hat{z}$] (d) $\frac{\mu_0 I^2}{4\pi d}\hat{x}$

6. A small loop of wire of area $A = 0.01\,\text{m}^2$, N = 400 turns and resistance $R = 2\Omega$ is initially kept in a uniform magnetic field B in such way that the field is normal to the plane of the loop. When it is pulled out of the magnetic field, a total charge of $Q = 2\times10^{-5}$C flows through the coil. The magnitude of the field B is:

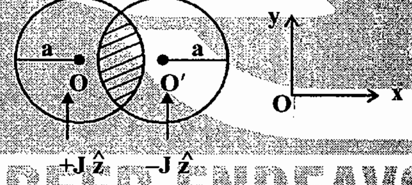
(a) $1\times10^{-3}\,T$ (b) $4\times10^{-3}\,T$

(c) Zero (d) $1\times10^{-5}\,T$

7. An electron with kinetic energy of 10 keV is moving in a circular orbit in a plane perpendicular to a uniform magnetic field (B) of strength 2 tesla. The period of rotation is:

[✓ (a) $2.8\times10^{-12}$ sec] (b) $5.6\times10^{-12}$ sec (c) $1.8\times10^{-11}$ sec (d) $1.1\times10^{-10}$ sec

8. Two thick wire each of radius 'a' in which one has uniform current density $\vec{J}$ in +z-direction and other has in −z -direction as shown in the figure below. The centres of two wires are separated by a distance 2b.



The magnetic field $(\vec{B})$ in the overlapping region is given by

[✓ (a) $\mu_0 Jb\hat{y}$] (b) $\mu_0 Jb\hat{x}$ (c) $-\mu_0 Jb\hat{y}$ (d) $-\mu_0 Jb\hat{x}$

**Linked Answer Q.9 and Q.10**

A metallic square loop of side 'a' meter and resistance $10\Omega$ is placed in 1st quadrant of y-z plane with one corner at origin. There is a time varying non-uniform magnetic field $(\vec{B})$ exists in the space.

9. Which one of the following could be a valid magnetic field $(\vec{B})$:

(a) $B_0e^{-(y+t)}\hat{x}$ (b) $B_0e^{-(x+t)}\hat{x}$ [✓ (c) $B_0e^{-t}\sin(kx)\hat{x}$] (d) $B_0e^{-(x+y+z+t)}\hat{x}$

10. The amount of net charge (Q), in coulombs, that flows past any point in the loop from $t = 0$ to $t = \infty$ is:

(a) $0.1B_0a(1-e^{-a})$ (b) $B_0a(1-e^{-a})^2$ (c) $0.1B_0a(e^{-a}-1)$ (d) $B_0a(e^{-a}-1)$

## PART-B

**Linked Answer for Q.11 and Q.12.**

A long straight wire of radius R has a current distribution

$$\boxed{\vec{J}(r)=\frac{J_0}{r}\cos\left(\frac{\pi r}{2R}\right)\hat{z}}$$ where 'r' is the distance from the axis.

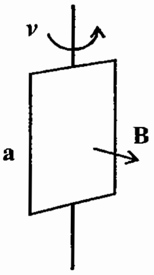
11. Total current (I) in the wire is:

(a) $2J_0R$ (b) $4J_0R$ [✓ (c) $J_0R$] (d) 0

12. Magnetic field inside the wire at a distance r < R from the axis is:

[✓ (a) $\frac{2\mu_0 J_0 R}{\pi r}\sin\left(\frac{\pi r}{2R}\right)\hat{\phi}$] (b) $\frac{2\mu_0 J_0 R}{\pi r}\sin\left(\frac{\pi r}{2R}\right)\hat{r}$ (c) $\frac{\mu_0 J_0 R}{r^2}\cos\left(\frac{\pi r}{2R}\right)\hat{\phi}$ (d) $2\mu_0 J_0 R\cos\left(\frac{\pi r}{2R}\right)\hat{r}$

13. A square loop (of conducting wire) of side 'a' and resitance 'R' is being rotated about a vertical axis as shown in the figure. A uniform horizontal magnetic field of strength B passes thorugh the loop. If the frequency of rotation is $v$. The average power dissipated in the loop is:

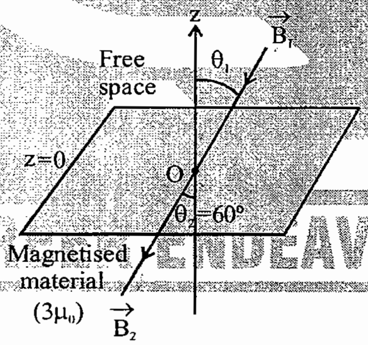


(a) $\frac{B^2a^2v^2}{4R}$ (b) $\frac{B^2a^4v^2}{2R}$ (c) 0 [✓ (d) $\frac{B^2a^2v^2}{R}$]

**Linked Answer Type Q.14 & Q.15**

z = 0 plane separates free space and magnetised material of permeability $3\mu_0$ and no free current at the plane.

For $z<0$ (magnetised material) the magnetic field is $\vec{B}_2=\mu_0\left[3\sqrt{2}\hat{x}-6\hat{y}+3\hat{z}\right]$



14. The magnetic field $(\vec{B}_1)$ for free space $(z>0)$ is:

(a) $\vec{B}_1=\mu_0\left[\sqrt{2}\hat{x}-2\hat{y}+3\hat{z}\right]$ (b) $\vec{B}_1=\mu_0\left[\hat{y}+3\hat{z}\right]$

(c) $\vec{B}_1=\mu_0\left[3\sqrt{2}\hat{x}-6\hat{y}+\hat{z}\right]$ (d) $\vec{B}_1=\mu_0\left[-\sqrt{2}\hat{x}+2\hat{y}+3\hat{z}\right]$

15. The angle $(\theta_1)$ magnetic field $(\vec{B}_1)$ making with z-axis is:

[✓ (a) 60°] (b) 75° (c) 45° (d) 30°

# CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
Best Institute for NET-JRF, GATE & IIT-JAM Exams

CSIR-UGC-NET/JRF | GATE - PHYSICS
Unit Test : ELECTROSTATICS

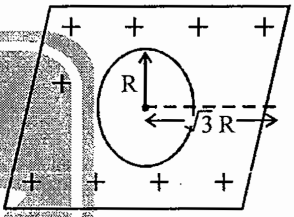
**Time : 01:00 Hour** **Date : 02-09-2014**
**M.M. : 70**

**Instructions:**

1. **Question Paper contains 20 objective type questions, each question carry 3.5 marks.**
2. **There is negative marking @ 25% mark will be deducted for each wrong answer.**
3. **Attempt all the questions, use of calculator is not allowed.**

1. An infinite dielectric sheet having charge density $\sigma$ has a hole of radius R in it. An electron is released on axis of the hole at a distance $\sqrt{3}$ R from the centre. The speed with which it crosses the centre of the hole is

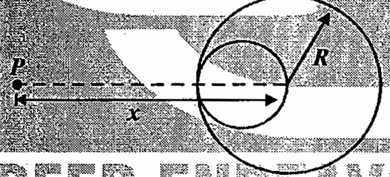
$E = -\frac{\sigma}{2\varepsilon_0}$

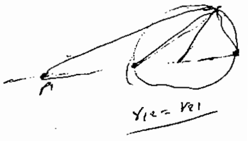


(a) $\sqrt{\frac{\sigma e R}{m\varepsilon_0}}$  (b) $-\sqrt{\frac{\sigma e R}{m\varepsilon_0}}$

(c) $-\sqrt{\frac{2\sigma e R}{m\varepsilon_0}}$  (d) $\sqrt{\frac{2\sigma e R}{m\varepsilon_0}}$

2. A solid spherical region having a spherical cavity whose diameter 'R' is equal to the radius of the spherical region, has a total charge 'Q'. The electric field at a point P is

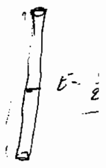




(a) $\frac{\sigma R^3}{3\varepsilon_0}\left[\frac{1}{x^2}+\frac{1}{2(2x-R)^2}\right]$  (b) $\frac{\sigma R^3}{3\varepsilon_0}\left[\frac{1}{x^2}-\frac{1}{2(2x-R)^2}\right]$

[✓ (c)] $\frac{\sigma R^3}{3\varepsilon_0}\left[\frac{1}{x^2}+\frac{1}{(2x-R)^2}\right]$  (d) $\frac{\sigma R^3}{3\varepsilon_0}\left[\frac{1}{x^2}-\frac{1}{(2x-R)^2}\right]$

3. A charge –Q is uniformly distributed over a non-conducting semi-circular rod of radius R. The potential at the centre is



(a) 0  (b) $\frac{-Q}{4\pi\varepsilon_0 R}$  (c) $\frac{Q}{4\pi\varepsilon_0 (2R)}$  (d) $\frac{2Q}{4\pi\varepsilon_0 R}$

4. An assembly of charge +q, –q, +q, –q, ........ are placed at distance $x = 1$m, $x = 2$m, $x = 4$m, $x = 8$m, ........, from the origin, in a plane. The potential at $x = 0$ due to the charges would be

$\frac{q}{4\pi\varepsilon_0 1} + \frac{q}{4\pi\varepsilon_0 4} - \frac{q}{4\pi\varepsilon_0 16}$

(a) $-\frac{q}{4\pi\varepsilon_0}$  (b) $-\frac{q}{6\pi\varepsilon_0}$  (c) $\frac{q}{6\pi\varepsilon_0}$  [✓ (d)] $\frac{q}{4\pi\varepsilon_0}$

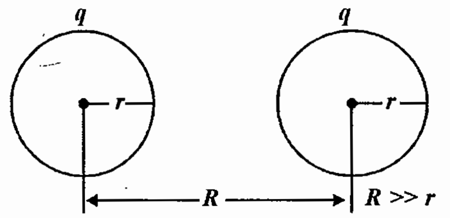
$1 + \frac{1}{4} - -$

$1 - \frac{1}{16}$

$\frac{16-1}{}$

$4 \frac{64}{16}$

5. The potential energy of the system of two identically charged spheres as shown in the figure is equal to (Assume the energy distribution to be uniform)



$E = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{r}$

$V = \frac{q}{4\pi\epsilon_0 r}$

(a) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{R}+\frac{1}{r}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$ [✓ (c) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r(R+r)}$] (d) none of these

$\frac{\epsilon_0 E^2}{2}$

6. A charge Q has to be divided between two solid spheres of radius 'R' which are distance 'd' from each other (d >> R). What should be the value of charge, which we should placed on spheres, so that the force of attraction between them is maximum?

$R \ll d$

(a) Q/4, 3Q/4 [✓ (b) Q/3, 2Q/3] (c) Q/2, Q/2 (d) Q/5, 4Q/5

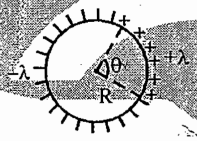
7. Two identical sheets of a metallic foil are separated by d and capacitance of the system is C and charged to a potential difference E. Keeping the charge constant, the separation is increased by '$l$'. Then, the new capacitance and potential difference will be

$C = \frac{Q}{V}$

$V_0 = E$

(a) $\frac{\varepsilon_0 A}{d}, E$ (b) $\frac{\varepsilon_0 A}{d+\ell}, E$ (c) $\frac{\varepsilon_0 A}{d+\ell}, \left(1+\frac{\ell}{d}\right)E$ [✓ (d) $\frac{\varepsilon_0 A}{d}, \left(1+\frac{\ell}{d}\right)E$]

8. In the figure shown some portion of the ring has $+\lambda$ charge density while remaining portion has $-\lambda$ charge density radius of the ring is R. Electric field at the centre of the ring is



$\frac{3}{8}$ $\frac{q}{4\pi\epsilon_0 R^2}$

$0 < r < a$

$a < r < c$

$a < r < b$

$b < r < c$

(a) $\frac{\lambda\sin\frac{\theta}{2}}{\pi\in_0 R}$ (b) $\frac{2\lambda\sin\frac{\theta}{2}}{\pi\in_0 R}$ [✓ (c) $\frac{\lambda\sin\frac{\theta}{2}}{2\pi\in_0 R}$] (d) $\frac{\lambda\sin\frac{\theta}{2}}{4\pi\in_0 R}$

9. A conducting sphere of radius 'a' has charge Q. It is surrounded by a conducting shell of inner and outer radii 'b' and 'c' respectively. The shell also has a total charge. If the sphere and the shell have equal surface charge density on outer surfaces, then which of the following is true

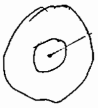
[✓ (a) $a = \frac{c}{\sqrt{2}}$] (b) $a = \frac{b}{\sqrt{2}}$ (c) $b = \frac{c}{\sqrt{2}}$ (d) $a = \frac{c}{2}$

10. A dielectric shell of inner and outer radii $R_1$ and $R_2$ hsa polarization $\vec{P} = \frac{P_0\hat{r}}{r^2}$. Electric potential at the centre of the shell is

(a) $\frac{P_0}{\varepsilon_0}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ [✓ (b) $\frac{P_0}{\varepsilon_0}(R_1+R_2)$] (c) $\frac{P_0}{\varepsilon_0}\left(\frac{1}{R_2}-\frac{1}{R_1}\right)$ (d) $\frac{P_0}{\varepsilon_0}(R_2-R_1)$

$\nabla\cdot D = \rho_f$

$E$ $D = \epsilon_0 E + P$ $P = \epsilon_0 E - D$

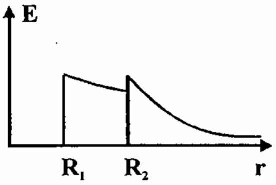


$P = \epsilon_0(1+\chi_e)E$

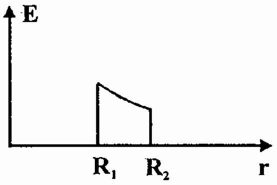
$(r_1 + r_2)$

11. In previous question, radial variation of electric field is given

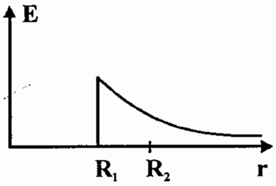
(a)



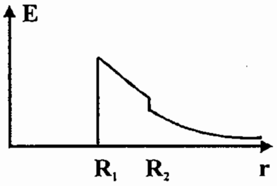
(b)



(c)



(d)



$\frac{R^2}{r^2}\hat{r}$

12. A solid sphere made of dielectric material has polarization $\vec{P} = P_0\vec{r}$. If R be its radius then dielectric field inside and outside at a distance 'r'r from centre

(a) $-\frac{P_0}{3\varepsilon_0}\vec{r}, \frac{-P_0}{3\varepsilon_0}\frac{R^2}{r^2}\hat{r}$ (b) $-\frac{P_0}{3\varepsilon_0}\vec{r}$, zero (c) $-\frac{P_0}{\varepsilon_0}\vec{r}, \frac{-P_0}{\varepsilon_0}\frac{R^2}{r^2}\hat{r}$ (d) $-\frac{P_0}{\varepsilon_0}\vec{r}, \frac{-P_0}{\varepsilon_0}\frac{R^2}{r^2}\hat{r}$

$\boxed{P = P_0 r}$

$E_{above} - E_{below}$

$D = E\epsilon_0$

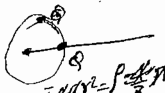
$E = \frac{D}{\epsilon_0}$

$D\epsilon_0 + P$

$= \frac{D}{\epsilon_0} + \frac{P}{\epsilon_0}$

13. A circular ring of radius R and charge Q is lying on a horizontal table. It is desired to keep a particle of charge Q on the axis of the ring so that the particle remains suspended in air. The maximum possible value of mass of the particle is

(a) $\frac{Q^2}{3\sqrt{3}\pi\varepsilon_0 R^2 g}$ (b) $\frac{2Q^2}{3\sqrt{3}\pi\varepsilon_0 R^2 g}$ (c) $\frac{Q^2}{6\sqrt{3}\pi\varepsilon_0 R^2 g}$ (d) $\frac{Q^2}{2\sqrt{3}\pi\varepsilon_0 R^2 g}$



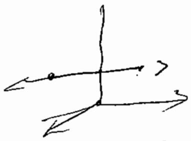
$E 4\pi r^2 = \frac{\rho \frac{4}{3}\pi R^3}{\epsilon_0}$

$E = \frac{\rho R^3}{\epsilon_0 r^2}$

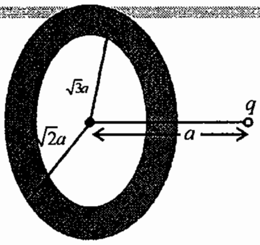
$\frac{\rho R^3}{E\epsilon_0 r^2}$

14. An electric dipole of dipole moment p is placed along x-direction. At a certain point on x-y plane whose distance from dipole is r, the direction of electric field is along +y direction. What is magnitude of electric field at that point

(a) $\frac{p}{2\pi \in_0 r^3}$ (b) $\frac{p}{2\sqrt{2}\pi \in_0 r^3}$ (c) $\frac{3p}{4\pi \in_0 r^3}$ (d) $\frac{3p}{4\sqrt{2}\pi \in_0 r^3}$



15. In the figure shown a point charge q is placed on the axis of a ring at a distance 'a' from its centre. Inner and outer radii of the ring are $\sqrt{3}a$ and $2\sqrt{2}a$. What is flux through the shaded region



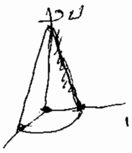
$\boxed{\phi = \int E\, ds}$

$2\pi \int_{\sqrt{3}a}^{8\sqrt{2}a} dr \quad \frac{a}{4\pi\epsilon_0}$

$\frac{\phi + \sqrt{3}\phi}{\phi + 2\sqrt{2}\phi}$

(a) $\frac{q}{6 \in_0}$ (b) $\frac{q}{3 \in_0}$ (c) $\frac{q}{24 \in_0}$ (d) $\frac{q}{12 \in_0}$

16. Two electric dipoles $P_1$ and $P_2$ are placed at (0, 0, 0) and (1, 0, 0) respectively, with both of them pointing in the +z direction. Without changing the orientations of the dipoles, $P_2$ is moved to (0, 2, 0). The ratio of the electrostatic potential energy of the dipoles after moving to that before moving is:



(a) $\frac{1}{16}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

17. If the electrostatic potential at a point (x, y) is given by $V = (2x + 4y)$ volts, the electrostatic energy density at that point (in J/m$^3$) is:

$\vec{E} = \frac{y}{2}$

(a) $5\varepsilon_0$ (b) $10\varepsilon_0$ (c) $20\varepsilon_0$ [✓ (d) $\frac{1}{2}\varepsilon_0 (2x+4y)^2$]

18. The electric charge distribution that creates an electric potential

$$V(r) = \frac{e^{-\lambda r}}{r} \text{ where } \lambda \text{ is a constant.}$$

Will be?

(a) $\frac{-\lambda^2}{r}\varepsilon_0 e^{-\lambda r}$ (b) $4\pi\varepsilon_0\delta^3(r) - \frac{\lambda^2}{r}\varepsilon_0 e^{-\lambda r}$

(c) $e^{-\lambda r}\varepsilon_0\left[4\pi\delta^3(r) + \frac{2\lambda}{r^2} - \frac{\lambda^2}{r}\right]$ [✓ (d) $4\pi\varepsilon_0\delta^3(r)$]

19. The electron cloud around the positively charged nucleus Q in the hydrogen atom is simply modeled as the spherically symmetric distribution

$$\rho(r) = -\frac{Q}{\pi a^3} e^{-2r/a}$$

where 'a' is called the Bohr radius. The charged enclosed in the sphere of radius equal to Bohr radius is:

(a) $-Q/2$ (b) $-Q$ [✓ (c) 0] (d) Q

20. An infinitely long wire placed along z-axis has the line charge density

$$\lambda(z) = \lambda_0 e^{-|z|/a} \quad \text{where a is some constant.}$$

The total charge (q) on the wire is:

[✓ (a) $2a\lambda_0$] (b) $-2a\lambda_0$ (c) $a\lambda_0$ (d) 0

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Unit Test : EMT

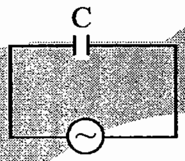
**Time : 01:00 Hour** **Date : 28-10-2014**
**M.M. : 77.5**

**Instructions:**

1. **There are Two Parts. Part-A contains 15 objective type questions, each question carry 3.5 marks and Part-B contains 5 objective type questions, each question carry 5 marks.**
2. **There is negative marking, @ 25% will be deducted for each wrong answer.**
3. **Attempt all the questions, use of calculator is not allowed.**

## PART-A

1. A parallel plate air gap capacitor is made up of two plates of area 10 cm$^2$ each kept at a distance of 0.88mm. A sine wave of amplitude 10V and frequency 50Hz is applied across the capacitor as shown in the figure. The amplitude of the displacement current density (in A/m$^2$) between the plates will be



(a) $3\times10^{-5}$ (b) $3\times10^{-6}$ (c) $5\times10^{-6}$ (d) $5\times10^{-5}$

**Common data for Q.2 and Q.3.**

The freshnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and dielectric of refractive index $\mu$ are

$$E_{||}^{\text{reflected}} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i} E_{||}^{\text{incident}}$$

and

$$E_{\perp}^{\text{reflected}} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i} E_{\perp}^{\text{incident}}$$

the subscripts | | and $\perp$ refer to polarisation, parallel and normal to the plane of incidence respectively. Here 'i' and 'r' are the angles of incidence and refraction respectively.

2. The condition for the reflected ray to be completely polarised?

(a) $\mu\cos i = \cos r$ (b) $\cos i = \mu\cos r$ (c) $\mu\cos i = -\cos r$ (d) $\cos i = -\mu\cos r$

3. For normal incidence at air-dielectric (non-magnetic) interface with dielectric constant $\varepsilon_r = 4$. The fraction of energy reflected in to air is given by

(a) 0.36 (b) 0.22 (c) 0.11 (d) 0.04

4. An electromagnetic with $\vec{E}(z,t) = 60\cos(\omega t - kz)\hat{i}$ V/m is travelling in free space and crosses a disc of radius 2m placed perpendicular to the z-axis. The average power, in watt, crossing the disc along the z-direction is

(a) 30 (b) 120 (c) 180 (d) 60

5. Two circular loops of conducting wires having radius R and 2R, carrying current $I_0\cos(100\pi t)$ and $I_0\cos(50\pi t)$ respectively. The ratio of power radiated by the loops is

(a) 1 (b) 4 (c) 32 (d) 16

**Linked answer type Q.6 and Q.7**

The vector $\vec{A} = \frac{1}{2}\alpha t\left(x\hat{j} - y\hat{i}\right)$, $\phi = \frac{1}{4}\alpha\left(x^2 + y^2\right)$

where '$\alpha$' is a constant and 't' is time.

6. The electric field $\left(\vec{E}\right)$ and magnetic fields $\left(\vec{B}\right)$ corresponding to these potentials are, respectively?

[✓ (a) $\frac{1}{4}\alpha\left[(x+y)\hat{i} + (x-y)\hat{j}\right], \frac{1}{2}\alpha t\,\hat{k}$] (b) $-\frac{1}{2}\alpha\left[(x-y)\hat{i} + (x+y)\hat{j}\right], \alpha t\,\hat{k}$

(c) $-\frac{1}{2}\alpha\left[x\hat{i} + y\hat{j}\right], \alpha t\,\hat{k}$ (d) $-\frac{1}{4}\alpha\left[(x+y)\hat{i} + (x-y)\hat{j}\right], \alpha t\,\hat{k}$

7. Condition on electric field $\left(\vec{E}\right)$ and magnetic field $\left(\vec{B}\right)$ to be Lorentz invariant is?

(a) $x^2 + y^2 - 2c^2t^2 = 0$ (b) $x^2 + y^2 - 4c^2t^2 = 0$

[✓ (c) $xy - c^2t^2 = 0$] (d) $x^2 + y^2 - c^2t^2 = 0$

**Linked answer type Q.8 and Q.9:**

A long straight solenoid of radius R and 'n' turns per unit length carries a current $I = \alpha t$, where '$\alpha$' is constant and 't' is time. The axis of the solenoid is along z-axis.

8. The electric field $\left(\vec{E}\right)$ at a distance $(s > R)$ from the axis of the solenoid is?

(a) $-\frac{\mu_0 n\alpha s}{2}\hat{\phi}$ [✓ (b) $\frac{\mu_0 n\alpha s}{2}\hat{\phi}$] (c) $\frac{\mu_0 n\alpha R^2}{2s}\hat{\phi}$ (d) $-\frac{\mu_0 n\alpha R^2}{2s}\hat{\phi}$

9. The poynting vector $\left(\vec{S}\right)$ at a distance $(s < R)$ from the axis of solenoid is?

[✓ (a) 0] (b) $-\frac{\mu_0 n^2\alpha^2 ts}{2}\hat{s}$ (c) $\frac{\mu_0 n^2\alpha^2 t\,s}{2}\hat{s}$ (d) $-\frac{\mu_0 n^2\alpha^2 R^2 t}{2s}\hat{s}$

10. The transformation equation relating the electric fields $\bar{E}$ and $\bar{E}'$ of an e.m. wave in two frames S and $S'$ moving with velocity $\bar{v}$ with respect to each other is:

(a) $E'_{||} = E_{||}\left(1 - {v^2}/{c^2}\right)^2$ (b) $E'_{||} = E_{||}$

[✓ (c) $E'_{||} = \dfrac{E_{||}}{\sqrt{1 - \dfrac{v^2}{c^2}}}$] (d) $E'_{||} = E_{||}\left(1 - \dfrac{v^2}{c^2}\right)^{3/2}$

**Linked Answer for Q.11 and Q.12.**

In an air filled rectangular wave guide y-component of electric field of a TE mode is given by

$$E_y = 5\sin\left(\frac{m\pi x}{2}\right)\cos\left(\frac{n\pi y}{3}\right)\sin\left(\frac{3\pi}{2}10^8 t - 12z\right)\text{V/m}$$

where x and y are in meters.

11. Dimensions of the wave guide are
(a) a = 2cm, b = 3 cm (b) a = 2m, b = 3m (c) a = 0.5 cm, b = 0.33 cm (d) a = 0.5 m, b = 0.33 m

12. Possible modes of operation are:
(a) $TE_{10}, TE_{01}$ & $TE_{11}$ (b) Only $TE_{10}$ [✓ (c) Only $TE_{01}$] (d) $TE_{10}$ & $TE_{01}$

13. The electric field of an electroamgnetic waves $\vec{E}(\vec{r},t) = \sqrt{2}\, E_0 \hat{z} \cos(x - y - \omega t)$. If 'c' stands for the velocity of the wave, the magnetic field $\vec{B}(\vec{r},t)$ is :

(a) $\vec{B}(\vec{r},t) = -\frac{1}{c\sqrt{2}}(\hat{x}+\hat{y})E_0\cos(x-y-\omega t)$ (b) $\vec{B}(\vec{r},t) = -\frac{1}{c}(\hat{x}-\hat{y})E_0\cos(x-y-\omega t)$

[✓ (c) $\vec{B}(\vec{r},t) = -\frac{1}{c}(\hat{x}+\hat{y})E_0\cos(x-y-\omega t)$] (d) $\vec{B}(\vec{r},t) = \frac{1}{c\sqrt{2}}(\hat{x}-\hat{y})E_0\cos(x-y-\omega t)$

14. **Group-I** contains x- and y-components of the electric field and **Group-II** contains the type of polarization of light:

| **Group-I** | **Group-II** |
|---|---|
| P. $E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz), E_y = E_0 \sin(\omega t + kz)$ | 1. Linearly polarized |
| Q. $E_x = E_0 \sin(\omega t + kz), E_y = E_0 \cos(\omega t + kz)$ | 2. Circularly polarized |
| R. $E_x = E_1 \sin(\omega t + kz), E_y = E_2 \sin(\omega t + kz)$ | 3. Unpolarized |
| S. $E_x = E_0 \sin(\omega t + kz), E_y = E_0 \sin\left(\omega t + kz + \frac{\pi}{4}\right)$ | 4. Elliptically polarized. |

The correct set of matches is:
(a) P→4, Q→2, R→4, S→1 (b) P→1, Q→3, R→1, S→4
(c) P→4, Q→2, R→1, S→4 (d) P→3, Q→1, R→3, S→2

15. In a lossy dielectric material of relative permitivity 12, the displacement current density is 25 times greater than conduction current at 1 MHz. The Conductivity of the dielectric is:

(a) $267\ \Omega^{-1}m^{-1}$ (b) $534\ \Omega^{-1}m^{-1}$ (c) $150\ \Omega^{-1}m^{-1}$ (d) $750\ \Omega^{-1}m^{-1}$

## PART-B

16. A charge particle with charge 'q' moves with constant velocity $\vec{v}$ along z-direction. Its scalar and vector potentials are

$$\phi = \frac{q}{\sqrt{(1-\beta^2)(x^2+y^2)+(z-vt)^2}} \text{ and } \vec{A} = \frac{\vec{v}}{c^2}\phi$$

where $\beta = \frac{v}{c}$ and c is speed of light. These potentials define electromagnetic waves under

(a) Coulomb Gauge only. (b) Lorentz-Gauge only.
[✓ (c) Both] (d) Neither

**Linked Answer Type Q. 17 and Q.18**

A uniform plane wave propagating in non-magnetic loss-less dielectric of dielectric constant $\varepsilon_r = 4$ has magnetic field:

$$\vec{B}(x,y,z,t) = B_0 e^{i\left[\sqrt{3}z+x+\omega t\right]}\,\hat{y}$$

17. Electric field $\vec{E}(x,y,z,t)$ of the wave is given by

(a) $\frac{B_0c}{4}\left(\sqrt{3}\,\hat{i}-\hat{k}\right)e^{i\left(\sqrt{3}\,z+x+\omega t\right)}$

[✓ (b) $\frac{B_0c}{4}\left(-\sqrt{3}\,\hat{i}+\hat{k}\right)e^{i\left(\sqrt{3}\,z+x+\omega t\right)}$]

(c) $\frac{B_0c}{2}\left(\sqrt{3}\,\hat{i}+\hat{k}\right)e^{i\left(\sqrt{3}\,z+x+\omega t\right)}$

(d) $\frac{B_0c}{2}\hat{k}\,e^{i\left(\sqrt{3}\,z+x+\omega t\right)}$

18. Frequency $(\omega)$ of the wave is (rad/sec):

$\omega = 2\pi\nu$

$\frac{\omega}{2\pi} = \nu$

(a) $9\times10^8$ [✓ (b) $6\times10^8$] (c) $10^8$ (d) $3\times10^8$

19. The instantaneous electric and magnetic fields created at a distance 'r' by a point source at the origin are given by

$$\vec{E} = \frac{A\cos\omega t}{2\pi\varepsilon_0 r}\hat{\theta} \qquad \hat{H} = \frac{B\cos\omega t}{\mu_0 r}\hat{\varphi}$$

where $\omega, A, B$ are constants, and the unit vectors $\left(\hat{r},\hat{\theta},\hat{\varphi}\right)$ form an orthonormal set. The time-averaged power radiated by the source is

(a) $\frac{\omega\varepsilon_0}{\mu_0}AB$ (b) $\frac{c^3}{2\pi}AB$ (c) $c^2AB$ [✓ (d) $\frac{2\pi\omega}{c}AB$]

20. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is CORRECT?

[✓ (a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.]

(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.

(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.

(d) There will be no reflected light and the transmitted light is circularly polarized.

## Assignment-I: Electromagnetic Theory

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (d) | 2. (a) | 3. (b) | 4. (a) | 5. (b) | 6. (a) | 7. (c) |
| 8. (b) | 9. (b) | 10. (c) | 11. (b) | 12. (a) | 13. (b) | 14. (a) |
| 15. (b) | 16. (a) | 17. (b) | 18. (b) | 19. (a) | 20. (d) | 21. (c) |
| 22. (b) | 23. (a) | 24. (a) | 25. (b) | 26. (a) | 27. (c) | 28. (a) |
| 29. (a) | 30. (b) | 31. (d) | 32. (c) | 33. (c) | 34. (c) | 35. (b) |
| 36. (c) | 37. (a) | 38. (b) | 39. (d) | 40. (a) | 41. (d) | 42. (c) |
| 43. (c) | 44. (b) | 45. (d) | 46. (b) | 47. (a) | 48. (c) | 49. (b) |
| 50. (c) | 51. (a) | 52. (b) | 53. (b) | 54. (b) | 55. (c) | 56. (c) |
| 57. (d) | 58. (c) | 59. (a) | 60. (d) | 61. (b) | 62. (c) | 63. (d) |
| 64. (a) | 65. (d) | 66. (c) | 67. (c) | 68. (b) | 69. (c) | 70. (b) |
| 71. () | 72. (c) | 73. (d) | 74. (c) | 75. (b) | 76. (d) | 77. (a) |
| 78. (b) | 79. (c) | 80. (d) | 81. (b) | 82. (a) | 83. (b) | 84. (c) |
| 85. (b) | 86. (a) | 87. (a) | 88. (b) | 89. (a) | 90. (a) | 91. (c) |
| 92. (b) | 93. (c) | 94. (a) | 95. (b) | 96. (d) | 97. (a) | 98. (b) |
| 99. (c) | 100. (a) | 101. (a) | 102. (d) | 103. (c) | 104. () | 105. (a) |
| 106. (d) | 107. (d) | 108. (b) | 109. (d) | 110. (a) | 111. (b) | |

# Assignment-II: Electromagnetic Theory

1. (b) 2. (c) 3. (c) 4. (d) 5. (d) 6. (d) 7. (d)
8. (c) 9. (d) 10. (a) 11. (c) 12. (a) 13. (c) 14. (b)
15. (b) 16. (b) 17. (a) 18. (a) 19. (b) 20. (a) 21. (b)
22. (a) 23. (b) 24. (a) 25. (a) 26. (b) 27. (c) 28. (d)
29. (b) 30. (d) 31. (b) 32. (d) 33. (a) 34. (a) 35. (b)
36. (b) 37. (b) 38. (d) 39. (a) 40. (b) 41. (c) 42. (c)
43. (c) 44. (b) 45. (a) 46. (d) 47. ( ) 48. (d) 49. (c)
50. (d) 51. (d) 52. (c) 53. (c) 54. (d) 55. (a) 56. (a)
57. (a) 58. (d) 59. ( ) 60. (d) 61. (c) 62. (d) 63. (d)
64. (c) 65. (d) 66. (b) 67. (c) 68. (b) 69. (c) 70. (b)
71. (d) 72. (a) 73. (a) 74. (b) 75. (a) 76. (c) 77. (c)
78. (a) 79. (d) 80. (a) 81. (b) 82. ( ) 83. (a) 84. (b)
85. (a) 86. (a) 87. (d) 88. (b) 89. (b) 90. (b) 91. (a)
92. ( ) 93. (c) 94. (d) 95. (a) 96. (a) 97. (d) 98. (b)
99_A(a) 99_B(d) 100. (c) 101. (c) 102. (d) 103. (d) 104. (d)
105. (c) 106. (a) 107. (d) 108. (b) 109. (b) 110. (a) 111. (c)
112. (c) 113. (b) 114. (b) 115. (a) 116. (d) 117. (d) 118. (a)
119. (d) 120. (c) 121. (c) 122. (d) 123. (b) 124. (d) 125. (d)
126. (a) 127. (a) 128. (b) 129. (a) 130. (a) 131. (b) 132. (c)
133. ( ) 134. (c) 135. (a) 136. (d) 137. (b) 138. (b) 139. (c)
140. (d) 141. (a) 142. (a) 143. (d) 144. (a) 145. (b) 146. (c)
147. (b) 148. (b) 149. (d) 150. (c) 151. (c) 152. (a) 153. (c)
154. (a) 155. (c) 156. (b) 157. (d) 158. (a) 159. (d) 160. (d)
161. (b) 162. (c) 163. (c) 164. (c) 165. (c) 166. ( ) 167. ( )
168. (c) 169. (b) 170. (c) 171. (a) 172. (a) 173. (d) 174. (c)
175. (c) 176. (d) 177. (b) 178. (c) 179. (c) 180. (a) 181. (c)
182. (0.15) 183. (d) 184. (c) 185. (c) 186. (a) 187. (d) 188. (c)
189. (b) 190. ( ) 191. ( ) 192. ( ) 193. ( ) 194. ( ) 195. ( )
196. ( ) 197. ( ) 198. ( ) 199. ( ) 200. ( ) 201. ( ) 202. (c)
203. (a) 204. (a) 205. (b) 206. (b) 207. (d) 208. (b) 209. (c)
210. (d) 211. (a) 212. (c) 213. (b) 214. (c) 215. ( ) 216. (a)
217. (a) 218. (d) 219. (b) 220. (a) 221. (c) 222. (d) 223. (b)
224. (b) 225. (1.49 to 1.51)) 226. (0.99 to 1.01) 227. (2.3 to 2.4)
228. (1.41 to 1.47) 229. (c) 230. (10.58 to 10.70)

# CSIR-NET/JRF & GATE

TIFR|BARC|JEST|IISc|DRDO|ISRO|SLET|ONGC & Ph.D Entrance

# Assignment

## Atomic & Molecular Physics

## PHYSICAL SCIENCES

CAREER ENDEAVOUR
CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED

South Delhi Centre: 28-A/11, Jia Sarai, Near-IIT Hauz Khas, New Delhi-110 016
T: 011-26851008, 26861009, (0) 9873744231, 9899628311

North Delhi Centre: 48, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T: 011-27241845, 65462244, (0) 9540292991
E: info@careerendeavour.com W: www.careerendeavour.com

**PHYSICAL SCIENCES (JRF-NET/GATE)**

## Assignment-I: Atomic Spectra

1. The linear Stark effect is possible in a hydrogen atom but not in a sodium atom because
(a) The principal quantum number for the ground state of the sodium atom is different from that of the hydrogen atm in the ground state
(b) Spin-orbit interaction is stronger in sodium than in hydrogen
(c) The electronic energy levels of sodium exhibit orbital-degeneracy
(d) The electronic energy levels of hydrogen exhibit orbital-degeneracy

2. The ground state of the deuteron is a
(a) pure ${}^3S_1$ state (b) pure ${}^3P_1$
(c) mixture of ${}^3S_1$ and ${}^3P_1$ (d) mixture of ${}^3S_1$ and ${}^3D_1$ states.

3. The possible values of the total angular momentum J resulting from the addition of two angular momenta $J_1 = 1$ and $J_2 = 2$ are
(a) 1, 2 (b) 1, 3 (c) 0, 1, 2 (d) 1, 2, 3

4. For a multi-electron atom, $\ell$, $L$ and S specify the one-electron orbital angular momentum, total orbital angular momentum and total spin angular momentum, respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by $\ell$, $L$ and S are
(a) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta \ell = 0, \pm 1$ (b) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta \ell = \pm 1$
(c) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta \ell = 0, \pm 1$ (d) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta \ell = \pm 1$

5. The frequencies of lines of a line spectrum of X-ray emission depend on
(a) The kinetic energy of the electron (b) The metal used for the anti-cathode
(c) The spin of the electron (d) The shape of the continuous spectrum

6. Sodium atom has 11 electrons. If the sequence in which the energy levels are filled in 1s, 2s, 2p, 3s, 3p, 4s, 3d...., the ground state of sodium is:
(a) ${}^3p_{1/2}$ (b) ${}^2p_{1/2}$ (c) ${}^1p_{1/2}$ (d) ${}^2s_{1/2}$

7. The orbital quantum numbers of two atomic electrons are $\ell_1 = 1$ and $\ell_2 = 2$. The possible values of the total angular momentum quamtum number J under LS coupling are
(a) –1, 1 only (b) 1, 2, 3 only (c) 0, 1, 2, 3, 4 (d) –2, –1, 0, 1, 2

8. The dependence of Doppler broadened line width of a laser transition on temperature T is given by
(a) T (b) $T^{-1/2}$ (c) $T^{1/2}$ (d) $T^2$

9. The continuous X-ray spectrum is the result of the
(a) Photoelectric effect (b) Inverse photoelectric effect
(c) Compton effect (d) Auger effect

10. The normal Zeeman effect is:
(a) Observed only in atoms with an even number of electrons
(b) Observed only in atoms with an odd number of electrons
(c) A confirmation of space quantization
(d) Not a confirmation of space quantization

11. Which of the following states exist?
(a) $2\,{}^2P_1$ (b) $2\,{}^2P_{3/2}$ (c) $2\,{}^2P_{5/2}$ (d) $2\,{}^2P_{7/2}$

12. The energy E of $K_\alpha X$-rays emitted from targets of different atomic number Z varies as

(a) $Z^2$ (b) $Z^{2/3}$ (c) Z (d) $Z^{1/2}$

13. In a Stern-Gerlach experiment the atomic beam whose angular momentum state is to be determined, must travel through

(a) Homogeneous radio frequency magnetic field

(b) Homogeneous static magnetic field.

(c) Inhomogeneous static magnetic field

(d) Inhomogeneous radio frequency magnetic field

14. Under the LS coupling scheme, the possible spectral terms $^{2s+1}L_J$ for the electronic configuration 2s3s are

(a) $^2S_{1/2}$, $^2P_{3/2}$, $^2P_{1/2}$ (b) $^1S_0$, $^3P_1$ (c) $^1S_0$, $^3S_1$ (d) $^3S_0$, $^3S_1$

15. Which of the following is the spectroscopic ground state for $Mn^{3+}$ ions of electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^4$ predicted by Hund's rule?

(a) $^5D_0$ (b) $^5D_4$ (c) $^5D_3$ (d) $^5D_2$

16. The Bohr model gives the value for the ionisation potential of $Li^{2+}$ ion as

(a) 13.6 eV (b) 27.2 eV (c) 40.8 eV (d) 122.4 eV

17. The number of hyperfine components observed in the electronic transition $^2p_{1/2} \rightarrow {}^2S_{1/2}$ of an atom with nuclear spin $\frac{1}{2}$ is :

(a) 3 (b) 4 (c) 6 (d) 5

18. The Lande g-factor for the $^3P_1$ level of an atom is:

(a) 1/2 (b) 3/2 (c) 5/2 (d) 7/2

19. The total number of Zeeman components observed in an electronic transition $^2D_{5/2} \rightarrow {}^2P_{3/2}$ of an atom in a weak field is:

(a) 4 (b) 6 (c) 12 (d) 10

20. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $|+\hat{z}\uparrow>$ state. Which of the following statements is true?

(a) The particle has a definite value of the y-component of the spin angular momentum

(b) The particle has a definite value of the square of the spin angular momentum

(c) The particle has a definite value of the x-component of spin angular momentum.

(d) The particle has definite values of x-and y-components of spin angular momentum

21. If the wavelength of the first line of the Balmer series in the hydrogen spectrum is $\lambda$, then the wavelength of the first line of the Lyman series is :

(a) $(27/5)\lambda$ (b) $(5/27)\lambda$ (c) $(32/27)\lambda$ (d) $(27/32)\lambda$

22. The spectral term for the atom with 70% filled subshell and only S = 3/2 is :

(a) $^3P_0$ (b) $^4F_{9/2}$ (c) $^3F_{1/2}$ (d) $^4P_{1/2}$

23. The hyperfine splitting of the spectral lines of an atom is due to :

(a) The coupling between the spins of two or more electron

(b) The coupling between the spins and the orbital angular momenta of the electron

(c) The coupling between the electron spins and the nuclear spin

(d) The effect of external electromagnetic field

24. The spin-orbit coupling constant for the upper state of sodium atom which emits D lines of wave numbers 16956.2 and 16973.4 $cm^{-1}$ is :

(a) 15 $cm^{-1}$ (b) 11.4 $cm^{-1}$ (c) 12.5 $cm^{-1}$ (d) 15.1 $cm^{-1}$

**Common data for Q.25 and Q.26:**

In the presence of an inhomogeneous weak magnetic field, spectral lines due to transitions between two sets of states were observed.

(1) $^5I_5 \to {}^5H_4$ and (2) $^2D_{5/2} \to {}^2P_{3/2}$

25. The types of Zeeman effect observed in (1) and (2) respectively are
(a) normal, normal (b) anomalous, anomalous
(c) anomalous, normal (d) normal, anomalous

26. The number of levels into which each of the above four terms split into respectively is:
(a) 6, 4, 10, 8 (b) 4, 6, 10, 12 (c) 11, 9, 6, 4 (d) 9, 5, 12, 10

27. In a hydrogen atom, the accidental or Coulomb degeneracy for the n = 4 state is:
(a) 4 (b) 16 (c) 18 (d) 32

28. The spectral terms for a certain electronic configuration are given by $^3D$, $^1D$, $^3P$, $^1P$, $^5S$, $^3S$. The term with the lowest energy is:
(a) $^5S$ (b) $^3P$ (c) $^3D$ (d) $^3S$

29. The degeneracy of the spectral term $^3F$ is:
(a) 7 (b) 9 (c) 15 (d) 21

30. The Lande g factor for the level $^3D_3$ is:
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

31. The L, S and J quantum numbers corresponding to the ground state electronic configuration of Boron(Z = 5) are
(a) $L = 1, S = \frac{1}{2}, J = \frac{3}{2}$ (b) $L = 1, S = \frac{1}{2}, J = \frac{1}{2}$
(c) $L = 1, S = \frac{3}{2}, J = \frac{1}{2}$ (d) $L = 0, S = \frac{3}{2}, J = \frac{3}{2}$

32. The degeneracies of the J-states arising from the $^3P$ term with spin-orbit inteaction are
(a) 1, 3, 5 (b) 1, 2, 3 (c) 3, 5, 7 (d) 2, 6, 10

33. Assuming that the L-S coupling scheme is valid, the number of permited transitions from $^2P_{3/2}$ to $^2S_{1/2}$ due to a weak magnetic field is:
(a) 2 (b) 4 (c) 6 (d) 10

34. Deuteron in its ground state has a total angular momentum J = 1 and a positive parity. The corresponding orbital angular momentum L and spin S combinations are
(a) L = 0, S = 1 and L = 2, S = 0 (b) L = 0, S = 1 and L = 1, S = 1
(c) L = 0, S = 1 and L = 2, S = 1 (d) L = 1, S = 1 and L = 2, S = 1

35. The $L_\beta$ line of X-rays emitted from an atom with principal quantum numbers n = 1, 2, 3, .... arises from the transition.
(a) $n = 4 \to n = 2$ (b) $n = 3 \to n = 2$ (c) $n = 5 \to n = 2$ (d) $n = 3 \to n = 2$

36. For an electron in hydrogen atom, the states are characterized by the usual quantum numbers n, l, $m_l$. The electric dipole transition between any two states requires that
(a) $\Delta\ell = 0, \Delta m_\ell = 0, \pm 1$ (b) $\Delta\ell = \pm 1, \Delta m_\ell = \pm 1, \pm 2$
(c) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 1$ (d) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 2$

37. The sodium doublet lines are due to transitions from $^2P_{3/2}$ and $^2P_{1/2}$ levels to $^2S_{1/2}$ level. On application of a weak magnetic field, the total number of allowed transitions becomes :
(a) 4 (b) 6 (c) 8 (d) 10

38. In a two electron atomic system having orbital and spin angular momenta $\ell_1, \ell_2$ and $s_1$ $s_2$ respectively, the coupling strengths are defined as $\Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}, \Gamma_{\ell_1s_1}, \Gamma_{\ell_2s_2}, \Gamma_{\ell_1s_2}$ and $\Gamma_{\ell_2s_1}$. For the jj coupling scheme to be applicable, the coupling strengths MUST satisty the condition.

(a) $\Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2} > \Gamma_{\ell_1s_1}, \Gamma_{\ell_2s_2}$
(b) $\Gamma_{\ell_1s_1}, \Gamma_{s_2s_2} > \Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}$
(c) $\Gamma_{\ell_1s_2}, \Gamma_{\ell_2s_1} > \Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}$
(d) $\Gamma_{\ell_1s_2}, \Gamma_{\ell_2s_1} > \Gamma_{\ell_1s_1}, \Gamma_{s_2s_2}$

39. The principal series of spectral lines of lithium is obtained by transitions between

(a) nS and 2P, $n > 2$
(b) nD and 2P, $n > 2$
(c) nP and 2S, $n > 2$
(d) nF and 3D, $n > 3$

40. In the linear Stark effect, the application of an electric field

(a) completely lifts the degeneracy of $n = 2$ level on hydrogen atom amd splits $n = 2$ level into four levels
(b) Partially lifts the degeneracy of $n = 2$ level on hydrogen atom amd splits $n = 2$ level into three levels
(c) Partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into two levels
(d) Does not affect the $n = 2$ levels

41. In hyperfine interaction, there is coupling between the electron angular momentum $\vec{J}$ and nuclear angular momentum $\vec{I}$, forming resultant angular momentum $\vec{F}$. The selection rules for the corresponding quantum number F in hyperfine transitions are

(a) $\Delta F = \pm 2$ only
(b) $\Delta F = \pm 1$ only
(c) $\Delta F = 0, \pm 1$
(d) $\Delta F = \pm 1, \pm 2$

42. The D1 and D2 lines of $Na\left(3^2P_{1/2} \to 3^2S_{1/2}, 3^2P_{3/2} \to 3^2S_{1/2}\right)$ will split on the application of a weak magnetic field into

(a) 4 and 6 lines respectively
(b) 3 lines each
(c) 6 and 4 lines respectively
(d) 6 lines each

43. The allowed states for He($2p^2$) configuration are

(a) ${}^1S_0, {}^3S_1, {}^1P_1, {}^3P_{0,1,2}$ ${}^1D_2$ and ${}^3D_{1,2,3}$
(b) ${}^1S_0, {}^3P_{0,1,2}$ and ${}^1D_2$
(c) ${}^1P_1$ and ${}^3P_{0,1,2}$
(d) ${}^1S_0$ and ${}^1P_1$

44. An atomic state of hydrogen is represented by the following wave function :

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1 - \frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where $a_0$ is a constant. The quantum-number of the state are:

(a) $\ell = 0, m = 0, n = 1$
(b) $\ell = 1, m = 1, n = 2$
(c) $\ell = 1, m = 0, n = 2$
(d) $\ell = 2, m = 0, n = 3$

45. In accordance with the selection rules of electric dipole transitions, the $4^3P_1$ state of helium can decay by photon emission to the states

(a) $2^1S_0, 2^1P_1$ and $3^1D_2$
(b) $3^1P_1, 3^1D_2$ and $3^1S_0$
(c) $3^3P_2, 3^3D_3$ and $3^3P_0$
(d) $2^3S_1, 3^3D_2$ and $3^3D_1$

46. If an atom is in the ${}^3D_3$ state, the angle between its orbital and spin angular momentum vectors $(\vec{L}$ and $\vec{S})$ is:

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$
(b) $\cos^{-1}\frac{2}{\sqrt{3}}$
(c) $\cos^{-1}\frac{1}{2}$
(d) $\cos^{-1}\frac{\sqrt{3}}{2}$

47. The hyperfine structure of $Na\left(3^2P_{3/2}\right)$ with nuclear spin $I = 3/2$ has

(a) 1 state
(b) 2 states
(c) 3 states
(d) 4 states

48. The radial wave function of the electrons in the state of n = 1 and $\ell = 0$ in a hydrogen atom is: $R_{10} = \frac{2}{a_0^{3/2}} \exp\left(\frac{-r}{a_0}\right)$, $a_0$ is the Bohr radius. The most probable value of r for electron is:

(a) $a_0$ (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

49. The last two terms of the electronic configuration of manganese(Mn) atom is $3d^5 4s^2$. The term factor of $Mn^{4+}$ ion is :'

(a) $^4F_{1/2}$ (b) $^4D_{3/2}$ (c) $^3F_{9/2}$ (d) $^3D_{7/2}$

50. Group I lists some physical phenomena while Group II gives some physical parameters. Match the phenomena with the corresponding parameter.

| **Goup I** | **Group II** |
|---|---|
| P. Doppler Broadening | 1. Moment of inertia |
| Q. Natural Broadening | 2. Refractive index |
| R. Rotational spectrum | 3. Lifetime of the energy level |
| S. Total interna reflection | 4. Pressure |

(a) P-4, Q-3, R-1, S-2 (b) P-3, Q-2, R-1, S-4
(c) P-2, Q-3, R-4, S-1 (d) P-1, Q-4, r-2, S-3

51. Cesium has a nuclear spin of 7/2. The hyperfine spectrum of the D lines of the Cesium atom will consist of

(a) 10 lines (b) 4 lines (c) 6 lines (d) 14 lines

**Common Data Q. 52 and Q.53:**

Consider the Zeeman splitting of a single electron system for the 3d → 3p electric dipole transition.

52. The Zeeman spectrum is :

(a) Randomly polarized (b) Only p polarized
(c) Only s polarized (d) Both p and s polarized

53. The fine structure line having the longest wavelength will split into :

(a) 17 components (b) 10 components
(c) 8 components (d) 4 components

54. An atom with one outer electron having orbital angular momentum $l$ is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is:

(a) $2\ell+2$ (b) $2\ell+1$ (c) $2\ell$ (d) $2\ell-1$

55. The minimum energy required to ionize hydrogen atom from its ground state is above

(a) 13.6 eV (b) −13.6 eV (c) 0.236 eV (d) 136 eV

56. The radius of a hydrogen atom in its ground state is:

(a) $10^{-4}$ cm (b) $10^{-6}$ cm (c) $10^{-8}$ cm (d) $10^{-10}$ cm

57. The electron in a hydrogen atom with a radius equal to first Bohr radius has a velocity equal to

(a) $\frac{C}{5}$ (b) $\frac{C}{10}$ (c) $\frac{C}{137}$ (d) $\frac{C}{8}$

58. The total energy of the electron in the nth orbit of hydrogen atom is:

(a) $\frac{e^2}{4\pi\varepsilon_0 r_n}$ (b) $\frac{e^2}{4\pi\varepsilon_0 r^2_n}$ (c) $\frac{e^2}{8\pi\varepsilon_0 r_n}$ (d) $-\frac{e^2}{8\pi\varepsilon_0 r_n}$

59. The kinetic energy of an electron in atom is:

(a) Half of its potential energy (b) Twice its potential energy
(c) Equal to its potential energy (d) Thrice its potential energy

60. The numerical value of the radius of the first orbit of hydrogen as

(a) 0.529 nm  (b) 0.0529 Å  (c) 5.29 Å  (d) 0.0529 nm

61. The total energy of the electron in hydrogen atom is:
(a) $-(13.6/n)$eV  (b) $(13.6/n)$eV  (c) $-(13.6/n^2)$eV  (d) $(13.6/n^2)$eV

62. The ratio of frequencies of the first line of the Lyman series and the first line of Balmer series is:

(a) $\frac{27}{5}$  (b) $\frac{27}{8}$  (c) $\frac{8}{27}$  (d) $\frac{4}{27}$

63. When an electron jumps from the fourth orbit to the second orbit, one gets
(a) The first line of P fund series  (b) Second line of Lyman series
(c) Second line of Paschen series  (d) Second line of Balmer series

64. The Bohr formula when utilised to calculate ionization energy. He is found to yield a result higher than experimental value because of
(a) Electron- electron interaction  (b) Spin orbit interaction
(c) Spin-spin interaction  (d) None of above

65. The first line of the Lyman series of a ten times ionized Na atom has a wavelength of about
(a) 0.1 Å  (b) 1Å  (c) 10Å  (d) 100Å

66. The range of frequencies of the lines of visible region in hydrogen spectrum is:

(a) $6000\times10^{14}\text{s}^{-1}$  (b) $600\times10^{14}\text{s}^{-1}$  (c) $60\times10^{14}\text{s}^{-1}$  (d) $6\times10^{14}\text{s}^{-1}$

67. The shortest wavelength of Balmer series in hydrogen spectrum is given by the equation

(a) $\frac{1}{\lambda}=R_H\left(\frac{1}{4}-\frac{1}{9}\right)$  (b) $\frac{1}{\lambda}=R_H\left(\frac{1}{2}-\frac{1}{\infty}\right)$  (c) $\frac{1}{\lambda}=R_H\left(1-\frac{1}{\infty}\right)$  (d) $\frac{1}{\lambda}=R_H\left(\frac{1}{4}-\frac{1}{\infty}\right)$

68. If v is the velocity of the electron in a stationary orbit of radius r, the orbital frequency of the electron is:

(a) $\frac{\upsilon}{2\pi r}$  (b) $\frac{\upsilon}{r}$  (c) $\frac{\upsilon}{\pi r}$  (d) $\frac{2\pi r}{\upsilon}$

69. Pauli exclusion principle state that
(a) Two electrons can have all the quantum numbers same
(b) No two electrons can have all the quantum numbers same
(c) Particles with integer and half integer spin cannot exist in the same state
(d) None of the above

70. Consider a spectal line resulting from the transition $n=2\rightarrow n=1$ in the atoms and ions given below. The shortest wavelength produced by
(a) singly ionised helium  (b) hydrogen atom
(c) doubly ionised lithium  (d) deuterium atom

71. One Bohr magneton is approximately

(a) $10^{23}$ Am$^2$  (b) $10^{-23}$ Am$^2$  (c) $10^{10}$ Am$^2$  (d) $10^{-10}$ Am$^2$

72. The spectroscopic symbol for the ground state of $_{13}$Al is $^2P_{1/2}$. Under the action of a strong magnetic field (when L-S coupling can be neglected) the ground state energy level will split into

(a) 3 levels  (b) 4 levels  (c) 5 levels  (d) 6 levels

73. The magnetic moment associated with the first orbit in hydrogen atom is given by

(a) $\frac{h}{4\pi me}$  (b) $\frac{4\pi me}{he}$  (c) $\frac{eh}{4\pi m}\times\sqrt{3}$  (d) $\frac{ehm}{4\pi}$

74. The length of the semi-major axis of an electron in an elliptical orbit is determined
(a) only by the principal quantum number
(b) only by the azimuthal quantum number
(c) both by principal quantum number and azimuthal quantum numbers
(d) none of these

75. When the azimuthal quantum number $n_\phi$ is zero

(a) The ellipse reduces to a straight line (b) The ellipse reduces into a circle

(c) The ellipse reduces to a parabola (d) none of these

76. If the $\phi$ – dependent part of the eigen function of an electron in a hydrogen atom is $e^{2i\phi}$ the minimum pricipal and orbital quantum numbers n and l respectively for the eigen function will be

(a) n2, 1, 1 (b) n3, 1, 2 (c) n4, 1, 3 (d) n3, 1, 1

77. Atoms with $\frac{1}{2}$ nuclear spin cannot have

(a) Hyperfine structure (b) Electric dipole interaction

(c) Fine structure (d) None of these

78. The doublets observed in alkali spectra are due to

(a) Screening of the K-electrons (b) Spin-~~orbit interaction~~ of the electron

(c) Pressure of isotopes (d) None of these

79. Possible values of the total angular momentum quantum number of a single f-electron are

(a) $j = \frac{7}{2}, \frac{5}{2}$ (b) $j = \pm 3, 2, 1, 0$ (c) $j = \pm\frac{7}{2}, \pm\frac{5}{2}$ (d) None of these

80. The electronic configuration $(np)^2$ can have only which one of the following term

(a) ${}^2D$, ${}^3P$ or ${}^1S$ (b) ${}^1D$, ${}^3P$ or ${}^1S$ (c) ${}^3D$, ${}^3P$ or ${}^2S$ (d) None of these

81. Which of the following has the order of increasing energy?

(a) ${}^1D_2, {}^3D_2, {}^3F_2$ (b) ${}^3F_2, {}^3D_2, {}^1D_2$ (c) ${}^3D_2, {}^3F_2, {}^1D_2$ (d) ${}^1D_2, {}^3F_2, {}^3D_2$

82. In outer atomic shells of atoms 4s level is filled first by electrons than 3d level because

(a) 4s electrons are less screened from the nucleus than 3d electrons

(b) Spin-spin interaction dominates for 3d level

(c) Spin orbit intersection between the electrons in the different levels reduces energy of 4s electrons below that of 3d electrons.

(d) None of these

83. In Zeeman effect, a spectral-line, upon the application of magnetic field, splits into more than three componets because of

(a) Energy levels split into $2J+1$

(b) In magnetic field $\Delta m_J = 0, \pm 1$ no longer holds

(c) Variation of Lande g-factor from one level to another

(d) None of the above

84. Two states of an atom have definite parities. An electric dipole transition between these states is:

(a) Allowed if both the states have even parity.

(b) Allowed if both the states have odd parity.

(c) Allowed if the two states have opposite parities.

(d) Not allowed unless a static electric field is applied.

**Common Data for Q.85 and Q. 86.**

In the presence of a weak magnetic field, atomic hydrogen undergoes the transition:

$${}^2p_{1/2} \to {}^1s_{1/2}, \text{ by emission of radiation.}$$

85. The number of distinct spectral lines that are observed in the resultant Zeeman spectrum is

(a) 2 (b) 3 (c) 4 (d) 6

86. The spectral line corresponding to the transition

$${}^2P_{1/2}\left(m_j = +\frac{1}{2}\right) \to {}^2S_{1/2}\left(m_j = -\frac{1}{2}\right)$$

is observed along the direction of the applied magnetic field. The emitted electromagnetic field is:
(a) Circularly polarized (b) Linearly polarized
(c) Unpolarized. (d) Not emitted along the magnetic field direction

87. Let $E_1, E_2, E_3$ be the first three energy levels of a hydrogen atom. Consider the ratio $(E_3 - E_2)/(E_2 - E_1)$. Neglecting the fine structure condition this ratio is approximately equal to
(a) $\frac{27}{5}$ (b) $\frac{1}{27}$ (c) $\frac{27}{4}$ (d) $\frac{5}{27}$

88. The electronic configuration of the ground state of the Na atom is $^2S_{1/2}$. This implies that
(a) $S=2,\ L=0, J=2$ (b) $S=0, L=1/2,\ \ J=1/2$
(c) $S=1/2,\ L=0,\ J=1/2$ (d) $S=0, L=2, J=2$

89. A proton and a neutron are both subject to a uniform magnetic field. Which of the following is true?
(a) Both particles undergo Larmor precession because they have non-zero intrinsic magnetic moments.
(b) The neutron does not precess because its intrinsic magnetic moment is zero.
(c) The proton does not precess because its intrinsic magnetic moment is zero.
(d) Both the particles precess and the direction of the precession is the same for the two particles.

90. The degeneracy of an excited state of nitrogen atom having electronic configuration $1s^2 2s^2 2p^2 3d^1$ is:
(a) 6 (b) 10 (c) 15 (d) 150

91. If we model the electron as a uniform sphere of radius $r_e$ spinning uniformly about an axis passing through its centre with angular momentum $L_e = \hbar/2$, and demand that the velocity of rotation at the equator connot exceed the velocity c of light in vacuum, then the minimum value of $r_e$ is
[✓ (a)] 19.2 fm (b) 0.192 fm (c) 4.8 fm (d) 1960 fm

92. The ratio of intensities of the $D_1$ and $D_2$ lines of sodium at high temperature
(a) 1:1 (b) 2:3 (c) 1:3 (d) 1:2

93. The Hydrogen $\alpha$ - line $H_\alpha$ of wavelength $\lambda = 656.3$ nm corresponds to a transition between the principal quantum states $n = 3$ and $n = 2$. The wavelength of the line $H_\beta$ corresponding to the transition between $n = 4$ and $n = 2$ is
(a) 486.1nm (b) 886.0 nm (c) 437.5 nm (d) 1093.7 nm

94. For a rigid diatomic molecule, if the transition energy between two rotational levels i and j is given by $E_{i\to j}$ then $\frac{E_{2\to 1}}{E_{3\to 2}}$ is:
(a) 1/3 (b) 3/2 (c) 1/2 (d) 2/3

95. What is the maximum number of electrons with spin +1/2 in a p orbital ?
[✓ (a)] 6 (b) 3 (c) 8 (d) 1

96. The shortest wavelength in the Balmer series of the Hydrogen atom occurs due to
(a) transition from the 3s to 2s (b) ionization from the 2s level
(c) ionization form th 1s level (d) transition from 2s to 1s level

97. The Helium atom is excited into the 1s 2s configuration. The following spectral terns $^{2s+1}L$ are the only ones allowed

(a) $^1S, {}^1P$ (b) $^1S, {}^3P$ (c) $^1S, {}^3S$ (d) $^2S, {}^2P$

98. In the j-j coupling scheme of an atom with two electrons in the 'p' orbital, possible mj values are

(a) $\pm 1/2$ only (b) $\pm 1, 0$ (c) $\pm 3/2, \pm \frac{1}{2}$ (d) $\pm 3, \pm 2, \pm 1$

99. In the Zeeman effect the light emitted along and perpendicular to the applied magnetic field are respectively

(a) linearly and circularly polarized (b) circularly and linearly polarized
(c) both linearly polarized (d) both circularly polarized

100. For the size of the electron orbit in a hydrogen atom to be comparable to the size of a baeterium $\left(\sim 10^{-7}\,\text{m}\right)$ its principal quantum number should be about

(a) 100 (b) 1000 (c) 10000 (d) 100000

101. Which of the following transitions give rise to the $K_{\alpha 1}$ line of X-ray spectra?

(a) $2\,{}^2S_{1/2} \rightarrow 1\,{}^2S_{1/2}$ (b) $2\,{}^2P_{1/2} \rightarrow 1\,{}^2S_{1/2}$ (c) $2\,{}^2P_{5/2} \rightarrow 1\,{}^2S_{1/2}$ (d) $2\,{}^2P_{3/2} \rightarrow 1\,{}^2S_{1/2}$

102. Consider a system of two spins $S_1 = 1/2, S_2 = 1/2,$ in a state with total spin zero. The correction to the energy due to the spin-spin interaction $\Delta E = A\vec{S}_1.\vec{S}_2$ is

(a) 3A/A (b) –3A/4 (c) A/4 (d) –A/4

103. The sodium D lines correspond to the transitions $^2P_{1/2} \rightarrow {}^2S_{1/2}$ and $^2P_{3/2} \rightarrow {}^1S_{1/2}$. The anomalous Zeeman effect splits the first of these lines into

(a) 2 lines (b) 4 lines (c) 1 lines (d) 3 lines

104. As a consequence of the interaction between the electron's orbital angular momentum and the nuclear spin, the $^2S_{1/2}$ electronic level of the hydrogen atom

(a) shifts up by a constant amount (b) splits into two hyperfine levels
(c) shifts down by a constant amount (d) splits into three hyperfine levels

105. A hydrogen atom is in the 2p state. To what state or states can it go by radiating a photon in an allowed transition?

(a) 1s, 2s, 3p, 3d (b) 1s, 2s, 3s, 3p (c) 2s, 3p, 3d (d) 1s, 2s

106. An atom in an excited sate $\psi_{n\ell m}$ can make a radiative transition to a state $\psi_{n'\ell' m'}$ if

(a) $\int_{-\infty}^{\infty} \psi^*_{n'\ell' m'}\, \bar{r}\, \psi_{n\ell m}\, dV \neq 0$ (b) $\int_{-\infty}^{\infty} \psi^*_{n'\ell' m'}\, \bar{r}^2\, \psi_{n\ell m}\, dV \neq 0$

(c) $\int_{-\infty}^{\infty} \psi^*_{n'\ell' m'}\, \psi_{n\ell m}\, dV = \delta_{nn'}\,\delta_{\ell\ell'}\,\delta_{mm'}$ (d) $\int_{-\infty}^{\infty} \psi^*_{n'\ell' m'}\, \bar{r}\, \psi_{n\ell m}\, dV = 0$

107. The longest wavelength in the spectral series of the Hydrogen atom occurs due to

(a) transition from $n = \infty$ to $n = 1$ (b) ionization from 1s level
(c) transition from 2s to 1s level (d) ionization from n = 100

108. The term symbol for a particular atomic state is ${}^4P_{5/2}$. The values of S (spin angular momentum), L (orbital angular momentum) J (total angular momentum) of the state are

(a) $S=\frac{3}{2}, L=0, J=\frac{3}{2}$ (b) $S=\frac{1}{2}, L=1, J=\frac{3}{2}$

(c) $S=\frac{1}{2}, L=2, J=\frac{5}{2}$ (d) $S=\frac{3}{2}, L=1, J=\frac{5}{2}$

109. The ground state of sodium atom $\left({}^{11}\text{Na}\right)$ is a ${}^2S_{1/2}$ state. The difference in energy levels arising in the presence of a weak external magnetic field B, given in terms of Bohr magneton, $\mu_B$, is

(a) $\mu_B B$ (b) $2\mu_B B$ (c) $4\mu_B B$ (d) $6\mu_B B$

110. The terms $\{j_1, j_2\}$, arising from $2s^1\ 3d^1$ electronic configuration in j – j coupling scheme are

(a) $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$ (b) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$

(c) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$ (d) $\left\{\frac{3}{2},\frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$

111. Consider the Bohr model of he hydrogen atom .If $\alpha$ is the fine-structure constant the velocity of the electron in its lowest orbit is

(a) $\frac{C}{1+\alpha}$ (b) $\frac{C}{1+\alpha^2}$ or $(1-\alpha)c$ (c) $\alpha^2 c$ (d) $\alpha c$

112. The spatial part of a two-electron state is symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is

(a) $|\uparrow\rangle|\uparrow\rangle$ (b) $|\uparrow\rangle|\downarrow\rangle$ (c) $|\downarrow\rangle|\uparrow\rangle-|\uparrow\rangle|\downarrow\rangle/\sqrt{2}$ (d) $|\downarrow\rangle|\uparrow\rangle+|\uparrow\rangle|\downarrow\rangle/\sqrt{2}$

113. The binding energy of the hydrogen atom (electron bound to proton) is 13.6eV. the binding energy of positronium (electron bound to positron) is

(a) $13.6/2\,\text{eV}$ (b) $13.6/1810\,\text{eV}$ (c) $13.6\times1810\,\text{eV}$ (d) $13.6\times2\,\text{eV}$

114. The ground state electronic configuration for a carbon atom is

$$(1s)^2(2s)^2(2p)^2$$

The first excited state of this atom would be achieved by

(a) re-alignment of the electron spins within the 2p orbital

(b) transition of an electron from the 2s orbital to the 2p orbital.

(c) transition of an electron from the 2p orbital to the 3s orbital

(d) transition of an electron from 2s orbital to the 3s orbital.

115. When a pure element is vaporised and placed in a uniform magnetic field $B_0$, it is seen that a particular spectral line of wavelength l, corresponding to a J = 1 ® J = 0 transition, gets split into three components $\lambda, \lambda \pm \Delta\lambda$. It follows that the Lande g-factor for the transition J = 1 ® = 0 is given by

(a) $g=\frac{hc}{\mu_B B_0}\frac{\Delta\lambda^2}{\lambda}$ (b) $g=\frac{hc}{\mu_B B_0}\frac{\lambda^2}{\Delta\lambda}$ (c) $g=\frac{hc}{\mu_B B_0}\frac{\lambda}{\Delta\lambda^2}$ (d) $g=\frac{hc}{\mu_B B_0}\frac{\Delta\lambda}{\lambda}$

116. Consider a system of three spins $S_1$, $S_2$ and $S_3$ each of which can take values +1 and −1. The energy of the system is given by $E = -J[S_1S_2 + S_2S_3 + S_3S_1]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively,
(a) J and 1 (b) −3J and 1 [✓ (c) −3J and 2] (d) −6J and 22

117. The number of spectral lines allowed in the spectrum for the $3\,^2D \rightarrow 3\,^2P$ transition in sodium is ______

118. In a normal Zeeman effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition $5\,^1D_2 \rightarrow 5\,^1P_1$ of cadmium atom is to be observed. The spectrometer has a resolution of 0.01 nM. The minimum magnetic field needed to observe this is ($m_e = 9.1 \times 10^{-31}\ kg$, $e = 1.6\times10^{-19}\ C$, $c = 3 \times 10^8\ m/s$)
(a) 0.26 T [✓ (b) 0.52 T] (c) 2.6 T (d) 5.2 T

119. The electronic energy levels in a hydrogen atom are given by $E_n = -13.6/n^2$ eV. If a selective excitation to the n = 100 level is to be made using a laser, the maximum allowed frequency line-width of the laser is
(1) 6.5 MHz (2) 6.5 GHz (3) 6.5 Hz (4) 6.5 kHz

120. Which one of the following would not cause the broadening in spectral lines?
(a) Doppler effect (b) Heisenberg's uncertainty
(c) Absorption of electromagnetic radiation (d) Collisions

121. If $E_H$ is the binding energy of the electron in the Bohr model of the Hydrogen atom, and both the charge and mass of the electron are doubled, the binding energy becomes
(a) 32 $E_H$ (b) 2 $E_H$ (c) 4 $E_H$ (d) 16 $E_H$

122. The electronic configuration of the ground state of a nitrogen is $1s^2 2s^2 2p^3$. The degeneracy of this configuration is:
(a) 3 (b) 20 (c) 6 (d) 0

**Statement for Linked Answer type Q. 123 and Q. 124.**

Positronium is an atom formed by an electron and a positron. The mass of a positron is the same as that of an electron and its charge is equal in magnitude but opposite in sign to that of an electron. The positronium atom is thus similar to the hydrogen atom with the positron replacing the proton.

123. The binding energy of a positronium atom is:
(a) 13.6 eV (b) 6.8 eV (c) 27.2 eV (d) 3.4 eV

124. If a positronium atom makes a transition from the state with n = 3 to a state with n = 2, the energy of the photon that is emitted in this transition is closest to
(a) 1.88 eV (b) 0.94 eV (c) 1.13 eV (d) 2.27 eV

125. The number of normal Zeeman splitting components of $^1P \rightarrow {}^1D$ transition is
(a) 3 (b) 4 (c) 8 (d) 9

126. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is
[✓ (a) 10] (b) 3 (c) 6 (d) 9

127. How much does the total angular momentum quantum number J change in the transition of $Cr(3d^6)$ atom as it ionizes to $Cr^{2+}(3d^4)$?
(a) increases by 2 (b) decreases by 2 (c) decreases by 4 (d) does not change

## ANSWER KEY

1. (d)  2. (d)  3. (d)  4. (a)  5. (b)  6. (d)  7. (c)

8. (c)  9. (b)  10. (c)  11. (b)  12. (a)  13. (c)  14. (c)

15. (a)  16. (d)  17. (a)  18. (b)  19. (c)  20. (b)  21. (b)

22. (b)  23. (c)  24. (d)  25. (b)  26. (c)  27. (d)  28. (a)

29. (d)  30. (d)  31. (b)  32. (a)  33. (c)  34. (c)  35. (a)

36. (c)  37. (d)  38. (b)  39. (c)  40. (b)  41. (c)  42. (a)

43. (b)  44. (c)  45. (d)  46. (a)  47. (d)  48. (a)  49. (a)

50. (a)  51. (a)  52. (d)  53. (b)  54. (b)  55. (a)  56. (c)

57. (c)  58. (d)  59. (a)  60. (d)  61. (c)  62. (a)  63. (d)

64. (a)  65. (c)  66. (d)  67. (d)  68. (a)  69. (b)  70. (c)

71. (b)  72. (c)  73. (c)  74. (c)  75. (a)  76. (b)  77. (b)

78. (b)  79. (a)  80. (b)  81. (b)  82. (c)  83. (c)  84. (c)

85. (c)  86. (a)  87. (d)  88. (c)  89. (a)  90. (d)  91. (a)

92. (d)  93. (d)  94. (d)  95. (a)  96.(b)  97. (c)  98. (d)

99. (b)  100. (a)  101. (d)  102. (b)  103. (b)  104. (b)  105. (d)

106. (a)  107. (d)  108. (d)  109. (b)  110. (b)  111. (d)  112. (d)

113. (a)  114. (a)  115. (d)  116. (c)  117. (3)  118. (b)  119. (b)

120. (c)  121. (a)  122. (b)  123. (b)  124. (b)  125. (a)  126. (a)

127. (c)

**PHYSICAL SCIENCES (JRF-NET/GATE)**

## Assignment-II: Molecular Spectra

1. A stable $H_2$ molecule is possible because .
   (a) The electronic ground state is spin triplet
   (b) The magnitude of the electronic charge density is maximal in the internuclear region
   (c) The electronic charge density is depleted in the internuclear region
   (d) Of strong spin-orbit interaction in the molecule.

2. The energy separation between two consecutive stokes lines in Raman Scattering depends on
   (a) Energy separation between vibrational levels in the excited state
   (b) Wavelength of the incident light
   (c) Energy separation between vibrational levels in the ground state
   (d) Intensity of the incident light

3. Typical energy of the rotational modes in a polyatomic molecule like $NH_3$ is:
   (a) $10^6$ eV (b) $10^3$ eV (c) $10^4$ eV (d) 1 eV

4. There is no infrared absorption for nitrogen molecule because:
   (a) Its polarizability is zero (b) It has no vibrational levels
   (c) It has no rotational levels (d) Its dipole moment is zero

5. The vibrational constants $\omega_0$ and $\omega_0 x_0$ of HCl are 2937.5 $cm^{-1}$ and 51.6 $cm^{-1}$. The first Raman stokes line will be observed at (in $cm^{-1}$)
   (a) 2989.1 (b) 2885.9 [✓ (c) 2834.3] (d) 3040.7

6. The intensity of the electronic 0-0 band of a diatomic molecule is very intense when the minimum of the potential curve for the upper electronic state lies (here r is the internuclear distance).
   (a) at the same value r as that of the lower potential curve
   (b) at a smaller value of r than that of the lower potential curve
   (c) at a larger value of r than that of the lower potential curve
   (d) above the dissociation level of the lower potential curve

7. Which one of the following molecules does not exhibit a rotational spectrum?
   (a) $H_2$ (b) CO (c) HCl (d) HBr

8. Resonance (NMR, ESR etc.) studies in solids provide information about
   (a) The electronic structure of single defects
   (b) The motion of the spin or of the surrounding
   (c) Collective spin excitations
   (d) Internal magnetic fields sampled by the spin

9. Infrared absorption can be observed in which of the following molecules?
   (a) $N_2$ (b) $O_2$ (c) HCl (d) $C_2$

10. The pure rotational levels of a molecule in the far-infrared region follows the formula $F(J) = BJ(J+1)$, where $F(J)$ is the energy of the rotational level with quantum number J and B is the rotational constant. The lowest rotational energy gap in rotational Raman spectrum is :
(a) 2B (b) 4B ~~(c) 6B~~ (d) 8B

11. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by
(a) $\frac{Ik_BT}{\hbar^2}$ (b) $\frac{2Ik_BT}{\hbar^2}$ (c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$

12. The vibrational spectrum of a molecule exhibits a strong line with P and R branches at a frequency $u_1$ and a weaker line at a frequency $u_2$. The frequency $u_3$ isnot shown up. Its vibrational Raman spectrum shows a strongly polarized line at frequency $u_3$ and no feature at $u_1$ and $u_2$
(a) The molecule could be linear
~~(b)~~ The molecule lacks a center of inversion
(c) $\mathbf{u}_1$ arises from a symmetric stretching mode
(d) $\mathbf{u}_3$ arises from a bending mode

13. Three values of rotational energies of molecules are given below in different units
**P:** 10 cm$^{-1}$ **Q:** $10^{-23}$ J **R:** $10^4$ MHz
Choose the correct arrangement in the increasing order of energy
(a) P, Q, R (b) R, Q, P (c) R, P, Q (d) Q, R, P

14. Consider the following statements about molecular spectra
P: $CH_4$ does not give pure rotational Raman lines
Q: $SF_6$ could be studied by rotational Raman spectroscopy
R: $N_2$ shows infrared absorption spectrum
S: $CH_3CH_3$ shows vibrational Raman and infrared absorptin lines
T: $H_2O_2$ shows pure rotational spectrum
Choose the right combination of correct statements
(a) P and Q (b) P, R and T (c) P, S and T (d) Q and R

15. All vibrations producing a change in the electric dipole moment of a molecule yield
(a) Raman spectra (b) Infrared spectra
(c) Ultra-violet spectra (d) X-ray spectra

16. Consider the pure rotational spectrum of a diatomic rigid rotor, The separation between two consecutive lines $(\Delta\bar{V})$ in the spectrum
(a) Is directly proportional to the moment of inertial of the rotor
(b) In inversely proportional to the moment of inertia of the rotor
(c) Depends on the angular momentum
(d) Is directly proportional to the square of the interatomic separation

17. Light of wavelength 1.5 μm incident on a material with a characteristic Raman frquency of $20\times10^{12}$ Hz results in a stokes-shifted line of wavelength $\left[\text{Given}: c = 3\times10^8 \text{m.s}^{-1}\right]$
(a) 1.47 μm (b) 1.57 μm (c) 1.67 μm (d) 1.77 μm

18. Match the following and choose the correct combination

| **Group-I** | **Group-2** |
|---|---|
| P. Atomic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$ | 1. Na |
| Q. Strongly electropositive | 2. Si |
| R. Strongly electronegative | 3. Ar |
| S. Covalent bonding | 4. Cl |

(a) P-1, Q-2, R-3, S-4 (b) P-3, Q-2, R-4, S-1
(c) P-3, Q-1, R-4, S-2 (d) P-3, Q-4, R-1, S-2

19. In a Raman scattering experiment, light of frequency u from a laser is scattered by diatomic molecules having the moment of inertia I. the typical Raman shifted frequency depends on
(a) u and I (b) only u (c) only I (d) neither u nor I

20. For a diatomic molecule with the vibrational quantum number n and rotational quantum number J, the vibrational level spacing $\Delta E_n = E_n - E_{n-1}$ and the rotational level spacing $\Delta E_J = E_J - E_{J-1}$ are approximately
(a) $\Delta E_n = \text{constant}, \Delta E_J = \text{constant}$ (b) $\Delta E_n = \text{constant}, \Delta E_J \propto J$
(c) $\Delta E_n \propto n, \Delta E_J \propto J$ (d) $\Delta E_n \propto n, \Delta E_J \propto J^2$

21. The typical wavelengths emitted by diatomic molecules in purely vibrational and purely rotational transitions are respectively in the region of
(a) Infrared and visible (b) Visible and infrared
(c) Infrared and microwave (d) Microwave and infrared

22. In the microwave spectrum of identical rigid diatomic molecules, the separation between the spectral lines is recorded to be 0.7143 $cm^{-1}$. The moment of inertia of the molecule, in kg $m^2$, is :
(a) $2.3\times10^{-36}$ (b) $2.3\times10^{-40}$ (c) $7.8\times10^{-42}$ (d) $7.8\times10^{-46}$.

23. A vibrational-electronic spectrum of homonuclear binary molecules, involving electronic ground state $\varepsilon''$ and excited $\varepsilon'$, exhibits a continuum at $\bar{\upsilon}\,cm^{-1}$. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by $E_{ex}cm^{-1}$, then dissociation energy of the molecule in the ground state is :
(a) $(\bar{\upsilon}+E_{ex})/2$ (b) $(\bar{\upsilon}-E_{ex})/2$ (c) $(\bar{\upsilon}-E_{ex})$ (d) $\sqrt{(\bar{\upsilon}^2-E_{ex}{}^2)}$

24. Match the following:
P: Franck-Hertz experiment 1. Electronic excitation of molecules
Q: Hartree-Fock method 2. Wave function of atom
R: Stern-Gerlach experiment 3. Spin angular momentum of atoms
S: Franck-Condon principle 4. Energy levels in atoms

| (a) | (b) | (c) | (d) |
|---|---|---|---|
| P-4 | P-1 | P-3 | P-4 |
| Q-2 | Q-4 | Q-2 | Q-1 |
| R-3 | R-3 | R-4 | R-3 |
| S-1 | S-2 | S-1 | S-2 |

25. The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.
In a system of such diatomic molecules of reduced mass $\mu$, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to $1.05\,\mu$. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition J = 4 → J = 5, is :
(a) 0.475B (b) 0.50 B (c) 0.95 B (d) 1.0 B

26. The number of fundamental vibrational modes of $CO_2$ molecule is :
(a) Four: 2 are Raman active and 2 are infrared active
(b) Four: 1 is Raman active and 3 are infrared active
(c) Three: 1 is Raman active and 2 are infrared active
(d) Three: 2 are Raman active and 1 is infrared active

(Rotational spectra)

27. Three consecutive absorption lines at $64.275\ \text{cm}^{-1}$, $77.130\ \text{cm}^{-1}$ and $89.985\ \text{cm}^{-1}$ have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia $I_A$ and $I_B$ are ($I_A$ is with respect to the bond axis passing through the centre of mass and $I_B$ is with respect to an axis passing through the centre of mass and perpendicular to bond axis)
(a) both equal to $\dfrac{\hbar^2}{12.855hc}\,\text{gm cm}^2$ [✓ (b) zero and $\dfrac{\hbar^2}{12.855hc}\,\text{gm cm}^2$]
(c) both equal to $\dfrac{\hbar^2}{6.427hc}\,\text{gm cm}^2$ (d) zero and $\dfrac{\hbar^2}{6.427hc}\,\text{gm cm}^2$

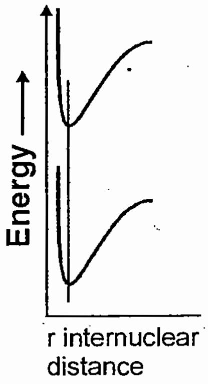
28. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency $u_e$. The frequency of two consecutive stokes lines are :
(a) $\upsilon_e - 10B, \upsilon_e - 14B$ (b) $\upsilon_e - 2B, \upsilon_e - 4B$
(c) $\upsilon_e + 10B, \upsilon_e + 14B$ (d) $\upsilon_e + 2B, \upsilon_e + 4B$

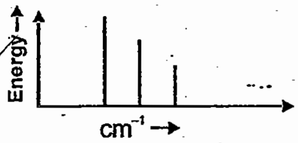
29. Which one of the following statement is INCORRECT in vibrational spectroscopy with anharmonicity?
(a) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, .....$
(b) Anharmonicity leads to multiple absorption lines
(c) The intensities of hot band lines are stronger than the fundamental absorption.
(d) The frequencies of hot band lines are smaller than the fundamental absorption.

30. The molecular spectra of two linear molecules O-C-O and O-C-S are recorded in the microwave region. Which one of the following statement is correct?
(a) Both the molecules would show absorption lines
(b) Both the molecules woulb not show absorption lines
(c) O-C-O would show absorption lines, but not O-C-S.
(d) O-C-S would show absorption lines, but not O-C-O.

31. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant, B is :
(a) 2B (b) 4B (c) 6B (d) 12B

32. In a diatomic molecule, the internuclear separation of the ground and first excited electronic state are the same as shown in the figure. If the molecule is initially in the lowest vibrational state of the ground state, then the absorption spectrum will appear as



(a)


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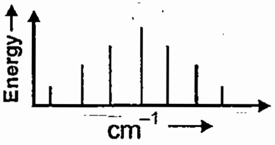

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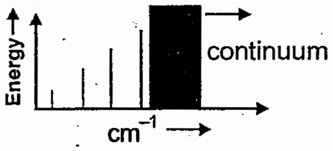
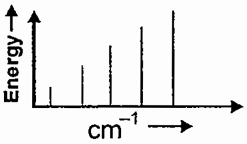


(d)


33. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the
(a) Visible range of frequencies
(b) Infrared range of frequencis.
(c) Ultraviolet range of frequencies
(d) Microwave range of frequencies.

34. The three principal moments of inertia of a methanol ($CH_3OH$) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are

(a) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$
(b) $\frac{\hbar^2}{2I}\ell(\ell+1)$
(c) $\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$
(d) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}+\frac{1}{I}\right)$

35. Match the typical spectra of stable molecules with the corresponding wave-number rnage

| | |
|---|---|
| 1. Electronic spectra | i. $10^6$ cm$^{-1}$ and above |
| 2. Rotational spectra | ii. $10^5$–$10^6$ cm$^{-1}$ |
| 3. Molecular dissociation | iii. $10^0$–$10^2$ cm$^{-1}$ |

(a) 1–ii, 2–i, 3–iii
(b) 1–ii, 2–iii, 3–i
(c) 1–iii, 2–ii, 3–i
(d) 1–i, 2–ii, 3–iii.

36. $CO_2$ molecule has the first few energy levels uniformly separated by approximately 2.5 meV. At a temperature of 300 K, the ratio of the number of molecules in the 4th excited state to the number in the 2nd excited state is about.
(a) 0.5
(b) 0.6
(c) 0.8
(d) 0.9

37. You are shown a spectrum consisting of a series of equally spaced lines. This could be
(a) The rotational spectrum CO.
(b) The vibrational spectrum of $N_2$.
(c) The NMR spectrum of $CH_4$
(d) The Mossbauer spectrum of $Fe_3O_4$.

38. The rotational energy levels of a rigid diatomic molecules are given by $E_J = B_e J(J+1)$ where J is the rotational quantum number and $B_e$ is the constant. The rotational absorption spectrum of the molecules therefore consists of
(a) One resonance line
(b) Lines that are equally spaced.
(c) Lines where the spacing increases with frequency
(d) Lines where the spacing decreases with frequency.

39. Consider the CO molecule as a diatomic rigid rotor with a bond length of 1.12 Å. The reduced mass of the system is obtained from the atomic masses of C and O. The rotational energies are defined in terms of B(the rotational constant) and J (the rotational quantum number). If $v_1$ and $v_2$ denotes the frequency of the first rotational resonance lines for the moleules $^{12}C$ $^{16}$O and $^{13}C$ $^{18}$O respectively, their ratio $v_1 / v_2$ is approximately.

(a) 1.5 (b) 1.1 (c) 0.9 (d) 1.01

40. The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where $\lambda_0$ is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

(a) Free particles (b) Particles in a box
(c) Harmonic oscillators [✓ (d) Rigid rotators]

41. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at $770\ \text{cm}^{-1}$, the approximate positions of the Stokes and the anti-Stokes lines are

(a) 481.5 nm and 520 nm (b) 481.5 nm and 500 nm
(c) 500 nm and 520 nm (d) 500 nm and 600 nm

42. Which of the diatomic molecules $H_2$, HD, $D_2$ and HT has the highest vibrational frequency?

(a) HT (b) $D_2$ (c) HD [✓ (d) $H_2$]

43. If a molecule has a centre of symmetry then
(a) Raman active vibrations are infrared active
(b) Raman active vibrations are infrared inactive and vice versa
(c) some, but not all, of the Raman active vibrations are infrared active
(d) all vibrations are Raman and infrared inactive.

44. In which of the following pairs do both molecules exhibit rotational spectra?

(a) $CCl_4$ and HF (b) $CCl_4$ and $O_2$ (c) HCN and HF (d) HCN and $CCl_4$

45. The sharpest sepctrum lines in the microwave rotational spectrum are observed when the sample is a
(a) solid (b) liquid (c) liquid crystal (d) gas

46. The unequal spacing in the vibrational energy levels of a diatomic molecule would be due to the
(a) Presence of anharmonic terms in the potential energy.
(b) Finite mass of the nuclei.
(c) Frank-Condon principle.
(d) Harmonic nature of the potential energy.

47. Match the typical spectroscopic regions specified in **Group I** With the corresponding type of transitions in **Group II.**

| **Group I** | **Group II** |
|---|---|
| (P) Infra-red region | (i) electronic transitions involving valence electrons |
| (Q) Ultraviolet- visible region | (ii) nuclear transitions |
| (R) X-ray region | (iii) vibrational transitions of molecules |
| (S) $\gamma$-ray region | (iv) transitions involving inner shell electrons |

(a) (P,i);(Q,iii);(R,ii);(S,iv) (b) (P,ii);(Q,iv);(R,i);(S,iii)

(c) (P,iii);(Q,i);(R,iv);(S,ii) (d) (P,iv);(Q,i);(R,ii);(S,iii)

48. The equilibrium vibration frequency for an oscillator observed at $2990\text{cm}^{-1}$. The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96. Considering the oscillator to be anharmonic, the anharmonicity constant is
(a) 0.005 (b) 0.02 (c) 0.05 (d) 0.1

49. NET The first absorption spectrum of $^{12}C^{16}O$ is at $3.842\,\text{cm}^{-1}$ while that of $^{13}C^{16}O$ is at $3.673\,\text{cm}^{-1}$. The ratio of their moments of inertia is
(a) 1.851 (b) 1.286 (c) 1.046 (d) 1.038

50. The spacing between vibrational energy levels in CO molecule is found to be $8.441 \times 10^{-2}$ eV. Given that the reduced mass of CO is $1.14 \times 10^{-26}$ kg, Planck's constant is $6.626 \times 10^{-34}$ Js and $1\text{eV} = 1.6 \times 10^{-19}$J. The force constant of the bond is CO molecule is
(a) 1.87 N/m (b) 18.7 N/m (c) 187 N/m (d) 1870 N/m

51. Consider the hydrogen deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is 938 MeV/$c^2$, then the energy difference $\Delta E$ between the two lowest rotational states is approximately
(1) $10^{-1}$eV (2) $10^{-2}$eV (3) $2 \times 10^{-2}$eV (4) $10^{-3}$eV

52. Raman effect is suppressed in
(a) semiconductors (b) dielectrics (c) metals (d) insulators

53. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers $J_A$ and $J_B$ are
(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

## ANSWER KEY

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (b)** | **2. (c)** | **3. (c)** | **4. (d)** | **5. (b)** | **6. (a)** | **7. (a)** |
| **8. (d)** | **9. (c)** | **10. (a)** | **11. (c)** | **12. (a, c)** | **13. (b)** | **14. (c)** |
| **15. (b)** | **16. (b)** | **17. (c)** | **18. (c)** | **19. (c)** | **20. (b)** | **21. (c)** |
| **22. (d)** | **23. (c)** | **24. (a)** | **25. (a)** | **26. (b)** | **27. (b)** | **28. (a)** |
| **29. (c)** | **30. (d)** | **31. (d)** | **32. (a)** | **33. (c)** | **34. (a)** | **35. (b)** |
| **36. (c)** | **37. (a)** | **38. (b)** | **39. (b)** | **40. (d)** | **41. (a)** | **42. (c)** |
| **43. (c)** | **44. (c)** | **45. (c)** | **46. (a)** | **47. (c)** | **48. (b)** | **49. (c)** |
| **50. (c)** | **51. (b)** | **52. ( )** | **53. (b)** | | | |

**PHYSICAL SCIENCES (JRF-NET/GATE)**

## Assignment-III: Laser Physics

1. The number of photons emitted per second from a 1 Watt Ar-ion laser operating at 488.0 nm is approximately

   (a) $10.23\times10^{19}$ (b) $2.46\times10^{18}$ (c) $10.23\times10^{17}$ (d) $2.46\times10^{15}$

2. Spectral line width of the He-Ne laser is 0.01 nm and the cross sectional area of the beam is 0.01 cm$^2$. If the output power is 1 milli Watt, the radiation intensity per unit wavelength (in Watt/cm$^3$) is:

   (a) $10^{10}$ (b) $10^{8}$ (c) $10^{-8}$ (d) $10^{-10}$

3. A laser beam emerging from a laser tube operating at 800 nm has a cross sectional diameter of 2 mm. The diameter of the beam at a distance of 1 km is approximately given by

   (a) 10 nm (b) 8 cm (c) 80 cm (d) 10 m

4. A laser beam of wavelength 600 nm with a cricular cross section having a radius of 10 mm falls normally on a lens of radius 20 mm and focal length 10 cm. The radius of the focussed spot is approximately

   (a) 0.3 μm (b) 0.6 μm (c) 3.0 μm (d) 6.0 μm

5. A three level system of atoms has $N_1$ atoms in level $E_1$, $N_2$ in level $E_2$, and $N_3$ in level $E_3$ $(N_2 > N_1 > N_3$ and $E_1 < E_2 < E_3)$. Laser emission is possible betwen the levels

   (a) $E_3 \to E_1$ (b) $E_2 \to E_1$ (c) $E_3 \to E_2$ (d) $E_2 \to E_3$.

6. Which one of the following electronic transitions in Neon is NOT responsible for LASER action in a helium-neon laser?

   (a) $6s \to 5p$ (b) $5s \to 4p$ (c) $5s \to 3p$ (d) $4s \to 3p$

7. In a He-Ne laser, the laser transition takes place in

   (a) He only (b) Ne only
   (c) Ne first, then in He. (d) He first, then in Ne

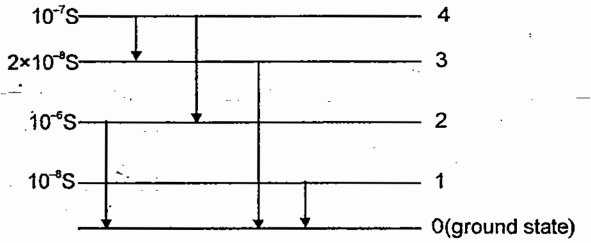
8. The coherence length of laser light is:

   (a) Directly proportional to the length of the active medium
   (b) Directly proportional to the width of the spectral line
   (c) Inversely proportional to the width of the spectral line
   (d) Inversely proportional to the length of the active lasing medium

9. When the refractive index m of the active medium changes by Dm in a laser resonator of length L, the change in the spectral spacing between the longitudinal modes of the laser is (c is the speed of light in free space)

   (a) $\dfrac{c}{2(\mu+\Delta\mu)L}$ (b) $\dfrac{c}{2\Delta\mu L}$ (c) $\dfrac{c}{2L}\dfrac{\Delta\mu}{\mu(\mu+\Delta\mu)}$ (d) zero

10. Five energy levels of a system including the ground state are shown below. Their lifetimes and the allowed electric dipole transitions are also marked.



Which one of the following transitions is the most suitable for a continuous wave (CW) laser?

(a) $1 \to 0$ [✓ (b) $2 \to 0$] (c) $4 \to 2$ (d) $4 \to 3$

11. The population inversion in a two level laser material CANNOT be achieved by optical pumping because
(a) The rate of upward transitions is equal to the rate of downward transitions.
(b) The upward transitions are forbidden but downward transitions are allowed.
(c) The upward transitions are allowed but downward transitions are forbidden.
(d) The spontaneous decay rate of the higher level is very low.

12. A collection of N atoms is exposed to a strong resonant electromagnetic radiation with $N_g$ atoms in the ground state and Ne atoms in the excited state, such that $N_g + N_e = N$. This collection of two–level atoms will have the following population distribution.

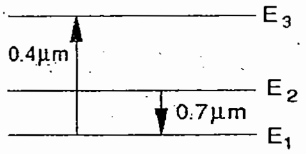
(a) $N_g << N_e$ (b) $N_g >> N_e$ (c) $N_g \approx N_e \approx N/2$ (d) $N_g - N_e \approx N/2$

13. For an atom with two energy levels placed in a cavity containing blackbody radiation, the ratio of the probabilities for spontaneous emission and stimulated emission is given by $\frac{A}{B} = \frac{8\pi\nu^3}{c^3}$, where A and B are the corresponding Einstein coefficients and $\nu$ is the frequency of the photon emitted. The probability of spontaneous emission.
(a) Is independent of the energy difference between the two levels.
(b) Increases with the energy difference between the two levels.
(c) Decreases with the energy difference between the two levels.
(d) Is zero.

14. The mirrors of a laser cavity are separated by a distance L. If T is the time taken by the light to travel from one mirror to the other and back, the mode separation is:

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$ (c) $\frac{1}{2T}$ (d) $\frac{1}{\sqrt{2}\,T}$

15. Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with $1.6\times10^{19}$ Chromium ions per cubic centimeter. All the atoms excited by the $0.4\ \mu m$ radiation decay rapidly to level $E_2$ which has a lifetime $\tau = 3ms$



**A.** Assuming that there is no radiation of wavelength $0.7\ \mu m$ present in the pumping cycle and that the pumping rate is R atoms per cm$^3$, the population density in the level $N_2$ builds up as:

(a) $N_2(t) = R\tau\left(e^{t/\tau} - 1\right)$ [✓ (b) $N_2(t) = R\tau\left(1 - e^{-t/\tau}\right)$]

(c) $N_2(t) = \frac{Rt^2}{\tau}\left(1 - e^{-t/\tau}\right)$ (d) $N_2(t) = Rt$

**B.** The minimum pump power required (per cubic centimeter) to bring the system to transparency, i.e. zero gain, is

(a) 1.52 kW (b) 2.64 kW (c) 0.76 kW (d) 1.32 kW

16. The central wavelength of a $0.6\,\mu m$ wavelength laser corresponds to the $m^{th}$ cavity mode of a resonator cavity of length 6 cm. The mode number m is

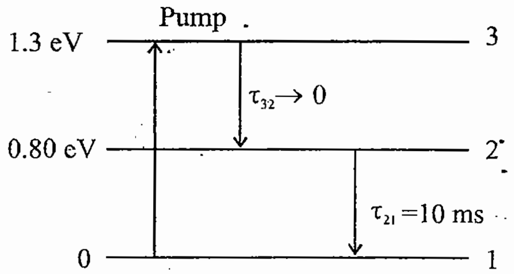
(a) 1000 (b) $2\times10^4$ (c) $2\times10^5$ (d) 200

17. A collection of spin-1/2, particles is in thermal equilibrium at temperature T in an applied magnetic field. The ratio of the population of the two allowed energy levels with separation $\Delta E$ is

(a) $\frac{N_{upper}}{N_{lower}}=\exp\left(\frac{-\Delta E}{k_BT}\right)$ (b) $\frac{N_{upper}}{N_{lower}}=1-\exp\left(\frac{-\Delta E}{k_BT}\right)$

(c) $\frac{N_{upper}}{N_{lower}}=\exp\left(\frac{-2\Delta E}{k_BT}\right)$ (d) $\frac{N_{upper}}{N_{lower}}=1+\exp\left(\frac{-\Delta E}{k_BT}\right)$

18. The typical energy level diagram for Erbium ions in a glass is shown below.



For an Erbium ion concentration of $\rho$ ions/cm$^3$, the pump power needed for population inversion is

(a) $80\,\rho\,\text{eV}/\text{m}^3\text{s}$ (b) $40\,\rho\,\text{eV}/\text{m}^3\text{s}$ (c) $130\,\rho\,\text{eV}/\text{m}^3\text{s}$ (d) $65\,\rho\,\text{eV}/\text{m}^3\text{s}$

19. In which of the following lasers can the wavelength not be tuned?

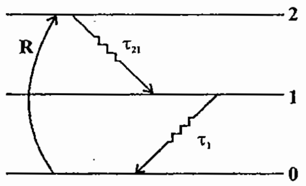
(a) semiconductor laser (b) Dye laser
(c) Ti: Sapphire laser (d) Ruby laser

20. Consider a hydrogen atom undergoing a $2P\rightarrow1S$ transition. The lifetime $t_{sp}$ of the 2P state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium $n_0=1$, the ratio of the Einstein coefficients for stimulated emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

(a) $0.683\times10^{12}\ \text{m}^3\text{J}^{-1}\text{s}^{-1}$ (b) $0.146\times10^{-12}\ \text{J s m}^{-3}$

(c) $6.83\times10^{12}\ \text{m}^3\ \text{J}^{-1}\ \text{s}^{-1}$ (d) $1.463\times10^{-12}\ \text{J s m}^{-3}$

21. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is $10^{20}$ atoms cm$^{-3}$ s$^{-1}$ and the decay routes are as shown with $\tau_{21}=20$ ns and $\tau_1=1\,\mu s$, the equilibrium populations of states 2 and 1 are, respectively,

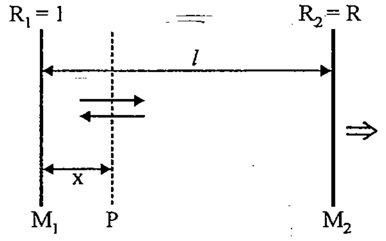
(a) $10^{14}$ cm$^{-3}$ and $2\times10^{12}$ cm$^{-3}$ (b) $2\times10^{12}$ cm$^{-3}$ and $10^{14}$ cm$^{-3}$
(c) $2\times10^{12}$ cm$^{-3}$ and $2\times10^{6}$ cm$^{-3}$ (d) zero and $10^{20}$ cm$^{-3}$.

22. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance d = 20 cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes $\delta v$ and the width $\Delta v_p$ of each mode of the laser cavity are:

(a) $\delta v = 75$ kHz, $\Delta v_p = 24$ kHz (b) $\delta v = 100$ kHz, $\Delta v_p = 100$ kHz

(c) $\delta v = 750$ MHz, $\Delta v_p = 2.4$ MHz (d) $\delta v = 2.4$ MHz, $\Delta v_p = 750$ MHz

23. Consider the laser resonator cavity shown in the figure.
If $I_1$ is the intensity of the radiation at mirror $M_1$ and $\alpha$ is the gain coefficient of the medium between the mirrors, then the energy density of photons in the plane P at a distance x from $M_1$ is



(a) $(I_1/c)e^{-\alpha x}$ (b) $(I_1/c)e^{\alpha x}$ (c) $(I_1/c)(e^{\alpha x} + e^{-\alpha x})$ (d) $(I_1/c)e^{2\alpha x}$

24. Two monochromatic sources, $L_1$, and $L_2$, emit light at 600 and 700 nm, respectively. If their frequency bandwidths are $10^{-1}$ and $10^{-3}$ GHz, respectively, then the ratio of linewidth of $L_1$ and $L_2$ is approximately
(a) 100 : 1 (b) 1 : 85 (c) 75 : 1 (d) 1 : 75

25. A gas laser cavity has been designed to operate at $\lambda = 0.5\mu m$ with a cavity length of 1m. With this set-up, the frequency is found to be larger than the desired frequency by 100 Hz. The change in the effective length of the cavity required to retune the laser is
(a) $-0.334 \times 10^{-12}$ m (b) $0.334 \times 10^{-12}$ m
(c) $0.167 \times 10^{-12}$ m (d) $-0.167 \times 10^{-12}$ m

26. A beam of atoms moving in a certain direction can be slowed down if they absorb photons from a laser beam moving in the opposite direction and subsequently spontaneously emit photons isotropically. For a beam of Sodium atoms (mass number A = 23) with speed 600 $ms^{-1}$, if a laser beam of wavelength 589 nm is used, the number of such absorption and emission cycles needed to bring a Sodium atom to rest would be approximately.
(a) $1.3\times10^5$ (b) $1.3\times10^4$ (c) $2.1\times10^3$ (d) $2.1\times10^4$

27. The emission wavelength for the transition ${}^1D_2 \to {}^1F_3$ is 3122 Å. The ratio of populations of the final to the initial states at a temperature 5000 K is ($h = 6.626\times10^{-34}$ $J.s$, $c = 3\times10^8$ $m/s$, $k_B = 1.380\times10^{-23}$ $J/K$)

(a) $2.03\times10^{-5}$ (b) $4.02\times10^{-5}$ (c) $7.02\times10^{-5}$ (d) $9.83\times10^{-5}$

## ANSWER KEY

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (b)** | **2. (b)** | **3. (c)** | **4. (a)** | **5. (b)** | **6. (c)** | **7. (b)** |
| **8. (c)** | **9. (c)** | **10. (b)** | **11. (a)** | **12. (c)** | **13. (a)** | **14. (a)** |
| **15_A(b)** | **15_B(a)** | **16. (b)** | **17. (d)** | **18. (c)** | **19.(d)** | **20. (a)** |
| **21. (b)** | **22. (c)** | **23. ( )** | **24. (c)** | **25. ( )** | **26. ( )** | **27. (c)** |

**PHYSICAL SCIENCES (JRF-NET/GATE)**

## Assignment-IV: Miscelleneous

1. A line arising from a transition of an atom from an excited state to the ground state has a width D of 0.5 cm$^{-1}$. If the width is almost entirely due to the width of the initial state, the lifetime of this state will be closest to :
(a) $6\times10^{-12}$ sec (b) $6\times10^{-15}$ sec (c) $6\times10^{-9}$ sec (d) $6\times10^{-17}$ sec

2. Unpolarized light of intensity $I_0$ falls on a combination of two linear polarizers where the second polarizer is having its axis rotated clockwise by 30° with respect to the first. The transmitted intensity will be:
(a) $3I_0/4$ (b) $3I_0/8$ (c) $\sqrt{3}I_0/4$ (d) $I_0/4$

3. A left circularly polarized light beam of wavelenfth 600 nm is incident on a crystal of thickness d and propagates perpendicular to its optic axis. The ordinary and extraordinary refractive indices of the crystal are $n_o = 1.54$ and $n_e = 1.55$ respectively. The emergent light will be right circularly polarized if d is :
(a) 120 μm (b) 60 μm (c) 30 μm (d) 15 μm

4. In a two beam interference pattern the maximum and minimum intensity values are found to be 25 $I_0$ and 9 $I_0$ respectively, where $I_0$ is a constant. The intensities of the two interfering beam are
(a) 16 $I_0$ and $I_0$ (b) 5 $I_0$ and 3 $I_0$ (c) 17 $I_0$ and 8 $I_0$ (d) 8 $I_0$ and 2 $I_0$

5. A parallel beam of infrared radiation of wavelength of $1.01\times10^{-6}$ m is incident normally on a screen with two slits $5\times10^{-6}$ m apart and the resulting interference pattern is observed on a distant screen. What is the largest number of maxima that can be observed on the screen?
(a) 4 (b) 9 (c) 13 (d) infinitely many

6. An atom emits a photon of wavelength $l = 600$ nm by transition from an excited state of life time $8\times10^{-9}$ s. If $\Delta\upsilon$ represents the minimum uncertainty in the frequency of the photon, the fractional width $\frac{\Delta\upsilon}{\upsilon}$ of the spectral line is of the order of
(a) $10^{-4}$ (b) $10^{-6}$ (c) $10^{-8}$ (d) $10^{-10}$

7. The lietime of an atomic state is 1 nanosecond. The Natural line widht of the spectral line in the emission spectrum of this state is of the order of
(a) $10^{-10}$ $eV$ (b) $10^{-9}$ $eV$ (c) $10^{-6}$ $eV$ (d) $10^{-4}$ $eV$

8. When light is emitted from a gas of excited atoms, the lines in the spectrum are Doppler-broadened due to the thermal motion of the emitting atoms.
The Doppler width of an emission line of wavelength 500 nanometres (nm) emitted by an excited atom of Argon $\left({}^{40}_{20}\mathrm{A}\right)$ at room temperature $(27°\mathrm{C})$ can be estimated as
(a) $5.8\times10^{-4}$ nm (b) $3.2\times10^{-4}$ nm (c) $3.2\times10^{-3}$ nm (d) $2.5\times10^{-3}$ nm

9. A sodium atom in the first excited 3P state has a lifetime of 16 ns for decaying to the ground 3S state. The wavelength of the emitted photon is 589 nm. The corresponding linewidth of the transition (in frequency units) is about
(a) $1.7\times10^{6}$ Hz (b) $1\times10^{7}$ Hz (c) $6.3\times10^{7}$ Hz (d) $5\times10^{14}$ Hz

10. The intensity of a laser in free space is $150\ mW/m^2$. The corresponding amplitude of the electric field of the laser is 10.63 V/m. $\left(\varepsilon_0 = 8.854\times10^{-12}\ C^2/N.m^2\right)$

## ANSWER KEY

1. (a) 2. (b) 3. (c) 4. (a) 5. (a) 6. (b) 7. (c)

8. (d) 9. (a) 10. (10.58 to 10.70)

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## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: ATOMIC & MOLECULAR PHYSICS

### Assignment 1: Bohr Atomic Model

1. An electron is initially in the n = 3 state and the average lifetime of this state is $10^{-8}$ sec. The number of revolution that the electron takes before making a transition to the n = 2 state, is
(a) $8.2\times10^4$ [✓ (b) $8.2\times10^5$] (c) $8.2\times10^6$ (d) $8.2\times10^7$

2. The wavelength of seventh line of the Balmer series of hydrogen atom is
(a) 384 nm (b) 424 nm (c) 456 nm (d) 512 nm

3. The energy required to remove an electron from a singly-ionized Helium atom which in the n = 8 state, is
(a) 0.213 eV (b) 0.51 eV (c) 0.85 eV (d) 1.11 eV

4. The momentum of the photon emitted by a hydrogen atom making a direct transition from an excited state n = 10 to the ground state, is
(a) $2.8\times10^{-27}\,kg\ m\ s^{-1}$ (b) $7.2\times10^{-27}\,kg\ m\ s^{-1}$ (c) $7.2\times10^{-26}\,kg\ m\ s^{-1}$ (d) $2.8\times10^{-26}\,kg\ m\ s^{-1}$

5. An accelarated beam of electrons bombards a sample of hydrogen and the first line of Paschen series is emiited. The electrons must have been accelarated through a minimum potential difference of
[✓ (a) 12.8 V] (b) 14.1 V (c) 16.7 V (d) 19.1 V

6. Suppose $E_H$ be the binding energy of hydrogen atom and $E_p$ be the binding energy of the positronium atom which consists of a positron and an electron. Then, which of the following statements is true?
(a) $E_p = E_H$ (b) $E_p = \frac{1}{3}E_H$ [✓ (c) $E_p = \frac{1}{2}E_H$] (d) $E_p = 2E_H$

7. Suppose $\lambda_H$ be the spectral wavelength of hydrogen atom and $\lambda_p$ be the spectral wavelength of the positronium atom which consists of a positron and an electron. Then, which of the following statements is true?
(a) $\lambda_P = \lambda_H$ (b) $\lambda_P = 2\lambda_H$ (c) $\lambda_P = \frac{1}{2}\lambda_H$ (d) $\lambda_P = \frac{1}{3}\lambda_H$

A $\mu^-$ meson (Charge -e, Mass $207m_e$) can be captured by a proton to form a hydrogen-like atom muonic atom.

8. The radius of the first bohr orbit is
[✓ (a) 0.285 pm] (b) 0.345 pm (c) 0.415 pm (d) 455 pm

9. The binding energy of the atom is
(a) -1750 eV (b) -2140 eV [✓ (c) -2530 eV] (d) -3120 eV

10. The first line of the Balmer series is
(a) 6.5 $A^0$ (b) 18.3 $A^0$ (c) 25.3 $A^0$ [✓ (d) 35.3 $A^0$]

11. A mixture of ordinary hydrogen and tritium is excited and its spectrum is studied. The shift in wavelength for the $H_\alpha$ lines of the two kinds of hydrogen, is
(a) 2.4 $A^0$ (b) 3.3 $A^0$ (c) 5.4 $A^0$ (d) 6.7 $A^0$

Electrons of energy 12.2 eV are fired at hydrogen atoms in a gas-discharge tube.

12. The principal quantum number of the highest state that the electron can reach, is
(a) 2 (b) 3 (c) 4 (d) 5

13. The number of transitions that the electron can make, is
(a) 1 (b) 2 (c) 3 (d) 4

14. In a transition to a state of excitation energy 10.19 eV, an hydrogen like atom emits a 4890 $A^0$ photon. The binding energy of the initial state is
(a) 7.66 eV (b) 8.92 eV (c) 10.19 eV (d0 13.6 eV

15. The temparature will the average molecular kinetic energy in the gaseous hydrgen be equal to the binding energy of the hydrogen atom, is
(a) $10^4 K$ (b) $10^5 K$ (c) $10^6 K$ (d) $10^7 K$

16. In the hydrogen spectrum, the ratio of the wavelengths for Lyman-$\alpha$ radiation to Balmer-$\alpha$ radiation is
(a) 5/48 (b) 1/3 (c) 5/27 (d) 3

17. The wavelength of one of the visible lines in the spectrum of the hydrogen atom is approximately 6560 $A^0$. It is known that this corresponds to a transition between the states with principal quantum number n = 2 and n = 3. The frequency corresponding to the transition from the ground state to the first excited state will be
(a) $4.56\times10^{14} Hz$ (b) $1.65\times10^{15} Hz$ (c) $2.47\times10^{15} Hz$ (d) $3.29\times10^{15} Hz$

18. Consider the spectral line resulting from the transition n = 2 and n = 1 in the atoms and ions given below. The shoertest wavelength will be produced by
(a) Hydrogen atom (b) Deutron atom
(c) Singly ionized helium (d) Doubly ionized lithium

19. A gas of monochromatic hydrogen is bombarded with a stream of electrons that have been accelerated from rest through a potenial difference 12.75 Volts. In the emission spectra one can't observe
(a) Lymam series (b) Balmer Series (c) Paschen Series (d) Pfund Series

20. If the principal quantum number and the azimuthal quantum number in the relativistic model of the atom are 3 and 1 respectively, then the magnitude of the semi-minor axis 'b' in terms of the semi major axis 'a' is given by
(a) b = a/3 (b) b = 2a/3 (c) b = a/2 (d) b = a

## ANSWER KEY

**1.(b)** **2.(a)** **3.(c)** **4.(b)** **5.(a)** **6.(c)**

**7.(b)** **8.(a)** **9.(c)** **10.(d)** **11.(a)** **12.(b)**

**13.(c)** **14.(a)** **15.(b)** **16.(c)** **17.(c)** **18.(d)**

**19.(d)** **20.(a)**

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**Subject: PHYSICAL SCIENCES (NET-JRF/GATE)**

**Section: ATOMIC & MOLECULAR PHYSICS**

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## Assignment 2: Vector atom model, Spin-orbit Interaction, Helium spectra

1. The two possible orientations of $\vec{S}$ with respect to an external magnetic field $\vec{B}$ along z-axis, are

(a) $\cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$ (b) $\cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$ (c) $\cos^{-1}\left(\pm\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\pm\frac{1}{2}\right)$

2. The magnitude of magnetic dipole moment corresponding to spin of an electron is

(a) $\mu_B$ (b) $2\mu_B$ (c) $\sqrt{2}\mu_B$ (d) $\sqrt{3}\mu_B$

3. A beam of electrons (each of rest mass m, charge e) enters in a uniform magnetic field B along z-axis. The energy separation between the electrons having spin parallel and anti-parallel with magnetic field, is

(a) $\frac{eB\hbar}{m}$ (b) $\frac{eB\hbar}{2m}$ (c) $\frac{eB\hbar}{4m}$ (d) 0

4. The number of possible spectroscopic terms of a one electron atom corresponding to n = 4 is

(a) 3 (b) 5 (c) 7 (d) 9

5. The angle between the orbital angular momentum and spin angular momentum for the term $^2D_{3/2}$ is

(a) $45^0$ (b) $62^0$ (c) $90^0$ (d) $135^0$

6. Which of the following spectroscopic term is not allowed?

(a) $^2D_{3/2}$ (b) $^2F_{5/2}$ (c) $^2P_{3/2}$ (d) $^2D_{1/2}$

7. Consider the state in which $l = 4, s = 1/2$. The orientation of total angular momentum w.r.t z-axis for the state with largest possible $j, m_j$, is

(a) $25.2^0$ (b) $39.8^0$ (c) $51.2^0$ (d) $74.8^0$

8. Which of the following statements is CORRECT?
The difference of spin-orbit correction to energy between the spin-down and spin-up electron in a hydrogen like atom will

(a) increase with increase in atomic number.
(b) decrease with the principal quantum number
(c) increase with principal quantum number
(d) decrease with orbital quantum number

9. In the quantum mechanical relativistic correction to energy of an electron in a hydrogen like atom, the change in the hamiltonian corresponding to relativity is

(a) $\frac{p^4}{8m_0^3c^2}$ (b) $-\frac{p^4}{8m_0^3c^2}$ (c) $-\frac{p^4}{16m_0^3c^2}$ (d) $-\frac{p^4}{4m_0^3c^2}$

10. The number of allowed transitions in the fine structure of $H_\alpha$ line is
(a) 3 (b) 4 (c) 5 (d) 6

11. Find the gound state term of the following atoms:
(i) ${}^{14}_{7}N$ (ii) ${}^{12}_{6}C$ (iii) ${}^{16}_{8}O$ (iv) ${}^{24}_{11}Na$ (v) $C^*$ (vi) $Na^*$

The electronic configuration of ${}_{25}Mn$ atom is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^2$.

12. The ground state term of Mn atom will be
(a) ${}^6S_{5/2}$ (b) ${}^6P_{5/2}$ (c) ${}^1S_0$ (d) ${}^3P_0$

13. The ground state term of $Mn^{3+}$ ion will be
(a) ${}^5D_0$ (b) ${}^5D_4$ (c) ${}^6S_{5/2}$ (d) ${}^6P_{5/2}$

14. How does the total angular quantum number *J* change in the transition $Cr\left(3d^6\right)$ atom ionizes to $Cr^{2+}\left(3d^4\right)$?
(a) Increases by 2 (b) Decreases by 2 (c) Decreases by 4 (d) Does not change

15. If the doublet splitting of the first excited state $2^2D_{5/2} - 2^2D_{3/2}$ of $He^+$ is 3.67 cm$^{-1}$, then the corresponding separation of H atom is
(a) 0.23 cm$^{-1}$ (b) 0.36 cm$^{-1}$ (c) 0.52 cm$^{-1}$ (d) 0.76 cm$^{-1}$

16. Which of the following is true for the correction to energy in the estimation of the ground state energy of the following atoms due to coloumbian interactions between electrons?
(a) $\Delta E_{He} < \Delta E_{Li^+} < \Delta E_{Be^{++}} < \Delta E_{B^{+++}}$ (b) $\Delta E_{He} > \Delta E_{Li^+} > \Delta E_{Be^{++}} > \Delta E_{B^{+++}}$
(c) $\Delta E_{He} = \Delta E_{Li^+} > \Delta E_{Be^{++}} > \Delta E_{B^{+++}}$ (d) $\Delta E_{He} > \Delta E_{Li^+} > \Delta E_{Be^{++}} = \Delta E_{B^{+++}}$

17. Why triplet energy states lie lower than the singlet energy states in He atom?

18. Why $1^1S_0$ is not the ground state of orthohelium?

19. Why $2^3S_1$ and $2^1S_0$ states in Helium are highly metastable?

20. The spin-orbit interaction in an atom is given by $H = a\vec{L}.\vec{S}$, where $\vec{L}$ and $\vec{S}$ denote the orbital and spin angular momenta respectively, of the electron. The splitting between the levels ${}^2P_{3/2}$ and ${}^2P_{1/2}$ is
(a) $\frac{3}{2}a\hbar^2$ (b) $\frac{1}{2}a\hbar^2$ (c) $3a\hbar^2$ (d) $\frac{5}{2}a\hbar^2$

## ANSWER KEY

**1.(b)** **2.(d)** **3.(a)** **4.(c)** **5.(d)** **6.(d)**

**7.(a)** **8.(c)** **9.(b)** **10.(c)** **12.(a)** **13.(a)**

**14.(c)** **15.(a)** **16.(b)** **20.( )**

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: ATOMIC & MOLECULAR PHYSICS

### Assignment 4: Rotational Spectra

**Common Data Questions Q.1 and Q.2**

The $J=0 \rightarrow J=1$ absorption line in CO occurs at a frequency of $1.153\times10^{11}\,s^{-1}$.

(Given: The reduced mass of CO molecule is $1.14\times10^{-26}\,kg$)

1. The moment of inertia of CO molecule is

(a) $0.8\times10^{-46}\,kg-m^2$ (b) $1.5\times10^{-46}\,kg-m^2$ (c) $0.8\times10^{-42}\,kg-m^2$ (d) $1.5\times10^{-42}\,kg-m^2$

2. The internuclear separation of CO molecule is

(a) $0.48A^0$ (b) $0.84A^0$ (c) $1.15A^0$ (d) $1.95A^0$

3. The $J=0 \rightarrow J=1$ transition line occurs at a frequency of $1.153\times10^{11}\,s^{-1}$ in ${}^{12}C^{16}O$ and at a frequency of $1.102\times10^{11}\,s^{-1}$ in ${}^{?}C^{16}O$. The mass number of the unknown isotope of carbon is

[✓ (a) 13] (b) 14 (c) 15 (d) 16

**Common Data Questions Q.4, Q.5 and Q.6**

The separation of lines in the far infra red spectrum of HCl is 20.68 $cm^{-1}$.

(Given: The reduced mass of HCl molecule is $1.62\times10^{-27}\,kg$)

4. The $J=14 \rightarrow J=15$ transition occurs at

(a) $103.4\,cm^{-1}$ (b) $206.8\,cm^{-1}$ [✓ (c) $310.2\,cm^{-1}$] (d) $413.6\,cm^{-1}$

5. The moment of inertia of the HCl molecule is

(a) $1.34\times10^{-40}\,kg-m^2$ (b) $2.7\times10^{-40}\,kg-m^2$ (c) $1.34\times10^{-47}\,kg-m^2$ (d) $2.7\times10^{-47}\,kg-m^2$

6. The internuclear distance of HCl molecule is

(a) $0.48A^0$ (b) $0.84A^0$ (c) $1.30A^0$ (d) $1.95A^0$

7. The wave numbers of the lines in a band are given by

$$\begin{aligned} \nu &= 1000(2n-1) \quad \text{for positive } 'n' \\ &= -1000(2n-1) \quad \text{for negative } 'n' \end{aligned}$$

The moment of inertia of the emitter molecule of the spectrum is

(a) $2.8\times10^{-49}\,kg-m^2$ (b) $5.6\times10^{-49}\,kg-m^2$ (c) $8.4\times10^{-49}\,kg-m^2$ (d) $11.2\times10^{-49}\,kg-m^2$

8. The rotational inertia of $H_2$ molecule is $4.5\times10^{-48}\,kg-m^2$. The temparature at which the average kinetic translational kinetic energy of an $H_2$ molecule will be equal to energy difference between the ground rotational state and the first excited rotational state, will be

(a) 60 K (b) 120 K (c) 180 K (d) 240 K

9. Which one of the following molecule does not exhibit rotational spectra?

(a) $CCl_4$ (b) $HF$ (c) $HBr$ (d) $O_2$

10. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the molecules are equal, then the corresponding values of the roational quantum numbers $J_A$ and $J_B$ are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

## ANSWER KEY

**1. (b)** **2. (c)** **3. (a)** **4. (c)** **5. (d)** **6. (c)**

**7. (a)** **8. (b)** **9. (d)** **10. (b)**

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: ATOMIC & MOLECULAR PHYSICS

### Assignment 5: Vibrational Spectra & Raman Spectra

**Common Data Questions Q.1, Q. 2 and Q. 3**

The force constant of the bond in CO molecule is 1870 $Nm^{-1}$.

(Given: The reduced mass of CO molecule is $1.14\times10^{-26}kg$)

1. The frequncy of vibration of the CO molecule is

(a) $6.45\times10^{13}s^{-1}$ (b) $9.45\times10^{13}s^{-1}$ (c) $9.45\times10^{11}s^{-1}$ (d) $6.45\times10^{11}s^{-1}$

2. The energy of lowest vibartional level of CO molecule is

(a) 0.07 eV (b) 0.13 eV (c) 0.26 eV (d) 0.39 eV

3. The spacing between the vibrational enrgy levels of CO molecule is

(a) 0.07 eV (b) 0.13 eV (c) 0.26 eV (d) 0.39 eV

4. The force constant of the vibrating HCl molecule is 480 $Nm^{-1}$. The amplitude of vibration of HCl molecule in the first excited vibrational level will be

(Given: The reduced mass of HCl molecule is $1.62\times10^{-27}kg$)

(a) 0.19 $A^0$ (b) 0.28 $A^0$ (c) 0.42 $A^0$ (d) 0.59 $A^0$

5. The vibrational frequency and the anharmonicity constant of HD molecule are $3817\ cm^{-1}$ and $95\ cm^{-1}$ respectively. The zero point energy of HD molecule in the units of kcal/mole, is

(Given: $1\ cm^{-1}=2.858\times10^{-3}$ kcal/mole)

(a) 1.25 (b) 2.47 (c) 3.39 (d) 5.39

**Common Data Questions Q. 4 and Q. 5**

The fundamental band and the first overtone for CO molecule is centered at 2143.3 $cm^{-1}$ and 4259.7 $cm^{-1}$ respectively. (Given: The reduced mass of CO molecule is $1.14\times10^{-26}kg$)

6. The vibrational frequency of CO molecule is

(a) $2170\ cm^{-1}$ (b) $2467\ cm^{-1}$ (c) $1108\ cm^{-1}$ (d) $3456\ cm^{-1}$

7. The anharmonicity constant of CO molecule will be

(a) $6.8\ cm^{-1}$ (b) $10.2\ cm^{-1}$ (c) $13.5\ cm^{-1}$ (d) $19.5\ cm^{-1}$

8. The fundamental mode of HCl occurs at $2886\ cm^{-1}$. The fundamental mode of DCl will occur

(a) $1367\ cm^{-1}$ (b) $2069\ cm^{-1}$ (c) $2778\ cm^{-1}$ (d) $3024\ cm^{-1}$

9. The moment of inertia of the $HCl^{35}$ molecule in the $v=0$ and $v=1$ levels is $20.8\times10^{-7}kg-m^2$. The wave number difference between the $R(0)$ and $P(1)$ lines of the fundamemtal band for $HCl^{35}$ molecule is

(a) 4164 $m^{-1}$ (b) 1041 $m^{-1}$ (c) 2082 $m^{-1}$ (d) 5205 $m^{-1}$

10. In an experiment, the exciting line of of a sample is observed at 5460 A$^0$ and stokes line is ob served at 5520 A$^0$. The wavelength of the anti-stokes line is

(a) 5420 A$^0$ (b) 5401 A$^0$ (c) 5380 A$^0$ (d) 5580 A$^0$

11. In th vibrational Raman spectrum of HF molecule, the Raman lines are observed at wavelengths 2670 A$^0$ and 3430 A$^0$ respectively. The fundamental vibrational frequency of the molecule is

(a) $0.62\times10^{14}$ *Hz* (b) $1.02\times10^{14}$ *Hz* [✓ (c) $1.24\times10^{14}$ *Hz*] (d) $1.62\times10^{14}$ *Hz*

12. In the Rotational Raman spectrum of *HCl* molecule, the displacemnts from the exciting line are represented by

$$\Delta\bar{v} = \pm(120+80J)\ cm^{-1}$$

The moment of inertia of the molecule is

(a) $1.4\times10^{-27}\ kg-m^2$ (b) $2.8\times10^{-27}\ kg-m^2$

(c) $4.2\times10^{-27}\ kg-m^2$ (d) $5.6\times10^{-27}\ kg-m^2$

## ANSWER KEY

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.(a)** | **2.(b)** | **3.(c)** | **4.(a)** | **5.(d)** | **6.(a)** | **7.(c)** |
| **8.(b)** | **9.(a)** | **10.(b)** | **11.(c)** | **12.(d)** | | |

# CSIR-NET/JRF & GATE

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# Assignment

## Nuclear & Particle Physics

## PHYSICAL SCIENCES

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South Delhi Centre: 28-A/11, Jia Sarai, Near-IIT Hauz Khas, New Delhi-110 016
T: 011-26851008, 26861009, (0) 9873744231, 9899628311

North Delhi Centre: 48, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T: 011-27241845, 65462244, (0) 9540292991
E: info@careerendeavour.com W: www.careerendeavour.com

### PHYSICAL SCIENCES (NET-JRF/GATE)
### Assignment-1: Nuclear Properties

1. The deviation of the charge distribution of a nucleus from spherical symmetry can be estimated by measuring its:
(a) electric charge
(b) electric dipole
(c) magnetic dipole moment
[✓ (d) electric quadrupole moment]

2. The ground state of the deutron is a:
(a) pure $^3S_1$ and $^3P_1$ states
(b) pure $^3P_1$ state
(c) mixture of $^3S_1$ and $^3P_1$ states
(d) mixture of $^3S_1$ and $^3D_1$ states

3. Nuclei which are β-emitters lie:
(a) below the line of β stability
(b) on the line of β stability
[✓ (c) above the line of β stability]
(d) below the N = z lines

4. The binding energy per nucleon is maximum for the nucleus:
[✓ (a) $^{56}$Fe]
(b) $^4$He
(c) $^{208}$Pb
(d) $^{101}$Mo

5. According to the nmeson theory of nuclear forces:
[✓ (a) a neutron emits a ($\pi^-$) meson and is converted into a proton.]
(b) a neutron emits a ($\pi^0$) meson and is converted into a proton.
(c) a neutron emits a ($\pi^+$) meson and is converted into a proton.
(d) a neutron cannot be converted into a proton.

6. The mean momentum of a nucleon in a nucleus with mass number A varies as:
(a) A
(b) $A^2$
[✓ (c) $A^{-2/3}$]
(d) $A^{-1/3}$

7. Identify the reaction which has the same transition probability as $\pi^+ + p \rightarrow \pi^+ + p$
(a) $\pi^+ + n \rightarrow \pi + + n$
(b) $p^- + p \rightarrow \pi^0 + n$
(c) $\pi + n \rightarrow \pi + n$
(d) $p^0 + p \rightarrow \pi^+ + n$

8. Nuclear forces are:
[✓ (a) spin dependent and have no non-centrla part]
(b) spin dependent and have a non-central part
(c) spin independent and have no non-central part
(d) spin independent and have a non-central part

9. The asymmetry terms in the Weizsacker semi-empirical mass formula is because of:
(a) non-spherical shape of the nucleus
(b) non-zero spoin of nucleus
[✓ (c) unequal number of protons and neutrons inside the nucleus]
(d) odd number of protons inside the nucleus

10. The masses of a hydroge atom, neutron and $^{238}U_{92}$ are given by 1.0078, 1.0087 and 238.0508 respectively. The binding energy of $^{238}U_{92}$ is therefore approximately equal to (taking 1 a.m.u. = 931.64 MeV)
(a) 120 MeV
(b) 1500 MeV
(c) 1600 MeV
[✓ (d) 1800 MeV]

11. As one moves along the line of stability from $^{56}$Fe to $^{235}$U nucleus, the nuclear binding energy per particle decreases frokm about 8.8 MeV to 7.6 MeV. This trend is mainly due to the:
(a) short range nature of the nuclear forces
(b) long range nature of the coulomb forces
(c) tensor nature of the nuclear forces
(d) spin dependence of the nuclear forces

12. Although mass-energy equivalence of special relatively allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because:
(a) the mass is not conserved (b) the energy is not conserved
(c) the momentum is not conserved (d) the charge is not conserved

13. Which of the following expressions for total binding energy B of a nucleus is correct ($a_1, a_2, a_3, a_4 > 0$)

(a) $B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

(b) $B = a_1 A + a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

(c) $B = a_1 A + a_2 A^{1/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

(d) $B = a_1 A + a_2 A^{1/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

14. With reference to nuclear forces which of the following statements is NOT true? The nuclear forces are:
Flase
(a) short range (b) charge independent
(c) velocity dependent (d) spin independent

15. The order of magnitude of the binding energy per nucleon in a nucleus is:
(a) $10^{-5}$ MeV (b) $10^{-3}$ MeV (c) 0.1 MeV (d) 10 MeV

16. The following gives a list of pairs containing (i) a nucleus (ii) one of its properties. Find the pair which is inappropriate:
(a) (i) $_{10}Ne^{20}$ nucleus; (ii) stable nucleus
(b) (i) A spheroidal nucleus; (ii) an electric qudrupole moment
(c) (i) $_8O^{16}$ nucleus; (ii) nuclear spin $J = \frac{1}{2}$
(d) $U^{238}$ nucleus; (ii) Binding energy = 1785 MeV (approximately)

17. The mass difference between the pair of minor nuclei $_6C^{11}$ and $_5B^{11}$ is given to be ΔMeV/c². According to the semi-empirical mass formula, the mass difference between the pair of mirror nuclei $_9F^{17}$ and $_8O^{17}$ will approximately be (rest mass of proton $m_p = 938.27$ MeV/c² and rest mass ofeutron
mn = 939.57 MeV/c²)
(a) 1.39Δ MeV/c² (b) (1.39Δ + 0.5) MeV/c²
(c) 0.86Δ MeV/c² (d) (1.6Δ + 0.78) MeV/c²

18. A heavy nucleus is found to contain more neutrons than protons. This fact is related to which one of the following statement?
(a) The nuclear force between neutrons is stronger than that between protons
(b) The nuclear force between protons is of a shorter range than those between neutrons, so that a smaller number of protons are held together by the nuclear force.
(c) Protons are unstable, so their number in a nucleus diminishes
[✓ (d) It costs more energy to add a proton to a (heavy) nucleus than a neutron because of the Coulomb repulsion between protons.]

19. Binding energy per nucleon is max. for
(a) $He^4$ (b) $Ca^{40}$ [✓ (c) $Fe^{56}$] (d) $Pb^{206}$

20. The nuclear density in kg/mm$^3$ is:
(a) $10^{17}$ (b) $10^{13}$ [✓ (c) $10^8$] (d) $10^{10}$

21. The binding energy per nucleon is approximately constant this leads to :
[✓ (a) saturation property of nuclear forces] (b) short range nature
(c) non-central nature (d) spin dependency.

22. For a nucleus to possen quadrupole moment :
(a) $I = 0$ (b) $I = \frac{1}{2}$ [✓ (c) $I > \frac{1}{2}$] (d) spherical shape of nucleus
I = nuclear spin

23. Number of nucleon/m$^3$ in a nucleus is :
(a) $10^{40}$ [✓ (b) $10^{44}$] (c) $10^{50}$ (d) $10^{49}$

24. The ratio of radii of the nuclei $He^4$ & $S^{32}$ is :
(a) $\frac{2}{1}$ (b) $\frac{3}{2}$ [✓ (c) $\frac{1}{2}$] (d) $\frac{4}{1}$

25. The expression of $a_c$ is:
(a) $\frac{3e^2}{4\pi \in_0 R_0}$ (b) $\frac{3e^2}{10\pi \in_0 R_0}$ [✓ (c) $\frac{3e^2}{20\pi \in_0 R_0}$] (d) $\frac{e^2}{20\pi \in_0 R_0}$

26. The energy of $e^-$ inside the nucleus (in $p - e^-$ hypothesis) is:
(a) 4 M eV [✓ (b) 20 M eV] (c) 13 M eV (d) 16 M eV.

27. Two protons are placed at a distane of about $10^{-13}$ cm from each other. The ratio of the strength of strong and electromagnetic forces between them is roughly
(a) 10 (b) 1 [✓ (c) $10^3$] (d) $10^{-5}$.

28. Which of the following is not an acceptable potential for the deutron $(V_0 > 0; k, \mu > 0)$?
(a) $V(r) = -V_0 e^{-\mu r}$
(b) $V(r) = -V_0$ for $r < R_0$
$= 0$ for $r > R_0$
[✓ (c) $V(r) = -\frac{1}{2}kr^2$]
(d) $V(r) = \frac{-V_0}{1+e^{\mu r}}$

29. The Coulomb repulsion term which contributes to the binding energy of a nucleus ${}^A X_Z$ is proportional to
(a) Z (b) $Z^2$ [✓ (c) Z(Z–1)] (d) It is independent of Z.

30. The binding energy of a nucleus with atomic number Z and atomic mass number A is given by the semi-empirical mass formula. $B(Z,A)=aA-bA^{2/3}-c\frac{(A-2Z)^2}{A}-d\frac{Z^2}{A^{1/3}}-\frac{\delta}{A^{1/2}}$

Where $a=15.8\ MeV, b=18.3, c=23.2\ MeV, d=0.7\ MeV$ and $\delta=(+11.2, 0, -11.2)\ MeV$ for nuclei which are (odd-odd-odd-even, even-even). If we consider all possible isobars of mass number 216, the most stable nuclide is

(a) ${}^{216}_{82}Pb$ (b) ${}^{216}_{87}Fr$ (c) ${}^{216}_{85}At$ (d) ${}^{216}_{86}Rn$

31. The semi-empirical mass formula for the binding energy of nucleus contains a surface correction term. This term depends on the mass number A of the nucleus as

(a) $A^{-1/3}$ (b) $A^{1/3}$ (c) $A^{2/3}$ (d) A

32. A neutron passing through a detector is detected because of
(a) The ionization it produces
(b) The scintillation light it produces
(c) The electron - hole pairs it produces
(d) The secondary particles produced in a nuclear reaction in the detector medium.

33. The radius of a ${}^{64}_{29}\mathrm{Cu}$ nucleus is measured to be $4.8\times10^{-13}$ cm.

**(A)** The radius of a ${}^{27}_{12}\mathrm{Mg}$ nucleus can be estimated to be

(a) $2.86\times10^{-13}$ cm (b) $5.2\times10^{-13}$ cm (c) $3.6\times10^{-13}$ cm (d) $8.6\times10^{-13}$ cm

**(B)** The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number A in its ground state varies as:

(a) $A^{4/3}$ (b) $A^{1/3}$ (c) $A^{-1/3}$ (d) $A^{-2/3}$

34. The difference in the Coulomb energy between the mirror nuclei ${}^{49}_{24}\mathrm{Cr}$ and ${}^{49}_{25}\mathrm{Mn}$ is 6.0 MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that $e^2$ is approximately 1.0 MeV-fm, the radius of the ${}^{49}_{25}\mathrm{Mn}$ nucleus is

(a) $4.9\times10^{-13}$m (b) $4.9\times10^{-15}$m (c) $5.1\times10^{-13}$m (d) $5.1\times10^{-15}$m

35. An atom of mass M can be excited to a state of mass $M+\Delta$ by photon capture. The frequency of a photon which can cause this transition is:

(a) $\Delta c^2/2h$ (b) $\Delta c^2/h$ (c) $\Delta^2c^2/2Mh$ (d) $\Delta(\Delta+2M)c^2/2Mh$

36. A proton is accelerated to a high energy E and shot at a nucleus of Oxegen $\left({}^{16}_{8}\mathrm{O}\right)$. In order to penetrate the Coulomb barrier and reach the surface of the Oxgen nucleus, E must be at least

(a) 3.6 MeV (b) 1.8 MeV (c) 45 keV (d) 180 eV

37. A experimental evidence for energy levels in nuclei is
(a) equality of binding energy of the mirror nuclei $H^3$ and $He^3$
(b) continuous energy spectrum in $\beta$-decay
(c) discrete energy specimen in $\alpha$ – decay
(d) different scattering lengths for spin triplet and spin singlet scattering

38. Which of the following provides an estimate of the size of a nucleus?
(a) high energy proton-proton scattering (b) low energy neutron-proton scattering
[✓ (c) scattering of high energy electrons by nuclei] (d) constant binding energy per nucleon

39. The experimentally measured nuclear charge density distribution $\rho$ is approximated by the function:
(a) $\rho=\rho_0\left(1+e^{(r-R)/a}\right)$ [✓ (b) $\rho=\frac{\rho_0}{1+e^{(r-R)/a}}$] (c) $\rho=\rho_0 e^{-r/R}$ (d) $\rho=\frac{\rho_0}{r}e^{-r/R}$

40. The large abundance of $^{56}Fe$ is because its nucleus has
[✓ (a) the maximum nuclear binding energy] (b) a large pairing energy
(c) a small symmetry energy (d) an even number of protons and neutrons

41. A deuteron having a binding energy $E_B$ can be dissociated into a proton and a neutron by a gamma ray photon of energy E. The following i correct.
(a) E should be less than $E_B$ [✓ (b) E should be exactly equal on $E_B$]
(c) does not depend on $E_B$ (d) E should be more than $E_B$

42. Which of the following does NOT provide evidence for the short range nature of the nuclear force?
(a) Constant binding energy per nucleon [✓ (b) High energy electron nucleus scattering]
(c) Low energy neutron proton scattering (d) Saturation property of nuclear forces

43. The ground state of deuteron is taken as an admixture of s and d states. The expression for the electric quadruple moment of the deuteron contains a cross term in s and d states because
(a) The nuclear force is short-ranged
(b) The nuclear force has a spin dependence
[✓ (c) The quadrupole operator has a terms of the form $Y_{2m}(\theta,\phi)$]
(d) The nuclear force has an exchange character

44. Deuteron has only one bound state with spin parity $1^+$, isospin 0 and electric quadrupole moment 0.286 efm$^2$. These data suggest that the nuclear forces are having
(a) only spin and isospin dependence (b) no spin dependence and no tensor components
(c) spin dependence but no tensor components [✓ (d) spin dependence along with tensor components]

45. Total binding energies of $O^{15}, O^{16}$ and $O^{17}$ are 111.96 MeV, 127.62 MeV and 131.76 MeV, respectively. The energy gap between $^1p_{1/2}$ and $^1d_{5/2}$ neutron shells for the nuclei whose mass number is close to 16, is:
(a) 4.1 MeV (b) 11.5 MeV (c) 15.7 MeV (d) 19.8 MeV

46. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A,Z)\approx 16A-20A^{2/3}-\frac{3}{4}Z^2A^{-1/3}\overset{-}{+}30\frac{(N-Z)^2}{A}$$

where N = A – Z is the neutron number. The value of Z of the most stable isobar for a given A is

[✓ (1) $\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$] (2) $\frac{A}{2}$ (3) $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ (4) $\frac{A}{2}\left(1+\frac{A^{4/3}}{64}\right)$

47. The binding energy of the *k*-shell electron in a Uranium atom ($Z = 92, A = 238$) will be modified due to (i) screening caused by other electrons and (ii) the finite extent of the nucleus as follows:
(a) increases due to (i), remains unchanged due to (ii).
(b) decreases due to (i), decreases due to (ii).
(c) increases due to (i), increases due to (ii).
(d) decreases due to (i), remains unchanged due to (ii).

48. Which one of the following is a fermion?

(a) $\alpha$ particle (b) ${}_4Be^7$ nucleus (c) hydrogen atom (d) deuteron

**CAREER ENDEAVOUR**

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**PHYSICAL SCIENCES (NET-JRF/GATE)**

**Assignment-1: Nuclear Properties**

## [ANSWERS]

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (d)** | **2. (d)** | **3. (c)** | **4. (a)** | **5. (a)** | **6. (d)** | **7. (a)** |
| **8. (a)** | **9. (c)** | **10. (d)** | **11. (a)** | **12. (c)** | **13. (a)** | **14. (c)** |
| **15. (d)** | **16. (c)** | **17. (b)** | **18. (d)** | **19. (c)** | **20. (c)** | **21. (a)** |
| **22. (c)** | **23. (b)** | **24. (c)** | **25. (b)** | **26. (b)** | **27. (c)** | **28. (c)** |
| **29. (b)** | **30. (c)** | **31. (c)** | **32. (b)** | **33_A(c)** | **33_B (c)** | **34. (b)** |
| **35. (d)** | **36. (a)** | **37. (c)** | **38. (c)** | **39. (b)** | **40. (a)** | **41. (b)** |
| **42. (b)** | **43. (c)** | **44. (d)** | **45. (d)** | **46. (b)** | **47. ()** | **48. (b)** |

## PHYSICAL SCIENCES (NET-JRF/GATE)

### Assignment-2: Nuclear Structure

1. The angular momentum and parity of $^{17}_{8}O$' nucleus, according toi the nuclear shell model (including spin-orbit coupling) is:
(a) $0^+$ (b) $1/2^-$ (c) 3/2+ (d) $5/2^+$

2. According to the shell model of the nucleus
(a) magic numbers exist.
(b) nucleons interact with their nearest neighbours only.
(c) nucleons in a nucleus interact with a general force field.
(d) large electron quadrupole moment exists for certain nuclei.

3. The nuclear spins of $_6C^{14}$ and $_{12}Mg^{25}$ nuclei and respectively:
(a) zero and half-integer (b) half-integer and zero
(c) an integer and half-integer (d) both half-integers

4. The spin and parity of $_4Be^9$ nucleus, as predicted by the shell model, are respectively:
(a) 3/2 and odd (b) 1/2 and odd (c) 3/2 and even (d) 1/2 and even

5. The ground state angular momentum and parity of $^{41}Ca$ are:
(a) $\left(\frac{7}{2}\right)^-$ (b) $\frac{3^+}{2}$ (c) $\frac{5^+}{2}$ (d) $\frac{5^-}{2}$

6. According to the shell model, the ground state of $^{15}_{8}O$ nucleus is:
(a) $\frac{3^+}{2}$ (b) $\frac{1^+}{2}$ (c) $\frac{3^-}{2}$ (d) $\frac{1^-}{2}$

7. According to the shell model the ground state spin of the $^{17}O$ nucleus is:
(a) $\frac{3^+}{2}$ (b) $\frac{5^+}{2}$ (c) $\frac{3^-}{2}$ (d) $\frac{5^-}{2}$

8. The four possible configurations of neutrons in the ground state of $_4Be^9$ nucleus, according to the shell model, and the associated nuclear spin are listed below. Choose the correct one:
(a) $(1s_{1/2})^2(1P_{3/2})^3;\ J=\frac{3}{2}$ (b) $1(s_{1/2})^2(1P_{1/2})^2(1p_{3/2})^1;\ J=\frac{3}{2}$
(c) $(1s_{1/2})^2(1P_{3/2})^4;\ J=\frac{1}{2}$ (d) $1(s_{1/2})^2(1P_{3/2})^2(1p_{1/2})^1;\ J=\frac{1}{2}$

9. The ground state of deutron is:
(a) pure $^3s_1$ state (b) pure $^3p_1$ state
(c) mixture of $^3s_1$ & $^3p_1$ (d) mixture of $^3s_1$ & $^3D_1$ states.

10. Which one of the following is not a magic no.
(a) 8 (b) 10 (c) 20 (d) 28

11. The angular momentum & parity for ${}_{20}\mathrm{Ca}^{41}$ nucleus, according to the nuclear shell model is:

(a) $\frac{5^-}{2}$ (b) $\frac{3^+}{2}$ (c) $\frac{1^-}{2}$ (d) $\frac{7^-}{2}$

12. According to the single particle nuclear shell model, the spin-parity of the ground state of ${}^{17}_{8}O$ is:

(a) $\frac{1^-}{2}$ (b) $\frac{3^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{5^+}{2}$

13. According to the shell model the spin and parity of the two nuclei ${}^{125}_{51}\mathrm{Sb}$ and ${}^{89}_{38}\mathrm{Sr}$ are, respectively.

(a) $\left(\frac{5}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$ (b) $\left(\frac{5}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$ (c) $\left(\frac{7}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$ (d) $\left(\frac{7}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$

14. A positive parity shell model state can accommodate 14 nucleons. Its total angular momentum j and orbital angular momentum *l* values will be, respectively

(a) 13/2 and 6 (b) 13/2 and 7 (c) 11/2 and 6 (d) 11/2 nd 5

15. Inclusion of a positive parity shell model state modifies the magic number from 40 to 50. The state has a total angular momentum j and orbital angular momentum *l*. The values of j , *l* are given by

(a) $\frac{9}{2}, 4$ (b) $\frac{7}{2}, 4$ (c) $\frac{9}{2}, 4$ (d) $\frac{11}{2}, 6$

16. The single particle enertgy difference between the p-orbitals (i.e. $p_{3/2}$ and $p_{1/2}$) of the nucleus ${}^{114}_{50}\mathrm{Sn}$ is 3 MeV. The energy difference between the states in its $1f$ orbital is

(a) –7 MeV (b) 7 MeV (c) 5 MeV (d) – 5 MeV

17. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range $r_0$. Given that, $\cot(kr_0 + \delta) \approx -\frac{\gamma}{k}$ where $\delta$ is the phase shift, $k$ the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

(a) $\delta \approx -\frac{k}{\gamma} - kr_0$ (b) $\delta \approx -\frac{\gamma}{k} - kr_0$ (c) $\delta \approx \frac{\pi}{2} - kr_0$ (d) $\delta \approx -\frac{\pi}{2} - kr_0$

18. The electromagnetic form factor $F(q^2)$ of a nucleus is given by, $F(q^2) = \exp\left|-\frac{q^2}{2Q^2}\right|$ where $Q$ is a constant.

Given that $F(q^2) = \frac{4\pi}{q}|_0^\infty\, r dr\, \rho(r) \sin qr \quad |d^3 r \rho(r) = 1$, where $\rho(r)$ is the charge density, the root mean square radius of the nucleus is given by

(a) $1/Q$ (b) $\sqrt{2}/Q$ (c) $\sqrt{3}/Q$ (d) $\sqrt{6}/Q$

**Linked Answer Questions**

**Statement for Linked Answer Q.19 and Q.20:** In the Schmidt model of nuclear magnetic moments, We have,

$$\vec{\mu} = \frac{e\hbar}{2Mc}(g_l\vec{l} + g_s\vec{S})$$

where the symbols have their usual meaning

19. For the case $J = l + 1/2$, where $J$ is the total angular momentum, the expectation value of $\vec{S}.\vec{J}$ in the nuclear ground state is equal to,

(a) $(J-1)/2$ (b) $(J+1)/2$ (c) $J/2$ (d) $-J/2$

20. For the $O^{17}$ nucleus (A = 17, Z = 8), the effective magnetic moment is given by,

$$\vec{\mu}_{eff} = \frac{e\hbar}{2Mc} g\vec{J}$$

where $g$ is equal to, ($g_s$ = 5.59 for proton and –3.83 for neutron)

(a) 1.12 (b) –0.77 (c) –1.28 (d) 1.28

21. A nucleus X undergoes a first forbidden $\beta$-decay to a nucleus Y. If the angular momentum (I) and parity (P), denoted by $I^P$ as $\frac{7^-}{2}$ for X, which of the following is a possible $I^P$ value for Y?

(a) $\frac{1^+}{2}$ (b) $\frac{1^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{3^-}{2}$

22. The recently-discovered Higgs boson at hte LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are 125 GeV/$c^2$ and 90 GeV/$c^2$ respectively, and athe decaying Higgs particle is at rest, the energy of the photon will approximately be

(a) $35\sqrt{3}\ GeV$ (b) 35 GeV (c) 30 GeV (d) 15 GeV

23. A permanently deformed even-even nucleus with $J^P = 2^+$ has rotational energy 93 keV. The energy of the next excited state is

(a) 372 keV (b) 310 keV (c) 273 keV (d) 186 keV

24. Let $y = \frac{1}{2}(x_1 + x_2) - \mu$, where $x_1$ and $x_2$ are independent and identically distributed Gaussian random variables of mean μ and standard deviation $\sigma$. Then $\frac{\langle y^4 \rangle}{\sigma^4}$ is

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

PHYSICAL SCIENCES (NET-JRF/GATE)

Assignment-2: Nuclear Structure

## [ANSWERS]

**1. (b)** **2. (a)** **3. (a)** **4. (a)** **5. (a)** **6. (d)** **7. (b)**

**8. (a)** **9. (d)** **10. (b)** **11. (d)** **12. (d)** **13. (c)** **14. (a)**

**15. (c)** **16. (b)** **17. (a)** **18. (c)** **19. (b)** **20. (b)** **21. (c)**

**22. (c)** **23. (b)** **24. (b)**

### PHYSICAL SCIENCES (NET-JRF/GATE)

### Assignment-3: Nuclear Reactions

1. Typical energies released in a nuclear fission and a nuclear fusion rection are respectively:
(a) 50 MeV and 1000 MeV (b) 200 MeV and 1000 MeV
(c) 1000 MeV and 50 MeV (d) 200 MeV and 10 MeV

**Data for Q.2 and Q.3**
An atomic bomb consisting of $^{235}U$ explodes and release an energy of $10^{14}$ J. It is known that each $^{235}U$ which undergoes fusion release 3 neutrons and about 200 MeV of energy. Further, only 20% of the $^{235}U$ atoms in the bomb undergo fission.

2. The total number of neutrons released is about:
(a) $4.7 \times 10^{24}$ (b) $9.7 \times 10^{24}$ (c) $1.9 \times 10^{25}$ (d) $3.7 \times 10^{25}$

3. The mass of $^{235}U$ in the bomb is about:
(a) 1.5 kg (b) 3.0 kg (c) 6.1 kg (d) 12 kg

4. The threshold temperature above which the thermonuclear reaction:
$$^3_2He +^3_2 He \to ^4_2 He + 2^1_1H + 12.86MeV$$
can occur is (use $e^2/4\pi\varepsilon_0 = 1.44 \times 10^{-15}$ MeVm)
(a) $1.28 \times 10^{10}$ K (b) $1.28 \times 10^{9}$ K (c) $1.28 \times 10^{8}$ K (d) $1.28 \times 10^{7}$ K

5. A neutron scatters elasticity from a heavy nucleus. The initial and final states of the neutron have the:
(a) same energy (b) same energy and linear momentum
(c) same energy and angular momentum (d) same linear and angular momentum

6. A neutral pi meson ($\pi^0$) has a rest-mass of approximately 140 MeV/$c^2$ and lifetime of $\tau$ sec. A $\pi^0$ produced in the laboratory is found to decay after 1.25 $\tau$ sec. into two photons. Which of the following sets represents a possible set of energies of the two photons as seen in the laboratory?
(a) 70 MeV and 70 MeV (b) 35 MeV and 100 MeV
(c) 75 MeV and 100 MeV (d) 25 MeV and 150 MeV

7. The energy released in the fission of 1 kg uranium (approximately [in joule]):
(a) $10^{14}$ (b) $10^{17}$ (c) $10^{16}$ (d) $10^{10}$

8. Which one of the following reaction is allowed?
(a) $p \to n + e^+$ (b) $p \to e^+ + \bar{\nu}$ (c) $p \to \pi^+ + \gamma$ (d) $\bar{p} + n \to \pi^- + \pi^0$

9. The condition for spontaneous fission of a nucleus is :
(a) $\frac{Z^2}{A} < 50$ (b) $\frac{Z^2}{A} > 45$ (c) $\frac{Z}{A} > 45$ (d) $\frac{Z}{A} < 45$

10. The transmission probability of $\alpha$ – decay is :
(a) $e^{-K_2L}$ (b) $e^{-2K_2L}$ (c) $e^{-3K_2L}$ (d) $e^{2K_2L}$

11. In a nuclear reactor, Plutonium $\left({}^{239}_{94}\mathrm{Pu}\right)$ is used as fuel, releasing energy by its fission into isotopes of Barium $\left({}^{146}_{54}\mathrm{Ba}\right)$ and Strontium $\left({}^{91}_{38}\mathrm{Sr}\right)$ through the reaction

$$ {}^{239}_{49}\mathrm{Pu} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{146}_{56}\mathrm{Ba} + {}^{91}_{38}\mathrm{Sr} + 3\times{}^{1}_{0}\mathrm{n} $$

The binding energy (B.E.) per nucleon of each of these nuclides is given in the table below :

| Nuclide | ${}^{239}_{94}\mathrm{Pu}$ | ${}^{146}_{54}\mathrm{Ba}$ | ${}^{91}_{38}\mathrm{Sr}$ |
|---|---|---|---|
| B.E. per nucleon (MeV) | 7.6 | 8.2 | 8.6 |

Using this information, One can estimate the number of such fission reactions per second in a 100 MW reactor as

(a) $3.9\times10^{18}$ (b) $7.8\times10^{18}$ (c) $5.2\times10^{19}$ (d) $5.2\times10^{18}$ (e) $8.9\times10^{17}$

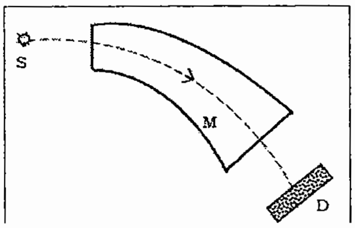
12. Muons are produced through the annihilation of particle a and its antiparticle, namely the process $a+\bar{a}\rightarrow\mu^{+}+\mu^{-}$. A muon has a rest mass of 105 MeV/c$^2$ and its proper life time is 2 $\mu$s. If the center of mass energy of the collision is 2.1 GeV in the laboratory frame that coincides with the center-of-mass frame, then the fraction of muons that will decay before they reach a detector placed 6km away from the interaction point is

(1) $e^{-1}$ (2) $1-e^{-1}$ (3) $1-e^{-2}$ (4) $e^{-10}$

13. A spin-1/2 particle A undergoes the decay $A\rightarrow B+C+D$ where it is known that B and C are also spin-1/2 particles. The complete set of allowed values of the spin of the particle D is

(1) $\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2},3,\ldots\ldots$ (2) 0, 1 (3) $\frac{1}{2}$ only (4) $\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},\ldots$

14. In a beta decay experiment, an electromagnetic M and a detector D are used to measure the energy of electrons $\left(\beta^{-}\right)$, as shown in the figure



The detector D is capable of detecting either electons $\left(\beta^{-}\right)$ or positrons $\left(\beta^{+}\right)$. Now the $\beta^{-}$ source is replaced with a $\beta^{+}$ source, and we would like to measure the energy of the positrons $\left(\beta^{+}\right)$ using the same setup. Which of the following is correct?

(a) This can be done quite easily, if the polarity of current in the coils of the electromagnet is reversed.

(b) This is can done trivially, without changing anything, since the detector D can detect either $\beta^{-}$ or $\beta^{+}$.

(c) There is no way to do this with the given set up, since $\beta^{+}$ will have to be converted into $\beta^{-}$, which is obviously not possible.

(d) This cannot be done since the magnet does not have a symmetric shape.

15. It is well-known that the energy of the Sun arises from the fusion of hydrogen nuclei (protons) inside the core of the Sun. This takes place through several mechanisms, each resulting in emission of energy. Which of the following reactions is NOT possible during the proton fusion inside the Sun?

(a) $^1_1H + ^1_1H \longrightarrow ^2_2He$
(b) $^2_1H + ^1_1H \longrightarrow ^3_2He$
(c) $^1_1H + ^1_1H \longrightarrow ^2_1H + e^+ + \nu_e$
(d) $^1_1H + ^1_1H + ^1_1H + ^1_1H \longrightarrow ^4_1He + 2e^+$

16. Which one of the following high energy processes is allowed by conservation laws?

(a) $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$
(b) $\pi + p \rightarrow \pi^0 + n$
(c) $n \rightarrow p + e^- + \nu_e$
(d) $\mu^+ \rightarrow e^- + \gamma$

17. Which one of the following three-quark states (qqqq), denoted by X, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript

(a) $X^{++}$
(b) $X^{+}$
(c) $X^{-}$
(d) $X^{--}$

18. Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons ($\mu$) in the centre of mass frame required to produce the pion $(\pi)$ pairs at rest is ________ MeV.

(Given: $m_\mu = 105\ MeV/c^2$, $m_\pi = 140\ MeV/c^2$ )

# CAREER ENDEAVOUR

ACADEMY PRIVATE LIMITED

Institute for NET-JRF/GATE & IIT-JAM

**PHYSICAL SCIENCES (NET-JRF/GATE)**

**Assignment-3: Nuclear Reactions**

## [ANSWERS]

**1. (d)** **2. (b)** **3. (c)** **4. (c)** **5. (a)** **6. (c)** **7. (a)**
**8. (d)** **9. (b)** **10. (b)** **11. (a)** **12. (d)** **13. (d)** **14. (a)**
**15. (d)** **16. (b)** **17. (d)** **18. (34.9 to 35.1)**

## PHYSICAL SCIENCES (NET-JRF/GATE)

## Assignment-4: Radioactivity

1. The decay of free neutron, $n \to p + e^- + \bar{\nu}$
   (a) occurs in Nature with a half-life of about 1,000 seconds
   (b) does not occur because of energy conservation
   (c) does not occur because of strangeness conservation
   (d) does not violate baryon number conservation

2. The evidence for the nonconservation of parity in $\beta$ decay has been obtained from the observation that the $\beta$ intensity:
   (a) antiparallel to the nuclear spin directions is same as that along the nuclear spin direction
   (b) antiparallel to the nuclear spin diretion is not the same as that along the nuclear spin direction
   (c) shows a continuous distribution as a function momentum
   (d) is independent of the nuclear spin direction

3. A nucleus having mass number 240 decays by a emission to the grond state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy (in MeV) of the $\alpha$ particle is:
   (a) 5.26 (b) 5.17 (c) 5.13 (d) 5.09

4. Which one of the following disintegration series of the heavy elements will give $^{209}$Bi as a stable nucleus?
   (a) Thorium series (b) Neptunium series
   (c) Uranium series (d) Actinium series

5. Fission fragments are generally radioactive as:
   (a) they have excess in neutrons (b) they have excess of protons
   (c) they are products of radioactive nuclides
   (d) their total kinetic energy is of the order of 200 MeV

6. Half life of a radio-isotope is $4\times10^8$ years. If there are $10^3$ radioactive nuclei in a sample today, the number of such nuclei in the sample $4\times10^9$ year ago were:
   (a) $128\times10^3$ (b) $256\times10^3$ (c) $512\times10^3$ (d) $1204\times10^3$

7. A relativistic particle travels a length of $3\times10^{-3}$ m in air before decaying. The decay process of the particle is dominated by:
   (a) strong interactions (b) electromagnetic interactions.
   (c) weak interactions (d) gravitational interactions

8. Which of the following configurations of the decay products correspond to the largest energy of the anti-neutrino $\bar{\nu}$? (rest mass of electron $m_e = 939.57$ MeV/c$^2$)
   (a) In the laboratory, proton is produced at rest
   (b) In the laboratory, momenta of proton, electron and the anti-neutrino all have the same magnitude.
   (c) In the laboratory, proton and electron fly-off with (nearly) equal and opposite momenta.
   (d) In the laboratory; electron is produced at rest.

9. Using the result of the above problem, answer the following. Which of the following represents approximately the maximum allowed energy of the anti-neutrino $\bar{\nu}$?
   (a) 1.3 MeV (b) 0.8 MeV (c) 0.5 MeV (d) 2.0 MeV

10. Which of the following phenomena can be explained by quantum mechanical tunneling through the barrier?

(i) $\alpha$ – Decay (ii) Field emission of $e^{-s}$ from a metal surface. (iii) $\beta$ – Decay

Select the correct answer using the code given below:

(a) i, ii, iii (b) i only (c) i & ii only (d) ii & iii only.

11. The ${}^{14}C$ is an unstable neucleus. Which one of the following correctly describes the $\beta$ - decay process for its decay. (e is $e^-$, $\bar{e}$ is positron, $\nu$ is neutron & $\bar{\nu}$ is antineutrino)

(a) ${}^{14}C \longrightarrow {}^{14}N + \bar{e} + \nu$ (b) ${}^{14}C \longrightarrow {}^{14}O + e + \bar{\nu}$

(c) ${}^{14}C \longrightarrow {}^{14}O + \bar{e} + \nu$ (d) ${}^{14}C \longrightarrow {}^{14}N + e + \bar{\nu}$

12. Half life of a radio-isotope is $4 \times 10^8$ yrs. If there are $10^3$ radioactive nuclei in a sample today, the no. of such nuclei in the sample $4 \times 10^9$ years ago were :

(a) $128 \times 10^3$ (b) $256 \times 10^3$ (c) $512 \times 10^3$ (d) $1024 \times 10^3$

13. The relation between the kinetic energy of $\alpha$ – particle & kinetic energy of daughter nucleus is :

(a) $\frac{K_\alpha}{K_d} = \frac{4}{A+4}$ (b) $\frac{K_\alpha}{K_d} = \frac{A+4}{4}$ (c) $\frac{K_\alpha}{K_d} = \frac{A-4}{4}$ (d) $\frac{K_\alpha}{K_d} = \frac{4}{A-4}$

14. A proton is moving in a circular orbit of radius R & in a constant magnetic field B. If B and R are kept const. The ratio of the K.E. of $\alpha$ particle and proton is:

(a) 1 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

15. The spin parity of the ground state of a nucleus is $0^-$. If absorbs a $\gamma$ – ray & go to the excited state. if the transition is mag. dipole, then spin - parity of excited state is :

(a) $0^-$ (b) $1^-$ (c) $1^+$ (d) $0^+$

16. ${}^{10}$Be in its first excited state has spin-parity $2^+$. It gets de-excited to the ground state, which has spin parity 0, by $\gamma$-emission. The multipoles carrried by $\gamma$ are

(a) E2 (b) M2 (c) E2, M2 (d) E4.

17. Which of the following is true for $\beta$-decay of the neutron? The process

(a) Violates both parity and charge conjugation symmetry.
(b) Violates parity but conserves charge conjugation symmetry.
(c) Conserves parity but violates charge conjugation symmetry.
(d) Conserves both parity and charge conjugation symmetry.

18. A 100 MeV proton and a 100 MeV alpha particle are detected in the same detector. What is the ratio of energy loss per unit path length i.e. $\left(\frac{dE}{dx}\right)_p : \left(\frac{dE}{dx}\right)_\alpha$ ?

(a) 1:1 (b) 1:2 (c) 1:4 (d) 1:16

19. Consider the alpha-decay reaction

$$P_{084}^{210} \rightarrow Pb_{82}^{206} + He_2^4$$

where atomic masses are

$$m(P_0) = 210.0483 \text{ amu}; \quad m(Pb) = 206.0386 \text{ amu}$$

$$m(He) = 4.0039 \text{ amu}; \quad 1 \text{ amu} = 931.141 \text{ MeV}$$

The kinetic energy of the alpha particle will be

(a) 5.4 keV (b) 2.7 keV. (c) 5.4 MeV (d) 10.8 MeV

20. The activity of a radioactive sample is defined as $\lambda N$ where $\lambda$ is the decay constant and N is the number of atoms. In an experiment, the activity of a sample of $^{55}_{24}Cr$ was found to change as

| After (min) | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| $\lambda N$ (m curie) | 19.2 | 7.13 | 2.65 | 0.99 | 0.37 |

The half life of $^{55}_{24}Cr$ is:

(a) 5.08 min (b) 3.52 min (c) 3.57 min (d) 5.16 min.

21. In the $\beta-$decay of neutron $n \to p + e^- + \overline{v}_e$, the anti-neutrino $\overline{v}_e$ escape detection. Its existence is inferred from the measurement of

(a) Energy distribution of electrons (b) Angular distribution of electrons.

(c) Helicity distribution of electrons (d) Forward-backward asymmetry of electrons.

22. Consider the decay process $\tau^- \to \pi^- + V_\tau$ in the rest frame of the $\tau^-$. The masses of $\tau^-, \pi^-$ and $\nu_\tau$ are $M_\tau, M_\pi$ and zero repectively.

**(A)** The energy of $\pi^-$ is:

(a) $\frac{\left(M_\tau^2 - M_\pi^2\right)c^2}{2M_\tau}$ (b) $\frac{\left(M_r^2 + M_\pi^2\right)c^2}{2M_\tau}$ (c) $\left(M_r - M_\pi\right)c^2$ (d) $\sqrt{M_\tau M_\pi}\ c^2$

**(B)** The velocity is $\pi^-$ is:

(a) $\frac{\left(M_\tau^2 - M_\pi^2\right)c}{M_\tau^2 + M_\pi^2}$ (b) $\frac{\left(M_\tau^2 - M_\pi^2\right)c}{M_\tau^2 - M_\pi^2}$ (c) $\frac{M_\pi c}{M_\tau}$ (d) $\frac{M_\tau c}{M_\pi}$

23. The energy spectrum of beta-particles is continuous because

(a) parity is violated in beta-decay (b) beta-particles are emitted in the continuum region

(c) beta- decay leads to a three body state (d) beta-decay is a weak interaction process

24. The $0 \to 0$ gamma transition is forbidden because

(a) photon has integer spin (b) photon is a massless particle

(c) photon has a definite polarization (d) parity will not be conserved

25. A fast moving proton (of rest mass $938.3\ \text{MeV/c}^2$ has an energy of 2.0 GeV. It is not possible for this to decay to a neutron (of rest mass $939.6\ \text{MeV/c}^2$) through the process $p \to n + X$ because

(a) angular momentum must be conserved

(b) this would voi late conservation of charge

(c) the neutron has a shorter lifetime than proton

(d) this would violate conservation of energy-momentum

26. Which if the following corresponds to an electric dipole gamma transition in a nucleus

(a) $(3/2)^+ \to (1/2)^-$ (b) $(3/2)^+ \to (1/2)^+$

(c) $1^+ \to 1^+$ (d) $3^+ \to 0^-$

27. If decay constant of a radioactive sample is $\lambda$ per minute, then the fraction that decays in the fifth minute is

(a) $e^{-4\lambda}$ (b) $e^{-5\lambda}$ (c) $e^{-4\lambda} - e^{-5\lambda}$ (d) $e^{-4\lambda} + e^{-5\lambda}$

28. In a Fermi transition in beta decay

(a) the angular momentum change is only 0 (b) the spins of $e^-$ and $\bar{\nu}$ are anti-parallel

(c) the spins of $e^-$ and $\bar{\nu}$ are parallel (d) the parity changes from initial to final state

29. A high-energy neutrino is detected in the laboratory through the process of

(a) capture by a nucleus (b) inverse beta decay

(c) annihilation with an anti-neutrino (d) scattering from electron clouds.

30. The ground state of $^{207}_{82}\text{Pb}$ nucleus has spin-parity $J^P = \frac{1^-}{2}$, while the first excited state has $J^P = \frac{5^-}{2}$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to the ground state are

(a) E2 and E3 (b) M2 and E3 (c) E2 and M3 (d) M2 and M3

31. The muon has mass 105 MeV/$c^2$ and mean lifetime 2.2 μs in its rest frame. The mean distance traversed by muon of energy 315 MeV/$c^2$ before decaying is approximately

(a) $3\times10^5$ km (b) 2.2 cm (c) 6.6 μm (d) 1.98 km

32. The decay process $n \rightarrow p^+ + e^- + \bar{\nu}_e$ violates

(a) baryon number (b) lepton number (c) isospin (d) strangeness

33. In the β decay process, the transition $2^+ \rightarrow 3^+$, is

(a) allowed both by Fermi and Gamow-Teller selection rule

(b) allowed by Fermi and but not by Gamow-Teller selection rule

(c) not allowed by Fermi but allowed by Gamow-Teller selection rule

(d) not allowed both by Fermi and Gamow-Teller selection rule

34. $^{238}$U decays with a half life of $4.51 \times 10^9$ years, the decay series eventually ending at $^{206}$Pb, which is stable. A rock sample analysis shows that the ratio of the numbers of atoms of $^{206}$Pb to $^{238}$U is 0.0058. Assuming that all the $^{206}$Pb has been produced by the decay of $^{238}$U and that all other half-lives in the chain are negligible, the age of the rock sample is

(a) $38 \times 10^6$ years (b) $48 \times 10^6$ years (c) $38 \times 10^7$ years (d) $48 \times 10^7$ years

35. A standard radioactive source is known to decay by emission of $\gamma$ rays. The source is provided to a student in a thick sealed capsule of unbreakable plastic and she is asked to find out the half-life. Which of the following would be the most useful advice to the student?

(a) The half-life cannot be measured because the initial concentration of the source is not given.

(b) Mount the source in front of a gamma ray detector and count the number of photons detected in one hour.

(c) Measure the mass of the source at different times with an accurate balance having a least count of 1 mg. Plot these values on a curve and fit it with an exponential decay law.

(d) Mount the source in front of a gamma ray detector and count the number of photons detected in a specific time interval. Repeat this experiment at different times and not how the count changes.

36. Cosmic ray muons generated at the top of the Earth's atmosphere decay according to the radioactive decay law

$$N(t) = N(0)\exp\left(-\frac{0.693t}{T_{1/2}}\right)$$

where N(t) is the number of muons at time t, and $T_{1/2} = 1.52\mu s$ is the proper half-life of the muon. Immediately after generation, most of these muons shoot down towards the Eearth's surface. Some of these muons decay on the way, but their interaction with the atmosphere is negligible.

An observer on the top of a mountain of height 2.0 km above mean sea level detects muons with the speed 0.98c over a period of time and counts 1000 muons. The number of muons of the same speed detected by an observer at mean sea level in the same period of time would be

(a) 232 (b) 539 (c) 839 (d) 983

# CAREER ENDEAVOUR

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Institute for NET-JRF/GATE & IIT-JAM

**PHYSICAL SCIENCES (NET-JRF/GATE)**

**Assignment-4: Radioactivity**

## [ANSWERS]

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (d)** | **2. (b)** | **3. (b)** | **4. (b)** | **5. (d)** | **6. (d)** | **7. (c)** |
| **8. (d)** | **9. (b)** | **10. (c)** | **11. (d)** | **12. (d)** | **13. (c)** | **14. (a)** |
| **15. (b)** | **16. (a)** | **17. (b )** | **18. (a)** | **19. (c)** | **20. (a)** | **21. (c)** |
| **22_A(b)** | **22_B(b)** | **23. (c)** | **24. (d)** | **25. (d)** | **26. (a)** | **27. (c)** |
| **28. (a)** | **29. (b)** | **30. (b)** | **31. (d)** | **32. (c)** | **33. (c)** | **34. ( )** |
| **35. (d)** | **36. (b)** | | | | | |

### PHYSICAL SCIENCES (NET-JRF/GATE)

### Assignment-5: Particle Physics

1. The quark structure of $\Delta^{++}$ is:
(a) uuu (b) udu (c) sss (d) ddd

2. Choose the particle with zero Baryon number from the list given below:
(a) pion (b) neutron (c) proton (d) $\Delta^+$

3. Which of the following decay is forbidden?
(a) $\mu^- \to e^- + \upsilon_\mu + \overline{\upsilon}_e$ (b) $\pi^+ \to \mu^+ + \upsilon_\mu$
(c) $\pi^+ \to e^+ + \upsilon_e$ (d) $\mu^- \to e^+ + e^- + e^-$

4. The interaction potential between two quarks, separated by a distance r inside a nucleon be described by (a, b and $\beta$ are positive constants)
(a) $ae^{-\beta r}$ (b) $\frac{a}{r} + br$ (c) $-\frac{a}{r} + br$ (d) $\frac{a}{r}$

5. The strangeness quantum number is conserved in:
(a) strong, weak and electromagnetic interactions
(b) weak and electromagnetic interactions only
(c) strong and weak interactions only
(d) strong and electromagnetic interactions only

6. The strange baryon $\Sigma^+$ has the quark structure:
(a) uds (b) uud (c) uus (d) uss

7. Choose the correct statement from the following:
(a) The reaction $K^+K^- \to p\overline{p}$ can proceed irrespective of the kinetic energies of $K^+$ and $K^-$
(b) The reaction $K^+K^- \to p\overline{p}$ is forbidden by the baryon number conservation
(c) The reaction $K^+K^- \to 2\gamma$ is forbidden by strangeness conservation
(d) The decay $K^0 \to \pi^+\pi^-$ proceeds via weak interactions

8. Which of the following particles are leptons?
(i) electron (ii) pion (iii) Neutron (iv) neutrino
Select the correct answer using the code given below:
(a) i, iii & iv (b) i & ii (c) ii & iii (d) i & iv only

9. If $m_\mu$, $m_\pi$, $m_p$ are the masses of a $\mu$-meson, pi-meson, & proton respectively, then which one of the following is correct?
(a) $m_\mu > m_p > m_\pi$ (b) $m_\mu > m_\pi > m_p$ (c) $m_p > m_\pi > m_\mu$ (d) $m_p > m_\mu > m_\pi$

10. The quark structure for $\sum^+$ is:
(a) uud (b) uus (c) uuu (d) dds

11. The Strangeness of $\Omega^-$ is:
(a) 1 (b) $-1$ (c) 2 (d) $-3$

12. Isospin & third component of isospin for proton are respectively:

(a) $\frac{1}{2}, -\frac{1}{2}$  (b) $\frac{1}{2}, \frac{1}{2}$  (c) 1, 1  (d) 0,0

13. If isospin T is 1, the no. of particles in the group will be:

(a) 1  (b) 2  (c) 3  (d) 4

14. Which of the following is heaviest particle:

(a) $e^-$  (b) H – atom  (c) n  (d) proton

15. Which is the lightest particle:

(a) $e^-$  (b) p  (c) n  (d) $\nu$

16. The reaction $p + \pi^- \to n$ is allowed by:

(a) strong inetraction  (b) E.M. interaction  (c) weak interaction  (d) gravitational interaction.

17. In the quark model, the state of $\pi^+$ is given by $|u\bar{d}\rangle$. The states for $\pi^-$ and $\pi^0$ are then given by

(a) $|\bar{u}d\rangle; \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$  (b) $|\bar{u}d\rangle; \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle$

(c) $|\bar{u}d\rangle; |u\bar{u}\rangle$  (d) $|\bar{u}d\rangle; |d\bar{d}\rangle$

18. The neutral pion $\pi^0$ at rest decays into two photons. One of the photons is right circularly polarized. The other photon is:

(a) Also right circularly polarized.  (b) Left circularly polarized.

(c) Unpolarized  (d) Plane polarized.

19. The isospin and the strangeness of $\Omega^-$ baryon are

(a) 1, –3  (b) 0, –3  (c) 1, 3  (d) 0, 3

20. A beam of pions $(\pi^+)$ is incident on a proton target, giving rise to the process $\pi^+ p \to n + \pi^+ + \pi^+$

**(A)** Assuming that the decay proceeds through strong interactions, the total isospin I and its third component $I_3$ for the decay products, are

(a) $I = \frac{3}{2}, I_3 = \frac{3}{2}$  (b) $I = \frac{5}{2}, I_3 = \frac{5}{2}$  (c) $I = \frac{5}{2}, I_3 = \frac{3}{2}$  (d) $I = \frac{1}{2}, I_3 = -\frac{1}{2}$

**(B)** Using isospin symmetry, the cross-section for the above process can be related to that of the process

(a) $\pi^- n \to p\pi^-\pi^-$  (b) $\pi^-\bar{p} \to \bar{n}\pi^-\pi^-$  (c) $\pi^+ n \to p\pi^+\pi^-$  (d) $\pi^+\bar{p} \to n\pi^+\pi^-$

21. An electron of energy 27 GeV collides with a proton of energy 820 GeV. The heaviest particle which can be produced in this collision has mass close to

(a) 300 GeV  (b) 821 GeV  (c) 850 GeV  (d) 1127 GeV

22. A muon participates is

(a) electromagnetic, weak and gravitational interactions only

(b) weak and gravitational interactions only  (c) weak and strong interaction only

(d) electromagnetic and gravitational interactions only

23. If parity P were conserved in all interactions, the number of $P = -1$ pions obtained in the decay of a $P = 1$ kaon $K^0$ would be

(a) 1  (b) 4  (c) 2  (d) 3

24. A delta baryon is found to have charge +2 and strangeness 0. Its isospin must be

(a) 2 (b) 3/2 (c) 1 (d) $\frac{1}{2}$

25. Within the framework of the quark model which of the following is not possible
(a) a baryon with spin 1
(b) a baryon with isospin $\frac{1}{2}$ and negative strangeness
(c) a baryon with isospin 1 and negative strangeness
(d) a meson with spin 1

26. The decay $\pi^0 \to 3\gamma$ is forbidden by
(a) energy conservation (b) angular momentum conservation
(c) charge conjugation invariance (d) parity invariance

27. A pion $\pi^+$ of rest mass 140 $MeV/c^2$ is accelerated to an energy of 2.5 GeV. This cannot decay to a proton of rest mass 938 $MeV/c^2$ and an anti-neutron of rest mass 939 $MeV/c^2$ because
(a) a moving pion cannot produce a proton at rest
(b) energy and momentum cannot be simultaneously conserved
(c) the spin of the pion is different from the spin of the proton
(d) it is forbidden by isospin conservation

28. Which of the following quantities of an elementary particle should be zero if parity is conserved?
(a) mass width (b) magnetic dipole moment
(c) electric quadrupole moment (d) electric dipole moment

29. The prinicipal decay mode of a neutral pion $\pi^0$ is through the process

(a) $\pi^0 \to \gamma\gamma$ (b) $\pi^0 \to e^+e^-$ (c) $\pi^0 \to \mu^+\mu^-$ (d) $\pi^0 \to \nu\bar{\nu}$

30. Which one of the following sets corresponds to fundamental particles?
(a) proton, electron and neutron (b) proton, electron and photon
(c) electron, photon and neutrino (d) quark, electron and meson

31. Choose the CORRECT statement form the following
(a) Neutron interacts through electromagnetic interaction
(b) Electron does not interact through weak interaction
(c) Neutrino interacts through weak and electromagnetic interaction
(d) Quark interacts through strong interaction but not through weak interaction.

32. The dominant interactions underlying the following processes

A. $K^- + p \to \Sigma^- + \pi^+$ B. $\mu^- + \mu^+ \to K^- + K^+$ C. $\Sigma^+ \to p + \pi^0$ are
(a) A: strong, B: electromagnetic and C: weak (b) A: strong, B: weak and C: weak
(c) A: weak, B: electromagnetic and C: strong (d) A: weak, B: electromagnetic and C: weak

33. If a Higgs boson of mass $m_H$ with a speed $\beta = \frac{v}{c}$ decays into a pair photons, then the invariant mass of the photon pair is

[**Note:** The invariant mass of a system of two particles, with four-momenta $p_1$ and $p_2$ is $(p_1 + p_2)^2$]

(a) $\beta m_H$ (b) $m_H$ (c) $m_H/\sqrt{1-\beta^2}$ (d) $\beta m_H/\sqrt{1-\beta^2}$

34. Consider the following particles: the proton p, the neutron n, the neutral pion $\pi^0$ and the delta resonance $\Delta^+$. When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

(a) $\pi^0$, n, p, $\Delta^+$ (b) p, n, $\Delta^+$, $\pi^0$ (c) p, n, $\pi^0$, $\Delta^+$ (d) $\Delta^+$, n, $\pi^0$, p

35. The isospin (I) and baryon number (B) of the up quark is

(a) I = 1, B = 1 (b) I = 1, B = 1/3 (c) I = 1/2, B = 1 (d) I = 1/2, B = 1/3

36. A *K* meson (with a rest mass of 494 MeV) at rest decays into a muon (with a rest mass of 106 MeV) and a neutrino. The energy of the neutrino, which can be taken to be massless, is approximately

(a) 120 MeV (b) 236 MeV (c) 300 MeV (d) 388 MeV

37. Consider the following ratios of the partial decay widths $R_1 = \dfrac{\Gamma(\rho^+ \to \pi^+ + \pi^0)}{\Gamma(\rho^- \to \pi^- + \pi^0)}$ and $R_2 = \dfrac{\Gamma(\Delta^{++} \to \pi^+ + p)}{\Gamma(\Delta^- \to \pi^- + n)}$. If the effects of electromagnetic and weak interactions are neglected, then $R_1$ and $R_2$ are, respectively,

(a) 1 and $\sqrt{2}$ (b) 1 and 2 (c) 2 and 1 (d) 1 and 1

38. The interaction strength of the recently-discovered Higgs boson (mass approximately 125 GeV/$c^2$) with any other elementary particle is proportional to the mass of that particle. Which of the following decay processes will have the greatest probability?

(a) Higgs boson decaying to a top quark + a top anti-quark

(b) Higgs boson decaying to a bottom quark + a bottom anti-quark

(c) Higgs boson decaying to an electron and a positron

(d) Higgs boson decaying to a neutrino-antineutrino pair.

## Assignment-5: Particle Physics

### [ANSWERS]

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (a)** | **2. (a)** | **3. (a)** | **4. (c)** | **5. (d)** | **6. (c)** | **7. (d)** |
| **8. (d)** | **9. (c)** | **10. (b)** | **11. (d)** | **12. (b)** | **13. (c)** | **14. (c)** |
| **15. (d)** | **16. (b)** | **17. (a)** | **18. (b)** | **19. (c)** | **20_A(c)** | **20_B(b)** |
| **21. (c)** | **22. (a)** | **23. (c)** | **24. (b)** | **25. (a)** | **26. (c)** | **27. (b)** |
| **28. (d)** | **29. (a)** | **30. (c)** | **31. (c)** | **32. (a)** | **33. (b)** | **34. (c)** |
| **35. (d)** | **36. ( )** | **37. (b)** | **38. ( )** | | | |

### PHYSICAL SCIENCES (NET-JRF/GATE)
### Assignment-6: Miscellaneous

1. A thermal neutron having speed v impinges on a $^{235}U$ nucleus. The reaction cross-section is proportional to:
(a) $v^{-1}$ (b) v (c) $v^{1/2}$ (d) $v^{-1/2}$

**Statement for linked answer Q.2 and Q.3**

A 16 μA beam of alpha particles, having cross-sectional area $10^{-4}m^2$, is incident on a rhodium target of thickness 1 μm. This produces neutrons through the reaction

$$a + {}^{100}Rh \rightarrow {}^{101}Pd + 3n$$

2. The number of alpha particles hitting the target per second is:
(a) $0.5 \times 10^{14}$ (b) $1.0 \times 10^{14}$ (c) $2.0 \times 10^{20}$ (d) $4.0 \times 10^{20}$

3. The neutrons are observed at the rate of $1.806 \times 10^8 s^{-1}$. If the density of rhodium is approximated as $10^4$ kg $m^3$ the cross-section for the reaction (in barns) is:
(a) 0.1 (b) 0.2 (c) 0.4 (d) 0.8

4. The total cross-section for $\mu^+ u^- \to e^+e^-$ at very high values of the centre of mass energies. $\sqrt{s} >> m_e$, is expected to behave as (s is impact parameter)
(a) $\frac{e^2}{s^2}$ (b) $\frac{e^4}{s^2}$ (c) constant (d) $\frac{e^4}{s}$

5. A relativistic particle of rest mass $m_0$ is moving with a speed $v$. The value of $v$ at which its kinetic energy is equal to its rest energy is:
(a) $v = c/2$ (b) $v = c/4$ (c) $v = c/\sqrt{2}$ (d) $v = 2(\sqrt{2}-1)c$

6. An elementary particle is produced in cosmic rays with a cross-section of 6 mb. It is found to dacay with a proper life time of 2 ms. It follows that the particle must be a
(a) electron (b) neutron (c) hyperon (b) proton

7. In case of a Geiger-Muller (GM) counter, which one of the following statements is CORRECT?
(a) Multiplication factor of the detector is of the order of $10^{10}$
(b) Type of the particles detected can be identified
(c) Energy of the particles detected can be distinguished
(d) Operating voltage of the detector is few tens of Volts.

8. The intrinsic electric dipole moment of a nucleus ${}^A_Z X$
(a) increases with Z, but independent of A (b) decreases with Z, but independent of A
(c) is always zero (d) increases with Z and A

9. According to the shell model, the total angular momentum (in units of $\hbar$) and the parity of the ground state of the ${}^7_3Li$ nucleus is
(a) $\frac{3}{2}$ with negative parity (b) $\frac{3}{2}$ with positive parity
(c) $\frac{1}{2}$ with positive parity (d) $\frac{7}{2}$ with negative parity

10. An alpha particle of energy E is shot towards a gold nucleus $\left({}^{197}_{79}Au\right)$. At the distances much larger than the nuclear size $R_N$, the dominant force is the Coulomb repulsion, but at distances comparable to the nuclear size the dominant force is the strong nuclear attraction. These combine to form a potential barrier of height $V_C$. If $E < V_C$, the probability that the alpha particle will fuse with the gold nucleus can be written (in terms of a dimensionless positive constant k) as

(a) zero (b) $\frac{kE}{\sqrt{k^2E^2+V_C^2}}$ (c) $k\left(1-\frac{E}{V_C}\right)$ (d) $\exp\left(-\frac{kV_C}{E}\right)$

11. A spin-2 nucleus absorbs a spin-½ electron and is then observed to decay to a stable nucleus in two stages, recoiling against an emitted invisible particle in the first stage and against an emitted spin-1 photons in the second stage. If the stable nucleus is spinless, then the set of all possible spin values of the invisible particle is

(a) {1/2, 5/2} (b) {3/2, 7/2} (c) {1/2, 3/2, 5/2} (d) {1/2, 3/2, 5/2, 7/2}

12. Given that binding energy of hydrogen atom in the ground state is 13.6 eV, the binding energy for n = 2 state of the positronium atom is

(a) 13.6 eV (b) 6.8 eV (c) 3.4 eV (d) 1.7 eV

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**PHYSICAL SCIENCES (NET-JRF/GATE)**

**Assignment-6: Miscellaneous**

## [ANSWERS]

**1.** (a) **2.** (a) **3.** (b) **4.** (d) **5.** (c) **6.** (c) **7.** (a)
**8.** ( ) **9.** ( ) **10.** (d) **11.** (d) **12.** (d)

# CSIR-UGC-NET/JRF | GATE

TIFR|BARC|JEST|IISc|DRDO|ISRO|SLET|ONGC-GT & Ph.D. Entrance

# Assignment

# Thermodynamics & Statistical PHysics

## PHYSICAL SCIENCES

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ACADEMY PRIVATE LIMITED

**South Delhi Centre:** 28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:** 48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

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### PHYSICAL SCIENCES (JRF-NET/GATE)

### Assignment-I: THERMODYNAMICS

1. Which of the following is not an exact differential?

(a) dQ(Q = heat absorbed) (b) dU(U = internal energy)

(c) dS(S = entropy) (d) dF(F = free energy)

2. If the entropy of a system remains constant in a thermodynamic process, the process is:

(a) isobaric (b) isochoric (c) isothermal (d) adiabatic

3. The sole effect of a hypothetical thermodynamic process $E_1$ is to convert an amount Q of heat entirely into work. A second hypothetical process $E_2$ converts an amount W of work entirely into heat. It can be said that the second law of thermodynamics is violated by

(a) $E_1$ but not $E_2$ (b) $E_2$ but not $E_1$ (c) both $E_1$ and $E_2$ (d) neither process.

4. Which of the following expresses the Second law of thermodynamics? (Symbols have their usual meanings).

(a) $\Delta F \le W$ (b) $\Delta S < Q/T$ (c) $\Delta U = Q + W$ (d) $\Delta S \le 0$

5. A carnot engine operates between a heat source at 500 K and a heat sink at 300 K. The temperature of the source is increased by 20 K. In order that the efficiecy of the engine remain unchanged, the temperature of the sink should be changed by

(a) + 20 K [✓ (b) +12 K] (c) –12 K (d) –20 K

6. A carnot cycle operates on a working substance between two reservoirs at temperatures $T_1$ and $T_2$, with $T_1 > T_2$. During each cycle, an amount of heat $Q_1$ is extracted from the reservoir at $T_1$ and an amount $Q_2$ is delivered to the reservoir at $T_2$. Which of the following statements is **INCORRECT?**

(a) Work done in one cycle is $Q_1 - Q_2$ (b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

(c) Entropy of the hotter reservoir decreases.

(d) Entropy of the universe (consisting of the working substance and the two reservoir) increases.

7. If the isothermal compressibility of a solid is $\kappa_T = 10^{-10}\,(Pa)^{-1}$, the pressure required to increase its density by 1% is approximately.

(a) $10^4\ Pa$ (b) $10^6\ Pa$ (c) $10^8\ Pa$ (d) $10^{10}\ Pa$

8. An ideal gas in a cylider is compressed adiabatically to one-third of its initial volume. During this process 20 J work is done on the gas by compressing agent. Which of the following statements is true for this case?

(a) Change in the internal energy in this process is zero.
(b) The internal energy increases by 20 J
(c) The internal energy decreases by 20 J
(d) Temperature of the gas decreases.

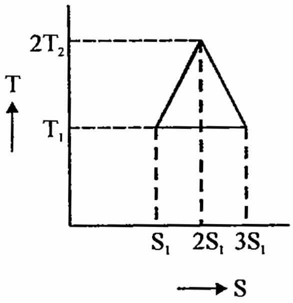
9. Mean total energy of a classical three-dimensional harmonical oscillator in equilibrium with a heat reservoir at temperature T is:

(a) $k_\beta T$ (b) $\frac{3}{2}k_\beta T$ (c) $2\,k_\beta T$ (d) $3\,k_\beta T$

10. An amount of heat Q is transferred from a heat reservoir at temperature $T_A$ to another heat reservoir at temperature $T_B$. What is the change in the entropy $\Delta S$ of the combined system?

(a) $Q\left(\frac{1}{T_B}-\frac{1}{T_A}\right)$ (b) $Q\left(\frac{1}{T_B}+\frac{1}{T_A}\right)$ (c) $\frac{Q}{\sqrt{T_B T_A}}\ln\frac{T_B}{T_A}$ (d) $Q\left(\frac{1}{T_A}-\frac{1}{T_B}\right)$

11. A reversible engine cycle is shown in the following T-S diagram. The efficiency of the engine is:



(a) 1/3 (b) 1/2 (c) 1/5 (d) 1/4

12. An engine absorbs heat at a temperature of 1000 K and rejects heat at 600 K. If the engine operates at maximum possible efficiency, the amount of work performed by the engine, for 2000 J heat input is:

(a) 1600 J (b) 1200 J (c) 800 J (d) 400 J

13. Boyle's law can be expressed in different form as

(a) $dV/dP = 1$ (b) $dV/dP = V/P$ (c) $dV/dP = P/V$ (d) $dV/dP = -V/P$

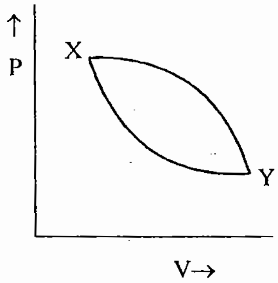
14. For any process, the second law of thermodynamics requires that the change of entropy of the universe be

(a) positive only (b) positive or zero (c) zero only (d) negative or zero

15. A sample of ideal gas with initial pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be $\Delta S$. The universal gas constant is R. Then the work done by the gas is given by

(a) $(PV\Delta S)/(nR)$ (b) $nR\Delta S$ (c) $PV$ (d) $(P\Delta S)/(nRV)$

16. A piston containing an ideal gas is originally in the state X (see figure). The gas is taken through a thermal cycle X→Y→X as shown:
The work done by the gas is positive if the direction of the thermal cycle is:



(a) clockwise
(b) counter - clockwise
(c) neither clockwise nor counter-clockwise
(d) clockwise from X→Y and counter-clockwise from Y→X.

17. Hydrogen molecules (mass m) are in thermal equilibrium at a temperature T. Assuming classical distribution of velocity, the most probable speed at room temperature is:

(a) $(k_B T)/m$ (b) $2k_B T/m$

(c) $\left(\sqrt{2}k_B T/m\right)$ (d) $mn/\left(\sqrt{2}k_B T\right)$

18. The root-mean square speed of a particle of mass m in the kinetic theory is given by

(a) $\sqrt{\frac{k_\beta}{m}}$ (b) $\sqrt{\frac{2k_\beta}{m}}$ (c) $\sqrt{\frac{3k_\beta}{m}}$ (d) $\sqrt{\frac{8k_\beta}{m}}$

19. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is:

(a) $pV^{1/3}$ = constant (b) $pV^{2/3}$ = constant (c) $pV^{4/3}$ = constant (d) $pV^{3/5}$ = constant

20. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is:

(a) $\frac{1}{2}k_BT$ (b) $k_BT$ (c) $\frac{3}{2}k_BT$ (d) $3\,k_BT$

21. A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22°C by suppling 450 $kJS^{-1}$. If the outside temperature is 0°C, the heat taken, in $kJs^{-1}$, from the outside air is approximately.
(a) 487 (b) 470 (c) 467 (d) 417

22. The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by in $p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in Kelvin) of the material is:
(a) 185 (b) 190 (c) 195 (d) 200

23. The internal energy of n moles of a gas is given $E = \frac{3}{2}nRT - \frac{a}{V}$, where V is the volume of the gas at temperature T and $a$ is a positive constant. One mole of the gas is state $(T_1, V_1)$ is allowed to expand adiabatically into vacuum to a final state $(T_2, V_2)$. The temperature $T_2$ is:

(a) $T_1 + Ra\left(\frac{1}{V_2} + \frac{1}{V_1}\right)$ (b) $T_1 - \frac{2}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

(c) $T_1 + \frac{2}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$ (d) $T_1 - \frac{1}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

24. The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is:
(a) the entropy of solid is zero in this temperature region.
(b) the entropy increases when the system goes from liquid to solid phase in this temperature region.
(c) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature.
(d) the change in entropy is zero in the liquid-to-solid transition region.

25. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium,
(a) the total entropy of the final system is the same as the sum of the entropy of the individual system always.
(b) the entropy of mixing is $2Nk_B \ln 2$.
(c) the entropy of the final system is less than that of sum of the initial entropies of the two gases.
(d) the entropy of mixing is non-zero when the atoms A and B are of the same type.

26. The mean free path of the particles of a gas at temperature $T_0$ and pressure $p_0$ has a value $\lambda_0$. If the pressure is increased to 1.5 $p_0$ and the temperature is reduced to 0.75 $T_0$, the mean free path
(a) remains unchanged. (b) is reduced to half
(c) is doubled (d) is equal to 1.125 $\lambda_0$

27. The boiling point of a liquid at pressure $P_0$ is $T_0$. Its molar latent heat of vaporisation is L and molar volume of the liquid phase is negligible as compared to the vapour phase. The vapour phase obeys perfect gas equation. The boiling point T at pressure P is given by

(a) $P/P_0 = (L/RT_0)(1-T_0/T)$  [✓ (b) $\ln(P/P_0) = (L/RT_0)(1-T_0/T)$]

(c) $\ln(P/P_0) = (L/RT_0)(1-T/T_0)$  (d) $P/P_0 = 1+(L/RT_0)\ln(T_0/T)$

28. The ratio $\gamma = C_P/C_V$ of $H_2$ gas is 7/5 at room temperature, and 5/3 at low temperatures (around 50 K). The reason for this change is

(a) Dissociation of $H_2$ molecules  (b) Suppression of rotational degrees of freedom

(c) Formation fo random clusters of molecules.  (d) Liquefaction of the gas.

29. For an isolated thermodynamical system p, V, T, U, S, and F represent the pressure, volume, temperature, internal energy, entropy, and free energy respectively. Then the following relation is true

(a) $\left(\frac{\partial F}{\partial T}\right)_V = -S$  (b) $\left(\frac{\partial F}{\partial T}\right)_P = -S$  (c) $\left(\frac{\partial U}{\partial T}\right)_P = T$  (d) $\left(\frac{\partial U}{\partial V}\right)_T = P$

30. A quantity of water is completely converted into steam by boiling.
(a) This is a second order phase transition.
(b) At the boiling point, the chemical potential in the vapour phase is less than that in the liquid phase.
(c) The boiling temperature would increse if the boiling were done under increased pressure.
[✓ (d) The entropy of the steam is greater than that of the water.]

31. In a first order phase transition, at the transition temperature, specific heat of the system
(a) Diverges and its entropy remains the same
[✓ (b) Diverges and its entropy has finite discontinuity.]
(c) Remains unchanged and its entropy has finite discontinuity.
(d) Has finite discontinuity and its entropy diverges.

32. Consider the transition of liquid water to steam as water boils at a temperature of 100°C under a pressure of 1 atmosphere. Which one of the following quantities does not change discontinuously at the transition?

[✓ (a) The Gibbs free energy]  (b) The internal energy

(c) The entropy  (d) The specific volume

33. The sublimation curve of solid ammonia is given by $\ln p=23-3750/T$ and the vaporization curve of the liquid ammonia is given by $\ln p=19.5-3050/T$, where p is in mm of Hg and T is in °K. The temperature of the triple point of ammonia is:

(a) 3750 K  (b) 3050 K  (c) 700 K  (d) 200 K

34. Which of the following is an example of a first order phase transition?
(a) A liquid - gas phase transition at the critical point.
(b) A paramagnet - ferromagnet phase transition
(c) A normal metal - superconductor phase transition.
[✓ (d) A liquid - gas phase transition away from the critical point.]

35. A second order phase transition is one in which
(a) the plot of entropy as a function of temperature shows a discontinuity.
[✓ (b) the plot of specific heat as a function of temperature shows a discontinuity]
(c) the plot of volume as a function of pressure shows a discontinuity.
(d) the plot of comprehensibility as a function of temperature is continuous.

36. Identify which one is a first order phase transition?
(a) A liquid to gas transition at its critical temperature.
[✓ (b) A liquid to gas transition close to its triple point.]
(c) A paramegnetic to ferromagnetic transition in the absence of a magnetic field.
(d) A metal to superconductor transition in the absence of a magnetic field.

37. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be ($k$ is the Boltzmann constant and $T$ is non zero)

(a) Zero (b) Infinite (c) $\frac{3}{2}kT$ ✓ (d) $kT$

↓ only for nonrelativistic particle

$K_B \neq 0$
Quantization
it is classically

## Answer Key (THERMODYNAMICS-I)

**1. (a)** **2. (d)** **3. (a)** **4. (b)** **5. (b)** **6. (d)** **7. (c)**

**8. (b)** **9. (d)** **10. (a)** **11. (a)** **12. (c)** **13. (d)** **14. (a)**

**15. (a)** **16. (a)** **17. (c)** **18. (c)** **19. (c)** **20. (b)** **21. (d)**

**22. (d)** **23. (c)** **24. (d)** **25. (b)** **26. (b)** **27. (b)** **28. (a)**

**29. (a)** **30. (d)** **31. (b)** **32. (a)** **33. (d)** **34. (d)** **35. (b)**

**36. (b)** **37. (d)**

**PHYSICAL SCIENCES (JRF-NET/GATE)**

## Assignment-II: THERMODYNAMICS

1. A thermodynamic system is classified as closed if it can
(a) Exchange energy with its surroundings, but not matter.
(b) Exchange both energy and matter with its surroundings.
(c) Exchange neither energy nor matter with its surroundings
(d) Exchange only matter, but not energy, with its surroundings.

2. The first law of thermodynamics states that $\Delta U = \Delta Q - \Delta W$, where the symbols have their usual meaning. Which one of the following quantity(ies) depend(s) only on the initial and the final state, and is (are) independent of the path of the system between these states?
(a) $\Delta Q$ (b) $\Delta W$ (c) $\Delta U$ (d) $\Delta U$ and $\Delta W$

3. In an infinitesimal reversible thermodynamic process, which of the following quantity depends only on the initial and the final states of the system and is independent of the path between these states?
(a) $T\Delta Q$ (b) $\Delta Q/T$ (c) $T/\Delta Q$ (d) $T^2/\Delta Q$

4. Which of the following statements is correct?
(a) You can construct an engine operating in cycles, extracting heat Q and performing exactly equivalent work W.
(b) You can construct an engine operating in cycles extracting heat Q and performing more than equivalent amount of work W.
(c) You can have an engine operating cyclically such that the entropy remains constant throughout the cycle.
(d) You cannot construct a cyclically operating engine that will extract heat Q and perform equivalent amount of work W.

5. Above a temperature $T_0$, the specific heat of a certain gas is approximate 3.5R, and is approximately 2.5R below $T_0$. The gas is most likely
(a) oxygen (b) ozone (c) carbon dioxide (d) argon

6. During the isothermal reversible expansion of an ideal gas,
(a) the entropy and the pressure both increase
(b) the entropy and the pressure both decrease
(c) the entropy increases and the pressure decreases
(d) the entropy decreases and the pressure increases

7. The isothermal compressibility, $\kappa$ of an ideal gas at temperature $T_0$ and volume $V_0$, is given by
(a) $-\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (c) $-V_0\frac{\partial P}{\partial V}\Big|_{T_0}$ (d) $V_0\frac{\partial P}{\partial P}\Big|_{T_0}$

8. Let $\Delta W$ be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about $\Delta W$ is correct?
(a) $\Delta W$ is a perfect differential if the process is isothermal
(b) $\Delta W$ is a perfect differential if the process is adiabatic
(c) $\Delta W$ is always a perfect differential
(d) $\Delta W$ cannot be a perfect differential.

9. An adiabat in the PV-plane is a curve of
(a) Constant temperature
(b) Constant entropy
(c) Costant pressure
(d) Constant volume

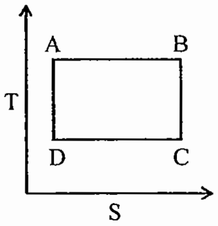
10. The volume of a thermodynamic system increases irreversibly by an incremental amount $\delta V$. If P is the pressure, the work done on the system is:
(a) $\delta W = P\,\delta V$
(b) $\delta W = -P\,\delta V$
(c) $\delta W < -P\,\delta V$
(d) $\delta W > -P\,\delta V$

11. Consider an elastic string of length L under tension $\tau$ at temperature T. Let U and S be its internal energy and entropy, respectively. Then the conjugate pairs of thermodynamic variables are
(a) $(\tau, T)$ and $(L, S)$
(b) $(L, \tau)$ and $(T, S)$
(c) $(T, L)$ and $(\tau, U)$
(d) $(L, S)$ and $(T, U)$

12. A carnot engine can be used to define an absolute temperature scale. This is made possible by the following property of the carnot engine.
(a) It involves an adiabatic as well as an isothermal process.
(b) It is a cyclic process.
(c) It is most efficient engine operating between two given temperatures.
(d) It conserves energy.

13. Which is the most effective way of increasing the efficiency of a carnot engine operating between temperatures $T_1$ and $T_2 (T_1 > T_2)$?
(a) Increasing $T_1$, keeping $T_2$ fixed.
(b) Increasing $T_2$, keeping $T_1$ fixed.
(c) Decreasing $T_2$ and increasing $T_1$
(d) Increasing $T_2$ and decreasing $T_1$.

14. Entropy - temperature (S-T) diagram of a Carnot cycle is shown below:



The adiabatic compression is represented by
(a) AB
(b) DA
(c) CD
(d) BC

15. The quantity that remains constant in Joule-Thomson expansion is
(a) $U + pV$
(b) $U + pV - TS$
(c) U
(d) $U - TS$

16. A steam turbine is operated with an intake temperature 407°C and exhaust temperature 117°C. Heat input to the turbine is Q. Then maximum work turbine can do is:
(a) 0.5Q
(b) 0.37Q
(c) Q
(d) 0.25 Q

17. For a system with constant volume and temperature, the thrmodynamic equilibrium corresponds to the state with minimum?
(a) Helmholtz free energy
(b) Gibbs thermodynamic potential
(c) Internal energy
(d) Enthalpy

18. A gas of argon atoms is kept in a container of volume V and pressure P and is in equilibrium with a heat bath at temperature T. The average energy of an argon atom in the gas is:
(a) RT
(b) $3k_nT$
(c) $3/2\, k_B T$
(d) 3 RT

Where $k_B$ is Boltzman constant.

19. A Carnot cycle operates on a working substance between two reservoirs at temperature $T_1$ and $T_2$, with $T_1 > T_2$. During each cycle, an amount of heat $Q_1$ is extracted from the reservoir at $T_1$ and an amount $Q_2$ is delivered to the reservoir at $T_2$. Which of the following statements is INCORRECT?

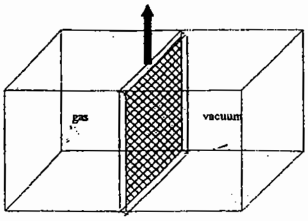
(a) Work done in one cycle is $Q_1 - Q_2$.

(b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_1}$

(c) Entropy of the hotter reservoir decreases

(d) Entropy of the universe (cosisting of the working substance and the two reservoirs) increases.

20. Consider a sealed but thermally conducting container of total volume V, Which is in equilibrium with athermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure)



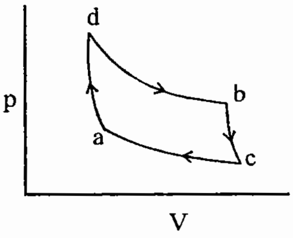
If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

(a) $2k_B$ (b) $k_B \ell n(1/2)$ (c) $k_B \ell n2$ (d) $(k_B \ell n2)/2$

21. The internal energy E of a system is given by $E = \frac{bS^3}{VN}$, where b is a constant and other symbols have their usual meaning. The temperature of this system is equal to

(a) $\frac{bS^2}{VN}$ (b) $\frac{3bS^2}{VN}$ (c) $\frac{bS^3}{V^3N}$ (d) $\left(\frac{S}{N}\right)^2$

22. In the Carnot cycle shown in the figure, the curves



(a) ab and cd are adiabatic, bc and da are isothermal

(b) ab and cd are isothermal, bc and da are adiabatic

(c) ab and cd are isothermal, bc and da are isobaric

(d) ab and cd are isobaric, bc and da are isothermal.

23. An ideal gas engine produces 1 kJ of useful work. If it operators between 400K and 300K, the amount of energy lost as waste heat is

(a) 4 kJ (b) 2kJ (c) 1kJ (d) 3kJ

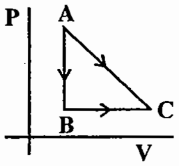
24. A monoatomic ideal gas at 17°C is adiabatically compressed to 1/8 of its original volume. The temperature after compression is

(a) 2.1°C (b) 17°C (c) –200.5°C (d) 887° C

25. Efficiency of perfectly reversible (Carnot) heat engine operation between absolute temperature T and zero is equal to

(a) 0 (b) 0.5 (c) .75 (d) 1

26. A given quantity of gas is taken from the state $A \to C$ reversibly, by two paths, $A \to C$ directly and $A \to B \to C$ as shown in the figure below.



During the $A \to C$ the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process $A \to B \to C$ the work done by the gas is 30 J, the heat absorbed is:

(a) 20J (b) 80J (c) 220 J (d) 280 J

27. The internal energy U is related to the Helmholtz free energy F by the relation:

(a) $U = -T\left\{\frac{\partial^2}{\partial T^2}\left(\frac{F}{T}\right)\right\}_V$ (b) $U = T\left\{\frac{\partial^2}{\partial T^2}\left(\frac{F}{T}\right)\right\}_V$

(c) $U = -T^2\left\{\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right\}_V$ (d) $U = T^2\left\{\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right\}_V$

28. Two insulated containers connected with a stop cock are initially evaluated. An ideal gas is filled in one of them and then allowed to expand into the other container by opening the cock. In this process the work done (W), the heat evolved (Q) and the change in internal energy $(\Delta U)$ are given by

(a) $W = 0, Q = 0$ and $\Delta U = 0$ (b) $W = +ve, Q = 0$ and $\Delta U = -ve$

(c) $W = -ve, Q = 0$ and $\Delta U = -ve$ (d) $W = -ve, Q = 0$ and $\Delta U = +ve$

29. A simple calculation for the entropy S of an ideal gas gives $S = \frac{3}{2} N k_B \ln V + S_0$ where N is the number of molecules in the gas, V is its volume, and $S_0$ an arbitrary constant. This expression is erroneous because

(a) It does not contain temperature T
(b) It does not contain pressure P
(c) It is not extensive
(d) It does not take into account the interaction between the molecules of the gas.

30. Imagine dividing a fluid with internal energy V, entropy S, volume V, particle number N, Temperature T, pressure P and chemical potential $\mu$ into two equal parts. The variables that remain unaffected by this division are

(a) U, V, S and N (b) U, V, T and P (c) T, P and $\mu$ (d) T, P and S

31. $N_1$ molecules of oxygen are confined in volume $V_1$ at temperature T and have entropy $S_1$; $N_2$ molecules of oxygen are confined in volume $V_2$ at temperature T and have entropy $S_2$. The gases are mixed in a volume $V = V_1 + V_2$. The entropy S of the mixture is:

(a) $S < S_1 + S_2$ (b) $S = S_1 + S_2$ (c) $S > S_1 + S_2$ (d) $RT\left(\ln\frac{V_1}{V_1+V_2} - \ln\frac{V_2}{V_1+V_2}\right)$

32. Which of the following expressions are correct?
(a) $dH = TdS + VdP$, Where H is enthalpy
(b) $dF = SdT + PdV$, Where F is Helmholtz free energy.
(c) $dG = SdT - VdP$, Where G is Gibbs function
(d) $SdT = dE + PdV$, where E is internal energy.

33. Consider a perfect gas in a vessel rushing into an evacuated vessel and let the whole system be isolated. Let $V_1$ and $V_2$ denote the initial and final volumes of the gas. The increase in entropy of the system is:

(a) Zero (b) $R\ln\frac{V_2}{V_1}$ (c) $\frac{1}{T}\int_{V_1}^{V_2} du + R\ln\frac{V_2}{V_1}$ (d) $\frac{1}{T}\int_{V_1}^{V_2} du$

34. In order to attain thermal equilibrium at a fixed volume, the system minimises,
(a) S (b) $H = E + pV$ (c) $G = U - TS + pV$ (d) $F = U - TS$
Where the symbols have their usual meaning.

35. The particle density fluctuations in a pure substance are maximum at
(a) Its freezing point (b) Its boiling point (c) At critical point (d) At triple point.

36. A thermally insulated cubical box has two chambers of equal volume. Initially, one mole of a mono atomic ideal gas is placed in one of the chambers while the other chamber is kept empty. The gas slowly leaks through a small hole and eventually occupies the whole box. The change in entropy of the gas is given by
(a) $R/\ln 2$ (b) $R\ln 2$ (c) zero (d) 2R

37. One mole of ideal monoatomic gas is kept in a volume V and is at temperature T. If the volume and temperature of the gas are respectively changed to $V'$ and $T'$. in such a way that the entropy of the gas is unchagned, then which of the following is true?
(a) $V' = 2V$, $T' = T/2$ (b) $V' = 8V$, $T' = T/4$
(c) $V' = 4V$, $T' = 8T$ (d) $V' = V/2$, $T' = 2T$

38. Gas molecules of mass 'm' are confined in a cylinder of radius R and height L (with $R \gg L$) kept vertically in the Earth's gravitational field. The average energy of the gas at low temperatures (such that $mgL >> k_BT$) is given by

(a) $Nk_BT/2$ (b) $3Nk_BT/2$ (c) $2Nk_BT$ (d) $5Nk_BT/2$

39. Consider a thermodynamic system of N particles with internal energy U, volume V, and entropy S. Its chemical potential is given by

(a) $\left(\frac{\partial U}{\partial S}\right)_{V,N}$ (b) $\left(\frac{\partial N}{\partial U}\right)_{S,V}$ (c) $\left(\frac{\partial U}{\partial V}\right)_{S,N}$ (d) $\left(\frac{\partial U}{\partial N}\right)_{S,V}$

40. The heat capacity at costant pressure of a certain gas is 1.4 times its heat capacity at constant volume. A best model for the molecules of this gas is
(a) rigid triangles (b) point particles
(c) rigid dumbbells (d) non- rigid dumbbells that can vibrate

41. A carnot engine operates between a source at temperature 300K and a sink at temperature 200 K. Let $\eta$ be the efficiency of the Carnot engine when the temperatures of both the source and the sink are decreased by the same amount. Then,

(a) $\eta < \frac{1}{3}$ (b) $\eta > \frac{1}{3}$ (c) $\eta = \frac{1}{3}$ (d) $\eta = \frac{1}{2}$

42. Consider the Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_{T,N}=\left(\frac{\partial P}{\partial T}\right)_{V,N}$ where the symbols have the usual meaning. this relation can be derived from

(a) Helmholtz free energy. F = U–TS
(b) Gibbs free energy : G = U–TS + PV
(c) Grand thermodynamic potential Z= U–TS – μN
(d) Enthaply H = U+ PV

43. We have $S\equiv(U,V,N)$, S = entropy , V= volume, U= internal energy, N= number of particles, P = Pressure, T = Temperature, μ = chemical Potential. Which of the following statements is correct?

(a) $\left(\frac{\partial S}{\partial V}\right)_{U,N}=-\frac{1}{T}$ (b) $\left(\frac{\partial S}{\partial V}\right)_{U,N}=-\frac{P}{T}$ (c) $\left(\frac{\partial S}{\partial V}\right)_{U,N}=\frac{P}{T}$ (d) $\left(\frac{\partial S}{\partial N}\right)_{V,U}=\frac{\mu}{T}$

44. Consider an ideal gas of mass m at temperature $T_1$ which is mixed isobarically ( i.e., at constant pressure) with an equal, mass of same gas at temperature $T_2$ in a thermally insulated container. What is the change of entropy of the universe ?

(a) $2mC_p\ \ln\left(\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right)$ (b) $2mC_p\ \ln\left(\frac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

(c) $2mC_p\ \ln\left(\frac{T_1+T_2}{2T_1T_2}\right)$ (d) $2mC_p\ \ln\left(\frac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

45. A thermally insulated ideal gas of volume $V_1$ and temperature T expends into another enclosure of volume $V_2$ through a porous plug. What is the change in the temperature of the gas ?

(a) 0 (b) $T\ \ln(V_1/V_2)$ (c) $T\ \ln(V_2/V_1)$ (d) $T\ \ln\left[(V_2-V_1)/V_2\right]$

46. The free energy of a gas N particled in a volume V and at a temperature T is $F=Nk_BT\ln\left[a_0V(k_BT)^{5/2}/N\right]$, where $a_0$ is a constant and $k_B$ denotes the Boltzmann constant. The internal energy of gas is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{5}{2}Nk_BT$

(c) $Nk_BT\ln\left[a_0V(k_BT)^{5/2}/N\right]-\frac{3}{2}Nk_BT$ (d) $Nk_BT\ln\left[a_0V/(k_BT)^{5/2}\right]$

47. Consider a Maxwellian distribution of the velocity of the molecules of an ideal gas. Let $V_{mp}$ and $V_{rms}$ denote the most probable velocity and the root mean square velocity, respectively. The magnitude of the ratio $V_{mp}/V_{rms}$ is:

(a) 1 (b) $2/3$ (c) $\sqrt{2/3}$ (d) $3/2$

48. Consider a sample of argon gas contained in a box of volume V. The root-mean square speed of the argon atoms is $v_m$. If the gas is allowed to expand at constant pressure to a volume of 2v, the root mean square speed of the gas molecules would be

(a) $\upsilon_m/2$ (b) $\sqrt{2}\upsilon_m$ (c) $\upsilon_m/\sqrt{2}$ (d) $2\upsilon_m$

49. A classical ideal gas in an enclosure is in equilibrium with a heat bath at a constant temperature. The ratio of the root-mean square speed $V_{rms}$, mean speed $\upsilon_{mean}$ and the most probable speed $\upsilon_{mp}$ namely $V_{rms} : V_{mean} : V_{mp}$ is:

(a) $\sqrt{3}:\sqrt{\frac{8}{\pi}}:\sqrt{2}$ (b) $\frac{\sqrt{3}}{2}:\sqrt{\frac{2}{\pi}}:1$ (c) $\sqrt{\pi}:\sqrt{3}:\sqrt{2}$ (d) $\sqrt{\frac{\pi}{3}}:\sqrt{\frac{\pi}{8}}:\frac{1}{\sqrt{2}}$

50. The average speed of molecules in a gas at 20°C is V. At what temperature it will be 2V?

(a) 899°C (b) 586°C (c) 80°C (d) 40°C

51. A classical gas of molecules each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are $v_x$, $v_y$ and $v_z$. The mean value of $(v_x + v_y)^2$ is:

(a) $\frac{k_BT}{m}$ (b) $\frac{3}{2}\frac{k_BT}{m}$ (c) $\frac{1}{2}\frac{k_BT}{m}$ [✓ (d) $\frac{2k_BT}{m}$]

52. At a certain temperature T, the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are, respectively,

(a) 355 m/s, 434 m/s (b) 820 m/s, 917 m/s
(c) 152 m/s, 301 m/s (d) 422 m/s, 600 m/s

53. At 100°C, water vapour and liquid water coexist in thermodyanics equilibrium in a closed container. If μ and S represent the chemical potential and entropy respectively, then at the interface.

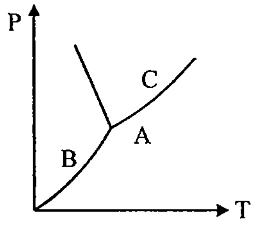
(a) $\mu_{vapour} = \mu_{liquid}$ (b) $S_{vapour} = S_{liquid}$
(c) $S_{vapour} < S_{liquid}$ (d) $\mu_{vapour} > \mu_{liquid}$

54. Finite changes in a second order phase transition occur in

(a) S (b) V (c) $C_P$ (d) T

Where the symbols have their standard meaning.

55. The pressure versus temperature diagram for a pure substance is shown below.



The region represented by A, B & C respectively are

(a) Solid, liquid and vapour (b) Vapour, solid and liquid
(c) Solid, vapour and liquid (d) Vapour liquid and solid.

56. The transition from ordinary liquid helium (He I) to superfluid helium (He II) is an example of a second order lambda point transition. The quantities that remain constant in such a transition at the transition point are

(a) entropy, volume, specific heat at constant pressure and Gibbs free energy
(b) pressure, volume and isothermal compressibility.
(c) specific heat at constant pressure, isothermal compressibility and pressure.
(d) pressure, volume, entropy and Gibbs free energy.

57. In a first order phase transition, at the transition temperature, specific heat of the system.

(a) Diverges and its entropy remains the same
(b) Diverges and its entropy has finite discontinuity
(c) Remains unchanged and its entropy has finite discontinuity.
(d) Has finite discontinuity and its entropy diverges.

58. When two phases 1 and 2 of a system are in thermodynamic equilibrium, their temperature T, pressure p and chemical potential μ satisfy:

(a) $T_1 = T_2$, $p_1 = p_2$ and $\mu_1 = \mu_2$ (b) $T_1 = T_2$, $p_1 = p_2$ and $\mu_1 \neq \mu_2$

(c) $T_1 = T_2$, $\mu_1 = \mu_2$ and $p_1 \neq p_2$ (d) $T_1 = T_2$ but $p_1 \neq p_2$, $\mu_1 \neq \mu_2$

59. When a crystal melts into a liquid at constant pressure, which of the following quantities changes continuously at the phase transition?
(a) Gibbs free energy (b) Volume (c) Internal energy (d) Entropy

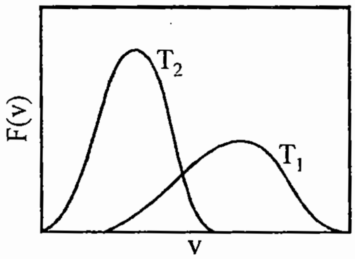
60. Consider a gas of atoms obeying Maxwell-Boltzman statistics. The average value of $e^{i\vec{a}.\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and *a* is its magnitude, *m* is the mass of each atom, *T* is temperature and *k* is Boltzmann's constant) is,

(a) One (b) Zero (c) $e^{-\frac{1}{2}a^2mkT}$ (d) $e^{-\frac{3}{2}a^2mkT}$

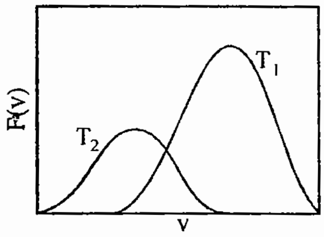
61. Ten grams of ice at 0°C is added to a beaker containing 30 grams of water at 25°C. What is the final temperature of the system when it comes to thermal equilibrium? (The specific heat of water is 1cal/gm/°C and latent heat of melting of ice is 80 cal/gm)
(a) 0°C (b) 7.5°C (c) 12.5°C (d) −1.25°C

62. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution F(v) of the speed v of a molecule in three dimensions is correctly represented by the following figure:

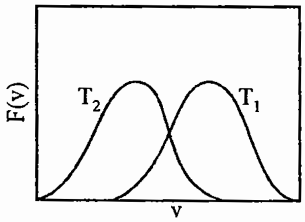
(a)



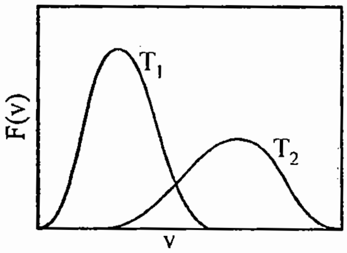
(b)



(c)



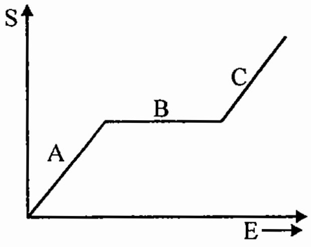
(d)



63. The speed v of the molecules of mass *m* of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let $(v_x, v_y, v_z)$ denote the components of the velocity and $k_B$ the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constants, is

(a) $(\alpha^2 - \beta^2)k_\beta T/m$ (b) $(\alpha^2 + \beta^2)k_\beta T/m$ (c) $(\alpha + \beta)^2 k_\beta T/m$ (d) $(\alpha - \beta)^2 k_\beta T/m$

64. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperatures of the phases A, B and C, denoted by $T_A$, $T_B$ and $T_C$, respectively, satisfy the following inequalities:

(a) $T_C > T_B > T_A$ (b) $T_A > T_C > T_B$ (c) $T_B > T_C > T_A$ (d) $T_B > T_A > T_C$

65. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are $T_1$ and $T_2$, respectively, and $T_1 > T_2$ then their common final temperature is

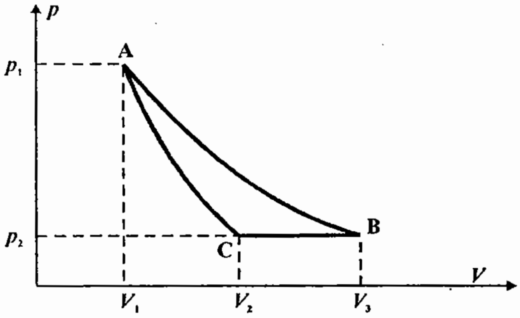
(a) $T_1^2/T_2$ (b) $T_2^2/T_1$ [✓ (c) $\sqrt{T_1T_2}$] (d) $\frac{1}{2}(T_1+T_2)$

66. Two different thermodynamic systems are described by the following equations of state:

$\frac{1}{T^{(1)}}=\frac{3RN^{(1)}}{2U^{(1)}}$ and $\frac{1}{T^{(2)}}=\frac{5RN^{(2)}}{2U^{(2)}}$ where $T^{(1,2)}, N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures, the mole numbers and the internal energies of the two systems, and $R$ is the gas constant. Let $U_{tot}$ denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{tot}}$ is

(a) $\frac{5N^{(2)}}{3N^{(1)}+5N^{(2)}}$ [✓ (b) $\frac{3N^{(1)}}{3N^{(1)}+5N^{(2)}}$] (c) $\frac{N^{(1)}}{N^{(1)}+N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)}+N^{(2)}}$

67. One mole of an ideal gas undergoes the cycle ACBA shown in the PV diagram below.



One of the curved lines in the cycle represents an isothermal change at temperature T, while the other represents an adiabatic change. The net heat gained by the gas in this cycle is

(a) $-p_2(V_3-V_2)+RT\ln\frac{V_2}{V_1}$ (b) $-p_2(V_3-V_2)+RT\ln\frac{V_3}{V_1}$

(c) $-p_2(V_3-V_2)+\gamma RT\left(V_2^{1-\gamma}-V_1^{1-\gamma}\right)$ (d) $(p_1V_1-p_2V_2)-RT\ln\frac{V_3}{V_1}$

68. An ideal gas at a temperature T is enclosed in a rigid container whose walls are initially at temperature $T_1$, where $T_1 < T$. The walls are covered on the outside with perfect thermal insulation and the system is allowed to come to equilibrium. The pressure exerted by the gas on the walls of the container

(a) remains constant throughout

(b) is lower at the initial state than at the final state.

(c) is higher at the initial stage than at the final state.

(d) is the same at the initial and final stages.

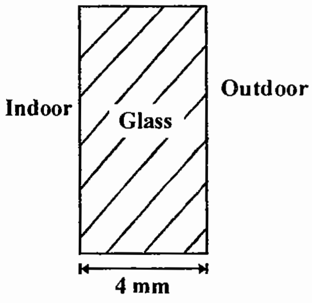
69. A thermally-insulated container of volume $V_0$ is divided into two equal halves by a non-permeable partition. A real gas with equation of state.

$$b^3\left(p+\frac{a^2}{V^3}\right)=nRT$$

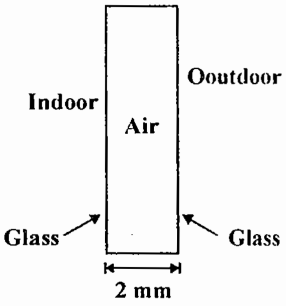
where 'a' and 'b' are constants, is confined to one of these halves at a temperature $T_0$. The partition is now removed suddenly and the gas is allowed to expand to fill the entire container. The final temperature of the gas, in terms of its specific heat $C_V$, will be

(a) $T_0-\frac{3a^2}{2C_VV_0^2}$ (b) $T_0-\frac{2a^2}{3C_VV_0^2}$ (c) $T_0+\frac{3a^2}{2C_VV_0^2}$ (d) $T_0+\frac{2a^2}{3C_VV_0^2}$

70. A manufacturer is able to offer two models of heat-conserving windows, as described below



Window A



Window B

Window A is a simple pane of glass, 4 mm thick. Window B, on the other hand, consists of two extremely thin panes of glass, separated by an air gap of 2mm, as shown in the figure below above. If the thermal conductivity of glass is known to be 0.8 $Wm^{-1}K^{-1}$ and that of air is 0.025 $Wm^{-1}K^{-1}$, then the ratio of heat flow $Q_A$ through window A to the heat flow $Q_B$ through Window B is given by

$$\frac{Q_A}{Q_B}=$$

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$ (c) 4 (d) 16

NPT 71. The free energy F of a system depends on a thermodynamics variable $\psi$ as

$$F=-\alpha\psi^2+b\psi^6$$

with a, b > 0. The value of $\psi$, when the system is in thermodynamic equilibrium, is

(a) zero (b) $\pm(a/6b)^{1/4}$ [✓ (c)] $\pm(a/3b)^{1/4}$ (d) $\pm(a/b)^{1/4}$

## Answer Key (THERMODYNAMICS-II)

1. (c) 2. (c) 3. (b) 4. (d) 5. (a) 6. (c) 7. (a)
8. (b) 9. (b) 10. (b) 11. (a) 12. (c) 13. (c) 14. (b)
15. (a) 16. (a) 17. (a) 18. (c) 19. (d) 20. (c) 21. (b)
22. (b) 23. (d) 24. (d) 25. (d) 26. (b) 27. (c) 28. (a)
29. (c) 30. (c) 31. (c) 32. (a) 33. (b) 34. (d) 35. (c)
36. (b) 37. (b) 38. (d) 39. (d) 40. (c) 41. (b) 42. (a)
43. (c) 44. (a) 45. (a) 46. (d) 47. (c) 48. (b) 49. (a)
50. (a) 51. (d) 52. (b) 53. (a) 54. (c) 55. (b) 56. (d)
57. (b) 58. (a) 59. (a) 60. (c) 61. (a) 62. (a) 63. (b)
64. (c) 65. (c) 66. (b) 67. (b) 68. (c) 69. (a) 70. (d)
71. (c)

# CAREER ENDEAVOUR

ACADEMY PRIVATE LIMITED

Institute for NET-JRF/GATE & IIT-JAM Exams

## PHYSICAL SCIENCES (NET-JRF/GATE)

## Assignment-I: STATISTICAL

1. A microcanonical ensemble represents
(a) a system in contact with a heat reservoir
(b) an isolated system in equilibrium
(c) a system that can exchange particles with its surroundings.
(d) a system under constant external pressure.

Consider a two level quantum system with energies $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon$

2. The Helmholtz free energy of the system is given by
(a) $-k_B T \ln\left(1+e^{-\varepsilon/k_B T}\right)$ (b) $k_B T \ln\left(1+e^{-\varepsilon/k_B T}\right)$
(c) $\frac{3}{2}k_B T$ (d) $\varepsilon - k_B T$

3. The specific heat of the system is given
(a) $\frac{\varepsilon}{k_B T}\frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$ (b) $\frac{\varepsilon^2}{k_B T}\frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)}$ (c) $\frac{\varepsilon^2\, e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$ (d) $\frac{\varepsilon^2}{k_B T}\frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$

4. The probability that an energy level $\varepsilon$ at a temperature T is unoccupied by a fermion of chemical potential $\mu$ is given by
(a) $\frac{1}{e^{(\varepsilon-\mu)/k_B T}+1}$ (b) $\frac{1}{e^{(\varepsilon-\mu)/k_B T}-1}$ (c) $\frac{1}{e^{(\mu-\varepsilon)/k_B T}+1}$ (d) $\frac{1}{e^{(\mu-\varepsilon)/k_B T}-1}$

**Common data for Q5 and Q6.**

Consider two dimentional electron gas of N electrons of mass 'm' each in a system of size L×L.

5. The density of states between energy $\varepsilon$ and $\varepsilon + d\varepsilon$ is
(a) $\frac{4\pi L^2 m}{h^2}$ (b) $\frac{4\pi L^2 m}{h^2}\frac{1}{\sqrt{\varepsilon}}$ (c) $\frac{4\pi L^2 m}{h^2}\sqrt{\varepsilon}$ (d) $\frac{4\pi L^2 m}{h^2}\varepsilon$

6. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0$, $\varepsilon$ and $2\varepsilon$ having degeneracies $g(E) = 1, 2$ and $4$ respectively. The mean energy of the system is:
(a) $2\varepsilon\left(\frac{4\exp\left(-\varepsilon/k_B T\right)+8\exp\left(-2\varepsilon/k_B T\right)}{1+2\exp\left(-\varepsilon/k_B T\right)+4\exp\left(-2\varepsilon/k_B T\right)}\right)$ (b) $\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_B T\right)+8\exp\left(-2\varepsilon/k_B T\right)}{1+2\exp\left(-\varepsilon/k_B T\right)+4\exp\left(-2\varepsilon/k_B T\right)}\right)$
(c) $\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_B T\right)+4\exp\left(-2\varepsilon/k_B T\right)}{1+2\exp\left(-\varepsilon/k_B T\right)+4\exp\left(-2\varepsilon/k_B T\right)}\right)^2$ (d) $\varepsilon\left(\frac{\exp\left(-\varepsilon/k_B T\right)+2\exp\left(-2\varepsilon/k_B T\right)}{1+\exp\left(-\varepsilon/k_B T\right)+\exp\left(-2\varepsilon/k_B T\right)}\right)$

7. Thermodynamic variables of a system can be volume V, pressure P, temperature T, number of particles N, internal energy E and chemical potential $\mu$, etc. For a system to be specified by microcanonical (MC), Canonical (CE) and Grand Canonial (GC) ensembles, the parameters required for the respective ensembles are
(a) $MC:(N,V,T); CE(E,V,N); GC:(V,T,\mu)$
(b) $MC:(E,V,N); CE:(N,V,T); GC:(V,T,\mu)$
(c) $MC:(V,T,\mu); CE:(N,V,T); GC:(E,V,N)$
(d) $MC:(E,V,N); CE:(V,T,\mu); GC:(N,V,T)$

8. The partition function of a single oscillator with energy levels $\left(n+\frac{1}{2}\right)\hbar\omega$ is given by

(a) $Z=\frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$ (b) $Z=\frac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$ (c) $Z=\frac{1}{1-e^{-\beta\hbar\omega}}$ (d) $Z=\frac{1}{1+e^{-\beta\hbar\omega}}$

9. The average number of energy quanta of the oscillators is given by

(a) $\langle n\rangle=\frac{1}{e^{-\beta\hbar\omega}-1}$ (b) $\langle n\rangle=\frac{e^{-\beta\hbar\omega}}{e^{-\beta\hbar\omega}-1}$ (c) $\langle n\rangle=\frac{1}{e^{\beta\hbar\omega}+1}$ (d) $\langle n\rangle=\frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

& ground state is non degenerate

10. A system has energy level $E_0, 2E_0, 3E_0,\ldots\ldots,$ where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is:

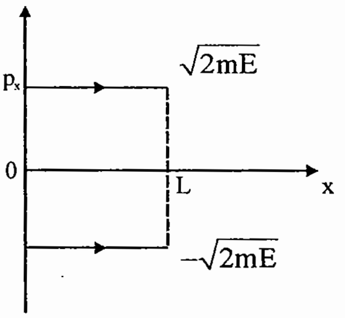
(a) 2 ~~(b) 3~~ (c) 4 (d) 5

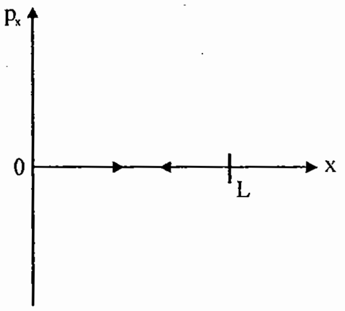
11. It is necessary to apply quantum statistics to a system of particles if
(a) There is substaintial overlap between the wavefunctions of the particles.
(b) The mean free path of the particles is comparable to the inter-particle separation.
(c) The particles have identical mass and charge.
(d) The particles are interacting.
Consider a radiation cavity of volume V at temperature T.

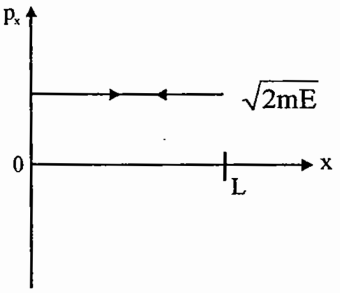
12. A monatomic crystalline solid comprises of N' atoms, out of which n atoms are in interstitial positions. If the available interstitial sites are N, then number of possible microstates is:

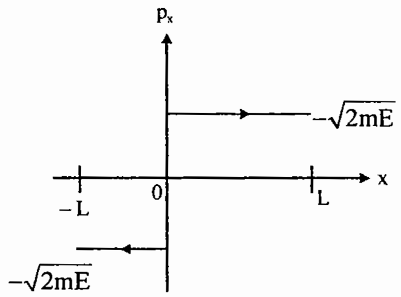
(a) $\frac{(N'+n)!}{n!N!}$ (b) $\frac{(N+n)!}{n!(N'-n)!}\,\frac{(N'+n)!}{n!(N'-n)!}$ (c) $\frac{N!}{n!(N'-n)!}$ ~~(d)~~ $\frac{N!}{n!(N-n)!}\,\frac{N'!}{n!(N'-n)!}$

13. The phase diagram of a free particle of mass m and kinetic energy E, moving in one-dimensional box with perfectly elastic walls at x = 0 and x = L, is given by

(a)


(b)


(c)


(d)


14. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperature of A and B are $T_1$ and $T_2$, respectively. The two vessels are brought into thermal contct. At equlibrium, the change in entropy is:

(a) $\frac{3}{2}Nk_B \ln\left[\frac{T_1^2+T_2^2}{4T_1T_2}\right]$ (b) $Nk_B \ln\left(\frac{T_2}{T_1}\right)$

(c) $\frac{3}{2}Nk_B \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (d) $2Nk_B$

15. The number of states for a system of N identical free particles in a three dimensional space having total energy between E and $E+\delta E(\delta E << E)$, is proportional to

[✓ (a) $\left(E^{\frac{3N}{2}-1}\right)\delta E$] (b) $E^{\frac{N}{2}}\delta E$ (c) $NE^{\frac{1}{2}}\delta E$ (d) $N^N\delta E$

16. The total number of accessible states of N non intracting particles of spin 1/2 is

(a) $2^N$ (b) $N^2$ (c) $2N^{N/2}$ (d) N

17. The dimension of phase space of ten rigid diatomic molecules is:

(a) 5 (b) 10 (c) 50 [✓ (d) 100]

18. Which of the following relations between the particle number density n and temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?

(a) $n/T^{1/2} << 1$ (b) $n/T^{3/2} << 1$

(c) $n/T^{3/2} >> 1$ (d) $n/T^{1/2}$ and $n/T^{3/2}$ can have any value.

19. In a classical micro-canonical ensemble for a system of N non-interacting particles, the fundamental volume in phase space which is regarded as "equivalent to one micro-state" is

[✓ (a) $h^{3N}$] (b) $h^{2N}$ (c) $h^N$ (d) h

where h is the Planck's constant.

20. Which of the following statements is true?
(a) In a micro-canonical ensemble the total number of particles N and the energy E are constants while in a canonical ensemble N and temperature T are constants.
(b) In a micro-canonical ensemble the total number of particles N is a constant but the energy E is variable while in a canonical ensemble N and T are constants.
(c) In a macro-canonical ensemble N and E are constants while in a canonical ensemble N and T both vary.
(d) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N is a constant but T varies.

21. Two particles are said to be distinguishable when
(a) the average distance between them is large compared to their de Broglie wavelength.
(b) the average distance between them is small compared to their de Broglie wavelengths.
(c) they have overlapping wavepackets.
(d) their total wave function is symmetric under particle exchange.

22. In a canonical ensemble
(a) the energy and the temperature are constants.
(b) the entropy and the energy are constans.
(c) the temperature and the density are constants.
(d) the density and the entropy are constants.

23. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V = \gamma T^3$ where $\gamma$ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is:

(a) $4\gamma T^4$ (b) $2\gamma T^4$ (c) $\frac{\gamma T^4}{4}$ (d) $\gamma T^4$

24. The distribution function $f(\varepsilon)$ for a photon gas is given by

(a) $\exp(-\varepsilon / kT)$ (b) $\frac{1}{\exp(\varepsilon / kT)+1}$

(c) $\frac{1}{\exp(\varepsilon / kT)-1}$ (d) $\frac{1}{\exp(-\varepsilon / kT)-1}$

25. Consider the electron gas in a simple metal at room temperature
(a) This is a degenerate Fermi-Dirac gas.
(b) The gas displays paramagnetism owing to the spins of the electrons.
(c) All the electrons are in the same single-particle energy level.
(d) The electrons have wave functions localized about the lattice positions of the positive ions.

26. A cavity of volume V contains blackbody radiation at an absolute temperature T.
(a) The internal energy of the radiation is proportional to $T^4$.
(b) The specific heat of radiation is proportional to $T^3$.
(c) The pressure of the radiation is proportional to $T^4$
(d) Although the radiation is in thermal equailibrium, the number of photons in the cavity is not constant in time.

27. Bose Einstein condensation temperature $T_B$ refers to the temperature below which
(a) an assembly of Bose gas condenses to the liquid state.
(b) there is an appreciable occupation of the ground state in an electron system.
(c) there is a significantly large occupancy of the ground state in a system of bosons
(d) the bosons essentially behave like fermious.

28. The energy of a single particle in an infinite potential box is $E_0 n^2$ where $E_0 = \hbar^2 \pi / 2ma^2, n = 1, 2, ....$ If three non-interacting bosons are in such a potential box, the ground state energy of the system will be
(a) $3E_0$ (b) $4E_0$ (c) $6E_0$ (d) $9E_0$

29. The Fermi energy of a free electron gas depends on the electron density $\rho$ as,

(a) $\rho^{1/3}$ (b) $\rho^{2/3}$ (c) $\rho^{-1/3}$ (d) $\rho^{-2/3}$

30. For an energy state E of a photon gas, the density of states is proportional to

(a) $\sqrt{3}$ (b) E (c) $E^{3/2}$ (d) $E^2$

31. Consider the Fermi-Dirac distribution function f(E) at room temperature (300 K) where E refers to energy. If $E_F$ is the Fermi energy, which of the following is true?
(a) f(E) is a step function (b) $f(E_F)$ has a value of 1/2
(c) States with $E < E_F$ are filled completely
(d) f(E) is large and tends to infinity as E decreases much below $E_F$.

32. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place?

(a) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{2/3}$ (b) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{2/3}$

(c) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$ (d) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

where m is the mass of each particle of the system, $k_B$ is the Boltzmann constant, h is the Planck's constant and $\varsigma$ is the well known Zeta function.

33. ✓ Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 cm$^3$ to 640 cm$^3$, the temperature of the cavity changes to

(a) 800 K (b) 700 K (c) 600 K (d) 500 K

34. The specific heat of an ideal Fermi gas in 3-dimension at very low temperatures (T) varies as

(a) T (b) $T^{3/2}$ (c) $T^2$ (d) $T^3$.

A gas of N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N\ln\left[(V-bN)(k_BT)^{3/2}\right]$ where b is a constant with appropriate dimensions.

35. If P is the pressure of the gas, the equation of state is given by

[✓ (a)] $P(V-bN) = Nk_BT$ (b) $P(V-bN) = k_BT$

(c) $P(V-b) = Nk_BT$ (d) $P(V-b) = Nk_BT$

36. The internal energy of the gas is given by

(a) $U = (1/2)k_BT$ (b) $U = (1/2)k_BT$ (c) $U = (3/2)Nk_BT$ (d) $U = 2Nk_BT$

37. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?

(a) Its energy and the number of photons both increase 8 times.

(b) Its energy increases 8 times and the number of photons increase 16 times.

[✓ (c)] Its energy increases 16 times and the number of photons increases 8 times.

(d) Its energy and the number of photons both increases 16 times.

38. The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3}aVT^4$ where *a* is a constant and T is the temperature of the gas. The chemical potential of the photon gas is

(a) 0 (b) $\frac{4}{3}aVT^3$ (c) $\frac{1}{3}aVT^3$ (d) $aVT^{-4}$

39. A one dimensional random walker takes steps to left or right with equal probability. The probability that the random starting from origin is back to origin after N even number of steps is:

[✓ (a)] $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$ (c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

40. If the partition function of a harmonic oscillator with frequency $\omega$ at a temperature T is $\frac{kT}{h\omega}$, then the free energy of N such independent oscillators is:

(a) $\frac{3}{2}NkT$ (b) $kT\ln\frac{\hbar\omega}{kT}$ (c) $NkT\ln\frac{\hbar\omega}{kT}$ (d) $NkT\ln\frac{\hbar\omega}{2kT}$

41. The partition function of two Bose particles each of which can occupy any of the two energy levels 0 and $\varepsilon$ is:

(a) $1+e^{-2\varepsilon/kT}+2e^{-\varepsilon/kT}$ (b) $1+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (c) $2+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (d) $e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

42. A system of non-interacting Fermi particles with Fermi energy $E_F$ has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature $T = 0$ is:

(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$ (c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$

43. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is:

(a) $P=\frac{3}{2}\frac{U}{V}$ (b) $P=\frac{2}{3}\frac{U}{V}$ (c) $P=\frac{3}{5}\frac{U}{V}$ (d) $P=\frac{1}{2}\frac{U}{V}$

An ensemble of N three-level systems with energies $E = -\epsilon_0, 0, +\epsilon_0$ is in thermal equilibrium at temperature T. Let $\beta=(k_\beta T)^{-1}$.

44. If $\beta\varepsilon_0 = 2$, the probability of finding the system in the level $\varepsilon = 0$ is:

(a) $(\cos h2)/2$ (b) $(\cos h2)^{-1}$ (c) $(2\cos h2)^{-1}$ (d) $(1+2\cos h2)^{-1}$

45. The free energy of the system at high temperature (i.e., $x \equiv \beta\varepsilon_0 << 1$) is approximately

(a) $-NK_B Tx^2$ (b) $-NK_B T[\ln 2+x^2/2]$

(c) $-NK_B T[\ln 3+x^2/3]$ (d) $-NK_B T\ln 3$

46. A system of N localized, non-interacting spin $\frac{1}{2}$ ions of magnetic moment $\mu$ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the system is:

(a) $Nk_BT\ln\left(\cos h\frac{\mu H}{k_BT}\right)$ (b) $-Nk_BT\ln\left(2\cos h\frac{\mu H}{k_BT}\right)$

(c) $Nk_BT\ln\left(2\cos h\frac{\mu H}{k_BT}\right)$ (d) $-Nk_BT\ln\left(2\sin h\frac{\mu H}{k_BT}\right)$

47. The free energy for a photon gas is given by $F=-\left(\frac{a}{3}\right)VT^4$, where *a* is a constant. The entropy S and the pressure P of the photon gas are

(a) $S=\frac{4}{3}aVT^3, P=\frac{a}{3}T^4$ (b) $S=\frac{1}{3}aVT^4, P=\frac{4a}{3}T^3$

(c) $S=\frac{4}{3}aVT^4, P=\frac{a}{3}T^3$ (d) $S=\frac{1}{3}aVT^3, P=\frac{4a}{3}T^4$

48. The density of states at energy E of the quantized radiation (photons) is:

(a) $\frac{8\pi V}{h^3C^3}E^2$ (b) $\frac{8\pi V}{h^3C^3}E^{3/2}$ (c) $\frac{8\pi V}{h^3C^3}E$ (d) $\frac{8\pi V}{h^3C^3}E^{1/2}$

49. The average number of photons in equilibrium inside the cavity is proportional to
(a) T (b) $T^2$ (c) $T^3$ (d) $T^4$.

50. A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is:
(a) $\exp\left(\frac{\hbar\omega}{k_B T}\right)+1$ (b) $\exp\left(\frac{\hbar\omega}{k_B T}\right)-1$ (c) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right)+1\right)^{-1}$ (d) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right)-1\right)^{-1}$

51. Two identical particles have to be distributed among three energy levels. let $r_B$, $r_F$ and $r_C$ represent the ratios of probability of finding two particles to that of finding one particle in a given enegy state. The subscripts B, E and C correspond to whether the particles are bosons, fermions and classical particles, respectively. The $r_B$:$r_F$:$r_C$ is equal to
(a) $\frac{1}{2}:0:1$ (b) $1:\frac{1}{2}:1$ (c) $1:\frac{1}{2}:\frac{1}{2}$ [✓ (d) $1:0:\frac{1}{2}$]

52. The ground state energy $E_0$ of the system in terms of the Fermi energy $E_F$ and the number of electrons N is given by
Q-S
(a) $\frac{1}{3}NE_F$ (b) $\frac{1}{2}NE_F$ (c) $\frac{2}{3}NE_F$ (d) $\frac{3}{5}NE_F$

53. Across a first order phase transition, the free energy is
(a) proportional to the temperature
(b) a discontinuous function of the temperature
(c) a continuous function of the temperature but its derivative is discontinuous
(d) such that the first derivative with respect to temperature is continuous

54. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same
(a) pressure and temperature (b) volume and temperature
(c) pressure and volume (d) volume and energy

55. The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where $a$ and $E_0$ are positive constants. The temperature of the system is
(a) negative for some energies (b) increases monotonically with energy
(c) decreases monotonically with energy (d) zero

56. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is ($k$ is the Boltzmann constant)
(a) Zero (b) $Nk$ (c) $\frac{1}{2}Nk$ (d) $Nk\,ln(2)$

57. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is ______

58. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to
(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

59. At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmoic oscillator is ____________ $k_B T$.

## Answer Key (STATISTICAL-I)

1. (b) 2. (a) 3. (d) 4. (c) 5. (a) 6. (b) 7. (b)
8. (b) 9. (a) 10. (b) 11. (a) 12. (d) 13. (a) 14. (c)
15. (a) 16. (a) 17. (b) 18. (c) 19. (a) 20. (a) 21. (a)
22. (c) 23. (d) 24. (c) 25. (a) 26. (d) 27. (c) 28. (a)
29. (b) 30. (d) 31. (b) 32. (a) 33. (d) 34. (a) 35. (a)
36. (c) 37. (c) 38. (a) 39. (a) 40. (c) 41. (b) 42. (d)
43. (b) 44. (d) 45. (d) 46. (b) 47. (a) 48. (a) 49. (c)
50. (d) 51. (d) 52. (b) 53. (b) 54. (a) 55. (a) 56. (d)
57. (70) 58. (d) 59. (1.99 to 2.01)

# CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
Institute for NET-JRF/GATE & IIT-JAM

## PHYSICAL SCIENCES (JRF-NET/GATE)

## Assignment-II: STATISTICAL

1. A system in thermal equilibrium consists of subsystems A and B that interact only weakly with each other. If $Z_A$ and $Z_B$ are the canonical partition functions of A and B respectively, the partition function of the total system is given, to a good approximation, by

(a) $Z_A + Z_B$ (b) $Z_A / Z_B$ (c) $Z_A - Z_B$ (d) $Z_A\, Z_B$

2. The standard deviation of the energy of a system in the canonical ensemble is equal to

(a) $\sqrt{C_v}$ (b) $k_B T^2 C_v$ [✓ (c) $\sqrt{k_B T^2 C_v}$] (d) $\sqrt{k_B T C_v}$

3. Let N be the total number of distinguishable particles in a system, out of which the number of particles with energies $E_1, E_2, \ldots\ldots\ldots E_n$ is $N_1, N_2 \ldots\ldots\ldots\ldots N_n$ respectively, $(N_1 + N_2 + \ldots\ldots N_n = N)$
The number of ways in which these particles can be distributed is

(a) $\dfrac{N!}{N_1!N_2!---N_n!}$ (b) $\dfrac{N_1!N_2!---N_n!}{N!}$

(c) $\dfrac{N!}{\sqrt{N_1!}+\sqrt{N_2!}---\sqrt{N_n!}}$ (d) $\dfrac{\sqrt{N!}}{N_1!N_2!---N_n!}$

4. The entropy of a system in the microcanonical ensemble formalism
(a) Is independent of the number of microslates.
(b) Is directly proportional to the number of microstates.
(c) Is inversely proportional to the number of microstates.
(d) Increases logarithmicaly with increase in number of microstates.

5. A degenerate electron gas is characterised by the following property:
(a) Each energy level has a very large occupancy.
(b) Each energy level has only a single occupancy.
(c) High density and low temperature.
(d) Low density and high temperature.

6. $Z_1$, $Z_2$ and $Z_3$ represent translational, vibrational and rotational partition functions of a diatomic molecule. The partition function of the molecule Z is given by

(a) $Z = Z_1\, Z_2\, Z_3$ (b) $Z = Z_1 + Z_2 + Z_3$

(c) $\ln Z = \ln(Z_1 + Z_2 + Z_3)$ (d) $e^Z = e^{Z_1} + e^{Z_2} + e^{Z_3}$

7. The principle of equal a priori probability asserts than
(a) All states of a system are equally probable.
(b) All microstates of a system compatible with a macrostate of energy between E and $E + \Delta E$ are equally probable.
(c) All microstates of a system which are incompatible with a macrostate energy E are equally probable.
(d) A-priori probability of all microstates has a normal distribution.

8. In a statistical description of a system in the canonical ensemble, the relative uncertainty in the energy of the system, $\Delta E/E$, varies with the number of particles N in the systems as

(a) $N^{-1}$ (b) $N^{-1/2}$ (c) $N^0$ (d) $N^{1/2}$

9. In a canonical ensemble description of a system, which of the following quantities remain fixed?
(a) Energy of the system (b) Square of the energy of the system
(c) Volume of the system (d) Number of particles in the system.

10. $E_1$ and $E_2$ is non degenerate energy then probability of $E_2$.

(a) $\dfrac{\exp(\beta E_1)}{\exp(\beta E_1)+\exp(\beta E_2)}$ (b) $\dfrac{\exp(\beta E_1)+\exp(\beta E_2)}{\exp(-\beta E_1)+\exp(-\beta E_2)}$

(c) $\dfrac{\exp(-\beta E_1)}{\exp(-\beta E_2)+\exp(-\beta E_1)}$ (d) $\dfrac{\exp(-\beta E_2)}{\exp(-\beta E_1)+\exp(-\beta E_2)}$

11. A system has two energy levels 0 and e > 0. The energy levels are populated according to the Boltzmann's distribution law. In the limit $T \to \infty$ the entropy per particle is:

(a) $+\infty$ (b) $k_B \ln 2$ (c) $2k_B$ (d) $k_B \ln e^2$

12. An ensemble of identical harmonic oscillators with a Hamiltonian, $H = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ have energies between $E$ and $E + \Delta E$. If the oscillators are initially, uniformly distributed in the phase space then which of the following is false?
(a) More particles are at the maximum amplitude than at the origin
(b) The mean displacement is zero.
(c) Mean kinetic energy is equal to the mean potential energy.
(d) More particles are at the origin than at maximum amplitude.

13. N molecules of an ideal gas are in a container of volume $V_0$. If a molecule has the same probability of being at any point in the container, the probability of M of them being in a smaller volume V in the container is:

(a) M/N (b) $\dfrac{V^M (V_0 - V)^{N-M}}{V_0^N}$

(c) $\dfrac{N!}{(N-M)!M!}\dfrac{V^M (V_0 - V)^{N-M}}{V_0^N}$ (d) $\dfrac{N!}{(N-M)!M!}\dfrac{V_0^M (V_0 - V)^{N-M}}{V^N}$

14. A non-interacting system has two energy levels, 0 and $\varepsilon$. The lower level is doubly degenerate while that of energy $\varepsilon$ is non-degenerate. If the system is in thermal equilibrium at temperature T, the single - particle partitin function is:

(a) $Z = 2e^{-\varepsilon/k_BT}$ (b) $Z = 1 + 2e^{-\varepsilon/k_BT}$ (c) $Z = 2 + e^{-\varepsilon/k_BT}$ (d) $Z = 2\left(1 + e^{-\varepsilon/k_BT}\right)$

15. A system of weakly interacting two-dimensional harmonic oscillators is in thermal conact with a heat bath of absolute temperature T. The average kinetic energy of an oscillator is:

(a) $k_BT$ (b) $2k_BT$ (c) $3\,k_BT/2$ (d) $k_BT/2$

16. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is:

(a) $2k_B \ln N$ (b) $3k_B \ln N$ (c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

17. A system has two energy levels with energies $\varepsilon$ and $2\varepsilon$. The lower level is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is:

(a) $\dfrac{1}{1+e^{-\varepsilon/k_BT}}$ (b) $\dfrac{1}{1+2e^{\varepsilon/k_BT}}$ (c) $\dfrac{1}{2e^{\varepsilon/k_BT}+4e^{2\varepsilon/k_BT}}$ (d) $\dfrac{1}{2e^{\varepsilon/k_BT}-4e^{2\varepsilon/k_BT}}$

18. Consider a system of N non-interacting spins, each of which has classifical magnetic moment of magnitude $\mu$. The Hamiltonian of this system in an external magnetic field $\vec{H}$ is $\mathcal{H} = -\sum_{i=1}^{N} \vec{\mu}_i . \vec{H}$ , where $\vec{\mu}_i$ is the magnetic moment of the $i^{th}$ spin. The magnetization per spin at temperature T is:

(a) $\frac{\mu^2 H}{k_B T}$ (b) $\mu\left[\coth\left(\frac{\mu H}{k_B T}\right) - \frac{k_B T}{\mu H}\right]$

(c) $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$ (d) $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$

19. A gas of N non-interacting particles is in thermal equilibrium at temperature T. Each particle can be in any of the possible non-degenerate states of energy 0, $2\varepsilon$ and $4\varepsilon$. The average energy per particle of the gas, when $\beta\varepsilon \ll 1$, is:

[✓ (a) $2\varepsilon$] (b) $3\varepsilon$ (c) $2\varepsilon/3$ (d) $\varepsilon$

20. A one-dimensional chain consists of a set of N rods each of length a. When stretched by a load, each rod can align either parallel or perpendicular to the length of the chain. The energy of a rod is $-\varepsilon$ when aligned parallel to the length of the chain and is $+\varepsilon$ when perpendicular to it. When the chain is in thermal equilibrium at temperature T, its average length is:

(a) Na/2 (b) Na (c) $Na/\left(1+e^{-2\varepsilon/k_B T}\right)$ (d) $Na/\left(1+e^{-2\varepsilon/k_B T}\right)$

21. Consider a system with two allowed microstates, one at energy zero and the other at energy $k_B T$ The system is in equilibrium at temperature T. What is the probability of finding the system in its ground state?

(a) 1 (b) $\frac{1}{e}$ (c) $\frac{1}{1+e}$ (d) $\frac{e}{1+e}$

22. Let $\sigma_v^2 \equiv \left\langle U - \langle U \rangle^2 \right\rangle$ and $\sigma_v^2 = \left\langle P - \langle P \rangle^2 \right\rangle$ be the variances of the internal energy U and the pressure P about their mean values $\langle U \rangle$ and $\langle p \rangle$ in a system of N particles at constant temperature. Then

[✓ (a) $\sigma_U^2 \propto N$ and $\sigma_p^2 \propto N$] (b) $\sigma_U^2 \propto \frac{1}{N}$ and $\sigma_p^2 \propto \frac{1}{N}$

(c) $\sigma_U^2 \propto \frac{1}{N}$ and $\sigma_p^2 \propto N$ (d) $\sigma_U^2 \propto N$ and $\sigma_p^2 \propto \frac{1}{N}$

23. In the canonical ensemble description, a system
(a) can only exchange energy with its environment
(b) is isolated from its environment
(c) can exchange energy as well as particles with its environment
(d) can only exchange particles with its environment

24. A closed system can be in any one of two nondegenerate microstates with energy $E_1$ and $E_2$. The system is in thermal equilibrium with a heat bath at temperature T. The probability that the system will be observed in the state with energy $E_2$ is given by

(a) $\exp\left(-\frac{E_2}{k_BT}\right)$ (b) $\exp\left(-\frac{E_1-E_2}{k_BT}\right)$

(c) $\frac{\exp\left(-\frac{E_2}{k_BT}\right)}{\exp\left(-\frac{E_1}{k_BT}\right)+\exp\left(-\frac{E_2}{k_BT}\right)}$ (d) $\frac{\exp\left(-\frac{E_1-E_2}{k_BT}\right)}{\exp\left(-\frac{E_1}{k_BT}\right)+\exp\left(-\frac{E_2}{k_BT}\right)}$

25. Consider a system whose three energy levels are given by 0, $\varepsilon$ and $2\varepsilon$. The energy level $\varepsilon$ is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta=\frac{1}{k_BT}$ is given by

(a) $1+2e^{-\beta\epsilon}$ (b) $2e^{-\beta\epsilon}+e^{-2\beta\epsilon}$ (c) $\left(1+e^{-\beta\epsilon}\right)^2$ (d) $1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}$

26. Consider a system of particles in three dimensions with momentum $\bar{p}$ and energy $E=c.\bar{P}|$.c being a constant. The system is maintained at inverse temperature $\beta$. volume V and chemical $\mu$. What is the grand particle partition function of the system?

(a) $\exp\left[e^{\beta\mu}8\pi V/(\beta ch)^3\right]$ (b) $e^{\beta\mu}6\pi V/(\beta ch)^2$

(c) $\exp\left[e^{\beta\mu}6\pi V/(\beta ch)^3\right]$ (d) $e^{\beta\mu}8\pi V/(\beta ch)^2$

27. A collection of N two- level systems with energies 0 and $E>0$ is in thermal equilibrium at temperature T. For $T\to\infty$, the specific. heat approaches

(a) 0 (b) $Nk_B$ (c) $3Nk_{B/2}$ (d) $\infty$

28. Consider a system maintained at temperature T, With two available energy states $E_1$ and $E_2$ each with degeneracies $g_1$ and $g_2$. If $p_1$ and $p_2$ are probabilities of occupancy of the energy states, what is the entropy of the system?

Canonical problem

(a) $S=-\kappa_B\left[p_1\text{In}(p_1/g_1)+p_2\text{In}(p_2/g_2)\right]$ (b) $S=-\kappa_B\left[p_1\text{In}(p_1g_1)+p_2\text{In}(p_2g_2)\right]$

(c) $S=-\kappa_B\left[p_1\text{In}\left(p_1^{g_1}\right)+p_2\text{In}\left(p_2^{g_2}\right)\right]$ (d) $S=-\kappa_B\left[(1/p_1)\text{In}(p_1/g_1)+(1/p_2)\text{in}(p_2/g_2)\right]$

29. A system has two normal modes of vibration, with frequencies $\omega_1$ and $\omega_2=2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\hbar\omega_1$? [In the following $x=e^{-\beta\hbar\omega}$ and Z is the partition function]

(a) $x^{3/2}\left(x+2x^2\right)/Z$ (b) $x^{3/2}\left(1+x+x^2\right)/Z$

(c) $x^{3/2}\left(1+2x^2\right)/Z$ (d) $x^{3/2}\left(1+x+2x^2\right)/Z$

30. The entropy of a system, S, is related to the accessible phase space volume $\Gamma$ by $S = k_B \ln \Gamma(E, N, V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that $\Gamma$

(a) does not change during evolution to equilibrium

(b) Oscillates during evolution to equilibrium

(c) Is a maximum in equilibrium

(d) Is a minimum in equilibrium

31. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2J. In a configuration with r spin flips, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^{N}C_r$; r varies from 0 to N. The partition function is

(a) $\left(\frac{J}{k_B T}\right)^N$ (b) $e^{-NJ/k_B T}$ (c) $\left(\sinh\frac{J}{k_B T}\right)^N$ (d) $\left(\cosh\frac{J}{k_B T}\right)^N$

32. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

(a) $\hbar\omega/2$ (b) $\hbar\omega$ [✓ (c) $3\hbar\omega/2$] (d) 0

33. Bose condensation occurs in liquid $He^4$ kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in $He^4$ in gaseous state, the density of which is 1000 times smaller than that of liquid $He^4$? (Assume that it is a perfect Bose gas)

(a) 2.17 mK [✓ (b) 21.7 mK] (c) 21.7 μK (d) 2.17 μK

34. A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that N/2 spins are aligned parallel and the remaining N/2 spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is,

(a) $2Nk_B \ln 2$ (b) $\frac{N}{2}k_B \ln 2$ (c) $\frac{3N}{2}k_B \ln 2$ (d) $Nk_B \ln 2$

35. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the CORRECT equation of state ( P being the pressure),

(a) $PV = \frac{1}{3}E$. [✓ (b) $PV = \frac{2}{3}E$] (c) $PV = E$ (d) $PV = \frac{5}{3}E$

36. If the volume of a three- dimensional free Fermi gas is decreased by a factor of 8 at zero temperature. its internal energy changes by a factor of

(a) 8 (b) 2 (c) 4 (d) $1/8$

37. An ideal Bose gas undergoes Bose-Einstein condensation at temperature $T_0$. At any finite temperature below $T_0$,

(a) the chemical potential is equal to zero (b) only the lowest energy level is occupied

(c) the internal energy is equal to zero (d) the pressure is equal to zero.

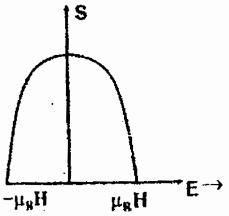
38. Two non-interacting bosons are distributed in three degenerate levels of energy zero. The canonical partition function of this system is
(a) 4.5 (b) 6 (c) 9 (d) 8

39. If the volume of a three-dimensional system of non-interacting fermions is decreased by a factor of 27 at zero temperature, its, internal energy change by a factor of
(a) 9 (b) 3 (c) 1/27 (d) 1/9

40. Consider a system of a large number N of very weakly interacting spin-1/2 particles, each with magnetic moment $\mu$. A magnetic field H is applied in the z-direction. The ratio between the numbers of up-and down-spin particles is given by
(a) $(1+\beta\mu H):1$ (b) $(1-\beta\mu H):1$ (c) $1:e^{\beta\mu H}$ (d) $e^{\beta\mu H}:1$

41. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temprature will
(a) increase by a factor of 4 (b) decrease by a factor of 4
(c) increase by a factor of 8 (d) decrease by a factor of 8

42. A system of N noninteracting spin $-\frac{1}{2}$ particles is placed in an external magnetic field H. The behaviour of the entropy of the system as a function of energy is given by

(a)


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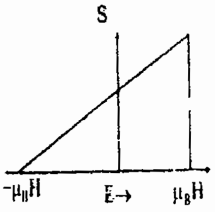

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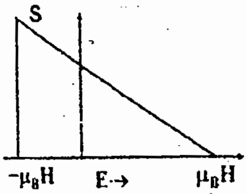

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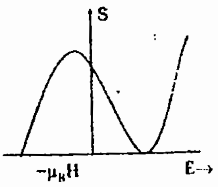


43. The energy density of black-body radiation at absolute temperature T in d dimensions proportional to
(a) $T^{(d^2-1)/2}$ [✓ (b) $T^{d+1}$] (c) $T^{(d+5)/2}$ (d) $T^{2d-2}$

44. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s > 0$. The range of s for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is:
(a) $1 < s < 3$ (b) $0 < s < 2$ [✓ (c) $0 < s < 3$] (d) $0 < s < \infty$

45. The chemical potential p is:
(a) Positive in Fermi-Dirac statistics (FD) and positive in Bose-Einstein statistics (BE) at room temperature (RT)
(b) Positive in FD and negative in BE at RT
(c) Negative in Maxwell Boltzmann statistics and negative in FD at RT.
(d) Always positive in all the three statistics.

46. Considering fermionic particles A and B with chemical potentials $\mu$ and $-\mu$ respectively moving in a box at temperature T. The number density of particle A is:
(a) greater than that of B (b) less than that of B
(c) equal to that of B (d) zero.

47. Fermi energy $E_F$ of an electron gas of N electrons confined in a volume V is proportional to:
(a) $(N/V)^{2/3}$ (b) $(N/V)^{1/3}$ (c) $(V/N)^{1/3}$ (d) $(V/N)^{2/3}$

48. A gas of N spin 1/2 particles and magnetic moment $\vec{\mu}$ each, is kept in uniform magnetic induction $\vec{B}$ at temperature T. Then, average energy of the gas is:
(a) $N\mu B\, e^{-\beta\mu B}$ (b) $-N\mu B\, e^{\beta\mu B}$
(c) $N\mu B\left(e^{\beta\mu B}+e^{-\beta\mu B}\right)$ (d) $\dfrac{-N\mu B\left(e^{-\beta\mu B}-e^{\beta\mu B}\right)}{\left(e^{-\beta\mu B}+e^{\beta\mu B}\right)}$

Where $\beta = 1/k_BT$; $k_B$ is Boltzmann constant.

49. It is observed that the highest occupied energy level in the ground state of the system of non-interacting identical particles enclosed in a box increases with the number of particles. From this observation, we can conclude that the particles have
(a) Zero spin (b) Odd integer spin (c) Even integer spin (d) Half integer spin

50. The difference between Maxwell-Boltzmann (MB), and Bose-Einstein (BE) distribution is that
(a) Both are identical except that one is for classical particles and the other for quantum particles.
(b) Density of occupied states at lower energies is more for BE than for MB.
(c) Density of occupied states at lower energies is less for BE than for MB.
(d) MB is applicable to particles with zero spin and BE to those of finite spin.

51. A system of particles can undergo a Bose Einstein condensation if all particles
(a) Have zero rest mass and zero spin (b) Have half-integer spins
(c) Have mass and spin zero (d) Are free electrons.

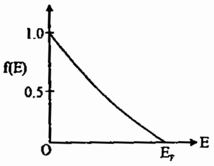
52. A particle of spin $\frac{1}{2}$ and magnetic moment $\mu$ is in a magnetic field B. Its partition function is:
(a) $2\tanh \mu B/kT$ (b) $2\cosh\dfrac{\mu B}{kT}$ (c) $2\sin h\dfrac{\mu B}{kT}$ (d) $e^{\frac{\mu B}{kT}}$

53. For blackbody radiation.
(a) The temperature of the photon gas is lower than the temperature of its container.
(b) The temperature of the photon gas is higher than the temperature of the container.
(c) The temperature of the electromagnetic radiation is a meaningless concept.
(d) The total number of photons in the container is not constant.

54. A system consists of two Bosons each of which can occupy two energy levels 0 and $E_0$. The partition function for the system is:
(a) $1+e^{-\beta E_0}$ (b) $1+e^{-2\beta E_0}$ (c) $1+e^{-\beta E_0}+e^{-2\beta E_0}$ (d) $e^{-\beta E_0}+e^{-2\beta E_0}$, Where $\beta = \dfrac{1}{kT}$.

55. According to Bose-Einstein Statistics, there exists a 'Bose Condensate' which is a collections of bosons. For this condense, which of the following statements is correct?
(a) As $T \to \infty$, all bosons reside in excited states
(b) For $T < T_c$ almost all bosons reside in the ground state.
(c) As $T \to \infty$, all bosons reside in the ground state.
(d) For $T > T_c$, bosons behave similar to fermions.

56. According to Fermi-Dirac Statistics, the probability of occupation of a quantum state of energy E is $f(E)=\frac{1}{e^{(E-E_T)/kT}+1}$ where $E_F$ is the Fermi energy. Which of the following grahs is true at T = 0 K.

(a)


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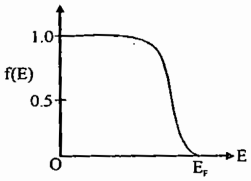
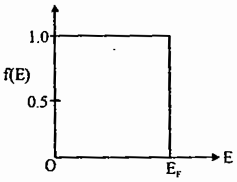


(c)


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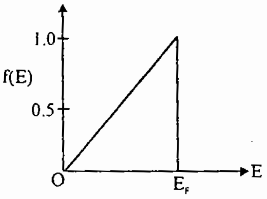

57. Plank's distribution formula black body radiation is given by

$$u_\lambda d_\lambda = \frac{8\pi hc}{-\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

For long wave lengths and high temperatures, $u_\lambda, d_\lambda$ becomes equal to

(a) $\frac{8\pi kT}{\lambda^4}d\lambda$ (b) $\frac{8\pi kT}{\lambda^5}d\lambda$ (c) $\frac{8\pi kT}{\lambda^5}e^{hc/\lambda kT}$ (d) $\frac{8\pi kT}{\lambda^5}\left(e^{-h/\lambda kT}+1\right)$

58. The number of ways in which 5 identical bosons can be distributed in 4 states is:

(a) $\frac{8!}{5!3!}$ (b) $\frac{9!}{5!4!}$ (c) $\frac{9!}{4!4!}$ (d) $\frac{8!}{4!4!}$

59. Blackbody radiation is enclosed inside a spherical cavity of radius r at a temperature T. What would be the temperature of the enclosure if the radius expands to 2r adiabatically?

(a) T/2 (b) T (c) $T/\sqrt{2}$ (d) 2T

60. Two localized non-interacting spin 1/2 magnetic ions of magnetic moment $\mu$ are placed in an external magnetic field H, at temperature T. If $k_BT >> \mu H$, then the entropy of the system is, to a good approximation.

(a) $S = k_B \ln 2$ (b) $S = 2k_B \ln 2$ (c) $S = 3k_B \ln 2$ (d) $S = 4k_B \ln 2$

61. If $P_1, P_2$ and $P_3$ represent the pressures of ideal B.E.; M.B. and F.D. gases at low temperature, respectively then

(a) $P_1 < P_2 ; P_1 < P_3$ (b) $P_1 < P_3 ; P_1 > P_2$ (c) $P_1 > P_2 ; P_1 > P_3$ (d) $P_1 = P_2 = P_3$

62. For an ideal Fermi gas at absolute Zero, the pressure is:

(a) zero (b) proportional to V
(c) proportional to $V^{-2/3}$ (d) proportional to $V^{-5/3}$.

63. Consider two independently diffusing non- interacting particles in 3-dimensional space, both placed at the origin at time t = 0. These particles have different diffusion constants $D_1$ and $D_2$. The quantity $\left\langle\left[\vec{R}_1(t)-\vec{R}_2(t)\right]^2\right\rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as:

(a) $6t(D_1+D_2)$ (b) $6t(D_1-D_2)$ (c) $6t\sqrt{D_1^2+D_2^2}$ (d) $6t\sqrt{D_1D_2}$

64. Consider a random walk of N steps, each of unit length but randomly and independently oriented with respect to each other in a two dimensional space. In the limit $N \to \infty$, the mean square distance of the end point from the starting point is proportional to

(a) $N^2$ (b) $\sqrt{N}$ (c) N (d) $\frac{1}{N}$

65. A random walker takes uncorrelated steps along the x-axis with probability $(P > 1/2)$ along the positive x-direction nd with the probability $(1-p)$ along the negative x-direction. After N steps, the mean distance of the walker from the starting position is proportional to :

(a) $N^{1/3}$　(b) $N^{1/2}$　(c) N　(d) $N^{4/3}$

**Common Data Questions**

**Common Data for Q. 66 and Q. 67:** There are four energy levels *E*, 2*E*, 3*E* and 4*E* (where *E*>0). The canonical partition function of two particles is, if these particles are

66. Two identical fermions

(a) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$　(b) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$ ✓

(c) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$　(d) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

67. Two distinguishable particles

(a) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$　(b) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$

(c) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$ ✓　(d) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

68. A vessel has two compartments of volume $V_1$ and $V_2$, containing an ideal gas at pressures $p_1$ and $p_2$, and temperatures $T_1$ and $T_2$ respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be

(a) $\dfrac{p_1T_1+p_2T_2}{p_1+p_2}$　(b) $\dfrac{V_1T_1+V_2T_2}{V_1+V_2}$

(c) $\dfrac{p_1V_1+p_2V_2}{(p_1V_1/T_1)+(p_2V_2/T_2)}$　(d) $(T_1T_2)^{1/2}$

69. A system of non-interacting spin-1/2 charged particles are placed in an external magnetic field. At low temperature T, the leading behavior of the excess energy above the ground state energy, depends on T as: (c is a constant)

(a) cT　(b) $cT^3$　(c) $e^{-c/T}$　(d) c(is independent of T)

70. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states D(E) depends on the energy as

(1) $1/\sqrt{E}$　(2) $\sqrt{E}$　~~(3)~~ E　(4) constant

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71. Three identical spin $-\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

~~(a)~~ 8　(b) 4　(c) 3　~~(d) 2~~ ✓

72. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition?

(a) internal energy　(b) Helmholtz free energy

(c) Gibbs free energy　(d) entropy

73. A system of N classical non-interacting particles, each of mass $m$, is at a temperature T and is confined by the external potential $V(r)=\frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is

[✓ (a) $3Nk_BT$]
(b) $\frac{3}{2}Nk_BT$

(c) $N(2mA)^{3/2}k_BT$
(d) $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_BT}{m}\right)$

74. Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat $C_V$ of CO gas is given in terms of the Boltzmann constant $k_B$ by

[✓ (a) $\frac{5}{2}k_B$]
(b) $2k_B$
(c) $\frac{3}{2}k_B$
(d) $\frac{1}{2}k_B$

75. A gas of photons is enclosed in a container of fixed volume at an absolute temperature T. Noting that the photon is massless particle (i.e. its energy and momentum are related by E = pc), the number of photons in the container will vary as

(a) T
(b) $T^2$
(c) $T^3$
(d) $T^4$

76. Which of the following statement best explains why the specific heat of electrons in metals is much smaller than that expected in a non-interacting (free) electron gas model?

(a) The mass of electron is much smaller than that of the ions in the crystal.
(b) The Pauli exclusion principle restricts the number of electrons which can absorb thermal energy
(c) Electron spin can take only two different values.
(d) Electrons in a metal cannot be modelled as non-interacting.

77. The root-mean square velocity $\bar{\upsilon}$ of helium atom at T degree K satisfies the relation

(a) $\bar{\upsilon}\propto\sqrt{T}$
(b) $\bar{\upsilon}\propto\frac{1}{\sqrt{T}}$
(c) $\bar{\upsilon}\propto T$
(d) $\bar{\upsilon}\propto\frac{1}{T}$

**Common data for Q.78, Q.79 and Q.80.**

A system consists of three spin-half particles, the z components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value $+1/2$ and $-1/2$. The total spin of the system is $S_z = S_z(1)+S_z(2)+S_z(3)$

78. The total number of possible micro-states of this system is:

(a) 3
(b) 6
(c) 7
[✓ (d) 8]

79. The total number of micro-states with $S_z = 1/2$ is:

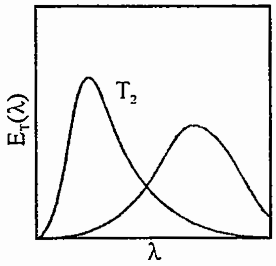
[✓ (a) 3]
(b) 5
(c) 6
(d) 7

80. Consider an ensemble of systems where each microstate has equal probability. The ensemble average of $S_z$ is:

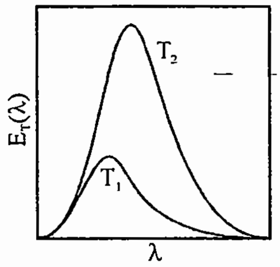
(a) $-1/2$
[✓ (b) 0]
(c) 1/2
(d) 3/2

81. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength $\lambda$ at two temperatures $T_1$ and $T_2$ $(T_1 < T_2)$?

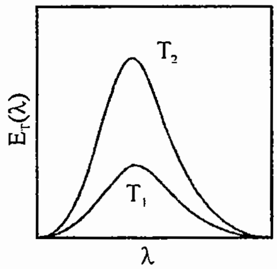
(a)



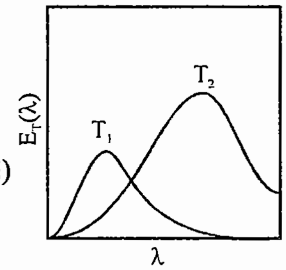
(b)



(c)



(d)



82. A system can have three energy levels:
$E = 0, \pm\varepsilon$. The level $E = 0$ is doubley degenerate, while the others are non-degenerate. The average energy at inverse temperature $\beta$ is

(a) $-\varepsilon \tanh(\beta\varepsilon)$

(b) $\dfrac{\varepsilon\left(e^{\beta\varepsilon} - e^{-\beta\varepsilon}\right)}{\left(1 + e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right)}$

(c) Zero

(d) $-\varepsilon \tanh\left(\dfrac{\beta\varepsilon}{2}\right)$

83. For a particular thermodynamics system the entropy S is related to the internal energy U and volume V by

$$S = c\, U^{3/4}\, V^{1/4}$$

where $c$ is a constant. The Gibbs potential $G = U - TS + pV$ for this system is

(a) $\dfrac{3pU}{4T}$ (b) $\dfrac{cU}{3}$ (c) zero (d) $\dfrac{US}{4V}$

84. The pressure of a non-relativistic free Fermi gas in three-dimensions depends, at T = 0, on the density of fermions $n$ as

(a) $n^{5/3}$ (b) $n^{1/3}$ (c) $n^{2/3}$ (d) $n^{4/3}$

85. The average local internal magnetic field acting on an Ising spin is $H_{\text{int}} = \alpha M$, where M is the magnetization and $\alpha$ is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature $T_c$, the magnetic susceptibility at zero external field is proportional to ($k_B$ is the Boltzmann constant)

(a) $k_B T - \alpha$ (b) $(k_B T + \alpha)^{-1}$ (c) $(k_B T - \alpha)^{-1}$ (d) $\tanh(k_B T + \alpha)$

200)

86. The van der Waals equation of state for a gas is given by

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

where, P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by

(a) $\frac{a}{9b}$  (b) $\frac{a}{27b^2}$  (c) $\frac{8a}{27bR}$  [✓ (d) $3b$]

87. For a system of two bosons, each of which can occupy any of the two energy levels 0 and $\varepsilon$, the mean energy of the system at a temperature T with $\beta=\frac{1}{k_B T}$ is given by

[✓ (a) $\frac{\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$]  (b) $\frac{1}{2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon e^{-\beta\varepsilon}+\varepsilon e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$  (d) $\frac{\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

88. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) $k_B \ln 2$  (b) $2k_B \ln 2$  [✓ (c) $2k_B \ln 2$]  (d) $3k_B \ln 4$

## Answer Key (STATISTICAL-II)

1. (d)  2. (c)  3. (a)  4. (d)  5. (c)  6. (a)  7. (b)
8. (b)  9. (c)  10. (d)  11. (b)  12. (d)  13. (c)  14. (c)
15. (b)  16. (d)  17. (b)  18. (b)  19. (a)  20. (c)  21. (d)
22. (a)  23. (a)  24. (c)  25. (c)  26. (a)  27. (a)  28. (b)
29. (d)  30. (c)  31. (d)  32. (c)  33. (b)  34. (d)  35. (b)
36. (c)  37. (a)  38. (a)  39. (a)  40. (d)  41. (a)  42. (d)
43. (b)  44. (a)  45. (c)  46. (c)  47. (a)  48. (d)  49. (d)
50. (a)  51. (c)  52. (b)  53. (c)  54. (c)  55. (a)  56. (c)
57. (a)  58. (a)  59. (a)  60. (b)  61. (b)  62. (d)  63. (a)
64. (c)  65. (b)  66. (b)  67. (c)  68. (c)  69. (c)  70. (c)
71. ()  72. (c)  73. (a)  74. (a)  75. (c)  76. (b)  77. (a)
78. (d)  79. (a)  80. (b)  81. (c)  82. (d)  83. (c)  84. (a)
85. (c)  86. (d)  87. (a)  88. (b)

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# Assignment

90

# Electronics

## PHYSICAL SCIENCES

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T : 011-65462244, 27241845, (0) 9899628311, 9540292991
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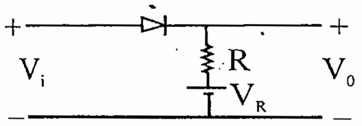
# Assignment : Network

1. For the given clipper-circuit shown in figure. If $V_i$ is the input voltage and $V_o$ is the output voltage.



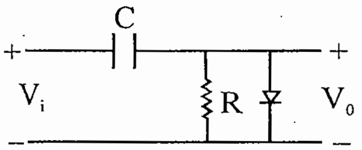
(a) the circuit clipps off the input for $V_i < V_R$

(b) The circuit clipps off the input for $V_i > V_R$ ✗

(c) The negative swings of $V_i$ are only clipped off

(d) The output $V_0 = V_R$ always.

2. Which statements are true about give ckt.



(a). It is clipper

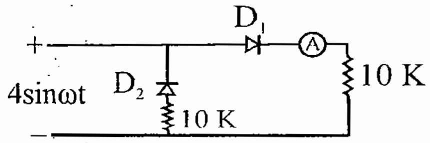
(b) It is rectifier

(c) It is a clamping circuit that would clamp positive peaks to near zero.

(d) It is a clamping circuit that would clamp negative peaks to near zero.

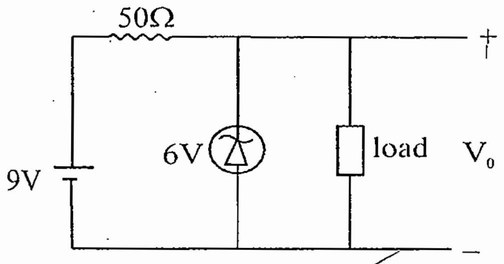
→ Tello (I avg)

3. In circuit of given figure assume that diodes are ideal and meter is an average indicating ammeter then ammeter will read.



(a) $0.4\sqrt{2}$ mA (b) $0.4$ mA (c) $\frac{0.8}{\pi}$ mA (d) $\frac{0.4}{\pi}$ mA

4. A Zener-diode shown in figure has a knee current of 5 mA and a maximum allowed power dissipation of 300 mw. What are the minimum and maximum load currents that can be drawn safely from circuit, keeping the output voltage constant at 6 volt



(a). 0 mA, 180 mA (b) 5 mA, 110 mA (c) 10 mA, 55 mA (d) 60 mA, 180 mA

# Assignment : Network

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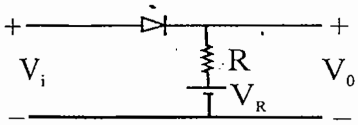

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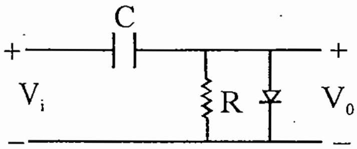

(a). It is clipper

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→ [illegible] (I avg)

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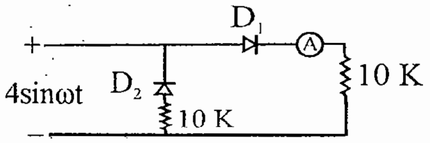

(a) $0.4\sqrt{2}$ mA

(b) 0.4mA

(c) $\frac{0.8}{\pi}$ mA

(d) $\frac{0.4}{\pi}$ mA

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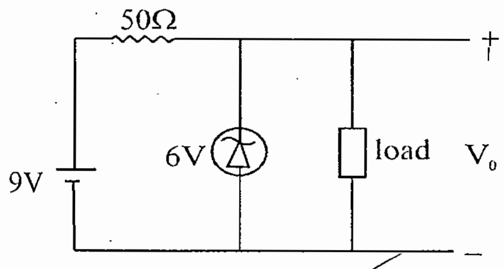


(a). 0 mA, 180 mA

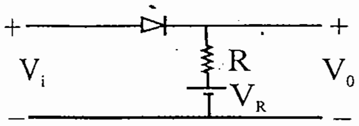
(b) 5 mA, 110 mA

(c) 10 mA, 55 mA

(d) 60 mA, 180 mA

# Assignment : Network

1. For the given clipper-circuit shown in figure. If $V_i$ is the input voltage and $V_o$ is the output voltage.



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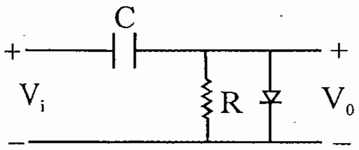

(a). It is clipper (b) It is rectifier

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→ Tells (I avg)

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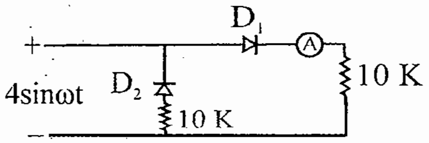
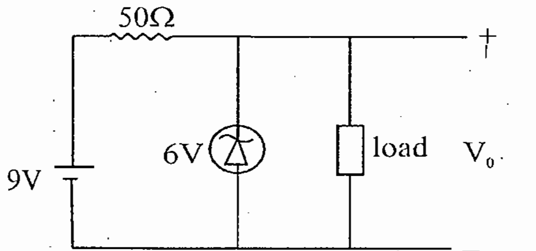


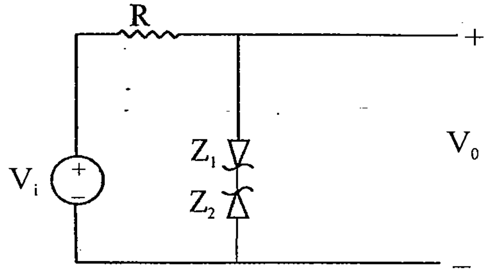
(a) $0.4\sqrt{2}$ mA (b) 0.4 mA (c) $\frac{0.8}{\pi}$ mA (d) $\frac{0.4}{\pi}$ mA

4. A Zener-diode shown in figure has a knee current of 5 mA and a maximum allowed power dissipation of 300 mw. What are the minimum and maximum load currents that can be drawn safely from circuit, keeping the output voltage constant at 6 volt



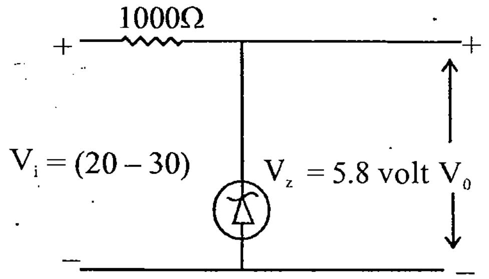
(a). 0 mA, 180 mA (b) 5 mA, 110 mA (c) 10 mA, 55 mA (d) 60 mA, 180 mA

5. In the circuit-shown in figure, zener voltage $V_{z_1} = V_{z_2} = 4.0$ volt, cut-involtage of diode $= 0.7$ volt



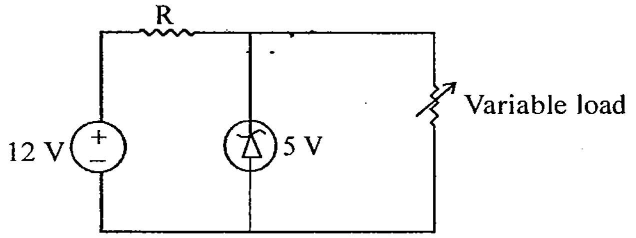
(a) If $|V_i| < 4.7V$, then $V_0 = V_i$
(b) If $|V_i| < 8V$, then $V_0 = V_i$
(c) If $|V_i| > 4.7\,V$, then $V_0 = V_i$
[✓ (d) $V_0 = 4.7$ volt for all values of $V_i$]

6. The zener-diode in regulator circuit shown in figure. has a zener-voltage of 5.8 volt and a zener knee current of 0.5 mA, then maximum load current drawn from circuit, ensuring proper functioning over, input voltage range b/w (20-30) volt is:



(a) 23.7 mA
(b) 14.2 mA
(c) 13.7 mA
[✓ (d) 24.2 mA]

7. In the voltage regulator shown in figure, the load current can vary from (100-500) mA, assuming that zener-diode is ideal (i.e. zener knee current is negligible & small, here zever resistance is zero in break-down region) Then what is the value of R?

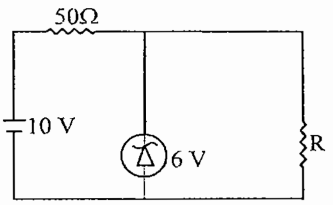


(a) 7Ω
(b) 70Ω
[✓ (c) 70 / 3Ω]
(d) 14Ω

8. The resistivity at room temperature of intrinsic Si is $2.3 \times 10^3\,\Omega - m$ and that of an extrinsic N- type Si, sample is $8.33 \times 10^{-2}\,\Omega -$ met. A bar of this extrinsic Si, sample is 3 mm long and has a rectangular cross-section $50 \times 100$ mm and a steady state current of $1\mu A$ exists in the bar, if voltage across the bar is found to be 50 mv. If same bar is of intrinsic Si, then voltage across bar will be about.

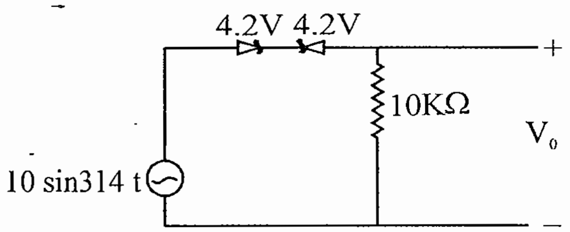
(a). 1400 volt
(b) 140 volt
(c) 14 volt
(d) 1.4 volt

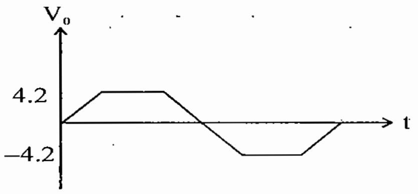
9. The 6V, Zener-diode shown in figure has zero zener resistance and a knee current of 5mA. The minimum value of R, so that voltage across it does not fall below 6 V is:

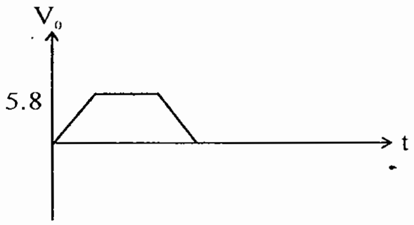


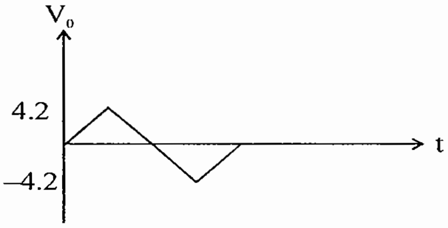
(a) 1.2kΩ (b) 80Ω (c) 50Ω (d) 0Ω

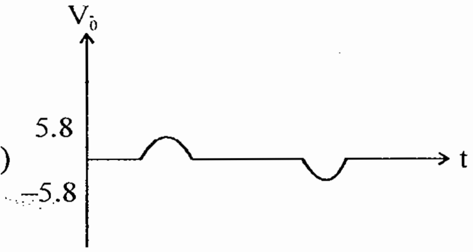
10. Draw wave shape $V_0$ for the given figure.



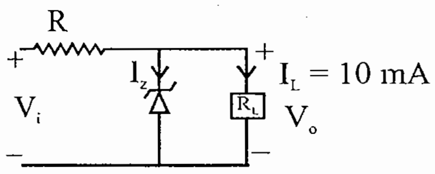
(a)


(b)


(c)


(d)


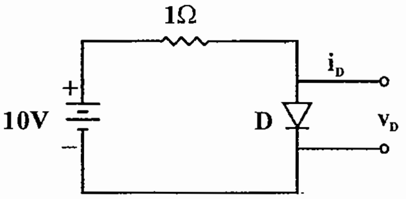
11. A zener-diode regulator shown in figure is to be designed to meet the specifications: $I_L = 10\text{mA}$, $V_0 = 10\text{volt}$ and $V_{in}$ varies from 30-50V if zener-diode has $V_z = 10$ volt & $I_{zk} = 1$ mA then for satisfactory operation



(a) $R \leq 1800\Omega$ (b) $1800\Omega \leq R3600\Omega$ (c) $3700\Omega \leq R \leq 4000\Omega$ (d) $R > 4000\Omega$

12. A diode D as shown in the circuit as an i–*v* relation which can be proximated by

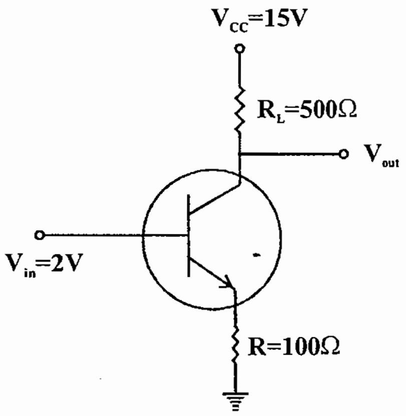
$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \le 0 \end{cases}$$



The value of $v_D$ in the circuit is:

(a) $\left(-1+\sqrt{11}\right)$ V (b) 8V (c) 5V (d) 2V

13. The circuit depicted on the right has been made with a silicon n-p-n transistor.
Assuming that there will be a 0.7V drop across a forward-biased silicon p-n junction, the power dissipated across the transistor will be, approximately,



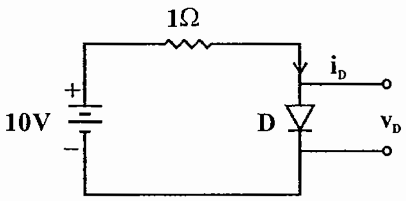
(a) 53 mW (b) 94 mW (c) 17 mW (d) 67 mW

14. A live music broadcast consists of a radio-wave of frequency 7 MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 KHz. The spectrum of modulated output will be zero outside the frequency band (freq Range)

(a) 7.00 MHz to 7.01 MHz (b) 6.99 MHz to 7.01 MHz
(c) 6.99 MHz to 7.00 MHz (d) 6.995 MHz to 7.005 MHz

15. A diode D as shown in the circuit as an i–*v* relation which can be proximated by

$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \le 0 \end{cases}$$

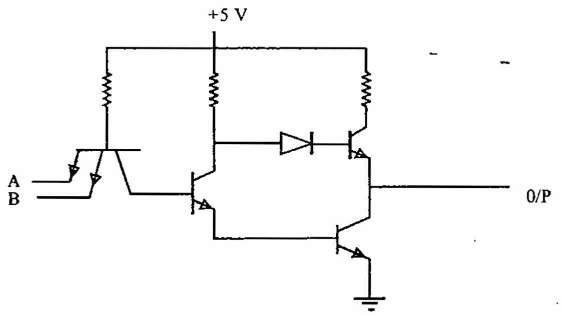


The value of $v_D$ in the circuit is:

(a) $\left(-1+\sqrt{11}\right)$ V (b) 8V (c) 5V (d) 2V

**Data for linked Q.16 and Q.17.**

In the I.C. logic gate, shown in figure. If thresold voltage $V_{BE}$ is 0.75 volt and $V_{CE}$ (sat) = 0.2 V, calculate value of output voltage.



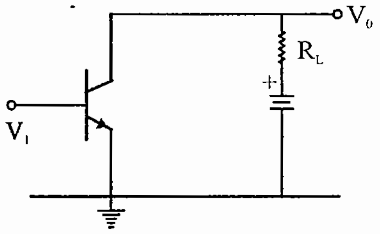
16. If $V_A = V_B = 4.5$ volt

(a) 0.75 V (b) 0.55 V (c) 5V [✓ (d) 0.2 V]

17. If $V_A = 4.5$ volt, $V_B = 0.2$ volt

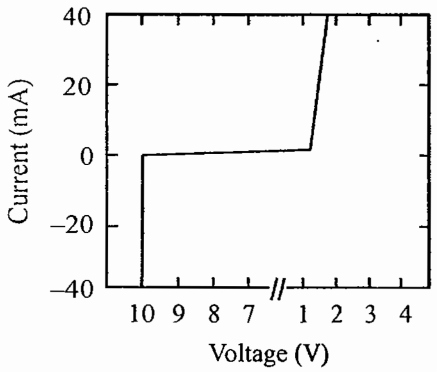
(a) 0.75 V (b) 0.55 V [✓ (c) 5V] (d) 0.2 V

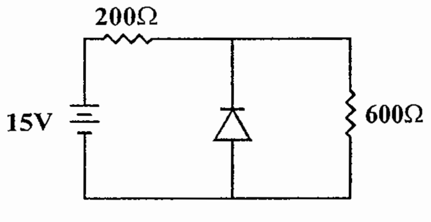
18. The transistor in the given circuit has $h_{fe} = 35\Omega$ and $h_{ie} = 1000\Omega$. If the load resistance $R_L = 1000\Omega$, the voltage and current gain are, respectively.



(a) –35, +35 (b) 35, 35 (c) 35, 0.97 (d) 0.98, 35

19. The figure on the right shows the current-voltage characteristics of a diode over a range of voltage and current where it is safe to operate the diode.





When this diode is used in the circuit on the extreme right, the approximate current, in mA, through the diode will be

[✓ (a) 0] (b) 8.3 (c) 16.7 (d) 25

20. Match the following:
(a) Hartley (b) Wien bridge (c) Crystal
1. Low frequency oscillator 2. High frequency oscillator
3. Stable frequency oscillator 4. Relaxation oscillator
5. Negative resistance oscillator.
(a) a-2 b-1 c-3 (b) a-1 b-2 c-3 (c) a-3 b-1 c-2 (d) a-5 b-4 c-1

21. In a common emitter BJT amplifier, the maximum usable supply voltage is limited by:
(a) Avalanche breakdown of Base-Emitter junction
(b) Collector-Base breakdown voltage with emitter open ($BV_{CBO}$).
(c) Collector-Emitter breakdown voltage with base open ($BV_{CBO}$)
(d) Zener breakdown voltage of the Emitter-Base junction.

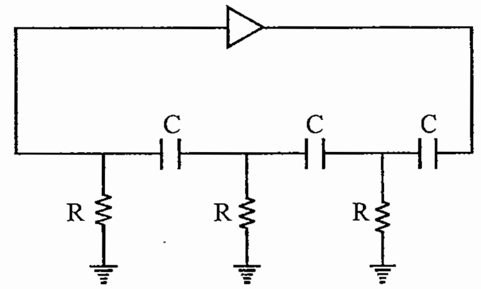
22. Choose the correct match for the input resistance of various amplifier configurations shown below:

| **Configuration** | **Input resistance** |
|---|---|
| CB: Common Base | LO: Low |
| CC: Common Collector | MO: Moderate |
| CE: Common Emitter | HI: High |

(a) CB-LO, CC-MO, CE-HI (b) CB-LO, CC-HI, CE-MO
(c) CB-MO, CC-HI, CE-LO (d) CB-HI, CC-LO, CE-MO

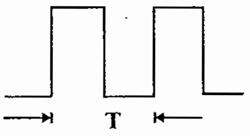
23. The cascode amplifier is a multistage configuration of: It is the configuration of - CE & CB
(a) CC-CB (b) CE-CB (c) CB-CC (d) CE-CC

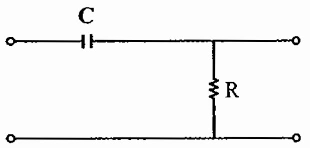
24. The oscillator circuit shown in figure has an ideal inverting amplifier. Its frequency of oscillation (in Hz) is:



(a) $\frac{1}{\left(2\pi\sqrt{6}\,RC\right)}$ (b) $\frac{1}{\left(2\pi\,RC\right)}$ (c) $\frac{1}{\left(\sqrt{6}\,RC\right)}$ (d) $\frac{1}{\sqrt{6}\left(2\pi\,RC\right)}$

25. For the circuit given in the figure, consider the following statements:
1. The output $V_0$ will consist of a positive and a negative spike if RC << T.
2. The output $V_0$ will be similar to $v_1$ if RC >> T.
3. The output will have a higher rise time if RC is made progressively smaller than T. Of these statements:





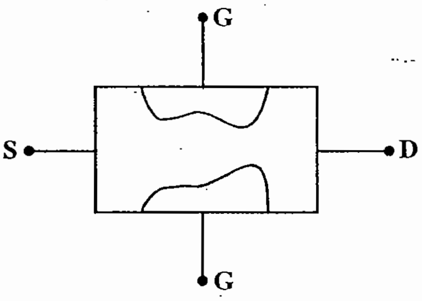
(a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 and 3 are correct (d) 1 and 3 are correct.

26. An ideal power supply has
(a) Zero internal resistance (b) Very large input resistance
(c) Very small output resistance (d) Very large output resistance

27. If $\alpha = 0.995$, $I_E = 10\text{ mA}$ and $I_{CBO} = 0.5\text{ mA}$, then $I_{CEO}$ will be
(a) 100 mA (b) 25 μA (c) 10.1 mA (d) 105 mA

28. The ripple factor of a power supply is given by (Symbols have the usual meaning)
(a) $\frac{P_{dc}}{P_{OC}}$ (b) $\sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$ (c) $\sqrt{\left(\frac{I_{dc}}{I_{rms}}\right)^2 - 1}$ (d) $\frac{I_{dc}}{I_{rms}}$

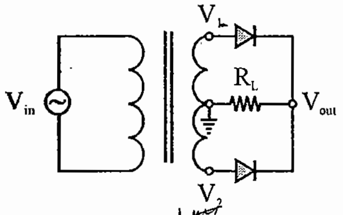
29. In a biased JFET, the shape of the channel is as shown in the given figure because



(a) It is the property of the material used
(b) The drain end is more reverse biased than source end
(c) The drain end is more forward biased than source end
(d) The impurity profile varies with the distance from source.

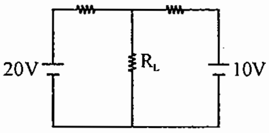
30. A Hartley oscillator is used for generation.
(a) Very low frequency oscillation (b) Radio frequency oscillation (High freq)
(c) Microwave oscillation (d) Audio-frequency oscillation.

31. For the rectifier circuit shown in the figure, the sinusoidal voltage ($V_1$ or $V_2$) at the output of the transformer has a maximum value of 10V. The load resistance $R_L$ is 1 kΩ. If $I_{av}$ is the average current through the resistor $R_L$ the circuit corresponds to a.



(a) Full wave rectifier with $I_{av} = 20/\pi$ mA not (b) Half wave rectifier with $I_{av} = 20/\pi$ mA
(c) Half wave rectifier with $I_{av} = 10/\pi$ mA (d) Full wave rectifier with $I_{av} = 10/\pi$ mA

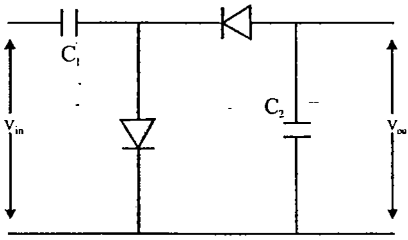
32. In the circuit shown in the figure the Thevenin voltage $V_{Th}$ and Thevenin resistance $R_{Th}$ as seen by the load resistance $R_L$ $(= 1\text{k}\Omega)$ are respectively.



(a) 15V, 1 kΩ (b) 30 V, 4 kΩ
(c) 20 V, 2 kΩ (d) 10V, 5 kΩ.

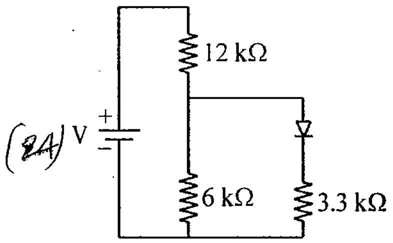
33. The high input impedance of field effect transistor (FET) amplifier is due to
(a) The pinch-off voltage (b) Its very low gate current.
(c) The source and drain being far apart (d) The geometry of the FET.

34. A sinusidal input voltage $v_{in}$ of frequency $\omega$ is fed to the circuit shown in the figure, where $C_1 >> C_2$. If $v_{in}$ is the peak value of the input voltage, then output voltage ($v_{out}$) is:



(a) $2v_{in}$ (b) $2v_0 \sin\omega t$ (c) $\sqrt{2}v_{in}$ (d) $\frac{v_{in}}{2}\sin\omega t$

35. In the following circuit, the voltage drop across the ideal diode in forward bias condition is 0.7V.



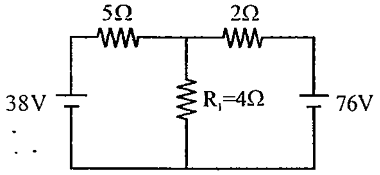
The current passing through the diode is

(a) 0.5 mA (b) 1.0 mA (c) 1.5 mA (d) 2.0 mA

36. In a typical n-p-n transistor the doping concentration in emitter, base and collector regions are $C_E$, $C_B$ and $C_C$ respectively. These satisfy the relation,

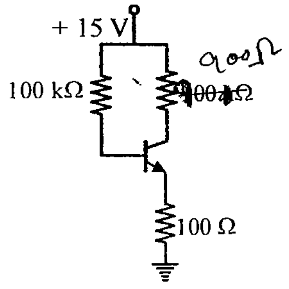
(a) $C_E > C_C > C_B$ (b) $C_E > C_B > C_C$ (c) $C_C > C_B > C_E$ (d) $C_E = C_C > C_B$

37. For the circuit shown, the potential difference (in Volts) across $R_L$ is:



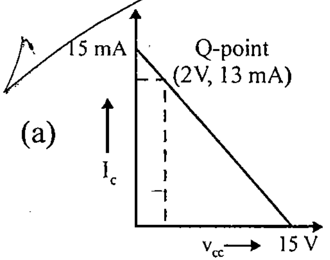
(a) 48 (b) 52 (c) 56 (d) 65

38. Consider the following circuit in which the current gain $\beta_{dc}$ of the transistor is 100.

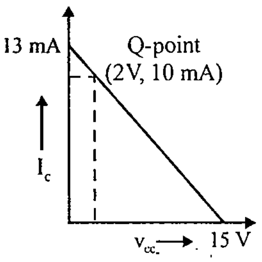


Which one of the following correctly represents the load line (collector current $I_c$ with respect to collectr-emitter voltage $V_{CE}$) and Q-point of this circuit?

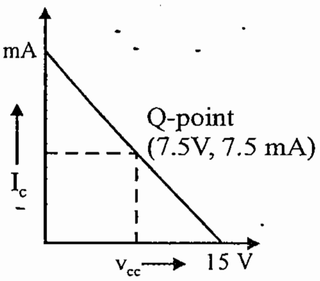
(a)



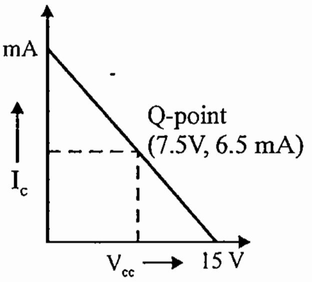
(b)



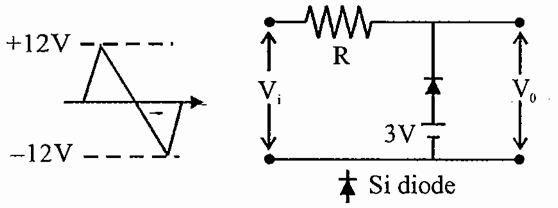
(c)



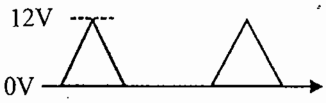
(d)



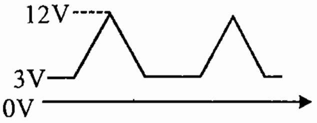
39. When an input voltage $V_i$, of the form shown, is applied to the circuit given below, the output voltage $V_0$ is of the form.



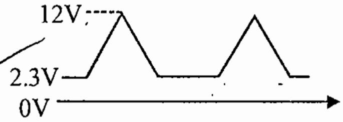
(a)



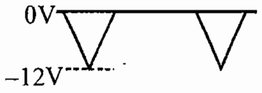
(b)



(c)



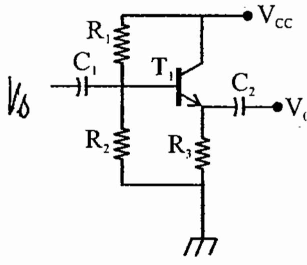
(d)



40. A common emitter transistor amplifier circuit is operated under a fixed bias. In this circuit, the operating point
(a) Remains fixed with an increase in temperature
(b) Moves towards cut-off region with an increase in temperature.
(c) Moves towards the saturation region with a decrease in temperature.
(d) Moves towards the saturation region with an increase in temperature

41. Under normal operating conditions, the gate terminal of an n-channel junction field effect transistor (JFET) and an n-channel metal oxide semiconductor field effect transistor (MOSFET) are
(a) Both biased with positive potentials.
(b) Both biased with negative potentials.
(c) Biased with positive and negative potentials, respectively.
(d) Biased with negative and positive potentials, respectively.

42. The circuit shown below

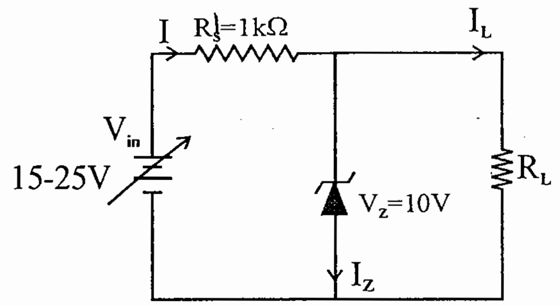


(a) Is a common-emitter amplifier
(b) Uses a pnp transistor
(c) Is an oscillator (with out i/p)
(d) Has a voltage gain less than one.

43. An amplifier of gain 1000 is made into a feedback amplifier by feeding 9.9% of its output voltage in series with the input opposing. If $f_L = 20$ Hz and $f_H = 200$ kHz for the amplifier without feedback, then due to the feedback.

(a) The gain decrease by 10 times
(b) The output resistance increases by 10 times
(c) The $f_H$ increases by 100 times
(d) The input resistance decreases by 100 times.

44. Pick the correct statement based on the below circuit.



(a) The maximum Zener current, $I_{Z(max)}$, when $R_L = 10k\Omega$ is 15mA
(b) The maximum Zener current, $I_{z(min)}$, when $R_L = 10k\Omega$ is 5mA
(c) With $V_{in} = 20V$, $I_L = I_Z$, when $R_L = 2k\Omega$
(d) The power dissipated across the Zener when $R_L = 10k\Omega$ and $V_{in} = 20V$ is 100 mW.

45. A current amplifier is characterised by
(a) low input impedance and high output impedance
(b) high input impedance and low output impedance.
(c) low impedance at both input and output terminals.
(d) high impedance at both input and output terminals.

46. Which of the following characteristics DOES NOT belong to a common collector transistor amplifer?
(a) Low voltage gain [≈]
(b) High current gain.
(c) High input impedance
[✓ (d)] High output impedance.

47. An amplifier has a voltage gain of 500 and an input impedance 20 KΩ, without any feedback. Now a negative feedback with $\beta = 0.1$ is applied. Its gain and input impedance with feedback will respectively be
(a) 9.8 and 392 K ohms
[✓ (b)] 9.8 and 1020 K ohms
(c) 50 and 1020 K ohms
(d) 50 and 2 K ohms

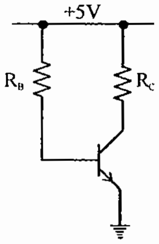
48. A bipolar junction transistor used in common collector configuration has
(a) High input impedance and high output impedance
(b) High input impedance and low output impedance.
(c) Low input impedance and high output impedance.
(d) Low input impedance and low output impedance.

49. A transistor is said to be in saturation region when
(a) Both collector and emitter junctions are forward biased.
(b) Both collector and emitter junctions are reverse biased.
(c) The collector junction is forward biased but the emitter junction is reverse biased.
(d) The collector junction is reverse biased but the emitter junction is forward biased.

50. A zener diode is used as a voltage regulator across an unregulated DC source. Overloading will damage
(a) DC source only [✓ (b)] Zener diode only (c) Both (a) and (b) (d) Neither (a) nor (b)

51. Which of the following are true for a field effect transistor (FET)?
(a) A FET has a very high power gain.
(b) A FET has a negative temperature coefficient of resistance.
(c) A FET has a very high input impedance.
(d) A FET is a current controlled device.

52. In the circuit on the right, $R_B = 1\ k\Omega$ and $R_C = 100\Omega$. If the transistor $\beta(h_{FE})$ is 100, the current through $R_C$ will be



(a) ≈ 0.43 A
(b) ≈ 50 mA
(c) zero
(d) Oscillating between 0 and 50 mA.

53. In an n-p-n transistor, the leakage current consists of
(a) Electrons moving from the base to the emitter
(b) Electrons moving from the collector to the base
(c) Electrons moving from the collector to the emitter.
(d) Electrons moving from the base to the collector.

54. A piece of semiconducting material is introduced into a circuit. If the temperature of the material is raised, the circuit current will
(a) Increase (b) Remain the same (c) Decrease (d) Cease to flow.

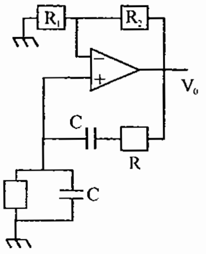
55. An n-channel silicon (dielectric constant = 12) FET with a channel width $a = 2\times10^{-6}$ m is doped with $10^{21}$ electrons/m$^3$. The pinch-off voltge is:
(a) 0.86 V (b) 0.68 V (c) 8.6 V (d) 6.8 V

56. An avalanche effect is observed in a diode when
(a) The forward voltage is less than the breakdown voltage.
(b) The forward voltage exceeds the breakdown voltage.
(c) The reverse voltage exceeds the breakdown voltage.
(d) The diode is heavily doped and forward biased.

57. A bipolar junction transistor with one junction forward biased and either the collector or emitter open, operates in the
(a) cut-off region (b) saturation region (c) pinch-off region (d) active region

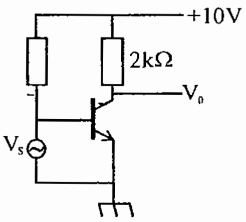
58. A field effect transistor is a
(a) Unipolar device
(b) Special type of bipolar junction transistor.
(c) Unijunction device
(d) Device with low input impedance

59. What should be the values of the components R and $R_2$ such that the frequency of the Wien Bride oscillator is 300 Hz? [Given: C = 0.01 μF and $R_1$ = 12 kΩ]



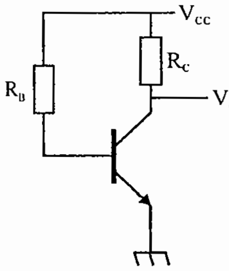
(a) $R = 48k\Omega$ and $R_2 = 12k\Omega$
(b) $R = 26k\Omega$ and $R_2 = 24k\Omega$
(d) $R = 530\ k\Omega$ and $R_2 = 1\ M\Omega$
(d) $R = 53k\Omega$ and $R_2 = 24k\Omega$

60. Figure shows a common emitter amplifier with $\beta = 100$. What is the maximum peak to peak input 500 kΩ signal ($v_s$) for which is distortion-free output may be obtained? [Assume $V_{BE} = 0$ and $r_e = 20\ \Omega$].



(a) 40 mV (b) 60 mV (c) 80 mV (d) 100 mV

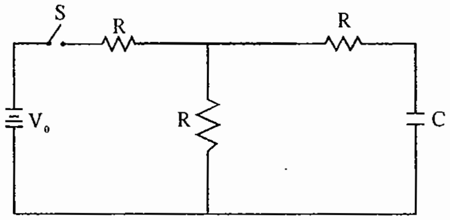
61. Calculate the collector voltage ($v_c$) of the transistor circuit shown in the figure
[Given: $\alpha = 0.96$, $I_{CBO} = 20\mu A$, $V_{BE} = 0.3V$, $R_B = 100k\Omega$, $V_{CC} = +10V$ and $R_C = 2.2k\Omega$]



(a) 3.78 V (b) 3.82 V (c) 4.72 V (d) 9.7 V

62. A capacitor C is connected to a battery $V_0$ through three equal resistors R and a switch S as shown above: The capacitor is initially uncharged. At time t = 0, the switch S is closed. The voltage across the capacitor as a function of time t for t > 0 is given by

TIFR
TEST
GATE

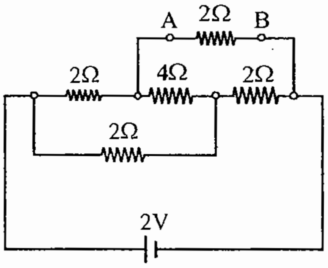


(a) $(V_0/2)(1-\exp(-t/2RC))$ (b) $(V_0/3)(1-\exp(-t/3RC))$

(c) $(V_0/3)(1-\exp(-3t/2RC))$ (d) $(V_0/2)(1-\exp(-2t/3RC))$

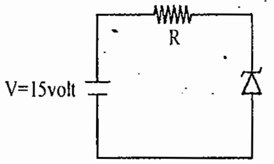
63. A junction field effect transistor behaves as a
(a) Voltage controlled current source (b) Voltage controlled voltage source
(c) Current controlled voltage source (d) Current controlled current source

64. For the circuit shown, the current passing through the 2Ω resistor (AB) is:



(a) 0.2 A (b) 0.5 A (c) 1.0 A (d) 2.0 A

65. The Zener diode in the following circuit has a power rating of 100 mw and the Zener break down voltage is. 10 volt. If the input supply voltage is 15 V dc, the minimum value of resistance R, in ohms, should be

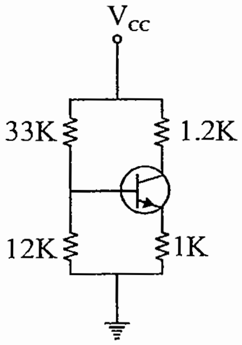


(a) 100 (b) 150 (c) 250 (d) 500

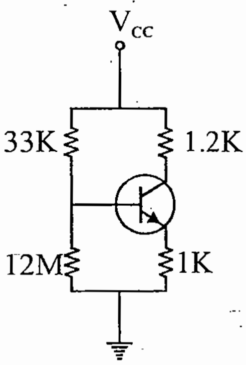
66. Transistor is acts as a switch in region
(a) Cut off (b) Saturation (c) Active region (d) None of these

67. Which of the following circuits has an appropriate voltage divider for biasing the transistor?

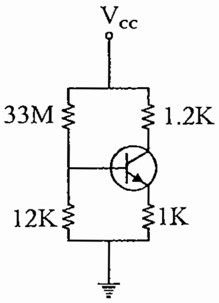
(a)



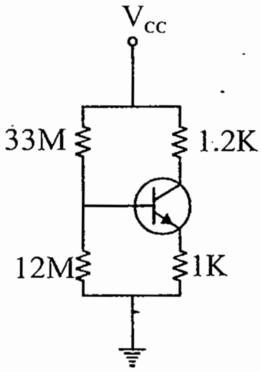
(b)



[✓ (c)]

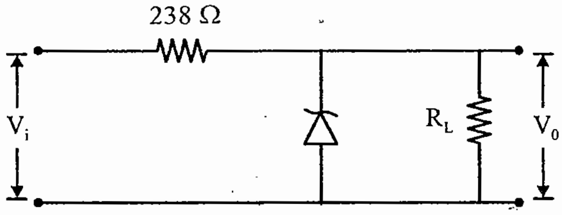


(d)



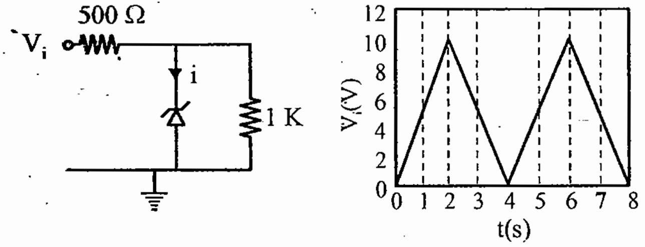
68. Which of the following statements is correct for a common emitter amplifier circuit?
(a) The output is taken from the emitter
[✓ (b) There is 180° phase shift between input and output voltages]
(c) There is no phase shift between input and output voltages
(d) Both p-n juntions are forward biased.

69. The voltage regulator circuit shown in the figure has been made with a Zener diode rated at 15V, 200 mW. It is required that the circuit should dissipate 150 mW power across the fixed load resistor $R_L$

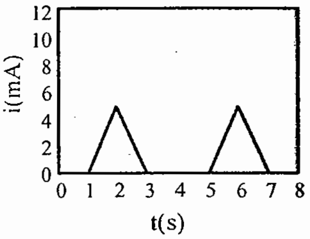


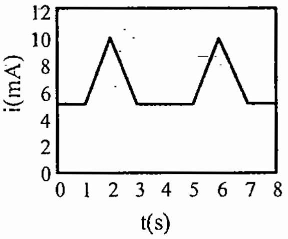
For stable operation of this circuit, the input voltage $V_i$ must have a range
[✓ (a) 17.5 V, -20.5 V] (b) 15.5 V, -20.5 V
(c) 15.5 V, -22.5 V (d) 17.5 V, -22.5 V

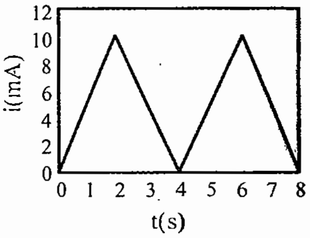
70. The figure below shows a voltage regulator utilizing a Zener diode of breakdown voltage 5V and a positive triangular wave input of amplitude 10V.

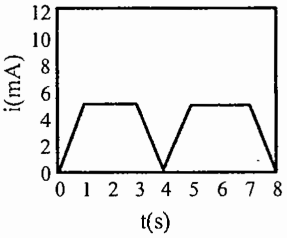


For $V_i > 5V$, the Zener regulates the output voltage by channeling the excess current through it self. Which of the following waveforms shows the current 'i' passing through the Zener diode?

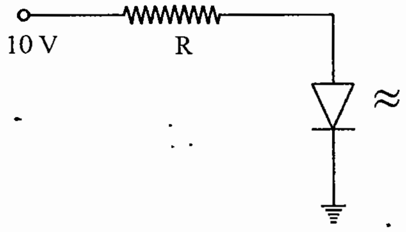
(a)


(b)


(c)


(d)


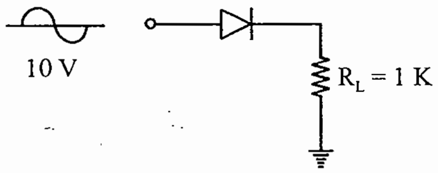
71. A red LED is used as a power indicator in an instrument powered by a 10V dc source. The appropriate value of the resistance R in the indicator bias circuit shown below is optimum current for LED 10mA

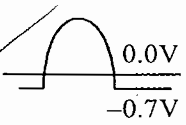


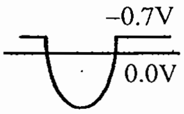
(a) 20kΩ [✓ (b) 1kΩ] (c) 200kΩ (d) 100 Ω

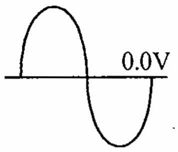
72. For operation as an amplifier, the base of an n-p-n transistor must be
(a) negative with respect to the emitter (b)positive with respect to the emitter
(c) positive with respect to the collector (d) at zero volt

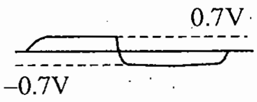
73. A sinusoidal voltage of amplitude 10V drives the following circuit. If the voltage across $R_L$ is viewed on an oscilloscope, which one of the representative waveforms you would expect to see?



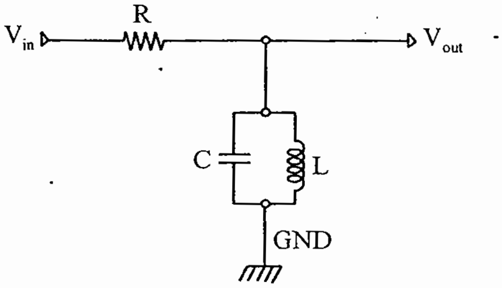
[✓ (a)]


(b)


(c)


(d)


74. The circuit



describes a

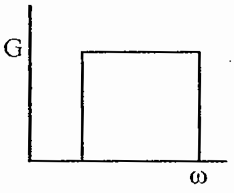
(a) band reject filter rejecting frequencies near $f = \frac{1}{2\pi\sqrt{LC}}$

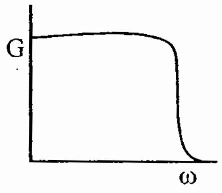
[✓ (b) band pass filter allowing frequencies near $f = \frac{1}{2\pi\sqrt{LC}}$]

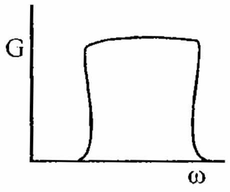
(c) high pass filter allowing $f > \frac{1}{2\pi\sqrt{LC}}$

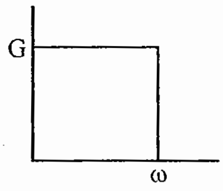
(d) low pass filter allowing $f < \frac{1}{2\pi\sqrt{LC}}$

75. Which of the following graphs correctly represents the frequency dependence of the gain G of a two-stage RC-coupled transistor amplifier?

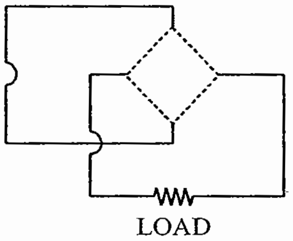
(a)


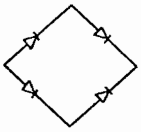
(b)


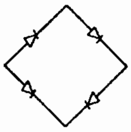
(c)


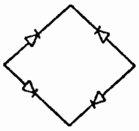
(d)


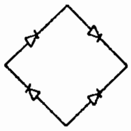
76. Given the full-wave rectifier circuit below, which is the correct configuration of the diodes?



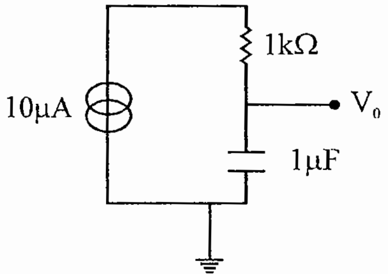
(a)


(b)


(c)


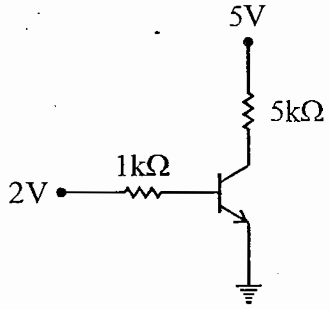
(d)


77. In the following circuit the time required for $V_0$ to reach 1 Volt, starting from $V_0 = 0$ at $t = 0$, is



(a) 100 millisecond (b) 10 millisecond [✓ (c) 1 millisecond] (d) 100 microsecond

78. The Si transistor in the circuit shown below has $\beta = 100$.



The transistor is

(a) in saturation (b) in deep saturation (c) in cut off (d) in the linear region

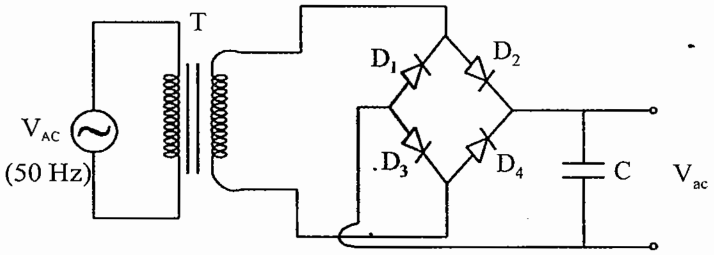
79. A comparator with hysteresis has

(a) no trigger points (b) one trigger point (c) two trigger points (d) a variable trigger point

80. The positive lead of an ohmmeter is connected to the anode of a diode and the negative lead is connected to the cathode. The diode then is

(a) short-circuited (b) reverse-biased (c) forward-baised (d) in avalanche breakdown

81. A DC power supply with a full wave bridge rectifier, a given in the figure below, shows a 50Hz ripple in the output voltage $V_{DC}$. The reason for the ripple is:



(a) the transformer secondary is shorted (b) the circuit is working properly.

(c) there is an open diode (d) the filter capacitor is leaky

82. A transistor can be used as a switching device when operated

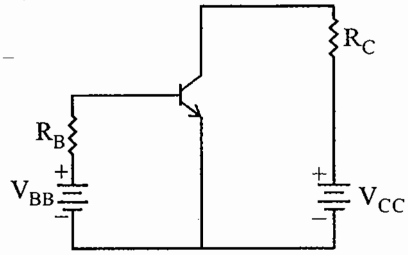
(a) in the active region only (b) with both junctions either reverse or forward biased

(c) in the saturation region only (d) in the cut off region only

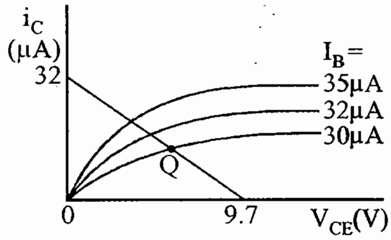
83. If the peak outoput voltage of a full wave rectifier is 10 V, its d.c. voltage is

(a) 10.0V (b) 7.07V (c) 6.36 V (d) 3.18V

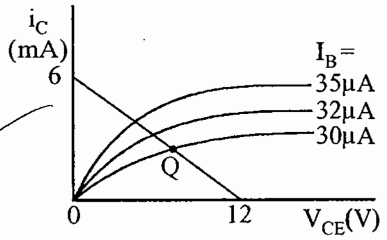
84. A silicon transistor with built-in voltage 0.7V is used in the circuit shown, with $V_{BB} = 9.7V$, $R_B = 300k\Omega$, $V_{CC} = 12V$ and $R_C = 2k\Omega$. Which of the following figures correctly represents the load line and the quiescent Q point?



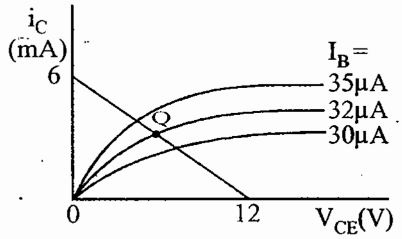
(a)



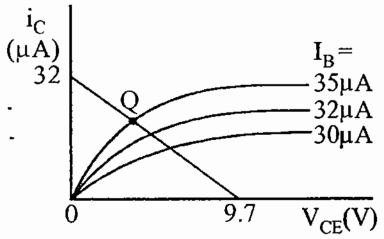
(b)



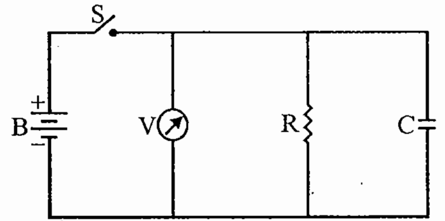
(c)



(d)



85. The insulation resistance R of an insulated cable is measured by connecting it in parallel with a capacitor C, a voltmeter, and battery B as shown. The voltage across the cable dropped from 150V to 15V, 1000 seconds after the switch S is closed. If the capacitance of the cable is 5 μF then its insulation resistance is approximately



(a) $10^9\,\Omega$ (b) $10^8\,\Omega$ (c) $10^7\,\Omega$ (d) $10^6\,\Omega$

86. Which of the following statements are correct for basic transistor amplifier configurations?
(a) CB amplifier has low input impedance and a low current gain.
(b) CC amplifier has low output impedance and a low current gain.
(c) CE amplifier has very poor voltage gain but very high input impedance.
(d) The current gain of CB amplifier is higher than the current gain CC amplifier.

87. A triangular-square wave generator uses
(a) A sine wave oscillation and a comparator (b) An integrator and a comparator
(c) A differentiator and a comparator (d) A sine wave oscillator and a clipper.

88. An $n$-channel junction field effect transistor has $5mA$ source to drain current at shorted gate ($I_{DSS}$) and -5V pinch off voltage ($V_p$). Calculate the drain current in $mA$ for a gate-source voltage ($V_{GS}$) of –2.5V. The answer should be up to two decimal places ________

89. In a dual slope type digital voltmeter, an unknown signal voltage is integrated over 100 cycles of clock. If the signal has a 50 Hz pickup, the maximum clock frequency can be.

(a) 50 Hz (b) 5 KHz (c) 10 KHz (d) 50 KHz

**Data for linked Q. 90 and Q. 91.**

A dual slope ADC uses a 16-bit counter with a 5MHz, clock. if maximum input voltage is +12 volt and maximum integrator output voltage for maximum count is -10 volt. If resistor used in integrator is 100 K$\Omega$. Then

90. What is conversion period of dual slope ADC

(a) 5ms (b) 10 ms (c) 13 ms (d) 26 ms

91. What is the value of capacitor used.

(a) 0.48 $\mu$F (b) 1 $\mu$F (c) 0.16 $\mu$F (d) 0.32 $\mu$F

92. The conversion time of an 8-bit successive approximation counter with 1MHz clock is nearly

(a) 8 $\mu$sec. (b) 128 $\mu$sec. (c) 256 $\mu$sec. (d) 512 $\mu$sec.

93. The largest analog output voltge from a 6-bit digital to analog converter (DAC) which produces 1.0 V output for a digital input of 010100, is:

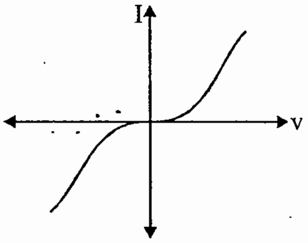
(a) 1.6 V (b) 2.9 V (c) 3.15 V (d) 5.0 V

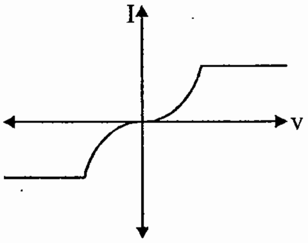
94. What should be the clock frequency of a 6-bit A/D converter so that its maximum conversion time is 32$\mu s$?

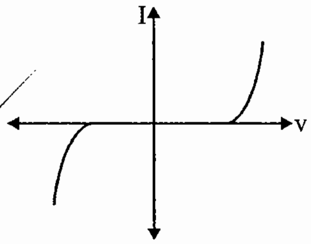
(a) 1 MHz (b) 2 MHz (c) 0.5 MHz (d) 4 MHz

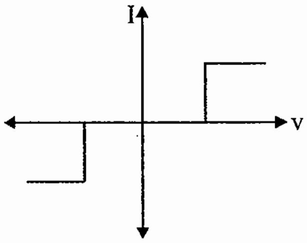
95. A voltage regulator has ripple rejection of –50dB. If input ripple is 1mV, what is the output ripple voltage in $\mu$V? The answer should be up to two decimal places _______

96. Two identical Zener diodes are placed back to back in series and are connected to a variable DC power supply. The best representation of the I-V characteristics of the circuit is

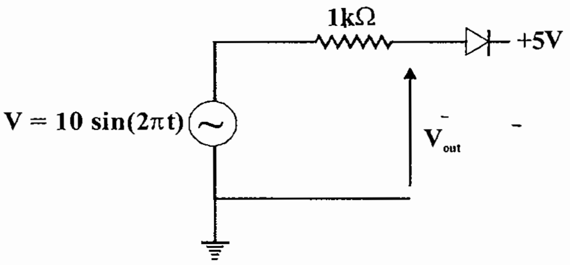
(a)


(b)


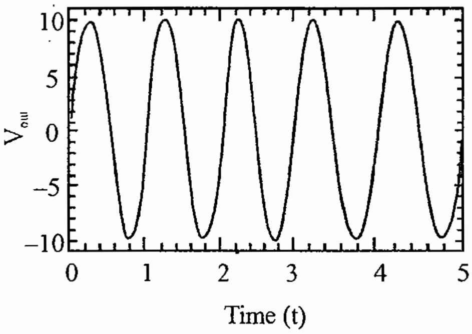
(c)


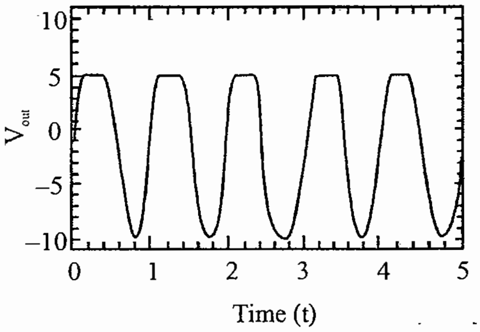
(d)


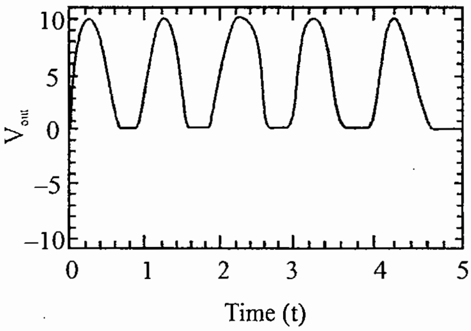
97. Consider the following circuit

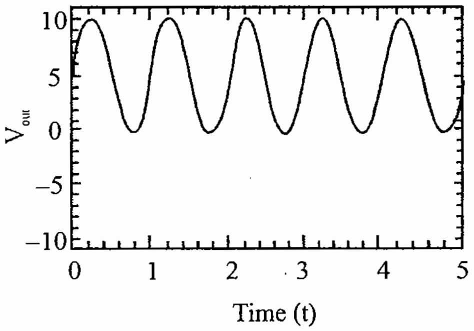


Which of the graphs given below is a correct representation of $V_{out}$?

(a)


(b)


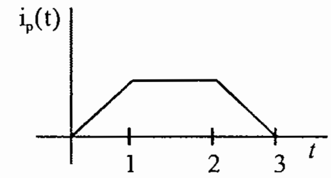
(c)


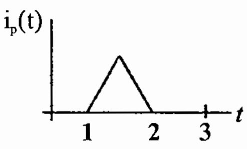
(d)


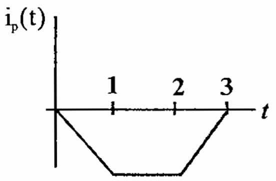
98. An RC network produces a phase-shift of 30°. How many such RC networks should be cascaded together and connected to a Common Emitter amplifier so that the final circuit behaves as an oscillator?

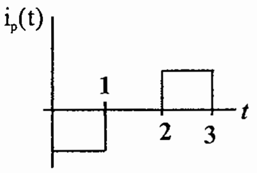
(a) 6 (b) 12 (c) 9 (d) 3

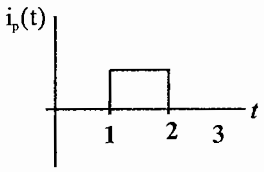
99. A current $i_p$ flows through the primary coil of a transformer. The graph of $i_P(t)$ as a function of time 't' is shown in figure below



(a)


(b)


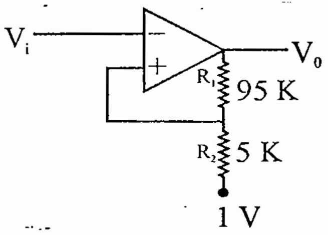
(c)


(d)


## Assignment: OP-AMP

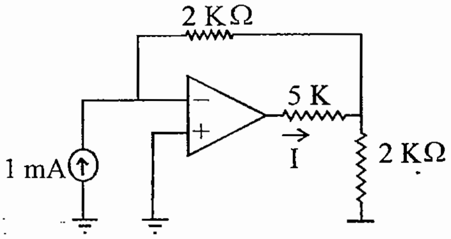
1. What is hysteresis voltage for a given Schmitt trigger circuit?

$V_{sat} = \pm 10V$



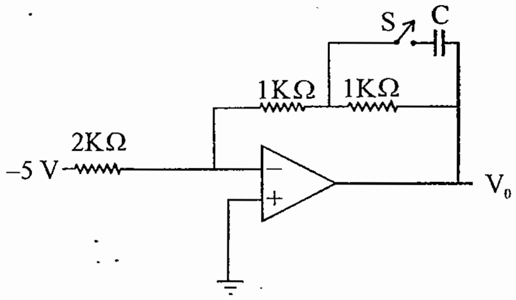
[✓ (a) 1 V]  (b) –1 V  (c) 2 V  (d) –2 V

2. Assume OP-AMP to be Ideal, what is value of current I, through 5KΩ resistor?



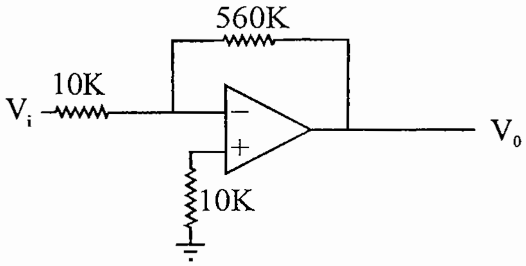
(a) 2mA  (b) –2mA  (c) 4 mA  (d) –4 mA

3. What is value of output voltage when switch is closed? It is assumed that system is in steady state position?



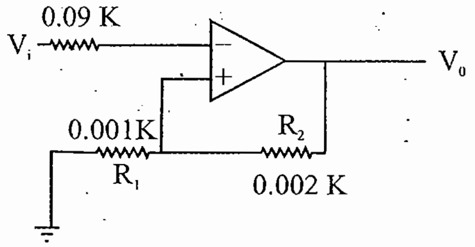
(a) +5  (b) –5  (c) –2.5  (d) +2.5

4. The OP-AMP used in inverting amplifier show in figure has an equivalent input offset voltage of 5 mV. The total offset voltage is:



(a) 5 mV  (b) –280 mV  [✓ (c) –285 mV]  (d) 560 mV

5. Feedback factor ($\beta$) of circuit is



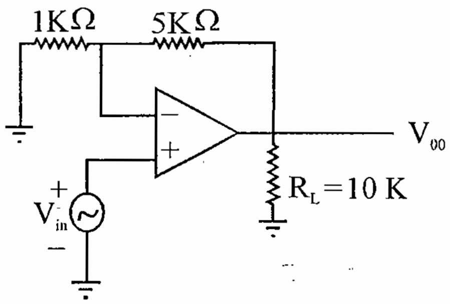
$\beta = \frac{R_1}{R_1+R_2}$

$= \frac{0.001}{0.003}$

$\simeq 0.33$

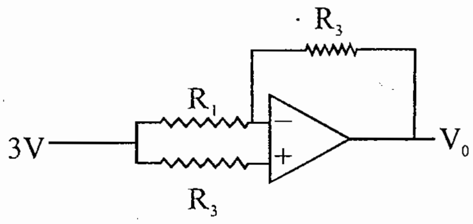
(a) 0.33 (b) 0.67 (c) Data is insufficient (d) 0.03

6. In case of circuit shown in figure, $V_{io} = 10$ mV D.C. The maximum possible output offset voltage $V_{00}$, caused by input voltage $V_{io}$, w.r.t. GND is:



(a) 60 volt D.C (b) 60 mV D.C (c) 110 volt D.C (d) 50 mV D.C

7. What is CMRR of given Op-Amp Circuit.



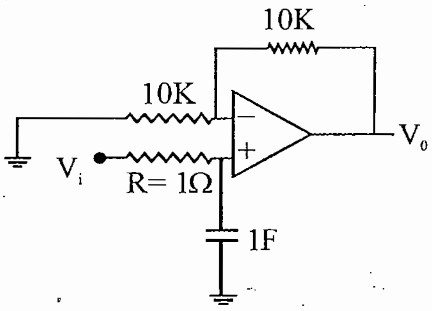
(a) 0 (b) 1 (c) $\infty$ (d) 3

8. For the above question, what is the value of output?

(a) 0 (b) 1

(c) 3 (d) Interms of $R_1, R_2$ & $R_3$

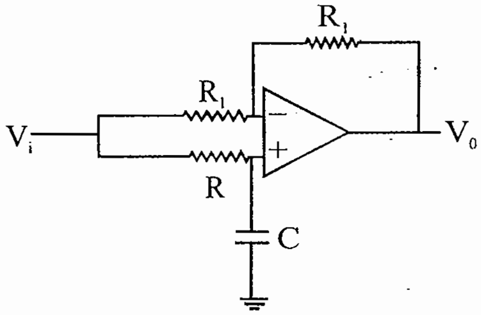
9. For given ideal OP-AMP if $V_i = \text{Sin}\, t$ What is the value of $V_0$?



(a) $\sqrt{2}\sin(t+45°)$ (b) $\sqrt{2}\sin(t-45°)$

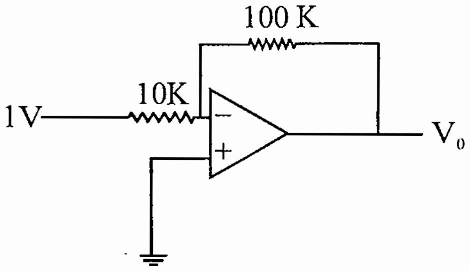
(c) $1/\sqrt{2}\sin(t+45°)$ (d) $1/\sqrt{2}\sin(t-45°)$

10. For the circuit shown in figure, with an ideal OP-AMP, what is the maximum phase shift of output with reference to input?



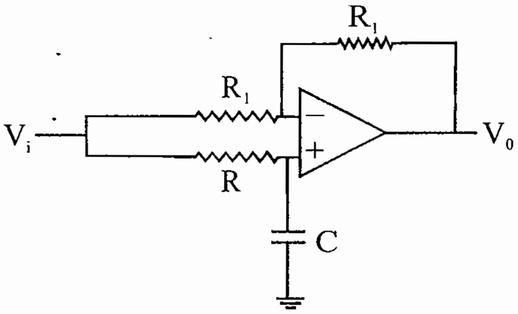
(a) 0° (b) 90° (c) –95° (d) ± 180°

11. The inverting OP-AMP, shown in figure has an open loop gain of 50, What is value of closed loop gain?



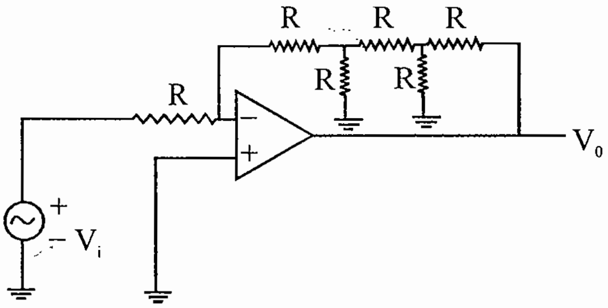
(a) –8 (b) –9 (c) –10 (d) +11

12. Calculate value of C if $\omega = 1000$ Rad /sec $R = 100K\Omega$, $R_1$ is $100\Omega$ and phase shift between $V_0$ and $V_i$ is 270°



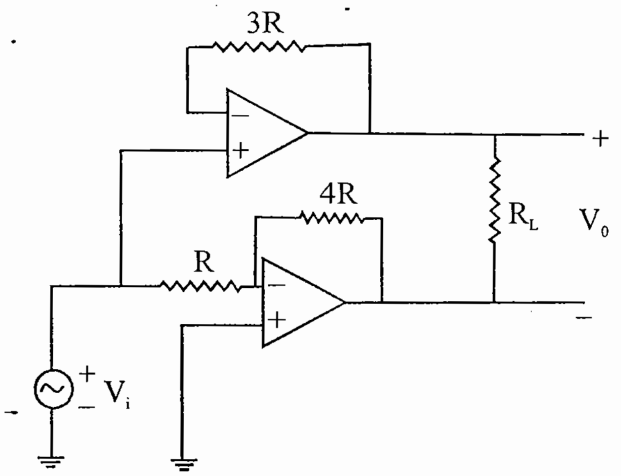
(a) 1μF (b) 0.1 μF (c) 0.2 μF (d) 0.01 μF

13. What is the value of gain of given circuit?



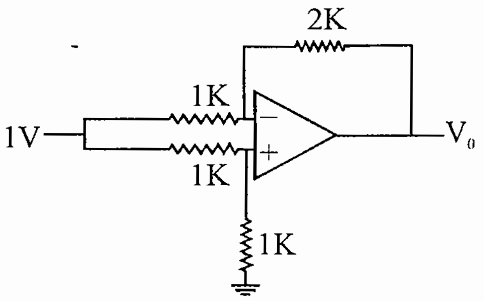
(a) –4 (b) –7 (c) –9 (d) –8

14. What is the value of gain $V_0/V_i$ of given circuit?



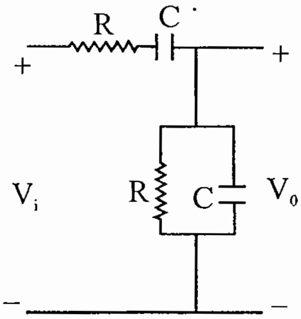
(a) +4  (b) 5  (c) 0  (d) $-3R_L/R$

15. For given OP-AMP what is value of output voltage $V_0$?



(a) –2V  (b) –0.5V  (c) –1V  (d) 0.5 V

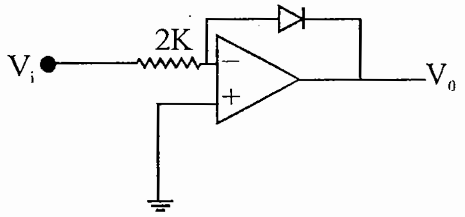
16. The RC, circuit shown in figure is?



(a) LPF (Low Pass Filter)  (b) HPF (High Pass Filter)

(c) BPF (Band Pass Filter)  (d) BSF (Band Stop Filter)

17. In the OP-AMP circuit shown, assume that diode current follows the equation $I = I_0 \exp(V/V_T)$. For $V_i = 2V$, $V_0 = V_{01}$ and for $V_i = 4V$, $V_0 = V_{02}$. Relationship between $V_{01}$ & $V_{02}$ is

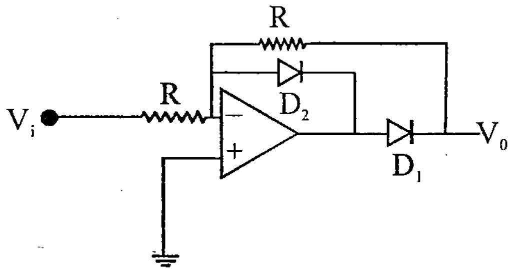


(a) $V_{02} = \sqrt{2}V_{01}$  (b) $V_{02} = e^2 V_{01}$  (c) $V_{02} = V_{01}\ln 2$  (d) $V_{01} - V_{02} = V_T \ln 2$

18. If OP-AMP is ideal and i/p is triangular wave then $V_0$ will be

(a) Sq. wave (b) Triangular (c) Parabolic (d) Sine

19. In circuit shown in figure, function of Diode $D_2$ is:



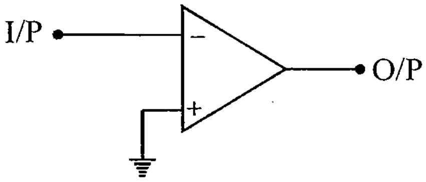
(a) to provide negative feedback when input in negative

(b) to avoid saturation of operational amplifier

(c) to reduce reverse berekdown voltage of diode $D_1$.

(d) In significant and can be removed.

20. If the input to the circuit of figure is sine wave the output will be



(a) A half-wave rectified sine wave (b) A full wave rectified sine wave

(c) A triangular wave (d) A square wave

21. The input to a lock-in amplifier has the form $V_i(t) = V_i \sin(\omega t + \theta_i)$ where $V_i$, $\omega$, $\theta_i$ are the amplitude, frequency and phase of the input signal respectively. This signal is multiplied by a reference signal of the same frequency $\omega$, amplitude $V_r$ and phase $\theta_r$. If the multiplied signal is fed to a low pass filter of cut-off frequency $\omega$, then the final output signal is

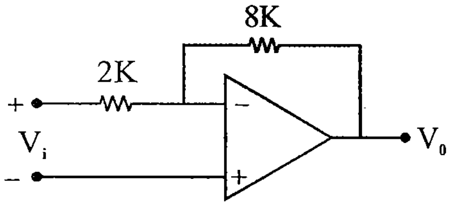
(1) $\frac{1}{2}V_iV_r\cos(\theta_i - \theta_r)$

(2) $V_iV_r\left[\cos(\theta_i - \theta_r) - \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)\right]$

(3) $V_iV_r\sin(\theta_i - \theta_r)$

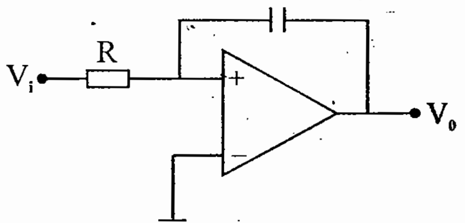
(4) $V_iV_r\left[\cos(\theta_i - \theta_r) + \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)\right]$

22. The OP-AMP of figure has a very poor open loop voltage gain of 45 but is otherwise ideal. The gain of the amplifier equals.



(a) 5 (b) 20 (c) 4 (d) 4.5

23. The circuit given below works as:



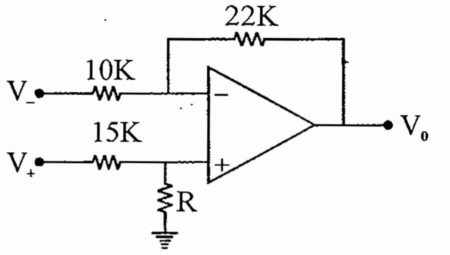
(a) Differentiator [✓ (b) Integrator]
(c) Both as integrator and differentiator. (d) None of above.

24. Two non-inverting amplifiers, one having a unity gain and the other having a gain of twenty are made using identical operational amplifiers. As a compared to the unity gain amplifier, the amplifier with gain twenty has:
(a) Less negative feedback (b) Greater input impedance
[✓ (c) Less bandwidth] (d) None of the above.

25. The input differential stage of OP-AMP 741 is biased at about 10 $\mu$A current. Such a low current of the input stage gives
1. High CMRR 2. High differential gain.
3. Low differential gain. 4. High input impedance
Which of these are correct?
(a) 1 and 2 (b) 1, 2 and 4 (c) 3 and 4 (d) 1, 2, 3 and 4

26. Negative feedback in amplifiers:
1. Improves the signal to noise ratio at the input.
2. Improves the signal to noise ratio at the output.
3. Does not affect the signal to noise ratio at the output.
4. Reduces distortion.
Then which of the following statements are correct:
(a) 1 and 2 only (b) 2 and 3 only (c) 2 and 4 only (d) 1, 2 and 4.

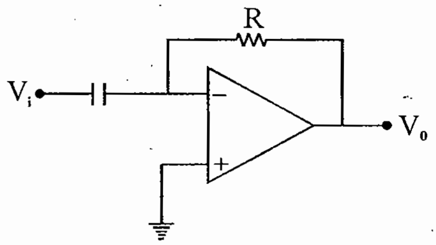
27. In the given circuit (figure) if the voltage inputs $V_-$ and $V_+$ are to be amplified by the same amplification factor, the value of R should be:



(a) 66 K (b) 99 K (c) 33 K (d) 11 K

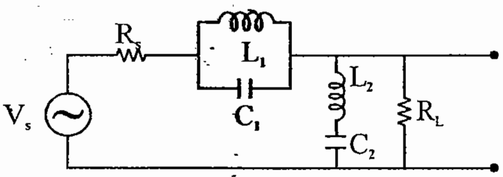
28. Negative feedback in voltage series configuration:
(a) Increase input impedance (b) Decease input impedance
(c) Increases closed loop gain (d) Leads to oscillation.

29. Assume that the OP-AMP of figure is ideal, if $V_i$ is a triangular wave, then $V_0$ will be:



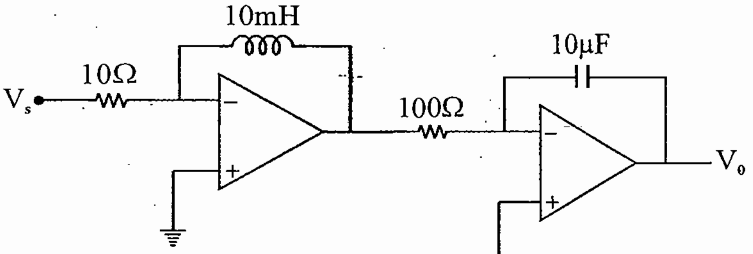
[✓ (a) Square wave] (b) Triangular wave (c) Parabolic wave (d) Sine wave

30. The circuit of figure represents a:



(a) Low pass filter (b) High pass filter (c) Band pass filter (d) Band reject filter.

31. In figure assume the Op-Amps to be ideal. The **output** $V_0$ of the circuit is:

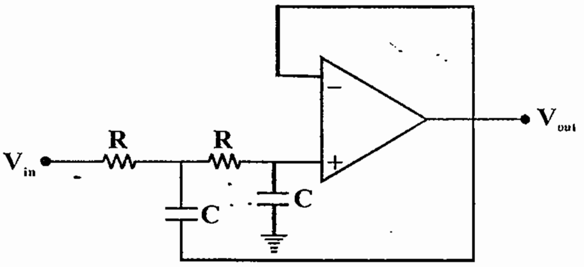


$V_s = 10\cos(100t)$

(a) $10\cos(100t)$ (b) $10\int_0^t \cos(100\tau)\,d\tau$ (c) $10^{-4}\int_0^t \cos(100\tau)\,d\tau$ (d) $10^{-4}\frac{d}{dt}\cos(100t)$

32. An ideal OP-AMP is an ideal:
(a) Voltage controlled current source (b) Voltage controlled voltage source
(c) Current controlled current source (d) Current controlled voltage source.

33. The circuit in figure is a:



[✓ (a) Low-pass filter] (b) High-pass filter (c) Band-pass filter (d) Band-reject filter

34. A differential amplifier has a differential gain of 20, 000. CMRR = 80 dB. The common mode gain is given by
[✓ (a) 2] (b) 1 (c) 0.5 (d) 0

35. Which of the following pairs is/are correctly matched?

| Waveform | Circuitry and input signal |
|---|---|
| 1. Triangular wave | Integrating circuit and square wave |
| 2. Impulsive wave (Dirac Delta) | Differentiating circuit and step signal |
| 3. Saw-tooth wave | Differentiating circuit and triangular wave |

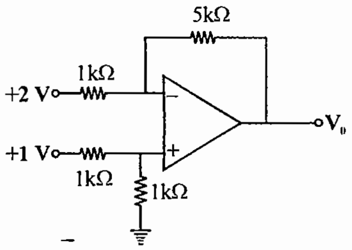
Select the correct answer using the codes given below:
(a) 2 and 3 [✓ (b) 1 and 2] (c) 1 alone (d) 1 and 3

36. An ideal OP-AMP has
(a) Infinite input and output impedance
(b) Very low input and output impedance
(c) Low input impedance and infinite output impedance
(d) Infinite input impedance and zero output impedance.

37. A differential amplifier is invariably used in the input stage of all OP-AMPs. This is done basically to provide the OP-AMPs with a very high

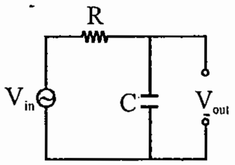
(a) CMRR (b) Bandwidth (c) Slew rate (d) Open-loop gain.

38. The output $V_0$ of the ideal OP-AMP circuit shown in the figure is:



(a) –7V (b) –5 V (c) 5 V (d) 7 V

39. The circuit shown in the figure can be used as a

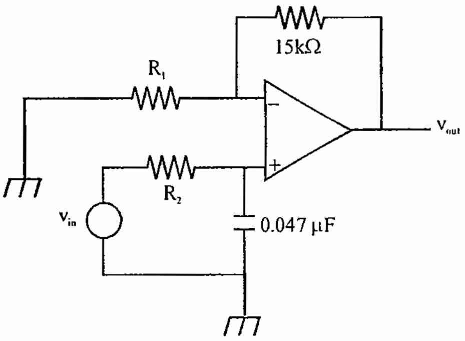


(a) High pass filter or a differentiator (b) High pass filter or an integrator

(c) Low pass filter or a differentiator (d) Low pass filter or an integrator.

40. The low-pass active filter shown in the figure has a cut-off frequency of 2kHz and a pass band gain of 1.5. The values of the resistors are

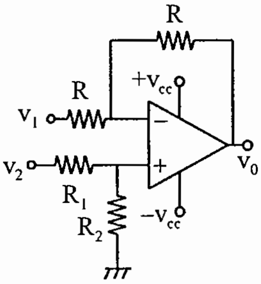
(or) mid band
(or) pass band gain



(a) $R_1 = 10k\Omega; R_2 = 1.3\Omega$ (b) $R_1 = 30k\Omega; R_2 = 1.3\Omega$

(c) $R_1 = 10k\Omega; R_2 = 1.7\Omega$ (d) $R_1 = 30k\Omega; R_2 = 1.7\Omega$

41. In the following circuit, for the output voltage to $V_0 = (-V_1 + V_2/2)$ the ratio $R_1/R_2$ is

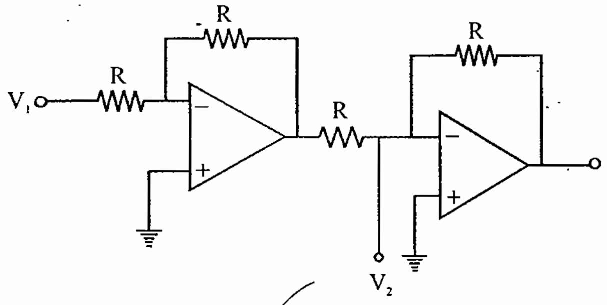


(a) 1/2 (b) 1 (c) 2 (d) 3

42. A power amplifier gives 150 W output for an input of 1.5W. The gain in dB, is:

(a) 10 (b) 20 (c) 54 (d) 100

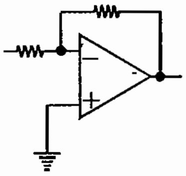
43. The circuit shown is based on ideal operational amplifiers. It acts as a



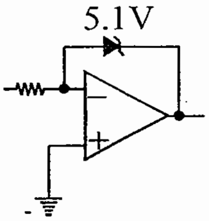
(a) Subtractor (b) Buffer amplifier [✓ (c) Adder] (d) Divider.

44. In one of the following circuits, negative feedback does not operate for a negative input. Which one is it? The opamps are running from $\pm 15V$ supplies.

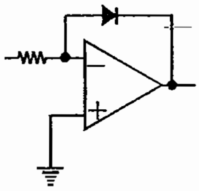
(a)



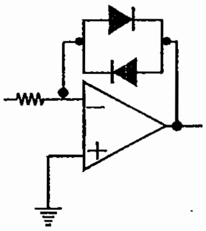
(b)



(c)

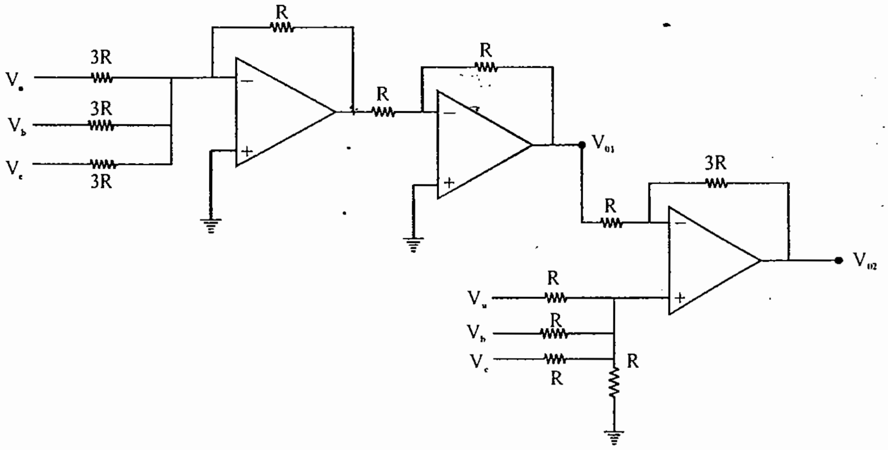


(d)



**Statement for Linked Answer Q.45 and Q.46**

The following circuit contains three operational amplifiers and resistors.



45. The output voltage at the end of second operational amplifier $V_{01}$ is:

(a) $V_{01} = 3(V_a + V_b + V_c)$

(b) $V_{01} = -\frac{1}{3}(V_a + V_b + V_c)$

[✓ (c) $V_{01} = \frac{1}{3}(V_a + V_b + V_c)$]

(d) $V_{01} = \frac{4}{3}(V_a + V_b + V_c)$

46. The output $V_{02}$ (at the end of third OP-AMP) of the above circuit is:

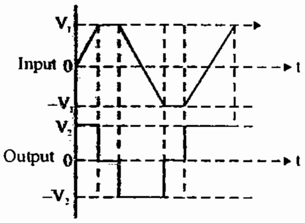
(a) $V_{02} = 2(V_a + V_b + V_c)$

(b) $V_{02} = 3(V_a + V_b + V_c)$

(c) $V_{02} = -\frac{1}{2}(V_a + V_b + V_c)$

(d) Zero.

47. The circuit for which the input and output waveforms are shown below is:



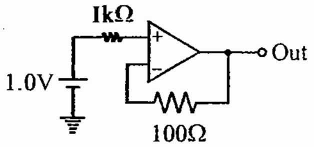
(a) Clipping circuit (b) Integrator [✓ (c) Differentiation] (d) Schmitt trigger.

48. An operational amplifier

[✓ (a) Has a very high gain] (b) Has a high input resistance

(c) Is a negative feedback amplifier (d) Has a very small input current.

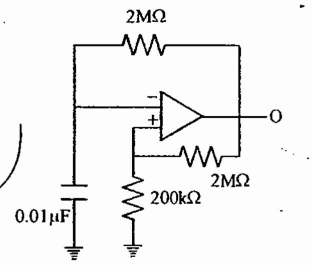
49. The output of the circuit on the right will be



[✓ (a) 1V] (b) 11V (c) −10V (d) 0 V

50. A multivibrator with a saturation voltage ±5V is shown in the diagram. The positive and negative threshold at the inverting terminal for which the multivibrator will switch to the other state are

(oscillator ckt)



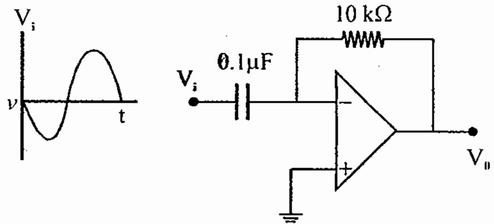
(a) ±5/11 V (b) ±10/11 V (c) ±5 V (d) ±11V

51. The inverting input terminal of an operational amplifier (OP-AMP) is shorted with the output terminal apart from being grounded. A voltage signal $v_i$ is applied to the non-inverting input terminal of the op-amp. Under this configuration, the op-amp functions as

(a) An open loop inverter (b) A voltage to current converter.

(c) A voltage follower (d) An oscillator.

52. Consider the following operational amplifier circuit with an input signal of frequency 10 kHz.

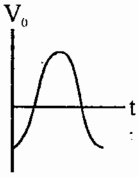


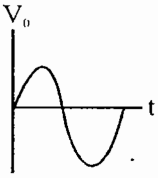
−sin

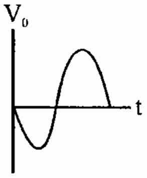
−cost

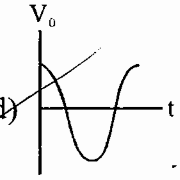
C 90

Which of the following represents the output waveform $V_0$?

(a)


(b)


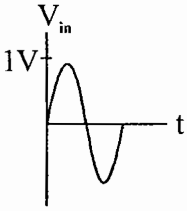
(c)


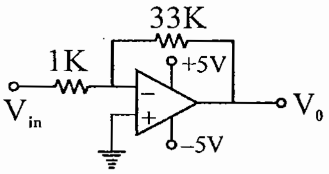
[✓ (d)]


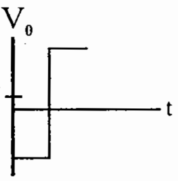
53. In an OP-AMP, when the input signal drives the output at a rate of voltage change greater than the slew rate, then the resulting signal.

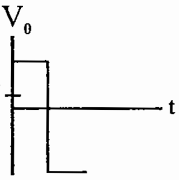
(a) Is enchanced
(b) Is clipped
(c) Is unaffected
(d) Remains the same, but with 90° phase difference.

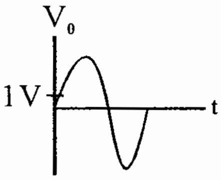
54. For the OP-AMP circuit shown in the figure below, which is the correct output waveform?

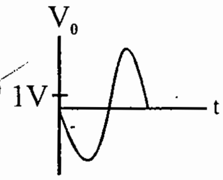




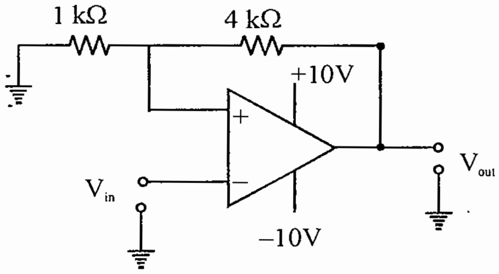
(a)


(b)


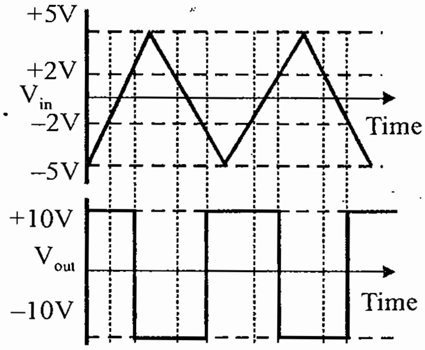
(c)


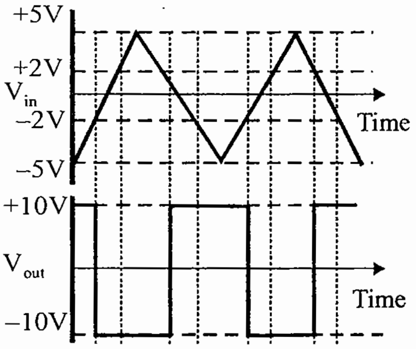
(d)


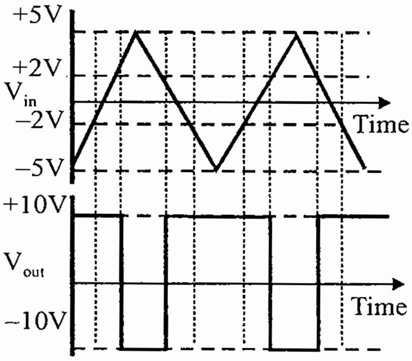
55. Consider the following circuit

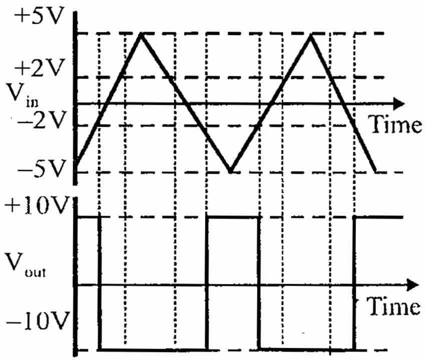


Which of the following represents the output $V_{out}$ corresponding to the input $V_{in}$?

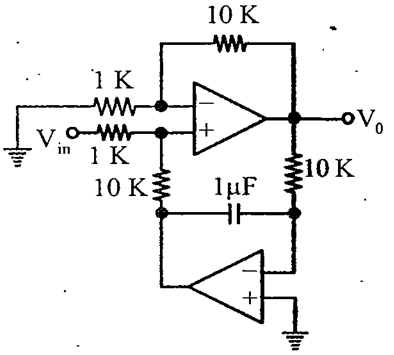
(a)


(b)


(c)


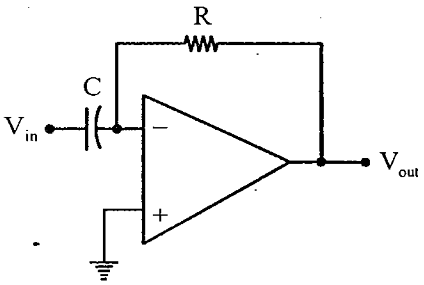
(d)


56. A time varying signal $V_{in}$ is fed to an op-amp circuit **with output** signal $V_0$ as shown in the figure below. The circuit implements a

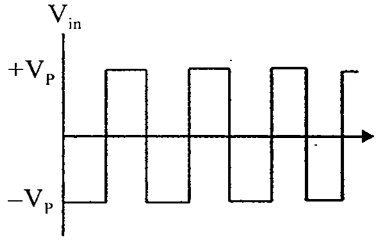


(a) High pass filter with cutoff frequency 16 Hz.
(b) High pass filter with cutoff frequency 100 Hz
(c) Low pass filter with cutoff frequency 16 Hz
(d) Low pass filter with cutoff frequency 100 Hz.

57. Consider the following circuit :



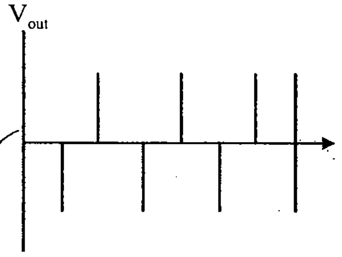
If the waveform given below is fed in at $V_{in}$



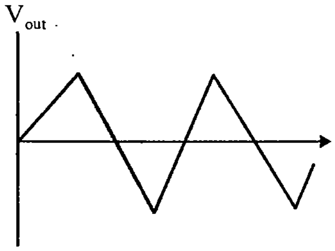
then the waveform at the output $V_{out}$ will be

D

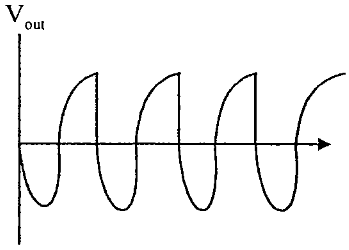
(a)



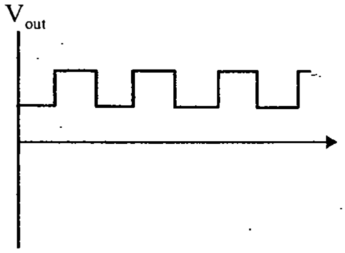
(b)



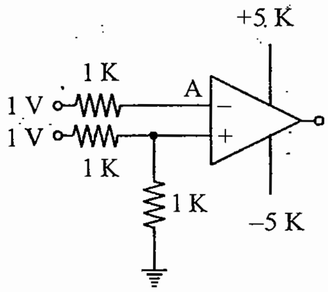
(c)



(d)

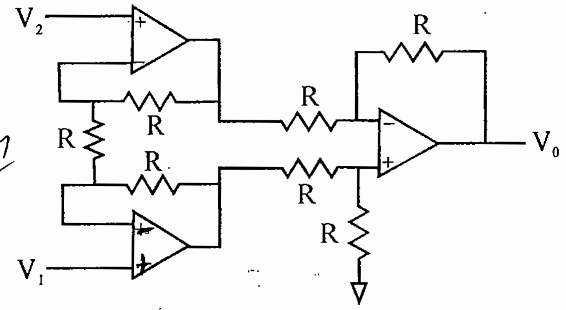


58. In the operational amplifier circuit below, the voltage at point A is



(a) 1.0 V (b) 0.5 V (c) 0 V (d) -5.0V

59. The classic there op-amp instrumentation amplifier configuration i shown below :



instrumentation Amplifier

The ratio of maximum to minimum resistance that can be obtained with N $1-\Omega$ resistors is

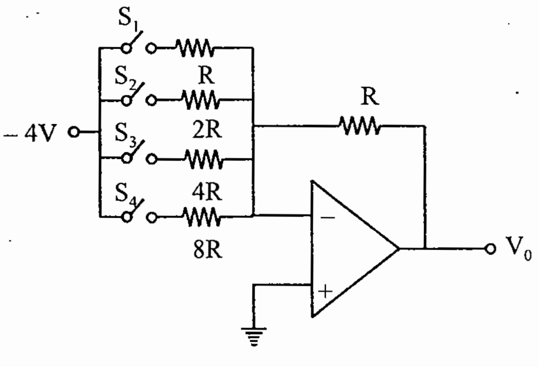
(a) N (b) $N^2$ (c) 1 (d) $\infty$

60. A triangular waveform is applied to the input of a differentiator. The output i

(a) a dc signal (b) a square waveform

(c) an inverted triangular waveform (d) the first harmonic of the triangular waveform

61. The circuit shown below implements a 4-bit DAC. Which of the switches S1,......., S4 should be closed to get an output voltge $V_0$ as close to 2.6V as possible?

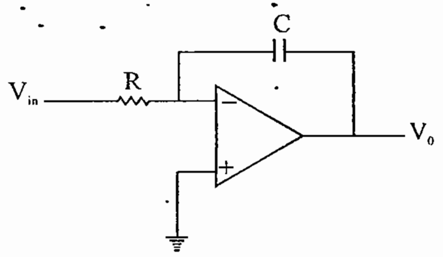


(a) S1 and S4 (b) S2 nd S3 (c) S3 nd S4 (d) S2 and S4

62. A sinusoidal signal $V_i = V_P \sin(\omega t)$ is applied to the input of an op-amp with slew rate $S_R$. The op-amp maximum frequency $\omega_{max}$ that the op-amp can sustain without output distortion is:

(a) $\sqrt{2\pi}\, S_R / V_p$ (b) $S_R / 2V_p$ (c) $2S_R / V_p$ (d) $S_R / V_p$

63. In the given operational amplifier circuit what is the correct expression for the output voltage?



(a) $V_0 = -RC\frac{dV_{in}}{dt}$ (b) $V_0 = -\frac{1}{RC}\frac{dV_{in}}{dt}$ (c) $V_0 = -RC\int_{t=0}^{t} V_{in}dt$ [✓ (d) $V_0 = -\frac{1}{RC}\int_{t=0}^{t} V_{in}dt$]

64. For a step input, the output of an integrator is a

(a) pulse (b) ramp (c) triangular wave from (d) spike

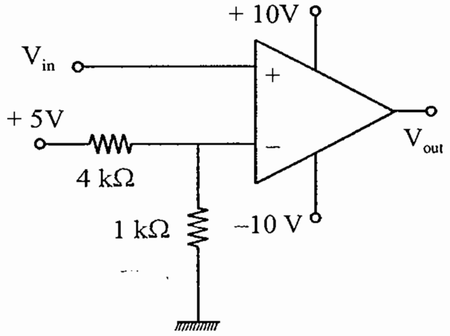
65. A certain inverting amplifier has a closed-loop gain of 25. The OP-AMP has an open-loop gain of 100000. If another OP-AMP with an open-loop gain of 300, 000 is substituted in the configuration, the closed-loop gain

(a) drops to 12.5 (b) doubles (c) remains at 25 (d) become $25\left(1-\frac{25}{200000}\right)$

66. A signal of frequency 10 kHz is being digitized by an A/D converter. Which of the following is an appropriate value for the sampling time?

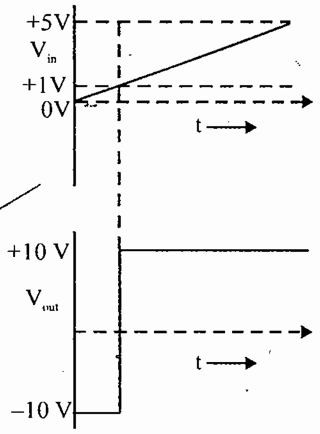
(a) 400 μs (b) 40 μs (c) 100 μs (d) 200 μs

67. Consider the following OP-AMP circuit.

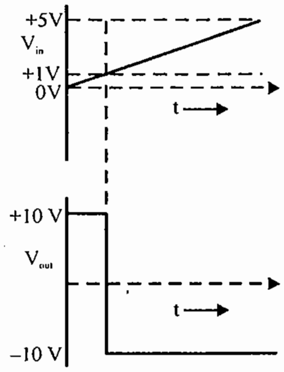


Which one of the following correctly represents the output $V_{out}$ corresponding to the input $V_{in}$ ?

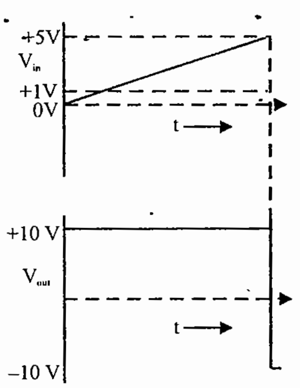
[✓ (a)]



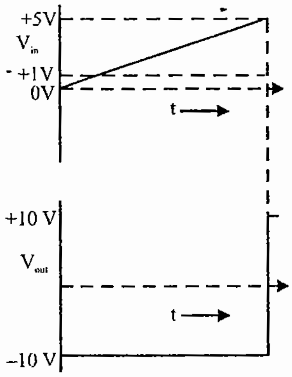
(b)



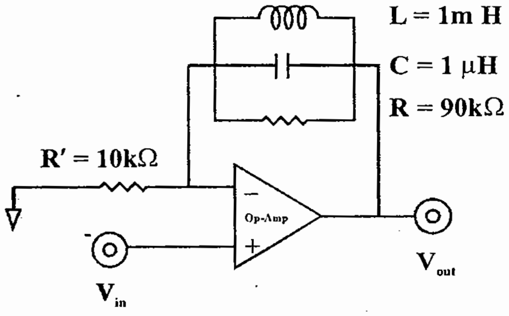
(c)



(d)

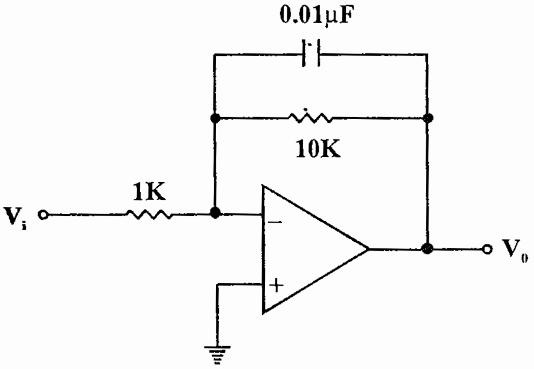


68. An input of 1.0 V DC is given to the ideal Op-Amp circuit depicted below. What will be the output voltage?



(a) 10.0 V (b) –9.0 V (c) 1.0 V (d) 0 V

69. In the op-amp circuit shown in the figure, $V_i$ is a sinusoidal input signal of frequency 10 Hz and $V_0$ is the output signal.



The magnitude of the gain and the phase shift, respectively, are close to the values

(a) $5\sqrt{2}$ and $\frac{\pi}{2}$ (b) $5\sqrt{2}$ and $\frac{-\pi}{2}$ (c) 10 and zero (d) 10 and $\pi$

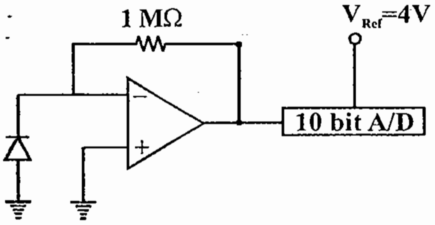
70. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and high pass filters are $\omega_0^{LP}$ and $\omega_0^{HP}$, respectively, the condition required to implement the band-pass and band-reject filters are respectively.

(a) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$ (b) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

(c) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$ (d) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

**Statement for Linked Q.71 and Q.72.**

Shown in the figure is a circuit to measure light intensity and convert it to a digital signal. The photodiode P has a responsivity of 0.1 A per watt of incident light intensity. The Op-amp converts the induced photocurrent to a voltage which is digitized by the 10-bit A/D converter with a reference voltage of 4V.



71. For a light intensity of 25 $\mu W$ incident on the photodiode, the voltage output of the OP-amp is:

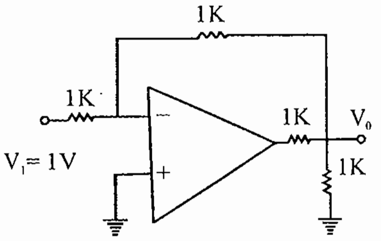
(a) 0.25 V (b) 1.0 V (c) 4.0 V (d) 2.5 V

72. The range of light intensity which can be measured by this set up is:

(a) 100 nW to 100$\mu W$ (b) 100 nW to 100$\mu W$

(c) 40 $nW$ to 40$\mu$W (d) 40 $nW$ to 40$\mu$W

73. In the op-amp circuit shown in the figure below, the input voltage $V_i$ is 1V. The value of the output $V_0$ is:



(a) −0.33 V (b) −0.50 V

(c) −1.00 V (d) −0.25 V

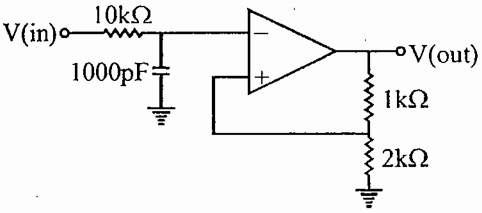
74. If the analog input to an 8-bit successive approximation ADC is increased from 1.0 V to 2.0 V, then the conversion time will:

[✓ (a) remain unchanged] (b) double

(c) decrease to half its original value (d) increase four times

**Statement for Linked Answer Q. 75 and Q. 76 :**

Consider the following circuit



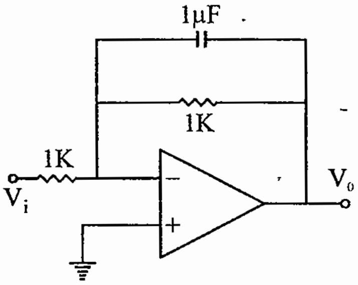
75. For this circuit the frequency above which the gain will decrease by 20$dB$ per decade is

(a) 15.9 kHz (b) 1.2 kHz (c) 5.6 kHz (d) 22.5 kHz

76. At 1.2 kHz the closed loop gain is

(a) 1 (b) 1.5 (c) 3 (d) 0.5

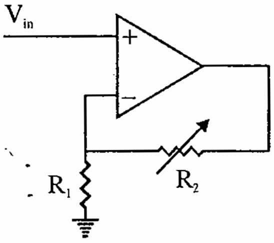
77. Consider the op-amp circuit shown in the figure.



If the input is a sinusoidal wave $V_i = 5\sin(1000t)$, then the amplitude of the output $V_0$ is

(a) $\frac{5}{2}$ (b) 5 (c) $\frac{5\sqrt{2}}{2}$ (d) $5\sqrt{2}$

78. In the following circuit, the resistance $R_2$ is doubled.



It follows that the current through $R_2$.

(a) remains the same (b) is halved

(c) is doubled (d) is quadrupled.

79. An op-amp based voltage follower

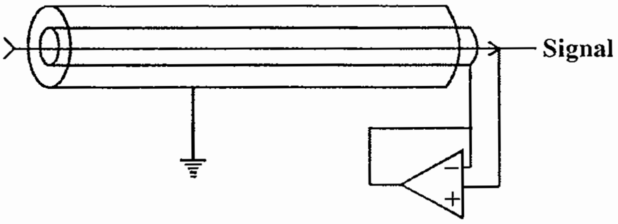
(a) is useful for converting a low impedance source into a high impedance source

(b) is useful for converting a high impedance source into a low impedance source

(c) has infinitely high closed loop output impedance

(d) has infinitely high closed loop gain

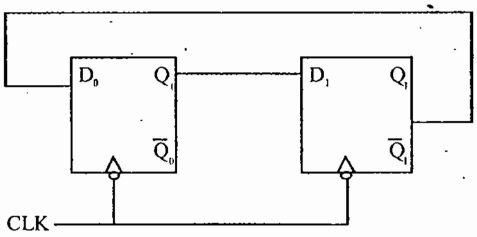
80. The inner shield of a triaxial conductor is driven by an (ideal) op-amp follower circuit as shown. The effective capacitance between the signal-carrying conductor and ground is

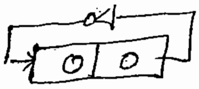


(a) unaffected (b) doubled (c) halved (d) made zero

## Assignment – Digital Electronics

1. In the circuit shown below initially

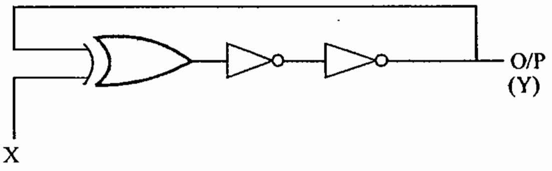

D₀
Q₀
Q̄₀
D₁
Q₁
Q̄₁
CLK



$Q_0 = Q_1 = 0$, then what are the values of $Q_0$ and $Q_1$ after 335th clock pulse.

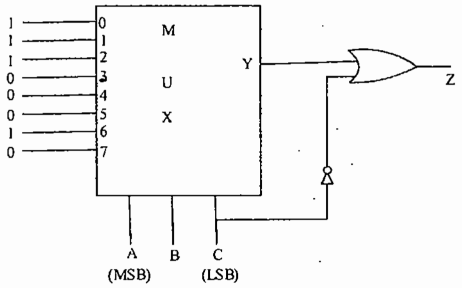
(a) 11 (b) 10 [✓ (c) 01] (d) 00

2. All the logic gates in circuit shown below have finite propagation delay. The circuit can be used as a clock generator, if



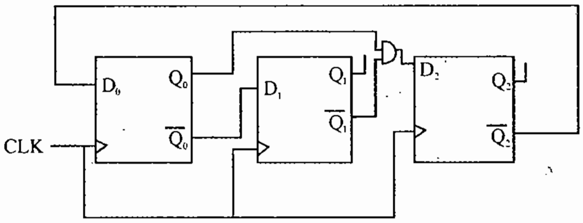
(a) $X = 0$ [✓ (b) $X = 1$] (c) $X = 0$ or 1 (d) $X = \bar{Y}$

3. A combinational circuit, using 8×1 MUX is shown in figure. The minimised expression for output Z is


1
1
1
0
0
0
1
0
0
1
2
3
4
5
6
7
M
U
X
Y
Z
A
(MSB)
B
C
(LSB)

(a) $C(\bar{A}+\bar{B})$ (b) $C(A+B)$ (c) $\bar{C}+\overline{AB}$ [✓ (d) $\bar{C}+\bar{A}\bar{B}$]

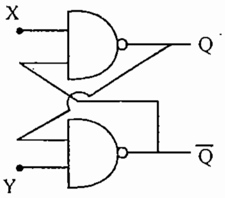
**Data for linked Q.4 and Q.5:**


D₀
Q₀
Q̄₀
D₁
Q₁
Q̄₁
D₂
Q₂
Q̄₂
CLK

4. Determine MOD of given counter

(a) 4 [✓ (b) 5] [✓ (c) 6] (d) 7

5. For a Flip-flop formed from 2 NAND gates as shown in figure, the unusable state corresponds to



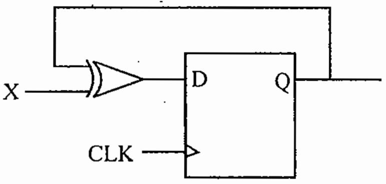
[✓ (a) $X = 0, Y = 0$] (b) $X = 1, Y = 0$ (c) $X = 0, Y = 1$ (d) $X = 1, Y = 1$

6. The minimal product of sums function described by K-Map, shown in figure is

| C \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| C 1 | 1 | 1 | × | 0 |
| C′ 0 | 0 | 0 | × | 0 |

(a) A'C' (b) A'+C' (c) A + C [✓ (d) A′C]

7. The Digital circuit shown in figure acts as a



(a) JK Flip flop (b) Clocked RS Flipflop [✓ (c) T Flipflop] (d) Ring Counter

8. The open collector outputs of 2 inputs NAND gate are connected to a common pull up resistor if the inputs to gate are P, Q and R, S respectively. Then what is the value of output.

(a) $\overline{PQ}.\overline{RS}$ (b) $\overline{PQ}+\overline{RS}$ (c) $PQ+RS$ (d) $PQ.RS$

9. The minimum no. of 2 inputs NAND gate required to implement boolean function f(A, B, C, D) = ABCD are

(a) 3 (b) 5 [✓ (c) 6] (d) 8

**Date for linked Q.10 and Q.11.**

The 4, variable function f is given in terms of min term as $f(A,B,C,D)=\sum(2,3,8,10,11,12,14,15)$. If by use of K-Map, function is minimised in sum of product forms, then SOP is

10. [✓ (a) $A\bar{D}+AC+\bar{B}C$] (b) $AD+\bar{A}C+BC$ (c) $\bar{A}D+AC+BC$ (d) $AC+\bar{B}C$

11. Above minimised SOP, can be implemented by how many minimum no. of 2 inputs NAND gate:

(a) 6 (b) 7 [✓ (c) 8] (d) 9

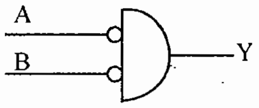
12. If 2's complement representation of a 16-bit number (one-sign bit and 15 magnitude bits) is FFFF, then its magnitude in decimal representation is

(a) 0 [✓ (b) 1] (c) 32767 (d) 65535

13. The initial state of MOD-16, down counter is 0110. What state will be it be after 37 clock pulses?

(a) Indeterminate (b) 0110 (c) 0101 [✓ (d) 0001]

14. The negative logic AND gate shown in figure is equivalent to a positive logic



(a) AND gate [✓ (b) OR gate] (c) NOR gate (d) NAND gate

15. How many min-terms (excluding redundant) does the minimal switching function

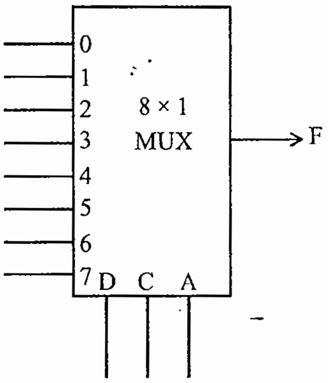
$f(A, B, C, D, E) = C + \bar{D}E$, originally have

(a) 16 [✓ (b) 20] (c) 24 (d) 32

**Data for linked Q.16 and Q.17.**

The logic function $f(A, B, C, D) = AC + ABD + ACD$ is to be realised using an $8 \times 1$ MUX

16. Shown in figure to realise this logic $I_3$, $I_4$, $I_5$ must be connected to



(a) 1, 0 , 0 [✓ (b) 1, 0, B] (c) 0, 1, B (d) B, B, B

17. Can the function be realise using a 4×1, MUX

(a) Yes always

(b) No

[✓ (c) Yes, using some combinational ckt. we can realise]

(d) Data not sufficient

18. The binary no. 110011 is to be converted to gray code, then no. of gates and type required are

(a) 6, AND (b) 6, XNOR (c) 6, XOR [✓ (d) 5, XOR]

19. What is F'S complement of $(2BFD)_{16}$

(a) E304 [✓ (b) D402] (c) D403 (d) C403

20. If x and y are boolean variables then which one of the following is equivalent to $x \oplus y \oplus xy$

[✓ (a) $x + y$] (b) $x + \overline{y}$ (c) $\overline{x} + \overline{y}$ (d) $\overline{x} + y$

21. Any combinational circuit can be built using

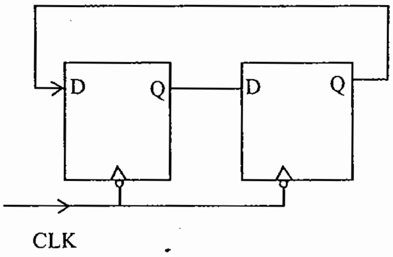
1. NAND 2. NOR 3. EX-OR 4. MUX

(a) 1, 2. (b) 1,2,3 [✓ (c) 1, 2, 4] (d) 1, 2, 3 4

22. A, MOD-11, ripple counter consists of 4 flipflops and a combinational circuit then economical combinational CKT. will be

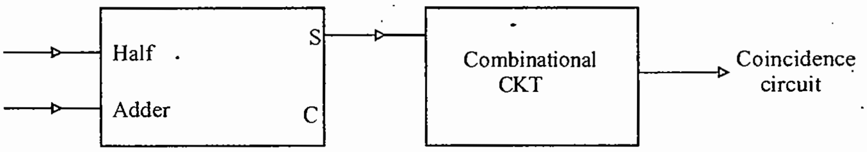
(a) 4 input AND gate (b) 4 input NAND gate (c) 3 input OR gate (d) 3 input NOR gate

23. Above circuit represent a



[✓ (a) MOD-2 counter] (b) MOD-3 counter (c) MOD-4 counter (d) MOD-4, ring counter.

24. For given combinational circuit if output is coincidence circuit, then unknown combinational circuit is



(a) AND gate (b) OR gate (c) EXNOR gate (d) NOR gate

25. A, shift register can be made by use of

1. DFF 2. RSFF 3. TFF 4. JKFF

(a) 1,2,3 (b) 1, 3, 4 (c) 1, 4 (d) 1,2,4

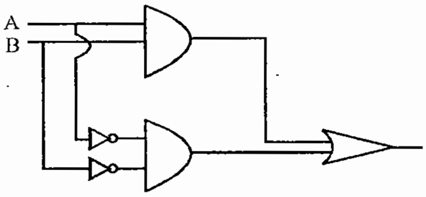
26. If an 8-bit 2's complement representation of an integer is FA (hex), then its decimal equivalent is

(a) –6 (b) +6 (c) –10 (d) +10

27. $(7E)_H$ and $(5F)_H$ are XOR. If output is multiplied by $(10)_H$ then result is:

(a) $(0210)_H$ (b) $(7EF5)_H$ (c) $(7FE5)_H$ (d) $(2100)_H$

28. The logic circuit shown in figure is:



(a) Half adder (b) X-OR gate (c) Equality detector (d) Full adder.

29. In a dual slope ADC, if reference voltage is 100 mV and the first integration period is set as 50 msec. For an input voltage of 120 mV, what is the second integration (de-integration) period.

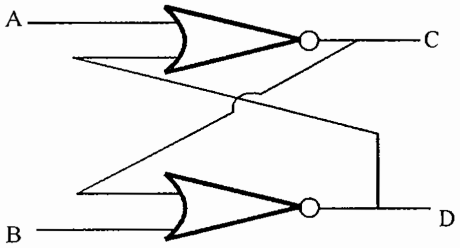
(a) 50 ms (b) 60 ms (c) 100 ms (d) 120 msec

30. If 2 4-bit 2's complement no's 1011 and 0110 are added, then result expressed in 2's complement notation is

(a) 0001 (b) 0010

(c) 1101 (d) cannot be expressed in 4 bit 2's complement

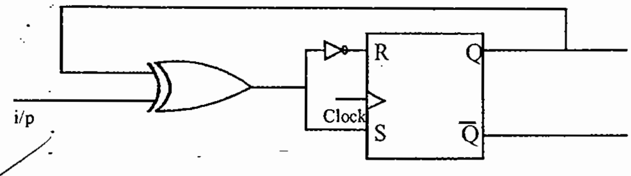
31. In circuit, shown in figure, when inputs A = B = 0, then possible logic states of C & D are



(a) C = 0, D = 1 or C = 1, D = 0 (b) C = 1, D = 1 or C = 0, D = 0

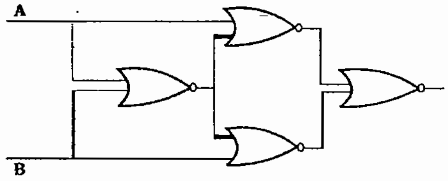
(c) C = 1, D = 0 (d) C = 0, D = 1

32. Given Digital circuit behaves as a



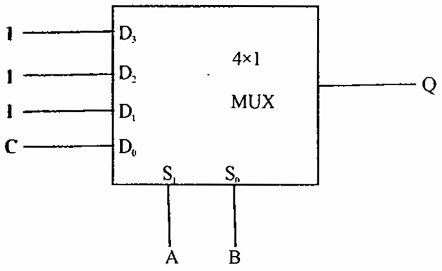
(a) D FF (b) T FF (c) JK FF (d) RS FF

33. Above given combinational circuit is equivalent to :



(a) NOR (b) OR (c) EX-OR (d) EX-NOR

34. The combinational circuit shown employs a 4×1, MUX, what is output Q of CKT.



(a) $\bar{A}+\bar{B}+C$ (b) $A+B+C$ (c) $A\oplus B\oplus C$ (d) $\overline{ABC}$

35. The boolean expression $f(P, Q, R) = \pi(0, 5)$ is to be realised using only two 2,input gates, what are these gates.

(a) AND & OR (b) NAND & OR (c) AND & XOR (d) OR & XOR

36. The boolean function $(x+y)(\bar{x}+z)(y+z)$ is equal to which one of the following expression

(a) $(x+y)(y+z)$ (b) $(\bar{x}+z)(y+z)$ (c) $(x+y)(\bar{x}+z)$ (d) $(x+y)(x+\bar{z})$

37. If output of a 2-level AND-OR network is F. What is the output when all gates are replaced by NOR gates.

(a) $F$ (b) $\bar{F}$ (c) $F_D$ (d) $\bar{F}_D$

38. The total no. of 1's in a 15-bit shift register is to be counted by clocking into a counter, which is present to O. The counter must have which one of the following.

(a) 4-bit (b) 5-bit (c) 6-bit (d) 16-bit

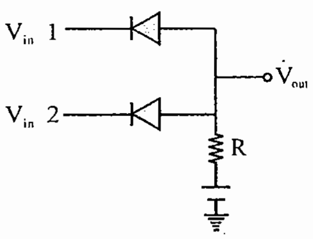
39. A number is expressed in binary 2's complement as 10011. Its decimal equivalent value is

(a) –19 (b) +19 (c) –13 (d) +13

40. The output voltage of a 5-bit DAC, binary ladder, which has a digital input of 11010 (assume O = 0V and 1 = +10 V) is:

(a) 3.4375 volt (b) 6.0 volt (c) 8.125 volt (d) 9.6875 volt

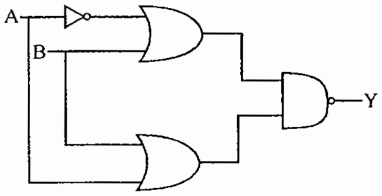
41. The circuit shown can be used as



(a) NOR gate (b) OR gate (c) NAND gate (d) AND gate

42. The Boolean expression: $B(A+B)+A.(\overline{B}+A)$ can be realized using minimum number of
(a) 1 AND gate (b) 2 AND gates (c) 1 OR gate (d) 2 OR gates.

43. In the given digital logic circuit, A and B form the input. The output Y is:

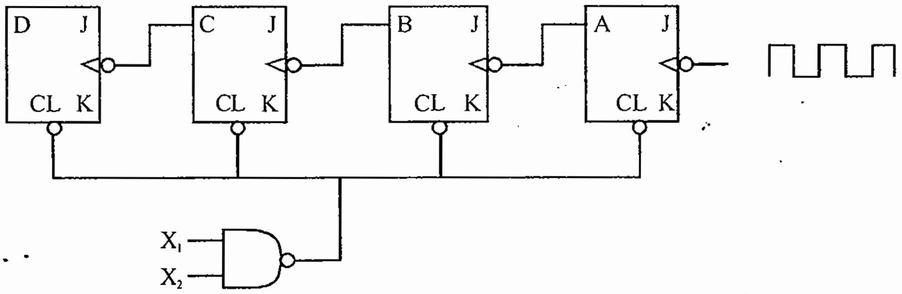


$\overline{A}B$ $\overline{A}+B$

A+B

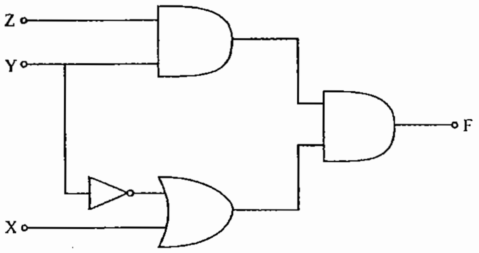
(a) $Y=\overline{A}$ (b) $Y=A\overline{B}$ (c) $Y=A+B$ (d) $Y=\overline{B}$

44. A ripple counter designed with JK flip-flops provided with CLEAR (CL) input is shown in the figure. In order that this circuit functions as a MOD-12 counter, the NAND gate input ($X_1$ and $X_2$) should be



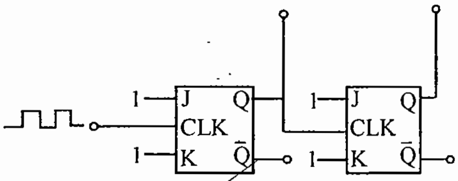
(a) A and C (b) A and D (c) B and D (d) C and D

45. Identify the function F generated by the logic network shown



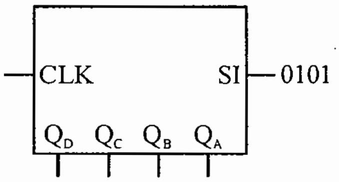
(a) $F=(X+Y)Z$ (b) $F=Z+Y+\overline{Y}X$ (c) $F=ZY(Y+X)$ (d) $F=XYX$

46. In the circuit shown, the ports $Q_1$ and $Q_2$ are in the state $Q_1 = 1$, $Q_2 = 0$. The circuit is now subjected to two complete clock pulses. The state of these ports now becomes



(a) $Q_2 = 1, Q_1 = 0$ (b) $Q_2 = 0, Q_1 = 1$ (c) $Q_2 = 1, Q_1 = 1$ (d) $Q_2 = 0, Q_1 = 0$

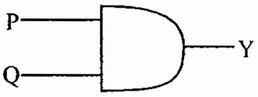
47. The registers $Q_D, Q_C, Q_B$ and $Q_A$ shown in the figure are initially in the state 1010 respectively. An input sequence SI=0101 is applied. After two clock pulses, the state of the shift registers (in the same sequence $Q_D\ Q_C\ Q_B\ Q_A$) is:



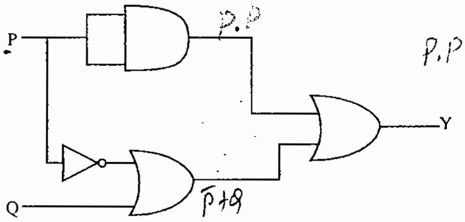
(a) 1001 (b) 0100 (c) 0110 (d) 1010

48. The simplest logic gate circuit corresponding to the Boolean expression, $Y = P + PQ$ is :

(a)



(b)



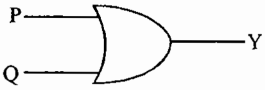
$P.P + (\bar{P}+Q)$

$P.(P+Q)$

$PP$ ...

$P+Q$

(c)



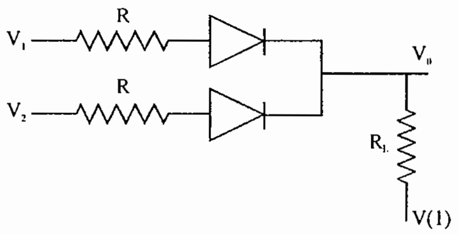
(d) None of these

49. An analog voltage V is converted into 2-bit binary number. The minimum number of comparators required and their reference voltages are

(a) $3, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}\right)$ (b) $3, \left(\frac{V}{3}, \frac{2V}{3}, V\right)$

(c) $4, \left(\frac{V}{5}, \frac{2V}{5}, \frac{3V}{5}, \frac{4V}{5}\right)$ (d) $4, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{5}, V\right)$

50. The following circuit (where $R_L >> R$) performs the operation of



(a) OR gate for a negative logic system (b) NAND gate for a negative logic system.

(c) AND gate for a positive logic system. (d) AND gate for a negative logic system.

**Statement for Linked Q.51 and Q.52.**

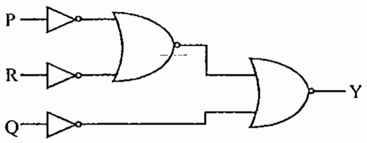
The karnaugh map of a logic circuit is shown below:

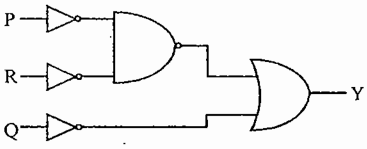
|  | $\bar{R}$ | R |
|---|---|---|
| $\bar{P}\bar{Q}$ | 1 | 1 |
| $\bar{P}Q$ | 1 |  |
| PQ |  |  |
| $P\bar{Q}$ | 1 | 1 |

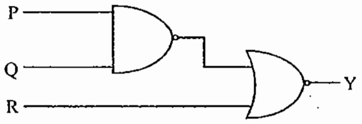
51. The minimized logic expression for the above map is:

(a) $Y = \bar{P}\bar{R} + \bar{Q}$ (b) $Y = \bar{Q}.PR$ (c) $Y = \bar{Q} + PR$ (d) $Y = Q.\overline{PR}$

52. The corresponding logic implementation using gates is given as:

(a)


(b)


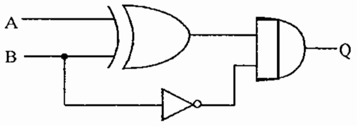
(c)


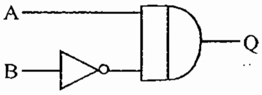
(d) None of the above

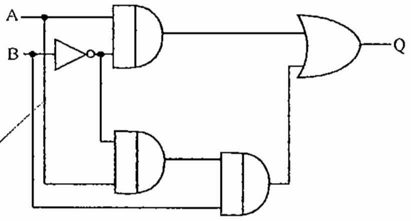
53. The voltage resolution of a 12-bit digital to analog converter (DAC), whose output varies from –10 V to + 10V is, approximately

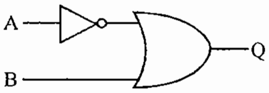
(a) 1 mV (b) 5 mV (c) 20 mV (d) 100 mV

54. For any set of inputs, A and B, the following circuits give the same output Q, except one. Which one is it?

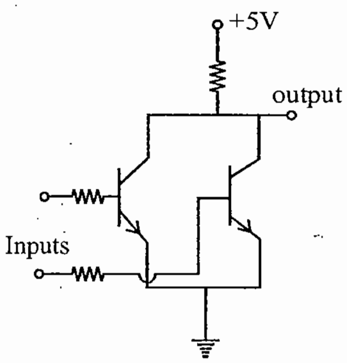
(a)


(b)


(c)


(d)


55. The circuit shown in figure below acts as a



(a) NOR gate (b) NAND gate

(c) AND gate (d) XOR gate

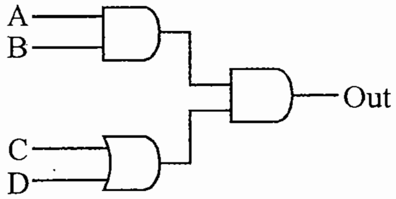
56. In a logic circuit the output Y in terms of inputs A, B, C is $Y = ABC + A\bar{B}C + AB\bar{C}$. This circuit can be replaced by another logic circuit having.

(a) $Y = ABC$ (b) $Y = A(B+C)$ (c) $Y = B(A+C)$ (d) $Y = C(A+B)$

57. 10101 binary number corresponds to decimal number
(a) 31 (b) 21 (c) 11 (d) 3

58. The basic memory element in a digital circuit
(a) Consists of a NAND gate (b) Consists of a NOR gate
(c) Is a FLIP-FLOP (d) Is a shift register.

59. If the output of the logic circuit shown in the figure is 1, the input could be
(a) A = 1, B = 1, C = 1, D = 0
(b) A = 1, B = 1, C = 0, D = 0
(c) A = 1, B = 0, C = 1, D = 1
(d) A = 0, B = 1, C = 1, D = 1.



60. Which of the following options is true for a two input XOR gate?

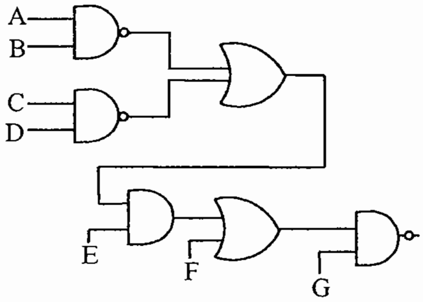
| | Input | | Output |
|---|---|---|---|
| | A | B | |
| (a) | 0 | 1 | 1 |
| (b) | 1 | 0 | 0 |
| (c) | 0 | 0 | 1 |
| (d) | 1 | 1 | 1 |

61. Which of the given relations between the Boolean variables P and Q is NOT correct? (In the notation used here, P′ denotes NOT P and Q′ denotes NOT Q)

(a) $PQ' + PQ = P$ (b) $(PQ)' = P' + Q'$ (c) $PQ' = (P' + Q)'$ (d) $PQ' + Q = P$

62. A half-adder is a digital circuit with
(a) Three inputs and one output (b) Three inputs and two outputs
(c) Two inputs and one output (d) Two input and two outputs.

63. The output 0, of the given circuit in cases I and II, where
**Case-I:** A, B = 1; C, D = 0; E, F = 1 and G = 0
**Case-II:** A,B = 0; C, D = 0; E, F = 0 and G = 1 are respectively



(a) 1, 0 (b) 0, 1 (c) 0, 0 (d) 1, 1

64. Which of the following logic families has the highest speed.
(a) DTL (b) TTL (c) CMOS (d) ECL

65. Five panelists are required to elect a sixth member to the panel. If any of the panelists votes againt a member, the member is disqualified. What would be the appropriate electronic circuit to be used in the electronic voting machine to implement the above rule?

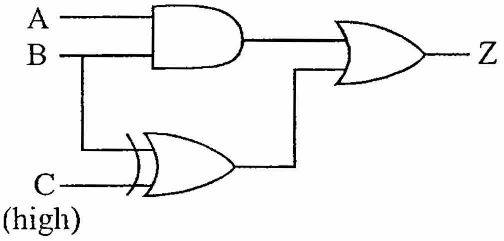
(a) XOR (b) XNOR (c) OR (d) AND

66. The following Boolean expression

$Y = A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{C} \cdot D + \bar{A} \cdot \bar{B} \cdot C \cdot D + \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D + \bar{A} \cdot B \cdot C \cdot D + A \cdot \bar{B} \cdot \bar{C} \cdot D$ can be simplified to

(a) $\bar{A} \cdot \bar{B} \cdot C + A \cdot \bar{D}$ (b) $\bar{A} \cdot B \cdot \bar{C} + A \cdot \bar{D}$ (c) $A \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot D$ (d) $A \cdot \bar{B} \cdot C + \bar{A} \cdot D$

67. Consider the digital circuit shown below in which the input C is always high (I).



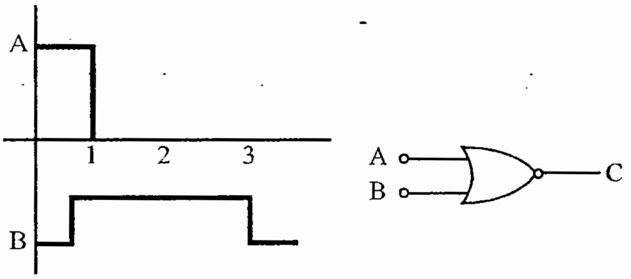
The truth table for the circuit can be written as

| A | B | Z |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | |
| 1 | 0 | |
| 1 | 1 | |

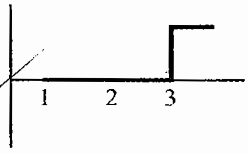
The entries in the Z column (vertically) are

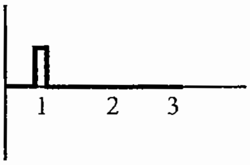
(a) 1011 (b) 0100 (c) 1111 (d) 1010

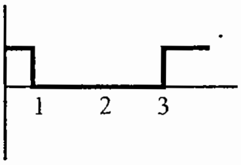
68. The pulses shown as A and B are fed into the input of the gate shown below.

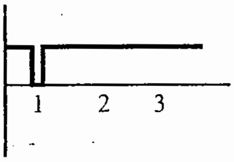


The output at C has the waveform

(a) 

(b) 

(c) 

(d) 

69. A J-K flip flop with J =1 and K=1 has a 10 kHz clock input. The Q output is

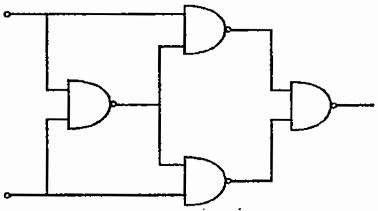
(a) a 10 kHz square wave (b) a 5 kHz square wave

(c) always LOW (d) always HIGH

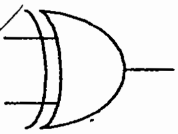
70. The binary number 1101 0100 0110 1111 can be written in hexadecimal as

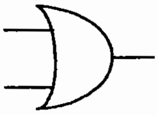
(a) D467 (b) D46F (c) E46F (d) C46F

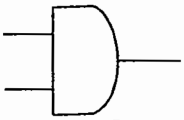
71. Consider the logic gate arrangement shown in the figure below

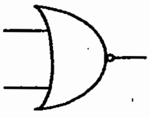


This arrangements is equivalent to the gate

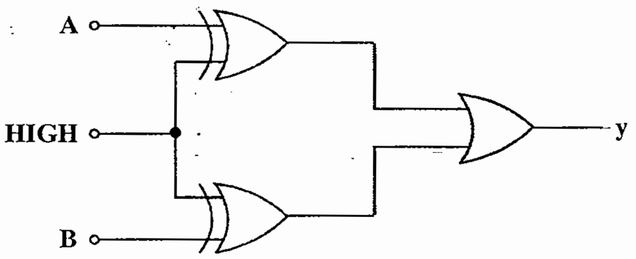
(a) 

(b) 

(c) 

(d) 

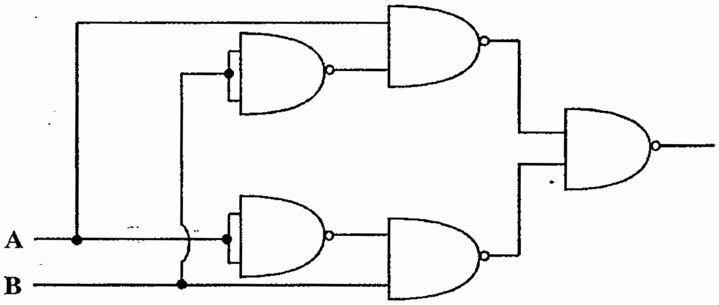
72. The logic circuit shown in the figure below.



implements the Boolean expression

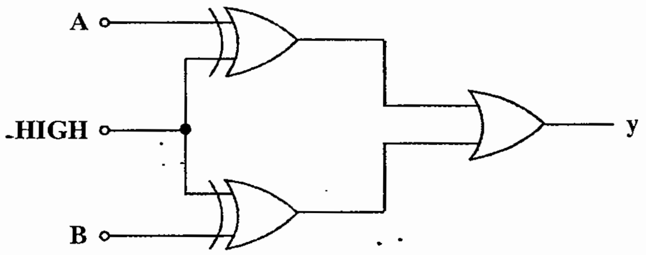
(a) $y=\overline{A.B}$ (b) $y=\bar{A}.\bar{B}$ (c) $y=A.B$ (d) $y=A+B$

73. The circuit shown below uses only NAND gates. Find the final output



(a) A XOR B (b) A AND B (c) A OR B (d) A NOR B.

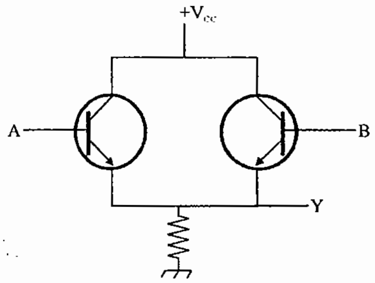
74. The logic circuit shown in the figure below.



implements the Boolean expression

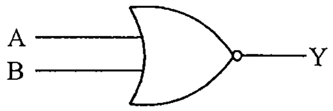
(a) $y=\overline{A.B}$ (b) $y=\bar{A}.\bar{B}$ (c) $y=A.B$ (d) $y=A+B$

75. The circuit shown in the figure function as



(a) an OR gate (b) an AND gate (c) a NOR gate (d) a NAND gate

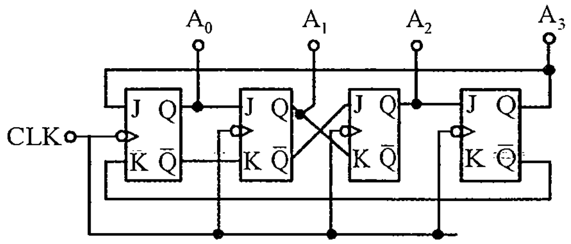
**76.** Consider the circuit shown below.



The minimum number of NAND gates required to design this circuit is

(a) 6 (b) 5 [✓ (c) 4] (d) 3

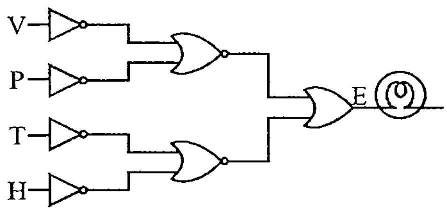
**77.** A counter consists of four fip-flops connected as shown in the figure.



If the counter is initialized as $A_0\ A_1\ A_2\ A_3 = 0110$, the state after the next clock pulse is

(a) 1000 (b) 0001 (c) 0011 (d) 1100

**78.** Four digital outputs V, P, T and H monitor the speed v, tyre pressure p, temperature t and relative humidity h of a car. These outputs switch from 0 to 1 when the values of the parameters exceed 85 km/hr, 2 bar, 40°C and 50%, respectively. A logic circuit that is used to switch ON a lamp at the output E is shown below.



Which of the following conditions will switch the lamp ON?

[✓ (a) $v < 85\text{km/hr}, p < 2\text{bar}, t > 40^\circ\text{C}, h > 50\%$]

(b) $v < 85\text{km/hr}, p < 2\text{bar}, t > 40^\circ\text{C}, h < 50\%$

(c) $v > 85\text{km/hr}, p < 2\text{bar}, t > 40^\circ\text{C}, h < 50\%$

(d) $v > 85\text{km/hr}, p < 2\text{bar}, t < 40^\circ\text{C}, h > 50\%$

**79.** If one of the inputs of a J-K flip flop is high and the other is low, then the outputs $Q$ and $\bar{Q}$

[✓ (a) oscillate between low and high in race-around condition]

(b) toggle and the circuit acts like a T flip flop

(c) are opposite to the inputs

(d) follow the inputs and the circuit acts like an R-S flip flop

**80.** A 4-variable switching function is given by $f = \Sigma(5,7,8,10,13,15) + d(0,1,2)$, where $d$ is the do-not-care-condition. The minimized form of $f$ in sum of products (SOP) form is

(a) $\bar{A}\bar{C}+\bar{B}\bar{D}$ (b) $A\bar{B}+C\bar{D}$ (c) $AD+BC$ (d) $\bar{B}\bar{D}+BD$

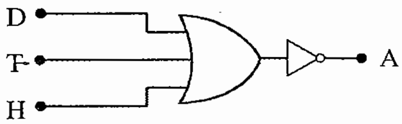
81: A control circuit needs to be designed to save on power consumption by an air-conditioning unit A in a windowless room with a single door. The room is fitted with the following devices:

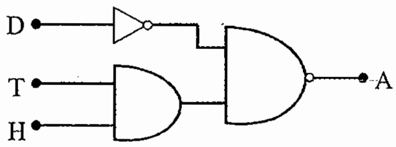
(1) A temperature sensor T, which is enabled (T = 1) whenever the temperature falls below a pre-set value;

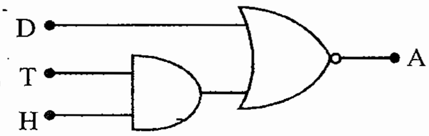
(2) A humidity sensor H, which is enabled (H = 1) whenever the humidity falls below a certain pre-set value;

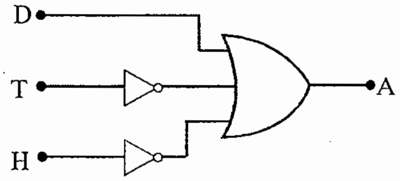
(3) A sensor D on the door, which is triggered (D = 1) whenever the door opens

Which of the following logical circuits will turn the air-conditioning unit off (A = 0) whenever the door is opened or when the both temperature and humidity are below their pre-set values?

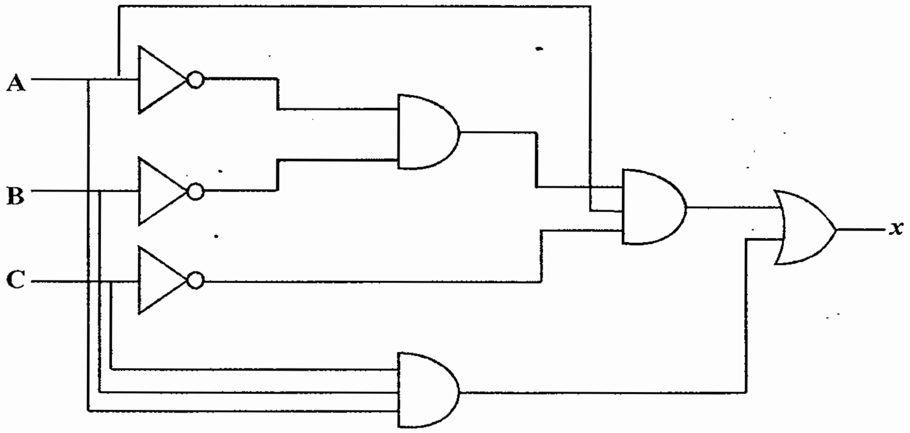
(a) 

(b) 

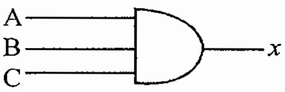
(c) 

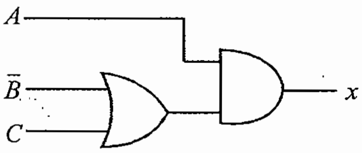
(d) 

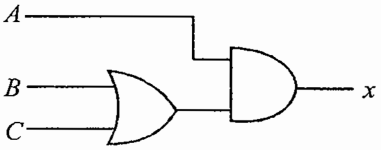
82. For the logic circuit shown in the figure below

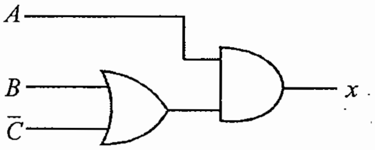


a simplified equivalent circuit is

(a) 

(b) 

(c) 

(d) 

37-49

## Answer Kye: NETWORKS

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (a) | 2. (c) | 3. (d) | 4. (c) | 5. (a) | 6. (a) | 7. (d) |
| 8. (a) | 9. (b) | 10. (d) | 11. (b) | 12. (d) | 13. (b) | 14. (d) |
| 15. (b) | 16. (d) | 17. (c) | 18. (a) | 19. ( ) | 20. (a) | 21. (c) |
| 22. (b) | 23. (b) | 24. (a) | 25. (b) | 26. (a) | 27. (a) | 28. (b) |
| 29. (b) | 30. (b) | 31. (a) | 32. (a) | 33. (b) | 34. (a) | 35. (b) |
| 36. (a) | 37. (d) | 38. (a) | 39. (c) | 40. (b) | 41. (b) | 42. (d) |
| 43. (c) | 44. (a) | 45. (a) | 46. (d) | 47. (b) | 48. (b) | 49. (a) |
| 50. (b) | 51. (c) | 52. (b) | 53. (a) | 54. (a) | 55. (b) | 56. (c) |
| 57. (a) | 58. (a) | 59. (d) | 60. (c) | 61. (b) | 62. (d) | 63. (a) |
| 64. (b) | 65. (d) | 66. (b) | 67. (c) | 68. (b) | 69. (a) | 70. (a) |
| 71. (b) | 72. (d) | 73. (a) | 74. (b) | 75. (b) | 76. (a) | 77. (c) |
| 78. (b) | 79. (c) | 80. (c) | 81. (d) | 82. (b) | 83. (c) | 84. (b) |
| 85. (b) | 86. (a) | 87. (b) | 88. (1.25) | 89. (b) | 90. (d) | 91. (c) |
| 92. (a) | 93. (c) | 94. (a) | 95. (3.15) | 96. (c) | 97. (b) | 98. (a) |
| 99. (c) | 100. (1.4 to 1.7) | | 101. (9.99 to 10.01) | | 102. (b) | |

## Answer Kye: OP-AMP

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (a) | 4. (c) | 5. (a) | 6. (b) | 7. (c) |
| 8. (a) | 9. (b) | 10. (d) | 11. (c) | 12. (d) | 13. (d) | 14. (b) |
| 15. (c) | 16. (c) | 17. (c) | 18. (a) | 19. (a) | 20. (d) | 21. (a) |
| 22. (d) | 23. (b) | 24. (c) | 25. (b) | 26. (c) | 27. (c) | 28. (a) |
| 29. (a) | 30. (d) | 31. (a) | 32. (b) | 33. (a) | 34. (a) | 35. (b) |
| 36. (d) | 37. (a) | 38. (a) | 39. (d) | 40. (c) | 41. (c) | 42. (b) |
| 43. (c) | 44. (c) | 45. (c) | 46. (b) | 47. (c) | 48. (b) | 49. (a) |
| 50. (a) | 51. (c) | 52. (d) | 53. (b) | 54. (d) | 55. (b) | 56. (c) |
| 57. (a) | 58. (b) | 59. (b) | 60. (b) | 61. (d) | 62. (d) | 63. (d) |
| 64. (b) | 65. (c) | 66. (b) | 67. (a) | 68. (c) | 69. (b) | 70. (c) |
| 71. (d) | 72. (b) | 73. (c) | 74. (a) | 75. (a) | 76. (b) | 77. (c) |
| 78. (a) | 79. (b) | 80. (d) | 81. (b) | | | |

## Anwer Key: DIGITAL ELECTRONICS

1. (c)
2. (b)
3. (d)
4. (b)
5. (d)
6. (d)
7. (c)
8. (c)
9. (a)
10. (a)
11. (b)
12. (d)
13. (b)
14. (c)
15. (c)
16. (b)
17. (a)
18. (d)
19. (b)
20. (a)
21. (c)
22. (c)
23. (a)
24. (c)
25. (d)
26. (a)
27. (a)
28. (c)
29. (b)
30. (a)
31. (a)
32. (b)
33. (d)
34. (a)
35. (d)
36. (c)
37. (c)
38. (d)
39. (c)
40. (c)
41. (d)
42. (c)
43. (d)
44. (d)
45. (d)
46. (d)
47. (a)
48. (d)
49. (a)
50. (a)
51. (a)
52. (d)
53. (b)
54. (d)
55. (a)
56. (b)
57. (b)
58. (c)
59. (a)
60. (a)
61. (d)
62. (d)
63. (d)
64. (d)
65. (d)
66. (c)
67. (a)
68. (a)
69. (b)
70. (b)
71. (a)
72. (a)
73. (a)
74. (d)
75. (a)
76. (c)
77. (c)
78. (a)
79. (d)
80. (d)
81. (c)
82. (d)
83. (6.99 to 7.01)

## CSIR-UGC-NET/JRF- PHYSICAL SCIENCES

### Unit Test: Electronics ( Digital)

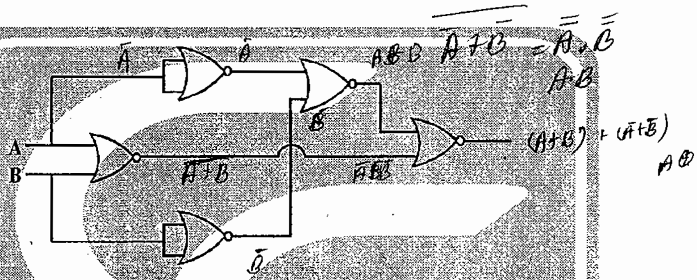
**Time : 00: 40 Hour** **Date : 13-08-2014**
**M.M. : 40**

**Instructions:**

1. **Question Paper contains 20 objective type questions, each question carry 2 marks.**
2. **There is negative marking, 0.5 mark will be deducted for each wrong answer.**
3. **Attempt all the questions, use of calculator is not allowed.**

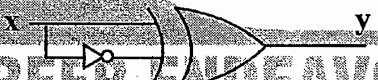
---

1. The given circuit will act as


A
B
$\bar{A}$
$\bar{A}$
$\overline{A+B}$
$\overline{AB}$
$\bar{B}$
$\bar{B}$
$\overline{\bar{A}+\bar{B}} = \bar{\bar{A}}.\bar{\bar{B}}$
$AB$
$(A+B)' + (\bar{A}+\bar{B})$
$A\oplus$

[✓ (a) Half adder] (b) Full adder
(c) Half substractor [✓ (d) Full substractor.]

2. The output y of ligic circuit given below is:



(a) 1 (b) 0 [✓ (c) x] (d) $\bar{x}$

$2^2$
2×2×2×2×2
5 5×5×2×2
10 1250
125×5
625 2

3. For the 5-variable total numbers of self dual expression possible is:
(a) 216 [✓ (b) 64] [✓ (c) 65536] (d) 680948

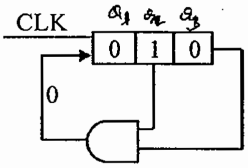
4. The present state output $Q_n$ of an edge triggered JK flip flop is logic 'O'. If J = 1, k = 0, then $Q_{n+1}$ is: 2×2×2×2×2
(a) Can't be determined [✓ (b) Logic 0]
[✓ (c) Logic 1] (d) Race around.

5. The given boolean represents f(A, B, C) = $\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$ is equivalent to A
001 010 100 111
(a) Sum of half adder (b) Binary to gray code converter
[✓ (c) Sum of full adder] (d) Gray to binary converter.

6. Numbers Demux required for the implementation of 1×16 from 1×4 demux. $2^4$
[✓ (a) 4] (b) 3 [✓ (c) 5] (d) 2

7. The function of EXOR can be utilised as inverter if
(a) By selecting any value at input
(b) Selecting one input pin = 0 and other pin having any input.
[✓ (c) Selecting one pin having input = 1 and other pin having any input.]
[✓ (d) All of the above.]

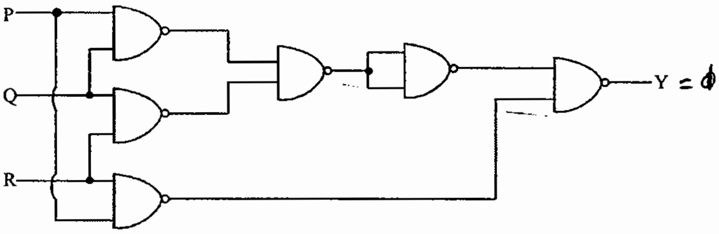
8. The content of shift register after 3rd CLK pulse is



(a) 011 (b) 001 (c) 011 (d) 000

$Q_0$ $Q_2$ $Q_1$
0 0 0
0 0 1
0 1 0
0 1 1
0 0 0
1 0 1
1 1 0
1 1 1

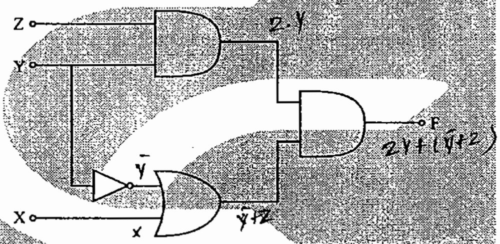
9. The output Y in the circuit below is always '1' when



x 0 1 0
0 0 1 0
1 1 0
0 1 1
d.0

(a) Two or more of the inputs, P, Q, R are '0'. (b) Two or more of the inputs P, Q, R are '1'
(c) Any odd number of the inputs P, Q, R is '0' (d) Any odd number of the inputs P, Q, R is '1'

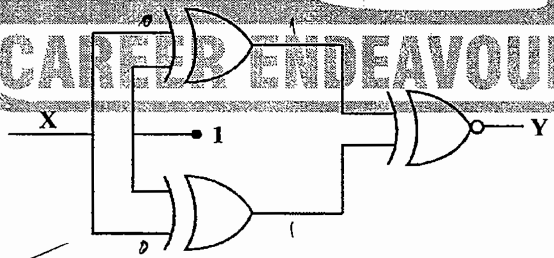
10 Identify the function F generated by the logic network shown



$z+y+(\bar{y}z)$

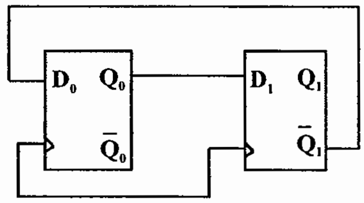
(a) $F = (X+Y)Z$ (b) $F = Z + Y + \bar{Y}X$ (c) $F = ZY(Y+X)$ (d) $F = XYZ$

11. For the given circuit calculate Y.

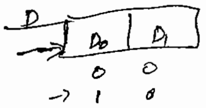


(a) 0 (b) 1 (c) $x$ (d) $\bar{x}$

12. For the given circuit identify contents after 373 clock pulse. If initially under reset condition



$D_0 = \bar{Q}_1$
$D_1 = Q_0$



(a) 00 (b) 01 (c) 10 (d) 11

13. For a race around condition to occur the necessary condition is
(a) $J = 0, K = 0$ (b) $J = 1, K = 1$ (c) $S = 1, R = 1$ (d) $D = 1$

14. For the given k-map the output minimized expression is

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | 1 | |
| 1 | | 1 | 1 | 1 |

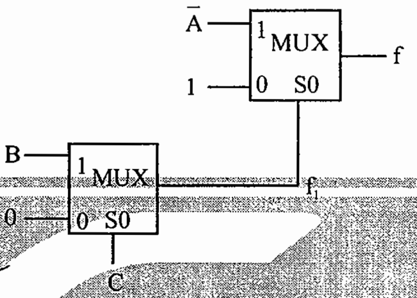
Ac + AB + BC

(a) $\bar{A}B+\bar{A}C+BC$ (b) $AB+BC+AC$ (c) $ABC+\bar{A}\bar{B}\bar{C}$ (d) $A\bar{B}\bar{C}+\bar{A}B\bar{C}$

15. A mod-39 counter is implemented by using J-K flip flop the number of flip-flops required.
(a) 6 (b) 5 (c) 4 (d) 7

xyz
mod 39
$2^5$
2×2×2×2×2 ×2

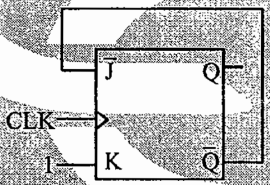
16. The network shown below implements



B

(a) NOR Gate (b) NAND Gate (c) XOR Gate (d) XNOR Gate

17. The J-K FF shown below is initially cleared and then clocked for 5 pulses, the sequence at the Q output will be



J K $Q_n$ $Q_{n+1}$

$J = \bar{Q}$

(a) 010000 (b) 011001 (c) 010010 (d) 010101

18. A combinational circuit has input A, B and C and its k-map shown below. The output of the circuit is given by

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | 1 |
| 1 | 1 | | 1 | |

$A\bar{B}\bar{C}+\bar{A}\bar{B}C+ABC+\bar{A}B\bar{C}$

(a) $(\bar{A}B+A\bar{B})\bar{C}$ (b) $(AB+\bar{A}\bar{B})\bar{C}$ (c) $\bar{A}\,\bar{B}\,\bar{C}$ (d) $A\oplus B\oplus C$

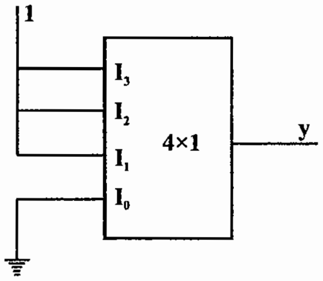
19. To count from 0 to 1024 the number of required flip-flop is
(a) 10 (b) 11 (c) 12 (d) 13

1024

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100

2 4 8 16 32 64 128 1024

20. The output 'f' of 4×1 multiplexer shown in figure is:



$I_3 = 1 \cdot 0$
$I_2 = 1 \cdot 0$
$I_0 = 0 \cdot 0$

(A+B)
$A+\bar{A}B$

(a) $\overline{xy}+x$ (b) $x+y$ (c) $\bar{x}+\bar{y}$ (d) $xy+\bar{x}$

# CSIR-UGC-NET/JRF | GATE

TIFR|BARC|JEST|IISc|DRDO|ISRO|SLET|ONGC-GT & Ph.D. Entrance

# Assignment

# Mathematical Physics

## PHYSICAL SCIENCES

**CAREER ENDEAVOUR**
ACADEMY PRIVATE LIMITED

**South Delhi Centre:** 28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:** 48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS

### Assignment I: Vector Alzebra & Vector Analysis

1. $\nabla^2\left(\frac{1}{r}\right)$ is:

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$

2. $\iiint \nabla^2\left(\frac{1}{r}\right)dV \quad r\neq 0$ is:

(a) 0 (b) $-4\pi$ (c) $4\pi$ (d) 1

3. $\operatorname{div}\left(\hat{i}+\hat{j}+\hat{k}\right)$ is

(a) 3 (b) 2 (c) 1 (d) 0

4. The value of t for which three vevtors $\left[(1-t),0,0\right],\left[1,(1-t),0\right]$ & $\left[1,1,(1-t)\right]$ are linearly dependent is

(a) 1 (b) 0 (c) 2 (d) $-1$

5. For what value of n vector $r^n\vec{r}$ is solenoidal

(a) 0 (b) 1 (c) $-1$ (d) $-3$

6. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by

(a) $\frac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{a^2}$ (b) $\frac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{b^2}$ (c) $\frac{\left(\vec{a}.\vec{b}\right)\vec{b}}{b^2}$ (d) $\frac{\left(\vec{b}.\vec{a}\right)\vec{a}}{a^2}$

7. A vector field is defined everywhere as $\vec{F}=\frac{y^2}{L}\hat{i}+z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z axes, is:

(a) $L^3$ (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

8. If $\vec{r}=x\hat{i}+y\hat{j}$, then:

(a) $\vec{\nabla}\bullet\vec{r}=0$ and $\vec{\nabla}|\vec{r}|=\vec{r}$ (b) $\vec{\nabla}\bullet\vec{r}=2$ *and* $\vec{\nabla}|\vec{r}|=\vec{r}$

(c) $\vec{\nabla}\bullet\vec{r}=2$ *and* $\vec{\nabla}|\vec{r}|=\frac{\vec{r}}{r}$ (d) $\vec{\nabla}\bullet\vec{r}=3$ *and* $\vec{\nabla}|\vec{r}|=\frac{\hat{r}}{r}$

9. Cosnider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is:

(a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$ (b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$ (d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

10. The value of $\oint \vec{A} \cdot \vec{d\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is:

(a) 0 (b) 2LA (c) 4LA (d) $L^2A$

11. The value of $\oint_S \frac{\vec{r} \cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is:

(a) 0 (b) $\pi$ (c) $4\pi$ (d) $8\pi$

12. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is

(a) 0 (b) $\hat{i}$ (b) $2\hat{j}$ (d) $3\hat{k}$

13. Given the four vectors, $u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \end{pmatrix}, u_4 = \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix}$, the linearly dependent pair is:

(a) $u_1, u_2$ (b) $u_1, u_3$ (c) $u_1, u_4$ (d) $u_3, u_4$

14. The unit normal to the curve $x^3 y^2 + xy = 17$ at the point (2, 0) is:

(a) $\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

15. For the function $\phi = x^2y + xy$ the value of $|\vec{\nabla}\phi|$ at $x = y = 1$ is

(a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

**Common data for Q.16 and Q.17.**

Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$

16. The surface integral of this vector over the surface of a cube of size and centered at the origin.
(a) 0 (b) $2\pi$ (c) $2\pi a^3$ (d) $4\pi$

17. Which one of the following is NOT correct?
(a) Value of the line integral of this vector around any closed curve is zero
(b) This vector can be written as the gradient of some scalar function
(c) The line integral of this vector from point P to point Q is independent of the path taken.
(d) This vector can represent the magnetic field of some current distribution

18. The curl of the vector $\vec{A} = z\,\hat{i} + x\,\hat{j} + y\,\hat{k}$ is given by:

(a) $\hat{i} + \hat{j} + \hat{k}$ (b) $\hat{i} - \hat{j} + \hat{k}$ (c) $\hat{i} + \hat{j} - \hat{k}$ (d) $-\hat{i} - \hat{j} - \hat{k}$

19. The two vectors $\vec{P} = \hat{i}, \vec{q} = (\hat{i} + \hat{j})\ /\ \sqrt{2}$ are:

(a) related by a rotation  (b) related by a reflection through the xy-plane

(c) related by an inversion  (d) not linearly independent

20. A vector $\vec{A} = (5x + 2y)\hat{i} + (3y - z)\hat{j} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value:

(a) 4  (b) –4  (c) 8  (d) –8

21. Which of the following vectors is orthogonal to the vector $\left(a\,\hat{i} + b\hat{j}\right)$, where a and b (a≠b) are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors?

(a) $-b\hat{i} + a\hat{j}$  (b) $-a\hat{i} + b\hat{j}$  (c) $-a\hat{i} - b\hat{j}$  (d) $-b\hat{i} - a\hat{j}$

22. The unit vector normal to the surface $3x^2 + 4y = z$ at the ponit (1, 1, 7) is:

(a) $\left(-6\hat{i} + 4\hat{j} + \hat{k}\right)/\sqrt{53}$  (b) $\left(4\hat{i} + 6\hat{j} - \hat{k}\right)/\sqrt{53}$

(c) $\left(6\hat{i} + 4\hat{j} - \hat{k}\right)/\sqrt{53}$  (d) $\left(4\hat{i} + 6\hat{j} + \hat{k}\right)/\sqrt{53}$

23. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\bar{r}$ is the positive vector, then the value of the following integral $\iint_S \vec{r}.\hat{n}dS$ is:

(a) V  (b) 2V  (c) 0  (d) 3V

24. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

(a) The three vectors are orthonormal  (b) The three vectors are linearly independent

(c) The three vectors cannot form a basis in a three-dimensional real vector space.

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$.

25. If $\bar{A}(t)$ is a vector of constant magnitude, which of the following is true?

(a) $\frac{d\vec{A}}{dt} = 0$  (b) $\frac{d^2\vec{A}}{dt^2} = 0$  (c) $\frac{d\vec{A}}{dt} \bullet \vec{A} = 0$  (d) $\frac{d\vec{A}}{dt} \times \vec{A} = 0$

26. The function $r^n\ \hat{r}$ $(r > 0)$, where r refers to spherical coordinate system, is:

(a) an irrotational vector for $n = -2$  (b) a solenoidal vector for $n = -2$

(c) an irrotational vector for $n = -3$  (d) a solenoidal vector for $n = -3$

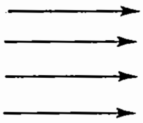
27. Two sets of vectors, one with m elements and the other with n elements $(m < n)$ span the same linear vector space. If k is the dimension of the vector space, then:

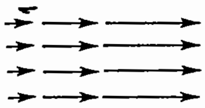
(a) $k \leq m$  (b) $k \geq m$  (c) $k = m + n$  (d) $k = n$

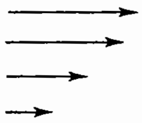
28. Given $\phi = x^2 - yz$, $\nabla\phi$ at (3, 4, 1) will be:

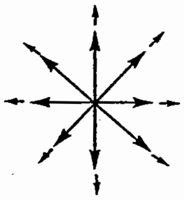
(a) $6\hat{i} - 4\hat{j} + \hat{k}$  (b) $6\hat{i} - \hat{j} + 4\hat{k}$  (c) $4\hat{i} - 6\hat{j} - \hat{k}$  (d) $6\hat{i} - 3\hat{j} - 2\hat{k}$

29. Identify the vector field given below which has a finite curl

(a)


(b)


(c)


(d)


30. If $\vec{A}$ and $\vec{B}$ are two unit vectors and $\theta \neq 0$ is the angle between them, then

(a) $\sin\theta = \frac{1}{2}\left|\vec{A}+\vec{B}\right|$ (b) $\sin\theta = \frac{1}{2}\left|\vec{A}-\vec{B}\right|$

(c) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{A}-\vec{B}\right|$ (d) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{A}+\vec{B}\right|$

31. Given any three non-zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$, their triple product $\vec{A}.\left(\vec{B}\times\vec{C}\right)$ vanishes if

(a) They are perpendicular to each other. (b) Any two of them are perpendicular

(c) Any two of them are parallel. (d) They are non-coplanar.

32. The unit vector normal to the surface $x^2+4y=z$ at the point (1, 1, 0) is:

(a) $\frac{1}{\sqrt{21}}\left(2\hat{i}+4\hat{j}-\hat{k}\right)$ (b) $\frac{1}{\sqrt{21}}\left(-2\hat{i}+4\hat{j}-\hat{k}\right)$ (c) $\frac{1}{\sqrt{21}}\left(2\hat{i}-4\hat{j}-\hat{k}\right)$ (d) $\frac{1}{\sqrt{21}}\left(2\hat{i}+4\hat{j}+\hat{k}\right)$

33. The necessary and sufficient condition that $\oint_c \vec{A}.d\vec{r} = 0$, for any closed curve C is

(a) $\vec{\nabla}.\vec{A}=0$ (b) $\vec{\nabla}\times\vec{A}=0$ (c) $\vec{\nabla}.\vec{\nabla}\times\vec{A}=0$ (d) $\vec{\nabla}\times\vec{\nabla}\times\vec{A}=0$

34. Any arbitrary vector in a three dimensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors:

(a) Arbitrary number (b) 1 (c) 2 (d) 3

35. Which one of the following vectors is normal to the surface $x^2y+z^3+xz^2=10$?

(a) $x^2y\hat{i}+z^3\hat{j}+xz^2\hat{k}$ (b) $2xy\hat{i}+2xz\hat{k}$

(c) $2y\hat{i}+\left(6z+2x\right)\hat{k}$ (d) $\left(2xy+z^2\right)\hat{i}+x^2\hat{j}+\left(3z^2+2xz\right)\hat{k}$

36. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to

(a) Curl of a vector function (b) Gradient of a scalar function

(c) Gradient of a vector function (d) Zero

37. When the fluid is incompressible, the equation of continuity can be reduced to $\frac{\partial \rho}{\partial t} = 0 = \rho\left(\vec{\nabla}.\vec{v}\right)$. Since the density $\rho$ is constant in this case. Here $\vec{v}$ is the velocity of a typical particle of the fluid. Further, if the flow is irrotational, then the equation can be rewritten as

(a) $\vec{\nabla}.\vec{\nabla}\phi = 0$ (b) $\vec{\nabla}\times\vec{\nabla}\phi = 0$ (c) $\vec{\nabla}\phi = 0$ (d) $\vec{v} = \text{constant}$

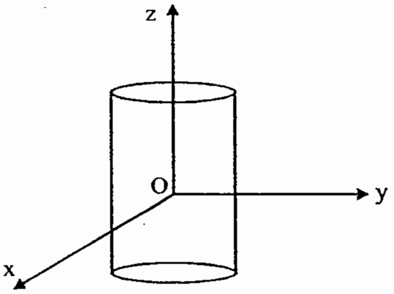
38. The vector perpendicular to $3\hat{i}+4\hat{j}-5\hat{k}$ is:

(a) $-2\hat{i}+4\hat{j}-2\hat{k}$ (b) $2\hat{i}+4\hat{j}-2\hat{k}$ (c) $2\hat{i}-4\hat{j}+2\hat{k}$ (d) $2\hat{i}-4\hat{j}-2\hat{k}$

39. The unit vector normal to the surface $x^2+y^2-z=1$ at the point P(1, 1, 1) is:

(a) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

40. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x+\hat{j}y+\hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is:



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) Zero.

41. Given the vector $\vec{A}(y,-x,0)$, the line integral $\oint_c \vec{A}.\vec{dl}$, where C is a circle of radius 5 units with its centre at the origin, (correct to the first decimal place) is

(a) 172.8 (b) 157.1 (c) −146.3 (d) 62.8

42. The value of the integral $I = \int_s \vec{r}.d\vec{s}$ where S is the surface enclosing the volume V is

(a) 3 (b) V (c) 3V (d) 0

43. The cross product of $\left(\hat{i}+\hat{j}\right)\times\left(\hat{j}+\hat{k}\right)$ where $\hat{i}, \hat{j}, \hat{k}$ are orthonormal unit vectors in 3-dimensions, is

(a) $\hat{i}-\hat{k}$ (b) $\hat{i}+\hat{k}$ (c) $\hat{i}+\hat{j}+\hat{k}$ (d) $\hat{k}-\hat{j}+\hat{i}$

44. Identify the CORRECT statement for the following vectors $\vec{a}=3\hat{i}+2\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}$

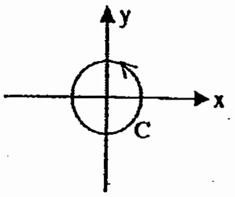
(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

45. Given $\vec{F}=\vec{r}\times\vec{B}$, where $\vec{B}=B_0\left(\hat{i}+\hat{j}+k\right)$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is,



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

46. A vector perpendicular to any vector that lies on the plane defined by $x+y+z=5$, is

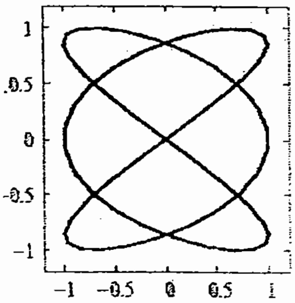
(a) $\vec{i}+\vec{j}$ (b) $\vec{j}+\vec{k}$ (c) $\vec{i}+\vec{j}+\vec{k}$ (d) $2\vec{i}+3\vec{j}+5\vec{k}$

47. A two-dimensional vector

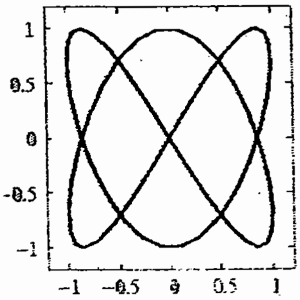
$$\vec{A}(r)=\hat{i}\sin 2t+\hat{j}\cos 3t$$

Which of the following graphs best describes the locus of the tip of the vector, as 't' is varied from 0 to $2\pi$?

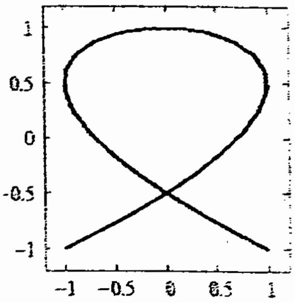
(a)

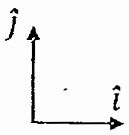


(b)

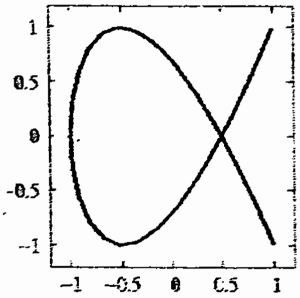


(c)





(d)



48. Consider the surface corresponding to the equation

$$4x^2+y^2+z=0$$

A possible unit tangent to this surface at the point $(1,2,-8)$ is

(a) $\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{j}$ (b) $\frac{1}{5}\hat{j}-\frac{4}{5}\hat{k}$ (c) $\frac{4}{9}\hat{i}-\frac{8}{9}\hat{j}+\frac{1}{9}\hat{k}$

49. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}},\frac{b}{\sqrt{3}},\frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is

(a) $\frac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$ (b) $\frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (c) $\frac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

50. If $A=\hat{i}yz+\hat{j}xz+\hat{k}xy$, then the integral $\oint_C A.dl$ (where $C$ is along the perimeter of a rectangular area bounded by $x=0, x=a$ and $y=0, y=b$) is

(a) $\frac{1}{2}(a^3+b^3)$ (b) $\pi(ab^2+a^2b)$ (c) $\pi(a^3+b^3)$ (d) 0

51. If $\vec{A}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.\vec{d\ell}$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{\pi}{4}$ (d) 0

## Vector Alzebra and Vector Analysis_Answer Key

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (c) | 2. (a) | 3. (d) | 4. (a) | 5. (d) | 6. (a) | 7. (a) |
| 8. (c) | 9. (a) | 10. (a) | 11. (c) | 12. (a) | 13. (d) | 14. (d) |
| 15. (d) | 16. (d) | 17. (d) | 18. (a) | 19. (a) | 20. (c) | 21. (a) |
| 22. (c) | 23. (d) | 24. (b) | 25. (c) | 26. (b) | 27. (b) | 28. (b) |
| 29. (c) | 30. (c) | 31. (c) | 32. (a) | 33. (b) | 34. (d) | 35. (d) |
| 36. (b) | 37. (b) | 38. (d) | 39. (d) | 40. (b) | 41. (b) | 42. (c) |
| 43. (d) | 44. (a) | 45. (c) | 46. (c) | 47. (b) | 48. (a) | 49. (a) |
| 50. (d) | 51. (d) | | | | | |

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### CSIR-UGC-NET/JRF | GATE Physics
### MATHEMATICAL PHYSICS

### Assignment II: Matrix Alzebra

1. The inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ is:

(a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) does not exist.

2. Which one of the following is not hermitian?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$

3. Eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(a) 2, 1, 0 (b) 1, 1, 1 (c) 3, 0, 0 (d) 1, 2, 1

4. The eigenvalues of a matrix are i, -2i and 3i. The matrix is:
(a) unitary (b) anti-unitary (c) hermitian (d) anti-hermitian

5. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are:

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

6. The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigenvalue is:
(a) −1 (b) 0 (c) 1 (d) 2

7. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\bar{x}$ from a three-dimensional real space to a two-dimensional real space. The transformation matrix T is:

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

**Common data for Q. 8 and Q. 9:**

One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5.

8. The other two eigen values are:
(a) 0 and 0 (b) 1 and 1 (c) 1 and $-1$ (d) $-1$ and $-1$

9. The normalized eigen vector corresponding to the eigen value 5 is:
(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

10. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are:
(a) $-2, -1, 1, 2$ (b) $-1, 1, 0, 2$ (c) $1, 0, 2, 3$ (d) $-1, 1, 0, 3$

11. The determinant of a $3 \times 3$ real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvlaue is:
(a) 4 (b) 6 (c) 8 (d) 9

12. A real traceless $4 \times 4$ unitary matrix has two eigen values $-1$ and 1. The other eigenvalues are:
(a) zero and +2 (b) $-1$ and +1 (c) zero and +1 (d) +1 and +1

13. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are:
(a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) $-1$ and +1

14. A $3 \times 3$ matrix has eigenvalues $0, 2+i$ and $2-i$. Which one of the following statements is correct?
(a) The matrix is Hermitian (b) The matrix is unitary
(c) The inverse of the matrix exists (d) The determinant of the matrix is zero

15. If two matrices A and B can be diagonalized simultaneously, which of the following is true?
(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$

16. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?
(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

17. For any operator A, $i(A^+ - A)$ is:
(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

18. A square matrix A is unitary if:
(a) $A^\dagger = A$ (b) $A^\dagger = A^{-1}$ (c) $Tr(A) = 1$ (d) $\det(A) = 1$

19. If A is an antisymmetric matrix and $A^T$ be the transpose of this matrix, then which one of the following relations does not hold good?

(a) $A^T = -A$  (b) $AA^T = A^TA$

(c) $A^2$ is an antisymmetric matrix  (d) $A^2$ is a symmetirc matrix

20. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are given by $\lambda_1$, $\lambda_2$ and $\lambda_3$. Which one of th following statement is NOT TRUE?

(a) $\lambda_1 + \lambda_2 + \lambda_3 = 0$  (b) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$

(c) All eigenvalues are real  (d) Product of the eigenvalues is 1

21. The matrix $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is:

(a) orthogonal  (b) hermitian  (c) symmetric  (d) anti-symmetric

22. A given (n×n) nilpotent matrix A satisfies the equation $A^k = 0$ for $1 < k < n$. Therefore,

(a) Exactly k eigenvalues of A must be zero.  (b) Exactly (n–k) eigenvalue of A must be zero.

(c) Every eigenvalue of A is zero.  (d) A can have (n–1) non-zero eigenvalues.

23. Consider the (n×n) matrix A with every element equal to unity. Which of the following statement is correct?

(a) The eigenvalues of A are all equal to unity.  (b) All the eigenvalues of A are zero.

(c) The largest eigenvalue of A is n.  (d) A cannot be diagonalised by a similarly transformation.

24. The eigenvalues of the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are

(a) 0, 0  (b) 1, 0  (c) 1, –1  (d) 1, 1

25. If L(x) is a linear differential operator and $y_1(x)$, $y_2(x)$ are two arbitrary functions. Then

(a) $L(x)(y_1 + y_2) = Ly_1 + Ly_2$  (b) $L(x)(y_1 + y_2) = y_1Ly_2 + y_2Ly_1$

(c) $L(x)(y_1y_2) = Ly_1 + Ly_2$  (d) $L(x)(y_1 - y_2) = Ly_1 + Ly_2$

26. For which of the following values of a, the inverse of the matrix, $M = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 3 & 5 \\ 0 & 4 & a \end{pmatrix}$, does not exist?

(a) –4  (b) 4  (c) 0  (d) 1

27. The eigenvalues of the matrix $M = \begin{pmatrix} 1 & 0 \\ 1 & i \end{pmatrix}$ are

(a) 1, 1  (b) 1, i  (c) 1, 0  (d) Cannot be determined.

28. Which one of the following matrices is orthogonal?

(a) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

29. The eigenvalues of a skew-Hermitian matrix are

(a) Real and positive (b) Real and negative

(c) Either zero or pure imaginary (d) Complex with nonzero real parts.

30. The number of independent real parameters of a most general hermitian matrix of order 4 is:

(a) 4 (b) 8 (c) 16 (d) 32

31. The eigen values of the matrix, $\begin{bmatrix} 0 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & 0 \end{bmatrix}$ are

(a) Purely imaginary (b) Complex (c) Real (d) Zero

32. If a matrix A satisfies the condition $A^2 = I$ and $\mathrm{Tr}(A) = 0$ (where I is a $n \times n$ identity matrix), then

(a) The determinant of A must be 0 (b) The determinant of A must be +1

(c) A must be odd dimensional (d) A must be even dimensional

33. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is **NOT TRUE**?

(a) Det A = Det B (b) Trace of A = Trace of B

(c) A and B have the same eigenvectors (d) A and B have the same eigenvalues.

34. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:

(a) 18 (b) 12 (c) 9 (d) 6

35. The matrix A, defined by $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$ is orthogonal if

(a) $a = 1, b = -1$ (b) $a = 1/\sqrt{2}, b = -1/\sqrt{2}$

(c) $a = 1/\sqrt{2}, b = -i/\sqrt{2}$ (d) $a = 1, \ b = 1$

36. Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

A. The eigenvalues of M are

(a) 0, 1, 2 (b) 0, 0, 3 (c) 1, 1, 1 (d) –1, 1, 3

**B.** The exponential of M simplifies to (I is the 3×3 identity matrix)

(a) $e^M = I + \left(\frac{e^3-1}{3}\right)M$ (b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$ (d) $e^M = (e-1)M$

37. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(a) 3,0, 1 (b) 3,0, 0 (c) 1,2, 0 (d) 1,1, 1

38. If A and B are two $n \times n$ matrices, then the trace of $C = [A,B]$ is

(a) $\mathrm{Tr}(AB)$ (b) $\mathrm{Tr}(A)\ \mathrm{Tr}(B)$ (c) 0 (d) $\mathrm{Tr}(A) + \mathrm{Tr}(B)$

39. Consider the matrix $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a and c are real and b may be complex. If M can be diagonalized by a matrix S, then S is

(a) diagonal (b) symmetric (c) orthogonal (d) unitary

40. The trace and the determinant of a 2×2 matrix are denoted by T and D. The eigenvalues of the matrix are given by

(a) $\frac{T \pm \sqrt{T^2 - 4D}}{2}$ (b) $\pm\frac{\sqrt{T^2 - 4D}}{2}$ (c) $\frac{T \pm D}{2}$ (d) $\pm\frac{T}{2}$

41. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0,1,1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

42. For an N×N matrix consisting of all ones,

(a) all eigenvalues = 1 (b) all eigenvalues = 0

(c) the eigenvalues are 1,2, ..., N (d) one eigenvalue = N, the others = 0

43. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

(a) (1, 4, 9) (b) (0, 7, 7) (c) (0, 1, 13) (d) (0, 0, 14)

44. The eigenvalues of the antisymmetric matrix, $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where $n_1, n_2$ and $n_3$ are the components of a unit vector, are

(a) 0, i, –i (b) 0, 1, –1 (c) 0, 1 + i, –1, –i (d) 0, 0, 0

45. A $2\times2$ matrix A has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = 1$ is:

(a) 20 (b) 30 (c) 60 (d) 120

46. Given a $2\times2$ unitary matrix U satisfying $U'U = UU' = 1$ with det $U = e^{i\varphi}$, one can construct a unitary matrix $V(V'V = VV' = 1)$ with det V = 1 from it by

(a) Multiplying U by $e^{-i\varphi/2}$

(b) Multiplying any single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column of U by $e^{-i\varphi/2}$

(d) Multiplying U by $e^{-i\varphi}$.

47. Consider an $n\times n(n > 1)$ matrix A, in which $A_{ij}$ is the product of the indices *i* and *j* (namely $A_{ij} = ij$). The matrix A

(a) has one degenerate eigenvalue with degeneracy $(n-1)$

(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) has one degenerate eigenvalue with degeneracy $n$

(d) does not have any degenerate eigenvalue

48. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that

(a) $\{M,N\}=0$ (b) $[M,N]=0$ (c) $M^\dagger = N$ (d) $M^\dagger = N^{-1}$

49. The coordinate transformation $x' = 0.8x + 0.6y$, $y' = 0.6x - 0.8y$ represents

(a) a translation (b) a proper rotation

(c) a reflection (d) none of the above

50. A $3\times3$ matrix M has $\mathrm{Tr}[M] = 6, \mathrm{Tr}[M^2] = 26$ and $\mathrm{Tr}[M^3] = 90$. Which of the following can be a possible set of eigenvalues of M ?

(a) $\{1,1,4\}$ (b) $\{-1,0,7\}$ (c) $\{-1,3,4\}$ (d) $\{2,2,2\}$

51. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

(a) $5, 2, -2$ (b) $-5, -1, -1$ (c) $5, 1, -1$ (d) $-5, 1, 1$

52. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are

(a) $-5, -2, 7$ (b) $-7, 0, 7$ (c) $-4i, 2i, 2i$ (d) $2, 3, 6$

53. The matrices

$$A=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\qquad B=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\text{ and } C=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

satisfy the commutation relations

(a) $[A,B]=B+C, [B,C]=0, [C,A]=B+C$

(b) $[A,B]=C, [B,C]=A, [C,A]=B$

(c) $[A,B]=B, [B,C]=0, [C,A]=A$

(d) $[A,B]=C, [B,C]=0, [C,A]=B$

## Matrix Alzebra_Answer Key

| | | | | | | |
|---|---|---|---|---|---|---|
| **1. (b)** | **2. (d)** | **3. (c)** | **4. (d)** | **5. (a)** | **6. (a)** | **7. (a)** |
| **8. (c)** | **9. (d)** | **10. (a)** | **11. (b)** | **12. (b)** | **13. (c)** | **14. (d)** |
| **15. (c)** | **16. (c)** | **17. (a)** | **18. (a)** | **19. (c)** | **20. (c)** | **21. (a)** |
| **22. (c)** | **23. (c)** | **24. (c)** | **25. (a)** | **26. (b)** | **27. (b)** | **28. (b)** |
| **29. (c)** | **30. (a)** | **31. (c)** | **32. (d)** | **33. (c)** | **34. (d)** | **35. (b)** |
| **36. (b,c)** | **37. (b)** | **38. (c)** | **39. (d)** | **40. (a)** | **41. (b)** | **42. (d)** |
| **43. (d)** | **44. (a)** | **45. (c)** | **46. (a)** | **47. (a)** | **48. (a)** | **49. (c)** |
| **50. (c)** | **51. (c)** | **52. (b)** | **53. (d)** | | | |

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### CSIR-UGC-NET/JRF | GATE Physics
### MATHEMATICAL PHYSICS

### Assignment III: Complex Analysis

1. The residue of $\frac{1}{1+z^2}$ at $z=i$ is :

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{2i}$ (d) $\frac{1}{-2i}$

2. The value of $\oint_C \frac{dz}{(z-3)}$ if C is the circle $|z-2|=5$ is

(a) $\pi i$ (b) $2\pi i$ (c) 0 (d) $2\pi$

3. The complex function $f(z)=e^{\frac{+1}{(z-1)^2}}$ has singular point at z=1 which is:

(a) a pole of order 1 (b) a pole of order 2
(c) an isolated essential singularity (d) a non-isolated essential singularity

4. The contour integral $\oint \frac{dz}{z^2+a^2}$ is to be evaluated on a circle of radius 2a centered at the origin. It will have contributions only from the points.

(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (b) ia and –ia

(c) ia, – ia, $\frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$ (d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

5. The value of $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where c is a circle defined by $|z|=3$, is:

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$ (c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^2$

6. Which of the following functions of the complex variable z is NOT analytic everywhere?

(a) $e^z$ (b) $|z|^3$ (c) $z^3$ (d) $\frac{|z|}{3}$

7. The value of the integral $\int_C \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z|=4$, is:

(a) 0 (b) $\pi i$ (c) $2\pi i$ (d) $4\pi i$

8. The inverse of the complex number $\frac{3+4i}{3-4i}$ is:

(a) $\frac{7}{25}+i\frac{24}{25}$ (b) $-\frac{7}{25}+i\frac{24}{25}$ (c) $\frac{7}{25}-i\frac{24}{25}$ (d) $-\frac{7}{25}-i\frac{24}{25}$

9. The value of $\int_C \frac{dz}{(z^2+a^2)}$, where C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is:

(a) $\pi$ for $a = 2$ (b) zero for $a = \frac{1}{2}$ (c) $4\pi$ for $a = 2$ (d) $\frac{\pi}{2}$ for $a = \frac{1}{2}$

10. The value of the integral $\int_C dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is:

(a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$

11. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true?

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$

(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$

(d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

12. The value of the integral $\int_C z^{10} dz$, where C is the unit circle with the origin as the centre is:

(a) 0 (b) $z^{11}/11$ (c) $2\pi i z^{11}/11$ (d) 1/11

13. If $Z_1$ and $Z_2$ are complex numbers, which of the following is always true?

(a) $Z_1^* Z_2 + Z_2^* Z_1 = 2 Z_1^* Z_2$
(b) $Z_1^* Z_2 + Z_2^* Z_1 = 2 Z_1 Z_2$
(c) $Z_1^* Z_2 + Z_2^* Z_1 \le 2|Z_1||Z_2|$
(d) $Z_1^* Z_2 + Z_2^* Z_1 \ge 2|Z_1||Z_2|$

14. Which of the following inequalities are valid in case of complex numbers?

(a) $|Z_1 + Z_2| \le |Z_1| + |Z_2|$
(b) $|Z_1 + Z_1| > |Z_2| + |Z_2|$
(c) $|Z_1 - Z_2| \ge |Z_1| - |Z_2|$
(d) $|Z_1 - Z_2| < |Z_1| - |Z_2|$

15. The function tan (1/z) does not have a pole at z =

(a) $\pi/2$ (b) $2/\pi$ (c) $2/(3\pi)$ (d) $-2/(3\pi)$

16. The real part of a complex function f(z), analytic in a region is given by $u(x, y) = x^2 - y^2$ If the function vainshes at $z = 0$, the imaginary part of the function is

(a) 2xy (b) −2xy (c) $x^2 + y^2$ (d) $y^2 - x^2$

17. The residue of the function, $f(x) = \frac{z}{(2z+1)(5-z)}$ at $z = -1/2$ is:

(a) 1/11 (b) −1/11 (c) 1/22 (d) −1/22

18. If $z_1$ and $z_2$ are two complex numbers then:

(a) $|z_1 + z_2| < |z_1| + |z_2|$
(b) $|z_1 + z_2| = |z_1| + |z_2|$
(c) $(z_1 + z_2)^2 > 0$
(d) $|z_1 + z_2|^2 = (z_1 + z_2)(z_1 + z_2)$

19. Two complex numbers $Z_1$ and $Z_2$ are given to satisfy $|Z_1| = |Z_2|$ necessarily follows that:

(a) $Z_1 = Z_2$ (b) $Z_1 = e^{i\phi} Z_2$ (c) $Z_1 = Z_2$ (d) $Z_1 = Z_2^{-1}$

20. The continuous complex function $f(z) = z^* = x - iy$ is differentiable

(a) only at $z = 0$ (b) only at $z = 1$ (c) nowhere (d) everywhere

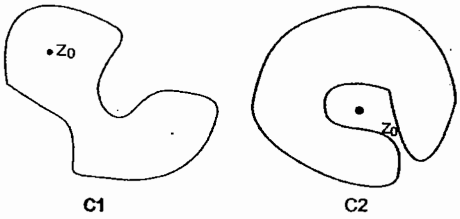
21. Two functions of a complex variable $z = x + iy$ are given as (i) $|z|$ and (ii) $z^2$. which of the following statements is correct?

(a) Both (i) and (ii) are analytic
(b) (i) is analytic and (ii) is nonanalytic
(c) Both are nonanalytic
(d) (i) is nonanalytic and (ii) is analytic

22. A function f(Z) of a complex variable Z is given as $f(Z) = Z\exp\left(\frac{1}{Z}\right)$. At $Z = 0$.

(a) It has no singularity
(b) It has a simple pole and residue is 1
(c) It has an essential sigularity and residue is 1/2
(d) It has a pole of order 2 and residue is 1/6

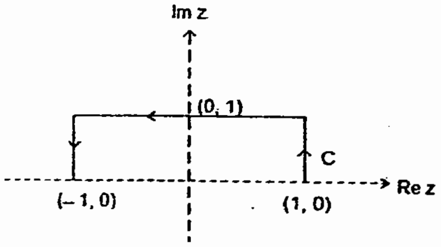
23. A function f(Z) of a complex variable Z is analytic everywhere in the complex plane except at $Z = Z_0$. Two contours C1 and C2 around $Z_0$ are shown in the figure.



The values of the integrals $I_1 = \oint_{C1} \frac{f(Z)dZ}{Z - Z_0}$, $I_2 = \oint_{C2} \frac{f(Z)dZ}{Z - Z_0}$ are

(a) $I_1 = 0, I_2 = 2\pi i f(Z_0)$
(b) $I_1 = 2\pi i f(Z_0), I_2 = 0$
(c) $I_1 = \pi i f(Z_0), I_2 = \pi i f(Z_0)$
(d) $I_1 = 2\pi i f(Z_0), I_2 = 2\pi i f(Z_0)$

24. The value of the integral $\int_C dz\, z^2\, e^z$, where C is an open contour in the complex z-plane as shown in the figure below, is:



(a) $\frac{5}{e} + e$
(b) $e - \frac{5}{e}$
(c) $\frac{5}{e} - e$
(d) $-\frac{5}{e} - e$

25. The residue of $e^{2/z^2}$ at $z = 0$ is:

(a) 0
(b) $\infty$
(c) $2\pi i$
(d) 2

26. The value of the integral $\oint_C \frac{z^3}{(z-1)^2}dz$, where C is the circle $|z|=4$ is

(a) $6\pi i$ (b) $-6\pi i$ (c) $2\pi i$ (d) Zero

27. The leading term in Laurent expansion of $f(z)=\frac{1}{z(1-z)^2}$ around z = 0 is

(a) $\frac{1}{z^3}$ (b) $\frac{1}{z^2}$ (c) $\frac{1}{z}$ (d) 1

28. The value of $\oint_c \frac{e^z(z+3)}{z-2}dz$ where c is a unit circle with centre at origin is:

(a) $5e^2$ (b) $\frac{5}{2\pi i}e^2$ (c) 0 (d) $10\pi ie^2$

29. If $f(z)=\frac{Z^2+a^2-2aZ}{(Z-a)^3(Z-b)}$, then Z = a is:

(a) Pole of order 1 (b) Pole of order 2 (c) Pole of order 3 (d) An essential singularity.

30. Which one of the following functions is not analytic?

(a) $f(z)=|z|$ (b) $f(z)=z^2$ (c) $f(z)=\cos z$ (d) $f(z)=\sin z$

31. The value of the integral $\oint_c \frac{\cos\pi z}{z(z+1)}$ where C is the circle $|z-1|=3$ is:

(a) 0 (b) $\pi i$ (c) $2\pi i$ (d) $4\pi i$

32. What is the type of singularity possessed by the function, $f(Z)=\exp\left[\frac{1}{(Z-1)^2}\right]$ at Z = 1?

(a) Simple pole (b) Branch point

(c) Pole of second order (d) Essential singularity

33. The residue of the function $f(z)=\frac{1}{z(z+2)^2}$ at z = −2 is:

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) $-\frac{\pi i}{2}$ (d) $4\pi i$

**Common data for Q. 34 and Q. 35.**

Consider a function $f(z)=\dfrac{z\sin z}{(z-\pi)^2}$ of a complex variable z.

34. Which of the following statements is TRUE for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z=\pi$

(c) $f(z)$ has a pole of order 2 at $z=\pi$ (d) $f(z)$ has a simple pole at $z=\pi$.

35. Consider a counterclockwise circular contour $|z|=1$ about the origin. The integral $\oint f(z)dz$ over this contour is:

(a) $-i\pi$ (b) Zero (c) $i\pi$ (d) $2i\pi$

36. The value of the contour integral, $\dfrac{1}{2\pi i}\oint\limits_{c} dz\, f(z)$. Where $f(z)=\dfrac{z}{2}+\dfrac{1}{z}+\dfrac{2z}{z^2-1}$ and the contour C is a circle of radius 2 centered at the origin, traversed in the counter-clockwise direction is:

(a) 2 (b) 1/2 (c) 1 (d) 3

37. Which of the following is an analytic function of the complex variable $z=x+iy$ in the domain $|z|<2$?

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

38. The first few terms in the Taylor series expansion of the function $f(x)=\sin x$ around $x=\dfrac{\pi}{4}$ are

(a) $\dfrac{1}{\sqrt{2}}\left[1+\left(x-\dfrac{\pi}{4}\right)+\dfrac{1}{2!}\left(x-\dfrac{\pi}{4}\right)^2+\dfrac{1}{3!}\left(x-\dfrac{\pi}{4}\right)^3\ldots\right]$

(b) $\dfrac{1}{\sqrt{2}}\left[1+\left(x-\dfrac{\pi}{4}\right)-\dfrac{1}{2!}\left(x-\dfrac{\pi}{4}\right)^2-\dfrac{1}{3!}\left(x-\dfrac{\pi}{4}\right)^3\ldots\right]$

(c) $\left[\left(x-\dfrac{\pi}{4}\right)-\dfrac{1}{3!}\left(x-\dfrac{\pi}{4}\right)^3\ldots\right]$ (d) $\dfrac{1}{\sqrt{2}}\left[1-x+\dfrac{x^2}{2!}-\dfrac{X^3}{3!}\ldots\right]$

39. The residue of the function $f(z)=\dfrac{1}{1+z^2}$ at z = i is

(a) $-i$ (b) $2i$ (c) $\dfrac{1}{2i}$ (d) 1

40. The value of the integral $\oint\limits_{c} e^{1/z}dz$, using the contour C of circle with unit radius $|z|=1$ is:

(a) 0 (b) $1-2\pi i$ (c) $1+2\pi i$ (d) $2\pi i$

41. The value of the integral $\int_0^{\infty} \frac{\ell nx}{\left(x^2+1\right)^2} dx$ is:

(a) 0 (b) $-\pi/4$ (c) $-\pi/2$ (d) $\pi/2$

42. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \le |z| \le 2$, and around $z=1$ is

(a) $\frac{1}{2}\left[1+z+z^2+z^3+......\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+.....\right]$

(b) $\frac{1}{1-z}-z-(1-z)^2+(1-z)^3+........$ (c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}-+......\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+.....\right]$

(d) $2(z-1)+5(z-1)^2+7(z-1)^3+......$

43. Let $u(x,y)=x+\frac{1}{2}\left(x^2-y^2\right)$ be the real part of an analytic function f(z) of the complex variable $z=x+iy$. the imaginary part of f(z) is

(a) $y+xy$ (b) $xy$ (c) $y$ (d) $y^2-x^2$

44. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{t^2-R^2} \cos\left(\frac{\pi t}{2R}\right) dt$ is

(a) $-2\pi/R$ (b) $-\pi/R$ (c) $\pi/R$ (d) $2\pi/R$

45. If $z=x+iy$ then the function,

$$f(x,y)=(1+x+y)(1+x-y)+a\left(x^2-y^2\right)-1+2iy(1-x-ax)$$

where 'a' is a real parameter, is analytic in the complex 'z' plane if $a=0$

(a) 1 (b) +1 (c) 0 (d) i

46. The Taylor expansion of the function $\ell n(\cosh x)$, where 'x' is real, about the point $x=0$ starts with the following terms:

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+..........$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+..........$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+..........$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+..........$

47. The value of the integral $\int_C \frac{z^3 dz}{z^2-5z+6}$, where C is a closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is:

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

48. Compute $\lim_{z\to 0} \frac{\mathrm{Re}(z^2)+\mathrm{Im}(z^2)}{z^2}$

(a) The limit does not exist (b) 1

(c) $-i$ (d) $-1$

49. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

(a) $x^2y$ (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2y - y - y^3$

50. The integral $\int_0^\infty \frac{dx}{4+x^4}$

evalautes to

(a) $\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

51. Given that the integral $\int_0^\infty \frac{dx}{y^2+x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{(y^2+x^2)^2}$ is

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

52. The partial differential equation, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

upon change of variables to $z = x + iy$ $z^* = x - iy$ becomes

(a) $\frac{\partial u}{\partial z} + \frac{\partial u}{\partial z^*} = 0$ (b) $z\frac{\partial u}{\partial z^*} + z^*\frac{\partial u}{\partial z} = 0$ (c) $z\frac{\partial u}{\partial z} + z^*\frac{\partial u}{\partial z^*} = 0$ (d) $z\frac{\partial u}{\partial z} - z^*\frac{\partial u}{\partial z^*} = 0$

53. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral

$$\oint_C \frac{dz}{\sin^2 z}$$

is

(a) $\infty$ (b) $2\pi i$ (c) 0 (d) $\pi i$

54. The function $\Phi(x, y, z, t) = \cos(z - vt) + \mathrm{Re}(\sin(x + iy))$ satisfies the equation

(a) $\frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$ (b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$ (d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

## Complex Analysis_Answer Key

1. (c)
2. (b)
3. (c)
4. (b)
5. (a)
6. (d)
7. (c)
8. (d)
9. (b)
10. (a)
11. (b)
12. (a)
13. (c)
14. (a,c)
15. (a)
16. (a)
17. (d)
18. (a)
19. (b)
20. (d)
21. (d)
22. (c)
23. (b)
24. (c)
25. (a)
26. (a)
27. (c)
28. (c)
29. (a)
30. (a)
31. (d)
32. (d)
33. (a)
34. (c)
35. (b)
36. (d)
37. (b)
38. (b)
39. (c)
40. (d)
41. (b)
42. (b)
43. (a)
44. (a)
45. (a)
46. (b)
47. (a)
48. (a)
49. (a)
50. (d)
51. (b)
52. (d)
53. (c)
54. (a)

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## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS

### Assignment-IV: Differential Equations

1. Solution of $\frac{d^2y}{dx^2}-\frac{5dy}{dx}+6=0$ is

(a) $C_1e^{-3x}+C_2e^{-2x}$ (b) $C_1\cos x+C_2\sin x$

(c) $C_1\cosh x+C_2\sinh x$ (d) $C_1e^{3x}+C_2e^{2x}$

2. Solution of $\frac{d^2x}{dt^2}+\frac{2dx}{dt}+1=0$ is

(a) $x=C_1e^{-t}+C_2e^{-t}$ (b) $x=C_1e^{t}+C_2e^{t}$ (c) $x=C_1e^{-t}+C_2te^{-t}$ (d) none

3. The value of $P_3(x)$ is

(a) $\frac{1}{2}(3x^2-1)$ (b) $3x$ (c) $\frac{1}{2}(5x^3-3x)$ (d) $\frac{1}{2}(5x^3-1)$

4. If $f(x)=x^4-2=\sum_{n=0}^{\infty}a_nP_n(x)$, then $a_3$ is

(a) $\frac{1}{2}$ (b) 1 (c) $-1$ (d) 0

5. The value of $\frac{J_{1/2}(x)}{J_{-1/2}(x)}$ is:

(a) 0 (b) $\tan x$ (c) $\cot x$ (d) $\tan h x$.

6. The points, where the series solution of the Legendre differential equation $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\frac{3}{2}\left(\frac{3}{2}+1\right)y=0$ will diverge, are located at:

(a) 0 and 1 (b) 0 and $-1$ (c) $-1$ and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

**Statement for Linked Answer Q. 7 and Q. 8:**

For the differential equaiton $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$

7. One of the solutions is:

(a) $e^x$ (b) $\ln x$ (c) $e^{-x^2}$ (d) $e^{x^2}$

8. The second linearly independent solution is:

(a) $e^{-x}$ (b) $xe^{x}$ (c) $x^2e^{x}$ (d) $x^2e^{-x}$

9. Consider the differential equation $d^2x/dt^2 + 2\,dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by:

(a) 1/e (b) 2/e (c) 1 (d) 3/e

10. The solution of the differential equation,

$$(1+x)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx} - y(x) = 0 \text{ is:}$$

(a) $Ax^2 + B$ (b) $Ax + Be^{-x}$ (c) $Ax + Be^{x}$ (d) $Ax + Bx^2$

where A and B are constants.

11. The solution of the system of differential equations,

$$\frac{dy}{dx} = y - z \text{ and } \frac{dz}{dx} = -4y + z$$

is given by (for A and B are arbitrary constants)

(a) $y(x) = Ae^{3x} + Be^{-x}$; $(x) = -2Ae^{3x} + 2Be^{-x}$ (b) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = 2Ae^{3x} + 2B^{-x}$

(c) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = 2Ae^{3x} - 2Be^{-x}$ (d) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = -2Ae^{3x} - 2Be^{-x}$

12. Legendre's differential equation: $(1-x^2)\,d^2y/dx^2 - 2x\,dy/dx + n(n+1)y = 0$ has an ordinary point at x =

(a) −1 (b) 0 (c) 1 (d) ∞

13. For the differential equation

$$z^2\frac{d^2y}{dz^2} + z\frac{dy}{dz} + y = 0$$

(a) $z = \infty$ is an ordinary point

(b) $z = 0$ is a regular singular and $z = \infty$ is an irregular singular point

(c) $z = 0$ and $z = \infty$ both are regular singular points

(d) $z = 0$ is an irregular singular point

14. The Legendre polynomial $P_n(x)$, where $-1 \le x \le +1$

(a) Is singular at $x = \pm 1$ (b) Satisfies $\int_{-1}^{1} dx P_n^2(x) = 1$

(c) Satisfies $\int_{-1}^{1} dx P_n(x) = 0$ for $n \ge 1$ (d) Is always an even function of x.

15. For the Bessel equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$, where n is an integer, the maximum number of linearly independent solutions, well-defined at x = 0 is

(a) zero (b) one (c) two (d) three

16. The general solution of the differential equation, $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ is:

(a) $C_1e^{2x}+C_2e^{-2x}$ (b) $C_1e^{2x}+C_2x\,e^{2x}$ (c) $C_1e^{-2x}+iC_2\,e^{2x}$ (d) $C_1e^{-2x}+C_2\,xe^{-2x}$

Here $C_1$ and $C_2$ are arbitrary constants.

17. In order to obtain the solution of the initial value problem of the equation of motion, $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+16x=0$.

How many initial conditions are required?

(a) 0 (b) 1 (c) 2 (c) 3

18. The general solution of the linear differential equation, $\frac{d^4u}{dx^4}=0$ is, u(x) equal to

(a) 0 (b) $c_3t+c_4$ (c) $c_2t^2+c_3t+c_4$ (d) $c_1t^3+c_2t^2+c_3t+c_4$

19. The regular singular points of the associated Legendre differential equation in the finite domain are

(a) $1,-1$ (b) 0 (c) $\infty$ (d) 2

20. The Wronskian of the linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}+4y=0$ is:

(a) Constant (b) 0 (c) $\infty$ (d) Undetermined

21. The solutions to the differential equation $\frac{dy}{dx}=-\frac{x}{y+1}$ are a family of

(a) Circles with different radii (b) Circles with different centres.
(c) Straight lines with different slopes (d) Straight lines with different intercepts on the y-axis

22. Given the Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)\frac{3x^2-1}{2}$, then the polynomial $(3x^2+x-1)$ can be expressed as

(a) $P_2(x)-P_1(x)$ (b) $2P_2(x)+P_1(x)$

(c) $P_2(x)+P_1(x)$ (d) $2P_2(x)+P_1(x)+P_0(x)$

23. The generating function $F(x,t)\equiv\sum_{n=0}^{\infty}P_n(x)t^n$ for the Legendre polynomials $P_n(x)$ is $F(x,t)=(1-2xt+t^2)^{-1/2}$. The value of $P_3(-1)$ is:

(a) 5/2 (b) 3/2 (c) +1 (d) -1

24. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+f(t)x=0$, and let $w(t)=x_1(t)\frac{dx_2(t)}{dt}-x_2(t)\frac{dx_1(t)}{dt}$ if $w(0)=1$, then $w(1)$ is given by

(a) 1 (b) $e^2$ (c) $1/e$ (d) $1/e^2$

25. If one expands $f(x)=x^4-2$ around $x=0$ in terms of the Legendre Polynomials $\left(f(x)=\sum_{n=0}^{\infty}a_nP_n(x)\right)$, the coefficient $a_3$ is:

(a) 1 (b) 0 (c) 3 (d) $e^2$

26. The most general solution of the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6=0$ in terms of arbitrary constants $C_1$ and $C_2$ is

(a) $c_1e^{-3x}+c_2e^{-2x}$ (b) $c_1\sin 3x+c_2\cos 2x$ (c) $c_1e^{3x}+c_2e^{2x}$ (d) $c_1e^{-3ix}+c_2e^{2ix}$

27. The solution of the differential equation $x\frac{dy}{dx}+y=1$, given the condition $y=0$ at $x=1$ is

(a) $y=(1-x^2)\ell n\, x$ (b) $y=\frac{x^2-1}{x}e^{-x}$ (c) $y=\frac{x-1}{x}$ (d) $y=\sinh(x)$

28. Let $y(x)$ be a continuous real function in the range 0 and $2\pi$, satisfying the inhomogeneous differential equaltion: $\sin x\frac{d^2y}{dx^2}+\cos x\frac{dy}{dx}=\delta\left(x-\frac{\pi}{2}\right)$. The value of $dy/dx$ at the point $x=\pi/2$

(a) is continuous (b) has a discontinuity of 3

(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

29. The differential equation,

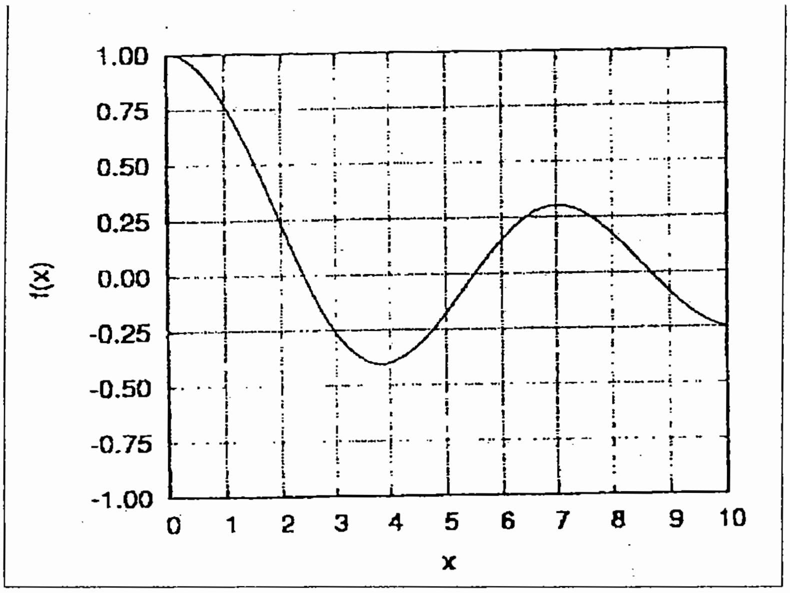
$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$$

has the complete solution, in terms of arbitary constants A and B,

(a) $A\exp x+B\,x\exp x$ (b) $A\exp x+B\exp(-x)$

(c) $A\exp x+B\,x\exp(-x)$ (d) $x\{A\exp x+B\exp(-x)\}$

30. The graph of the function f(x) as shown below is best described by



(a) The bessel function $J_0(x)$ (b) cos x

(b) $e^{-x}\cos x$ (d) $\frac{1}{x}\cos x$

31. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2}-(3-4i)f=0$ and satisfies the conditions $f(0)=1$ and $f(x)\to 0$ as $x\to\infty$. The value of $f(\pi)$ is :

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi i}$

32. Consider the differential equation $\frac{dG(x)}{dx}+k\,G(x)=\delta(x)$, where $k$ is a constant. Which of the following statements is true?

(a) Both $G(x)$ and $G'(x)$ are continuous at $x=0$.

(b) $G(x)$ is continuous at $x=0$ but $G'(x)$ is not.

(c) $G(x)$ is discontinuous at $x=0$.

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x=0$ depend on the value of $k$.

33. Given,

$\sum_{n=0}^{\infty}P_n(x)t^n=(1-2xt+t^2)^{-1/2}$, for $|t|<1$, the value of $P_5(-1)$ is

(a) 0.26 (b) 1 (c) 0.5 (d) –1

34. Consider the differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$$

with the initial conditions $x(0)=0$ and $x(0)=1$. The solution $x(t)$ attains its maximum value when 't' is

(a) 1/2 (b) 1 (c) 2 (d) $\infty$

## Differential Equations_Answer Key

1. (d) 2. (c) 3. (c) 4. (d) 5. (b) 6. (d) 7. (a)

8. (b) 9. (b) 10. (b) 11. (a) 12. (b) 13. (a) 14. (c)

15. (c) 16. (b) 17. (c) 18. (d) 19. (a) 20. (a) 21. (a)

22. (b) 23. (d) 24. (d) 25. (b) 26. (c) 27. (c) 28. (d)

29. (a) 30. (a) 31. (c) 32. (c ) 33. (d) 34. (b)

## CSIR-UGC-NET/JRF | GATE Physics
## MATHEMATICAL PHYSICS

### Assignment V: Fourier Series, Fourier Transform & Laplace Transform

1. Inverse Laplace transform of $\frac{a}{s^2+a^2}$ is

   (a) sin at (b) sinh at (c) cos at (d) cos h at

2. Using the fourier series of $f(x)=x^2, -\pi<x<\pi$. The value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is

   (a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{12}$ (d) $\pi^2$

3. Fourier transorm of Gaussian function $e^{-x^2}$ is:

   (a) $\frac{1}{\sqrt{2}}e^{-\omega^2/4}$ (b) $\frac{1}{\sqrt{2\pi}}e^{-\omega^2/4}$ (c) $e^{-\omega^2}$ (d) $e^{-\omega^2/2}$

4. Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is:

   (a) $\cos 2x+\frac{1}{2}\sin 2x$ (b) $\cos x+\frac{1}{2}\sin x$

   (c) $\cosh x+\frac{1}{2}\sinh x$ (d) $\cosh 2x+\frac{1}{2}\sinh 2x$

5. The kth Fourier component of $f(x)=\delta(x)$ is:

   (a) 1 (b) 0 (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

6. If $\bar{f}(s)$ is the Laplace transform of f(t) the Laplace transform of f(at), where a is a constnat, is

   (a) $\frac{1}{a}\bar{f}(s)$ (b) $\frac{1}{a}\bar{f}(s/a)$ (c) $\bar{f}(s)$ (d) $\bar{f}(s/a)$

7. The Laplace transform of $f(t)=\sin\pi t$ is $F(s)=\frac{\pi}{(s^2+\pi^2)}, s>0$. Therefore, the Laplace transform of $t\sin\pi t$ is:

   (a) $\frac{\pi}{s^2(s^2+\pi^2)}$ (b) $\frac{2\pi}{s^2(s^2+\pi^2)^2}$ (c) $\frac{2\pi s}{(s^2+\pi^2)^2}$ (d) $\frac{2\pi}{(s^2+\pi^2)^2}$

8. A periodic function f(x) = x for $-\pi < x < +\pi$ has the Fourier series representation $f(x) = \sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin nx$. Using this, one finds the sum $\sum_{n=1}^{\infty} n^{-2}$ to be:

(a) 2 ln 2  (b) $\frac{\pi^2}{3}$  (c) $\frac{\pi^2}{6}$  (d) π ln2

9. The Fourier transform F(k) of a function f(x) is defined as:

$F(k) = \int_{-\infty}^{\infty} dx\, f(x) \exp(ikx)$. Then F(k) for $f(x) = \exp(-x^2)$ is:

[Given: $\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}$ ]

(a) $\pi \exp(-k)$  (b) $\sqrt{\pi} \exp\left(\frac{-k^2}{4}\right)$  (c) $\frac{\sqrt{\pi}}{2} \exp\left(\frac{-k^2}{2}\right)$  (d) $\sqrt{2\pi} \exp(-k^2)$

10. The Fourier transform of the function f(x) is $F(k) = \int e^{ikx} f(x)\, dx$. The Fourier transform of df(x)/dx is:

(a) dF(k) / dk  (b) $\int F(k)/dk$  (c) $-ikF(k)$  (d) ikF(k)

11. Fourier transform of which of the following functions does not exist?

(a) $e^{-|x|}$  (b) $xe^{-x^2}$  (c) $e^{x^2}$  (d) $e^{-x^2}$

12. If f(x) be an even, period function of x which is continuous. Then
(a) It can be represented by a cosine series.  (b) It can be represented by a sine series
(c) It cannot be represented by a Fourier series
(d) It needs both sine and cosine terms in its Fourier expansion

13. Laplace transform L{F(t)} of a function F(t) is f(s). Which of the following expression is FALSE?

(a) $\frac{d^n}{ds^n} f(s) = (-1)^n L\{t^n F(t)\}$  (b) $f(s+a) = L\{e^{-at}F(t)\}$

(c) $L\left\{\sum_{i=1}^{n} a_i F_i(t)\right\} = \sum_{i=1}^{n} a_i L\{F_i(t)\}$  (d) $L\{F(at)\} = \frac{1}{a} f(s)$

14. Which of the following is NOT a periodic function of $\theta$?

(a) $\sin\theta + \pi\cos\theta$  (b) $\sin(\pi\theta) + \cos(\pi\theta)$

(c) $\sin\theta + \cos(\pi\theta)$  (d) $\sin\theta + \cos(\theta + \pi)$

15. The Fourier series expansion of the function $f(x) = x(\pi - x), 0 \le x \le t,$ is:

$\frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots\dots\right)$. The value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is deduced as

(a) $\frac{\pi^2}{6}$  (b) $\frac{2\pi^2}{6}$  (c) $-\frac{\pi^2}{6}$  (d) Zero

16. The Fourier coefficients $a_n$ in the Fourier series expansion.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx,\text{ for } f(x)=\begin{cases}-1, & -\pi\le x\le 0\\ +1, & 0\le x\le \pi\end{cases}\text{, are}$$

(a) $\frac{2}{n\pi}\left[1-(-1)^n\right]$ (b) $\frac{4}{n\pi}$ (c) $\frac{2}{n\pi}\left[1+(-1)^n\right]$ (d) 0

17. Coefficient of cosx in the Fourier series expansion of x over 0 to $2\pi$ is:

(a) 0 (b) 1 (c) 2 (d) 1/2

18. The function $f(x)=x^2, 0<x<2$ has a Fourier series representation

$$x^2=\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos nx-\frac{4\pi}{n}\sin nx\right)$$

At $x=0$ and $2\pi$, the series converges to $2\pi^2$. Then the value of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is:

(a) $2\pi^2$ (b) $\pi^2$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi^2}{6}$

19. Consider the Fourier series corresponding to the function

$$f(x)=\begin{cases}3, & 0<x<5\\ -3, & -5<x<0\end{cases}$$

Then the fourier series contains.

(a) Cosine terms only (b) Both cosine and sine terms

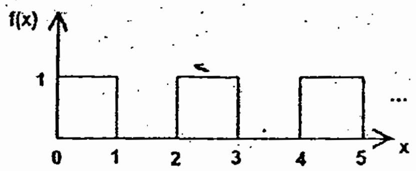
(c) Sine terms only (d) Sine terms and non-zero constant terms.

20. The constant coefficient in the Fourier series of the function

$$f(x)=\begin{cases}0 & -5<x<0\\ 3 & 0<x<5\end{cases}\text{ is:}$$

(a) 3 (b) $\frac{3}{2}$ (c) 0 (d) 5

21. The graph of the function $f(x)=\begin{cases}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{cases}$ (where $n=0,1,2,\ldots\ldots.$) is shown below.



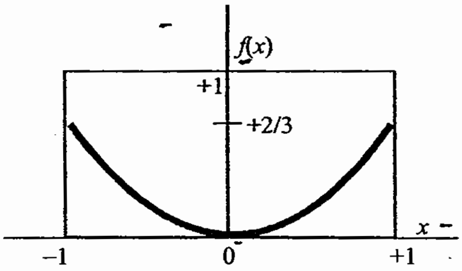
Its Laplace transform $\tilde{f}(s)$ is

(a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s(1+e^{-s})}$ (d) $\frac{1}{s(1-e^{-s})}$

22. Fourier transform of the derivative of the Dirac $\delta$-function, namely $\delta'(x)$, is proportional to

(a) 0 (b) 1 (c) sink (d) ik

23. A student is asked to find a series approximation for the function $f(x)$ in the domain $-1\le x\le +1$, as indicated by the thick line in the figure below.



The student represents the function by a sum of three terms

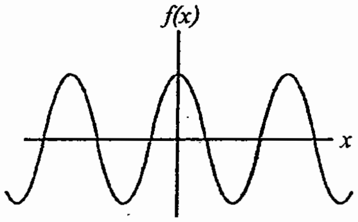
$$f(x)\approx a_0+a_1\cos\frac{\pi x}{2}+a_2\sin\frac{\pi x}{2}$$

Which of the following would be the best choices for the coefficients $a_0$, $a_1$ and $a_2$?

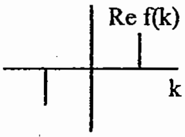
(a) $a_0=1, a_1=-\frac{1}{3}, a_2=0$ (b) $a_0=\frac{2}{3}, a_1=-\frac{2}{3}, a_2=0$

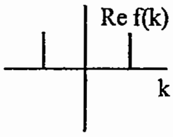
(c) $a_0=\frac{2}{3}, a_1=0, a_2=-\frac{2}{3}$ (d) $a_0=-\frac{1}{3}, a_1=0, a_2=-1$

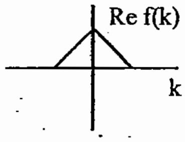
24. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below

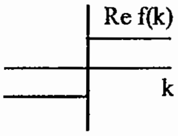


Which of the following graphs represents the real part of its Fourier transform?

(a)


(b)


(c)


(d)


## Fourier Series, Fourier Transform & Laplace Transform_Answer Key

**1. (a)** **2. (b)** **3. (a)** **4. (d)** **5. (c)** **6. (b)** **7. (c)**
**8. (c)** **9. (b)** **10. (c)** **11. (c)** **12. (a)** **13. (d)** **14. (c)**
**15. (a)** **16. (d)** **17. (a)** **18. (d)** **19. (c)** **20. (b)** **21. (c)**
**22. (b)** **23. (b)** **24. (b)**

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## CSIR-UGC-NET/JRF | GATE Physics

## Assignment VI: Probability

1. The probability of getting m heads out of n trials of a coin is

(a) $\frac{m!}{n!(n-m)!}\left(\frac{1}{2}\right)^m$ (b) $\frac{n!}{m!(n-m)!}\left(\frac{1}{2}\right)^n$ (c) $\frac{n!}{m!(n-m)!}\left(\frac{1}{2}\right)$ (d) $\frac{m!}{n!(n-m)!}\left(\frac{1}{2}\right)^n$

2. There are 3 cows and 4 bulls in a field, the probabiliy of taking birth of a calf by the combination of a cow and a bull is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

3. The probability of throwing more than 4 by one dice in a throw is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

4. An unbiased dice is thrown three times successively. The probability that the numbers of dots on the uppermost surface add up to 16 is

(a) $\frac{1}{16}$ (b) $\frac{1}{36}$ (c) $\frac{1}{108}$ (d) $\frac{1}{216}$

5. An unbiased coin is tossed n times. The probability that exactly m heads will come up is.

(a) $\frac{1}{2^m}\frac{n!}{m!(n-m)!}$ (b) $\frac{1}{2^n}\frac{n!}{m!(n-m)!}$ (c) $\frac{n}{2^m}$ (d) $\frac{m}{2^n}$

6. If a three digit number is chosen at random, the probability for the middle digit to be 3 is

(a) $1/10$ (b) $(1/10)^3$ (c) $1/3$ (d) $1/30$

7. Suppose a team is equally likely to win or lose a match. In a series of two matches, if we know that it has won at least one match, what is the probability that it has won both matches?

(a) $3/4$ (b) $1/2$ (c) $1/3$ (d) $1/4$

8. A coin and a die are thrown independently. The probability that a 'head' appears on the coin and an even integer on the die (assuming both are unbiased )is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

9. An unbiased die is cast twice. The probability that the positive different (bigger– smaller) between the two numbers is 2 is

(a) ${}^{1}/_{9}$ (b) ${}^{2}/_{9}$ (c) ${}^{1}/_{6}$ (d) ${}^{1}/_{3}$

10. A ball is picked at random from one of two boxes that contain 2 black and 3 black and 4 white balls respectively. What is the probability that it is white?
(a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

11. In a series of five cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is
(a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

12. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to
(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

13. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the $x$ axis. The probability that they meet after $n$ steps is
(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

14. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one?
(a) 0.99 (b) 0.925 (c) 0.075 (d) 0.01

15. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6, respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?
(a) $\frac{144}{441}$ (b) $\frac{225}{441}$ (c) $\frac{221}{441}$ (d) $\frac{220}{441}$

16. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0.255, 0.255, 0.245, and 0.245, respectively. After a large number of steps, N, the expected position of the child with respect to the starting point is at a distance
(a) $\sqrt{2}\times10^{-2}Na$ in the north-east direction (b) $\sqrt{2N}\times10^{-2}a$ in the north-east direction
(c) $2\sqrt{2}\times10^{-2}Na$ in the south-east direction (d) 0

17. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?
(a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

## Probability_Answer Key

**1. (b)** **2. (d)** **3. (b)** **4. (b)** **5. (b)** **6. (a)** **7. (d)**
**8. (a)** **9. (b)** **10. (b)** **11. (d)** **12. (d)** **13. (a)** **14. (c)**
**15. (b)** **16. (a)** **17. (a)**

## CSIR-UGC-NET/JRF | GATE Physics

## MATHEMATICAL PHYSICS

### Assignment VII: Miscelleneous Problems

**(Tensors, Numerical Techniques, Group Theory, Beta-Gamma Function, Series, Dirac-Delta Function)**

1. Given that $T_{i,j} = \frac{1}{2}\left(x_i y_j - x_j y_i\right)$ and $S_{ij} = \frac{1}{2}\left(x_i y_j + x_j y_i\right)$. Then $T_{ij}\, S_{ij}$ is:

(a) 0 (b) $\sum x_i^{\ 4} - y_i^{\ 4}$ (c) $\sum x_i^2 y_i^2$ (d) $\sum\left(x_i^2 - y_i^2\right)$

2. The number of independent components of symmetric tensor of rank 2 in n-dimensional space is:

(a) 2n (b) $n^2$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

3. The number of independent components of antisymmetric tensor of rank 2 in n-dimensional space is:

(a) n (b) $n^2$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

4. $\int_0^\infty x\, e^{-x^2} dx$ is

(a) $\frac{1}{2}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\frac{2}{\pi}}$ (d) 1

5. The integral formula $\int_{x_0}^{x_0+nh} y dx = \frac{h}{3}\left[\left(y_0 + y_n\right) + 4\left(y_1 + y_3 + + \ldots\ldots + y\right) + 2\left(y_2 + y_4 + \ldots\ldots + y_{n-2}\right)\right]$

(a) simpson's $\frac{3}{8}$ rule (b) simpson's $\frac{1}{3}$ rule (c) trapezoidal rule (d) weddle's rule.

6. The average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is:

(a) 15 (b) 20 (c) 40 (d) 80

7. The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is:

(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$

8. $S_{ij}$ and $A_{ij}$ represent a symmetric and an antisymmetric real-valued tensor respectively in three dimensions. The number of independent components of $S_{ij}$ and $A_{ij}$ are:

(a) 3 and 6 respectively (b) 6 and 3 respectively
(c) 6 and 6 respectively (d) 9 and 6 respectively.

9. Consider the four statements given below about the function $f(x) = x^4 - x^2$ in the range $-\infty < x < +\infty$. Which one of the following statements is correct?

P: the plot of f(x) versus x has two maxima and two minima
Q: the plot of f(x) versus x cuts the x axis at four points
R: the plot of f(x) versus x has three extrema

S: no part of the plot f(x) versus x lies in the fourth guadrant

Pick the right combination of correct choices from those given below.

(a) P and R (b) R only (c) R and S (d) P and Q

10. If $u(x, y, z, t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions, is a solution of the wave equation, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, then $\beta$ is

(a) $\left(1 - \frac{v}{c}\right)^{1/2}$ (b) $\left(1 - \frac{v}{c}\right)$ (c) $\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1 - \frac{v^2}{c^2}\right)$

11. The Dirac delta function $\delta(x)$ can be represented by:

(a) $\frac{1}{\pi}\lim_{N \to 0}\frac{\sin(Nx)}{x}$

(b) $\frac{1}{\pi}\lim_{\varepsilon \to 0+}\frac{\varepsilon}{x^2 + \varepsilon^2}$

(c) $\frac{1}{\sqrt{x}}\lim_{\varepsilon \to +\infty}\frac{1}{\sqrt{\varepsilon}}\exp\left(-x^2/\varepsilon\right)$

(d) $\frac{1}{2p}\int_{-\infty}^{+\infty}\exp\left(i\left|x - x^4\right|u\right)du$

12. The error function is defined by $\operatorname{erf}(x) = \left(2/\sqrt{\pi}\right)\int_0^x \exp\left(-t^2\right)dt$

Choose the correct alternatives:

(a) Error function is an even function

(b) Error function is an odd function

(c) $\operatorname{erf}(0) = 1$

(d) $\operatorname{erf}(\infty) = 1$

13. A wave $f(r, z, t) = J_0(\alpha r)\cos(\omega t - kz)$, where r and z refer to cylindrical coordinate system, satisfies the wave equation

$$\nabla^2 f - \left(1/v^2\right)\left(\partial^2 f/\partial t^2\right) = 0$$

when

(a) $v^2 = \omega^2/k^2$ (b) $v^2 = \omega^2/(k^2 + \alpha^2)$ (c) $v^2 = \omega^2/(k^2 - \alpha^2)$ (d) $v^2 = \omega^2/\alpha^2$

14. The volume element in the spherical polar coordinates is given by:

(a) $r^2 \sin\theta\, d\theta\, d\phi dr$

(b) $r^2 \sin\theta \sin\phi d\theta d\phi dr$

(c) $r \sin\theta d\theta d\phi dr$

(d) $r^2 d\theta d\phi dr$

15. From dimensional argument, the Fourier transform of Coulomb potential in three dimension is:

(a) $\frac{C}{k^3}$ (b) $\frac{C}{k}$ (c) C (d) $\frac{C}{k^2}$

where C is a constant, and k is the magnitude of the wave vector.

16. Which of the following is a valid solution of the differential equation, $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

(a) $x^2 - y^2$ (b) $ax^2 - by^2$ (c) $x^2 + y^2$ (d) $x^4 + y^4$

17. The Dirac delta function $\delta(x)$ is defined as $\int_{-\infty}^{\infty}\delta(x)dx = 1$. Which of the following integration is **INCORRECTLY** evaluated?

(a) $\int_{0}^{2\pi}\cos x\,\delta(x-\pi)\,dx = -1$ (b) $\int_{-2}^{+2}x^3\delta(x+1)dx = -1$

(c) $\int_{-2}^{2}(2x+3)\delta(3x)dx = 1$ (d) $\int_{0}^{2}(x^3+3x+2)\delta(x-1)dx = 0$

18. The nonzero independent components of an antisymmetric tensor $A_{ij}$ of rank 2 in three dimensions are

(a) $A_{11}, A_{12}, A_{13}$ (b) $A_{12}, A_{13}, A_{22}$

(c) $A_{12}, A_{23}, A_{13}$ (d) $A_{13}, A_{23}, A_{33}$

19. Which of the following matrices is an element of the group SU(2)?

(a) $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}\frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}}\end{pmatrix}$ (c) $\begin{pmatrix}2+i & i\\ 3 & 1+i\end{pmatrix}$ (d) $\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$

20. The number of independent components of a symmetric and an antisymmetric tensor of rank 2 (in 3–dimensions) are, respectively,

(a) 6, 6 (b) 9, 3 (c) 6, 3 (d) 3, 6

21. The trapezoidal, Simpson's 1/3 and Simpson's 3/8 rules are exact for polynomials of order

(a) 1, 2, 3 respectively (b) 1, 3, 3 respectively

(c) 1, 3, 4 respectively (d) 2, 3, 4 respectively.

22. The sum of $1-\frac{\pi^2}{2!4^2}+\frac{\pi^4}{4!4^4}-\frac{\pi^6}{6!4^6}+........$is

(a) Not convergent (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

23. The integral $\int_{-1}^{1}x^{-2}dx$ is:

(a) 0 (b) 1 (c) 2 (d) $\infty$

24. $\int_{-1}^{1}\sin mx\cos nx\,dx,\ (m\neq 0, n\neq 0)$ is:

(a) $\delta_{mn}$ (b) 2 (c) $2\,mn\,\pi$ (d) 0

25. Given $f(x)=\frac{1}{1-x^2}, x<1$, the series expansion for f(x) is:

(a) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\ldots\ldots$ (b) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots\ldots\ldots\ldots$

(c) $1+x+x^2+x^3+\ldots\ldots$ (d) $1+x^2+x^4+x^6+\ldots.$

26. Consider the function f such that $f(\alpha+\beta)=f(\alpha)+f(\beta)$ for any real $\alpha,\beta$. If m and n are integers, then $f\left(\frac{m}{n}\right)$ is

(a) $\frac{m}{n}f(1)$ (b) $\frac{m}{n}$ (c) $\frac{f(m)}{f(n)}$ (d) $f\left(\frac{1}{m}\right)+f\left(\frac{1}{n}\right)$

27. Given functions $\cos 2x, \sin^2 x, \cos^2 x$ are

(a) Linearly independent (b) Linearly dependent

(c) Orthogonal to each other (d) Unrelated to each other

28. The value of the integral $\int_0^{\infty} x\, e^{-x^2}\, dx$ is equal to

(a) $\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\pi}$ (c) 0 (d) $\frac{1}{2}$

29. Under a small rotation $\delta\theta$ about the x–axis, the component of a second rank tensor $\Gamma_{yz}$ transforms as

(a) $\Gamma'_{yz}=(1+\delta\theta)\Gamma_{yz}$ (b) $\Gamma'_{yz}=\Gamma'_{yz}+\delta\theta(\Gamma_{yy}-\Gamma_{zz})$

(c) $\Gamma'_{yz}=\Gamma_{yz}+\delta\theta(\Gamma_{yx}-\Gamma_{zx})$ (d) $\Gamma'_{yz}=\Gamma'_{yz}+\delta\theta\Gamma_{xx}$

30. In spherical polar coordinates $\vec{r}=(r,\theta,\varphi)$ the delta function $\delta(\vec{r}_1-\vec{r}_2)$ can be written as

(a) $\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$

(b) $\frac{1}{r_1^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$

(c) $\frac{1}{|\vec{r}_1-\vec{r}_2|^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$

(d) $\frac{1}{r_1^2\cos\theta_1}\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$

31. The character table of $C_{3v}$, the group of symmetries of an equilateral triangle is given below

|  | $\chi^{(0)}$ | $\chi^{(1)}$ | $\chi^{(2)}$ |
|---|---|---|---|
| $1\mathcal{C}_1$ | 1 | 1 | b |
| $3\mathcal{C}_2$ | 1 | a | c |
| $2\mathcal{C}_3$ | 1 | 1 | d |

In the above $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ denotes the three classes of $C_{3v}$, containing 1, 3 and 2 elements repectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters o f the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of $C_{3v}$.

A. The entries a, b, c and d in this table are, respectively

(a) $2, 1, -1, 0$ (b) $-1, 2, 0, -1$ (c) $-1, 1, 0, -1$ (d) $-1, 1, 1, -1$

B. The reducible representation $\Gamma$ of $C_{3v}$ with character $\chi = (4, 0, 1)$ decomposes into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(2)}$

32. The function $f(x)$ represents the nearest integer less than $x$, e.g. $f(3.14) = 3$.

The derivative of this function (for arbitrary $x$) will be given in terms of the integers $n$ as $f'(x) =$

(a) 0 (b) $\sum_n \delta(x-n)$ (c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

33. Consider the integral, $\int_{p^2}^{+p^2} \frac{dx}{\sqrt{x^2 - p^2}}$

where p is a constant. This integral has a real, nonsingular value if

(a) $p < -1$ (b) $p > 1$ (b) $p = 1$ (d) $p \to 0$

34. The value of the real integral $I = \int_{-3}^{+3} \frac{dx}{x^2 + 3x + 2}$ is:

(a) $\frac{3\pi}{2}$ (b) $\ell n\left(\frac{2}{5}\right)$ (c) $\infty$ (d) 0

35. Let $\varphi(x)$ be an arbitrary smooth function of x in $x_1 \le x \le x_2$. The condition that the integral $J = \int_{x_1}^{x_2} \left(\frac{d\varphi}{dx}\right)^2 dx$ has an extremum is given by

(a) $\frac{d^2\varphi}{dx^2} > 0$ (b) $\frac{d\varphi}{dx} = x$ (c) $\frac{d^2\varphi}{dx^2} < 0$ (d) $\frac{d^2\varphi}{dx^2} = 0$

36. Let $f(x) = e^{-x} \cos x$, what is the value of $f(x)$ at $x = 0.01$ up to four decimal places

(a) 0.0001 (b) 0.0101 (c) 0.9900 (d) 0.9999

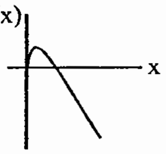
37. The value of the integral $\int_0^{\infty} xe^{-x^2} dx$ is equal to

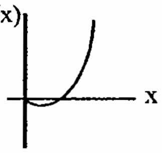
(a) $\sqrt{\frac{x}{2}}$ (b) $\sqrt{x}$ (c) $\frac{1}{2}$ (d) 0

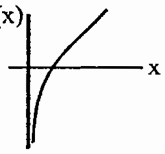
38. If $T_{ij}=\frac{1}{2}\left(x_iy_j-x_jy_i\right)$ and $S_{ij}=\frac{1}{2}\left(x_iy_j-x_jy_i\right)$ are components of tensor T and $S=\left(i,j=1,2,3\right)$ then $\sum_{i,j}T_{ij}S_{ij}$ is :

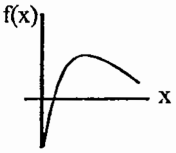
(a) 0 (b) $\sum_i\left(x_i^2-y_i^2\right)$ (c) $\sum_i x_i^2y_i^2$ (d) $\sum_i\left(x_i^4-y_i^4\right)$

39. Which ofhe following plots represents the function $f\left(x\right)=x\,\ell n\,x$ in the range $0\le x<\infty$ ?

(a)


(b)


(c)


(d)


40. The values of a function $f\left(x\right)$ are tabulated for three value of x below:

| x | $f(x)$ |
|---|---|
| 1.0 | 1.76 |
| 2.0 | 0.41 |
| 3.0 | –0.16 |

What is the approximate value of x at which $f\left(x\right)$ vanishes?

(a) 2.3 (b) 2.9 (c) 2.1 (d) 1.9

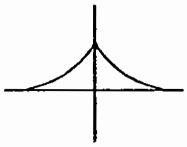
41. The number of independent components of symmetric second rank tensor $S_{ij},\ i,j=1,2,......,d$ is

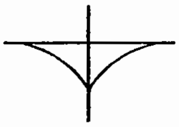
(a) 2 (b) $d^2$ (c) $d^2/2$ (d) $d\left(d+1\right)/2$

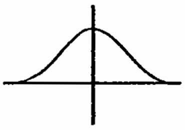
42. The sum of the infinite series $1+\frac{3}{2}+\frac{3.4}{2^2.2!}+\frac{3.4.5}{2^3.3!}+\frac{3.4.5.6}{2^4.4!}+.......$ is

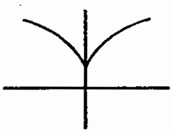
(a) 8 (b) $e^2$ (c) $2\pi$ (d) $\infty$

43. Which of the following plots represents the function $f\left(x\right)=e^{-|x|}$ for $-\infty<x<\infty$?

(a)


(b)


(c)


(d)


44. The maximum value of the function $f\left(x\right)=xe^{-x}$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) 1 (d) $e^2$

45. Given the integral $\int_0^\infty e^{-ax^2}dx=\frac{1}{2}\sqrt{\frac{\pi}{a}}$, the value of the integral $\int_0^\infty x^2e^{-2x^2}dx$ is

(a) $\frac{1}{4}\sqrt{\frac{\pi}{2}}$ (b) $2\sqrt{2\pi}$ (c) $2\sqrt{\pi}$ (d) 0

46. The numerical value of a function $f(x)$ at three points are tabulated below:

| $x$ | $f(x)$ |
|---|---|
| 0 | 0.0000 |
| 0.1 | 0.0998 |
| 0.2 | 0.1986 |

The approximate value $\dfrac{d^2f(x)}{dx^2}$ at $x=0.1$ is

(a) −0.02 (b) −0.01 (c) −0.1 (d) +0.1

47. The number of independent components of the symmetric tensor $A_{ij}$ with indices $i, j = 1, 2, 3$ is

(a) 1 (b) 3 (c) 6 (d) 9

48. As $x \to 1$, infinite series $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + .....$

(a) diverges
(b) converges to unity
(c) converges to $\pi/4$
(d) none of the above

49. If $\lfloor x \rfloor$ denotes the greatest integer not exceeding x, then $\int_0^\infty \lfloor x \rfloor e^{-x}dx$ is

(a) $\dfrac{1}{e-1}$ (b) 1 (c) $\dfrac{e-1}{e}$ (d) $\dfrac{e}{e^2-1}$

50. What is the value of the following series ?

$$\left(1+\frac{1}{2!}+\frac{1}{4!}+...\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+...\right)^2$$

(a) 0 (b) e (c) $e^2$ (d) 1

51. Which of the following limits exists?

(a) $\lim\limits_{N\to\infty}\left(\sum\limits_{m=1}^{N}\frac{1}{m}+\ln N\right)$
(b) $\lim\limits_{N\to\infty}\left(\sum\limits_{m=1}^{N}\frac{1}{m}-\ln N\right)$
(c) $\lim\limits_{N\to\infty}\left(\sum\limits_{m=1}^{N}\frac{1}{\sqrt{m}}-\ln N\right)$
(d) $\lim\limits_{N\to\infty}\sum\limits_{m=1}^{N}\frac{1}{m}$

52. A bag constain many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to

(a) $\dfrac{2N+1}{3}$ (b) $\dfrac{N}{2}$ (c) $\dfrac{N+1}{2}$ (d) $\dfrac{N(N+1)}{2}$

53. The value of the integral

$$\int_0^\infty dx\, x^9 \exp\left(-x^2\right)$$

is

(a) 20160 (b) 12 (c) 18 (d) 24

54. The integral

$$\int_{-\infty}^{\infty} dx\, \delta\left(x^2-\pi^2\right)\cos x$$

evaluates to

(a) $-1$ (b) 0 (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

55. What is the value of the following series? $\left(1-\frac{1}{2!}+\frac{1}{4!}-\ldots\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}-\ldots\right)^2$

(a) 0 (b) $e$ (c) $e^2$ (d) 1

56. If the distribution function of $x$ is $f(x)=xe^{-x/\lambda}$ over the interval $0<x<\infty$, the mean value of $x$ is

(a) $\lambda$ (b) $2\lambda$ (c) $\lambda/2$ (d) 0

57. The expression

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$$ is proportional to

(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$ (d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

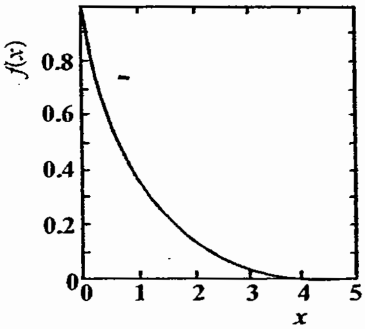
58. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij}=A_iB_j$

(a) reduces to a direct sum of three 3-dimensional representations
(b) is an irreducible 9-dimensional representation
(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations
(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

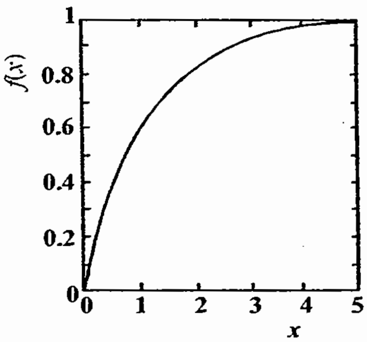
59. The solution of the integral equation $f(x)=x-\int_0^x dt\, f(t)$

has the graphical form

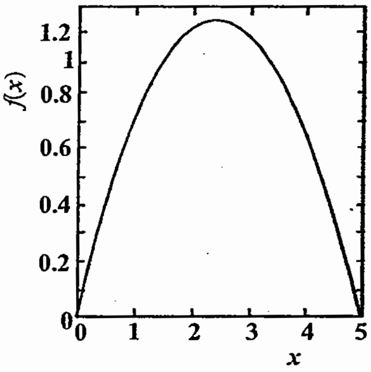
(a)



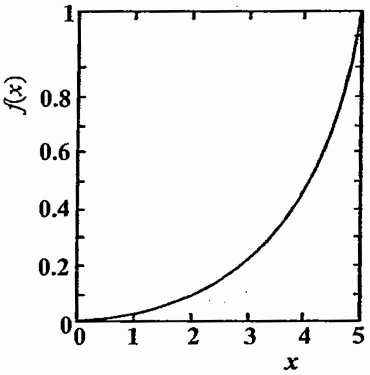
(b)



(c)



(d)



60. In the most general case, which one of the following quantities is NOT a second order tensor?
(a) Stress (b) Strain (c) Moment of inertia (d) Pressure

61. The following data is obtained in an expriment that measures the viscosity $\eta$ as a function of molecular weight M for a set of polymers.

| $M(Da)$ | $\eta(kPa-s)$ |
|---|---|
| 990 | $0.28 \pm 0.03$ |
| 5032 | $30 \pm 2$ |
| 10191 | $250 \pm 10$ |
| 19825 | $2000 \pm 200$ |

The relation that best describes the dependence of $\eta$ on M is
(a) $\eta \sim M^{4/9}$ (b) $\eta \sim M^{3/2}$ (c) $\eta \sim M^2$ (d) $\eta \sim M^3$

62. The integral $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is
(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

## Miscellaneous_Answer Key

1. (a) 2. (c) 3. (d) 4. (a) 5. (b) 6. (c) 7. (b)
8. (b) 9. (b) 10. (c) 11. (b) 12. (b,d) 13. (b) 14. (a)
15. (b) 16. (a) 17. (d) 18. (c) 19. (b) 20. (c) 21. (a)
22. (d) 23. (a) 24. (d) 25. (d) 26. (a) 27. (b) 28. (d)
29. (c) 30. (b) 31. (b), (c) 32. (b) 33. (d) 34. (d) 35. (d)
36. (c) 37. (c) 38. (a) 39. (b) 40. (b) 41. (d) 42. ( )
43. (a) 44. (a) 45. (a) 46. (d) 47. (c) 48. (c) 49. (a)
50. (d) 51. (b) 52. (a) 53. (b) 54. (d) 55. (d) 56. (b)
57. (b) 59. (b) 60. (d) 61. (d) 62. (c)

Σ

# CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
Best Institute for NET-JRF/GATE & IIT-JAM Exams

## Course Structure of MATHEMATICAL PHYSICS

- *No. of Questions asked in Part B of CSIR-NET exam : (4-5 ques.) x 3.5 marks each*
- *No. of Questions asked in Part C of CSIR-NET exam : (4-5 ques.) x 5 marks each*
- *There will approximately 35-40 marks questions out of 200 marks.*
- *For the last few years, the qualifying marks for (i) JRF is 90-100 marks (approx.)*
  *(ii) NET is 80-90 marks (approx.)*

## Course Details:

*This course is divided into three parts and there will be a test after completion of each part.*

### PART I :

- ***Vector Algebra & Vector Calculus***
  Dot & Cross Product, Triple Product, Vector differentiation, Gradient, Divergence, Curl, Vector Integration, Divergence Theorem, Stokes Theorem, Green's Theorem, Spherical & Cylindrical co-ordinate system.
- ***Matrices***
  Various types of Matrices, Eigenvalues & Eigenvectors, Cayley Hamilton Theorem, Similarity Transformation, Applications
- ***Complex Analysis***
  Basics of Complex numbers and Complex functions, C-R equations, Harmonic functions, Milne Thomson Method, Cauchy's Integral Theorem, Taylor Series, Laurent Series, Singular Points, Poles, Residues, Cauchy Residue Theorem, Evaluation of Integrals, Branch points

### PART II :

- ***Differential Equations & Special Functions***
  Separation of variables, Linear first order differential equations, Second order homogeneous and non-homogeneous differential equations, Legendre, Bessel, Hermite & Laguerre Differential Equations.
- ***Fourier Series***
  Periodic Functions, Dirichlet's Conditions, Fourier series in different forms, Even & Odd Function, Perseval's Formula
- ***Fourier Transform & Laplace Transform***
- ***Probability***
  Elementary idea, Random Variables, Binomial distribution, Poisson distribution, Gaussian (Normal) Distribution, Uniform Distribution, Exponential Distribution, Central Limit Theorem

- PART III :
    - *Partial Differential Equations*

      Laplace, Heat & Wave equations in 2-D & 3-D
    - *Numerical Techniques*

      Significant figures, Discrete rule of signs, Newton Raphson Method, Bisection Method, Interpolation, Extrapolation, Integration by Trapezoidal rule, Simpson 1/3 rd rule, Runge-Kutta Method, Finite difference Methods.
    - *Tensors*

      Basic Preview, Einstein Summation Convention, Contra-variant & Covariant Tensor, Levi Civita Tensor, ~~Metric~~ Tensor, Chrystoffer Symbols
    - *Introductory Group Theory*

      Basic Preview, Abelian Group, Sub-group, Group of symmetries, Group multiplication Table, Conjugate elements & Classes, Isomorphism, Homomorphism, SU(2) Group, O(3) Group

## Reference Books:

- *Mathematical Physics: H. K. Dass, Dr. Rama Verma (S. Chand Publications, 6th Edition)*
- *Advanced Engineering Mathematics: Erwin Kreyszig (Wiley Publications, 9th Edition)*
- *Vector Analysis: Murray R. Spigel (Schaum's Series, 2nd Edition)*
- *Functions of Complex Variables: Goyal & Gupta (Pragati Publications)*
- *Laplace & Fourier Transforms: Goyal & Gupta (Pragati Publications)*
- *Probability: Dr. K. C. Sinha (Eduwiser Publishing Group)*
- *Class Notes and Assignments*

**CAREER ENDEAVOUR**
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## Subject: PHYSICAL SCIENCES (NET-JRF/GATE)

## Section: MATHEMATICAL PHYSICS

## Assignment 1: VECTOR ALGEBRA & VECTOR CALCULUS

1. The angle between the vectors $2\hat{i}+2\hat{j}-\hat{k}$ and $6\hat{i}-3\hat{j}+2\hat{k}$ is equal to

(a) $\cos^{-1}(4/11)$ (b) $\sin^{-1}(4/11)$ [✓ (c) $\cos^{-1}(4/21)$] (d) $\sin^{-1}(4/21)$

2. If the projection of the vector $\vec{A}=\hat{i}+m\hat{j}+\hat{k}$ on vector $\vec{B}=4\hat{i}-4\hat{j}+7\hat{k}$ is equal to 19/9, then the value of 'm' is equal to

[✓ (a) 1] (b) -1 (c) 2 (d) -2

3. For any vector $\vec{A}$, which of the following statements is true?

(a) $\vec{A}=(\vec{A}\times\hat{i})\hat{i}+(\vec{A}\times\hat{j})\hat{j}+(\vec{A}\times\hat{k})\hat{k}$

(b) $\vec{A}=(\vec{A}\times\hat{i}).\hat{i}+(\vec{A}\times\hat{j}).\hat{j}+(\vec{A}\times\hat{k}).\hat{k}$

[✓ (c) $\vec{A}=(\vec{A}.\hat{i})\hat{i}+(\vec{A}.\hat{j})\hat{j}+(\vec{A}.\hat{k})\hat{k}$]

(d) $\vec{A}=(\vec{A}.\hat{i})\times\hat{i}+(\vec{A}.\hat{j})\times\hat{j}+(\vec{A}.\hat{k})\times\hat{k}$

4. Given: $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-4\hat{k}$ are the sides of

(a) an isosceles triangle [✓ (b) a right-angled triangle]
(c) an equilateral triangle (d) none of these

5. The value of the constant 'm' for which $\vec{A}=m\hat{i}+\hat{j}+\sqrt{5}\hat{k}$ subtends an angle $60^0$ with $\vec{B}=4\hat{i}-5\hat{j}+\sqrt{5}\hat{k}$ is equal to

(a) $\sqrt{46/3}$ (b) $\sqrt{29/4}$ (c) $\sqrt{41/3}$ (d) $\sqrt{17/3}$

6. The range of x for which the angle between the vectors $\vec{A}=2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $\vec{B}=7\hat{i}-2\hat{j}+x\hat{k}$ is obtuse, is equal to

(a) $x<0$ (b) $x>1/2$ [✓ (c) $0<x<1/2$] (d) None of these

7. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\cos^{-1}(15/22)$ (b) $\cos^{-1}(225/2)$ (c) $\pi/2$ (d) $\pi/3$

8. $|\vec{a}\times\vec{b}|$ can be written as

(a) $a^2b^2-(\vec{a}.\vec{b})^2$ (b) $\sqrt{a^2b^2-(\vec{a}.\vec{b})^2}$ (c) $a^2b^2+(\vec{a}.\vec{b})^2$ (d) $a^2b^2-\vec{a}.\vec{b}$

9. If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, then which of the following statements is true?

(a) $\vec{a}$ is parallel to $(\vec{b}-\vec{c})$ (b) $\vec{a}$ is perpendicular to $(\vec{b}-\vec{c})$
[✓ (c) either $\vec{a}=0$ or $\vec{b}=\vec{c}$.] (d) None of these

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ , $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ , then $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$ if 't' is equal to

(a) 8 (b) 4 (c) 6 (d) 2

11. If $\vec{a}$ and $\vec{b}$ are unit vectors, which of the following statements is true?

(a) $\sin \theta/2 = \frac{1}{2}|\vec{a} - \vec{b}|$ (b) $\sin \theta/2 = \frac{1}{2}|\vec{a} + \vec{b}|$ (c) $\cos \theta/2 = \frac{1}{2}|\vec{a} + \vec{b}|$ (d) $\cos \theta/2 = \frac{1}{2}|\vec{a} - \vec{b}|$

12. The area of the triangle PQR having vertices $P(1,3,2), Q(2,-1,1), R(-1,2,3)$ is equal to

(a) $\frac{1}{2}\sqrt{107}$ (b) $\frac{1}{2}\sqrt{77}$ (c) $\frac{1}{2}\sqrt{117}$ (d) $\frac{1}{2}\sqrt{117}$

13. Given : $\vec{P} = 2\hat{i} - 3\hat{j}$, $\vec{Q} = \hat{i} + \hat{j} - \hat{k}$, $\vec{R} = 3\hat{i} - \hat{k}$ , the vector $\vec{V}$, which is orthogonal to $\vec{P}$ and $\vec{Q}$ and having unit scalar product with $\vec{R}$ , can be estimated to be

(a) $(3\hat{i} + 2\hat{j} + 5\hat{k})/4$ (b) $(3\hat{i} + 2\hat{j} - 5\hat{k})/4$ (c) $(3\hat{i} - 2\hat{j} + 5\hat{k})/4$ (d) $(3\hat{i} - 2\hat{j} - 5\hat{k})/4$

14. Given: $\vec{A} = \hat{i} - \hat{j} - 2\hat{k}, \vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}, \vec{C} = -\hat{i} + 4\hat{j} + m\hat{k}$ . The value of 'm' such that the vectors will be co-planar is equal to

(a) 13/2 (b) 7/2 (c) 5/2 (d) 11/2

15. The equation of the plane that contains the points P (2, -1, 1), Q(3, 2, −1), R(−1, 3, 2) is equal to

(a) $11x + 5y + 13z = 30$ (b) $11x + 7y + 9z = 30$

(c) $4x + 7y - 9z = 30$ (d) $4x + 11y - 3z = 30$

16. The unit vector which is perpendicular to vector $2\hat{i} + \hat{j} - 3\hat{k}$ and lie in the plane of vectors $7\hat{i} - \hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} - 3\hat{k}$ is equal to

(a) $(3\hat{i} - 3\hat{j} - \hat{k})/\sqrt{19}$ (b) $(3\hat{i} - 3\hat{j} + \hat{k})/\sqrt{19}$ (c) $(3\hat{i} - 3\hat{j} + \hat{k})$ (d) $(3\hat{i} - 3\hat{j} - \hat{k})$

17. If $\vec{b} = \hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$, then $\vec{b}$ can be simplified to

(a) 0 (b) $\vec{a}$ (c) $2\vec{a}$ (d) None of these

18. Which of the following relations is true?

(a) $(\vec{A} \times \vec{B}).(\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = 0$ (b) $(\vec{A} \times \vec{B}).(\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A}.\vec{B} \times \vec{C})^2$

(c) $(\vec{A} \times \vec{B}).(\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A} \times (\vec{B} \times \vec{C}))^2$ (d) $(\vec{A} \times \vec{B}).(\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A} \times (\vec{B} \times \vec{C}))$

19. Three unit vectors $\vec{a}, \vec{b}, \vec{c}$ ($\vec{b}$ and $\vec{c}$ are not parallel) are such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}\vec{c}$ . The angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$ , respectively are

(a) $30^0, 90^0$ (b) $150^0, 90^0$ (c) $60^0, 90^0$ (d) $90^0, 30^0$

20. A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$. The components of its acceleration at $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$, is equal to

(a) $2/\sqrt{14}$ (b) $-2/\sqrt{14}$ (c) $16/\sqrt{14}$ (d) $-16/\sqrt{14}$

21. The unit tangent vector on the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at the point where $t = 2$ is equal to

(a) $\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$ (b) $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$ (c) $\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$ (d) $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$

22. The unit vector perpendicular to the surface $x^2 + y^2 - z^2 = 11$ at the point $(4, 2, 3)$ is equal to

(a) $4\hat{i} + 2\hat{j} + 3\hat{k} / \sqrt{29}$ (b) $4\hat{i} + 2\hat{j} - 3\hat{k} / \sqrt{29}$

(c) $4\hat{i} + 2\hat{j} - 3\hat{k}$ (d) $4\hat{i} + 2\hat{j} + 3\hat{k}$

23. The equation of a surface of revolution is $z = \pm\sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$. The unit normal to the surface at the point $A\left(\sqrt{\frac{2}{3}}, 0, 1\right)$ is

(a) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$ (b) $\sqrt{\frac{3}{10}}\hat{i} + \frac{2}{\sqrt{10}}\hat{j}$ (c) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$ (d) $\sqrt{\frac{3}{5}}\hat{i} - \frac{2}{\sqrt{10}}\hat{k}$

24. The angle between the two surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$ is

(a) $\sin^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ (b) $\cos^{-1}\left(\frac{8}{\sqrt{21}}\right)$ (c) $\sin^{-1}\left(\frac{8}{\sqrt{21}}\right)$ (d) $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$

25. The directional derivative of $1/r$ in the direction of $\vec{r}$ is equal to

(a) $-1/r$ (b) $-1/r^2$ (c) $-1/r^3$ (d) $-1/r^4$

26. The maximum value of direction derivative of $\varphi = x^2 - 2y^2 + 4z^2$ at the point (1,1,-1) is equal to

(a) $6\sqrt{7/3}$ (b) $\sqrt{7/3}$ (c) 84 (d) $3\sqrt{3/7}$

27. The surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. Which of the following relations is true for 'a' and 'b'?

(a) $a - b = 4$ (b) $2a - b = 4$ (c) $a = 3b$ (d) $3a + b = 9$

28. The directions along which there is no change in the value of the function $f(x, y) = e^{(x^2 + xy)}$ at the point (3,-2) are equal to

(a) $0.6\hat{i} - 0.8\hat{j}, -0.6\hat{i} + 0.8\hat{j}$ (b) $-0.6\hat{i} - 0.8\hat{j}, 0.6\hat{i} + 0.8\hat{j}$

(c) $-0.6\hat{i} - 0.8\hat{j}, 0.6\hat{i} - 0.8\hat{j}$ (d) $0.6\hat{i} + 0.8\hat{j}, -0.6\hat{i} + 0.8\hat{j}$

29. If $\bar{F}$ is a constant vector and $\bar{r}$ is the position vector then $\bar{\nabla}\left(\bar{F}.\bar{r}\right)$ would be

(a) $\left(\bar{\nabla}\bar{r}\right)\bar{F}$ (b) $|\bar{r}|\bar{F}$ (c) $\left(\bar{\nabla}.\bar{F}\right)\bar{r}$ (d) $\bar{F}$

30. The value of 'm' for which $\bar{V}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+mz)\hat{k}$ is a solenoidal vector is equal to

[✓ (a) −1] (b) 1 (c) −2 (d) 2

31. If $\bar{a}$ and $\bar{b}$ are irrotational in nature, then $\bar{a}\times\bar{b}$ will be

(a) irrotaional [✓ (b) solenoidal] (c) rotational (d) none of these

32. If $f=\tan^{-1}(y/x)$, then div (grad f) is equal to

(a) 1 (b) −1 (c) 0 (d) 2

33. The directional derivative of $\Omega(x,y)=\dfrac{x}{x^2+y^2}$ along a line making an angle $30^0$ with the positive x-axis at (0,2)

[✓ (a) $\dfrac{\sqrt{3}}{2}$] (b) $\dfrac{2}{\sqrt{3}}$ (c) $\dfrac{\sqrt{3}}{8}$ (d) $\dfrac{1}{2\sqrt{2}}$

34. For a constant vector $\bar{a}$ and $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$, consider the following statements:

$I.\,curl\left(\bar{a}\times\bar{r}\right)=2\left[grad\left(\bar{a}.\bar{r}\right)\right]$, $II.\,div\left[\left(\bar{a}.\bar{r}\right)\bar{r}\right]=2\left(\bar{a}.\bar{r}\right)$, then

(a) both statements are true (b) Only statement I is true
(c) Only statement II is true (d) Both statements are false

35. The value of the line integral $\int grad(x+y+z).\bar{dr}$ from (0,1,-1) to (1,2,0) is

(a) −1 (b) 3 (c) 0 (d) None of these

36. The work done by a force in moving a particle of mass 'm' form any point (x,y) to a neighbouring point (x+dx, y+dy) is given by $dW=2xy\,dx+x^2\,dy$. The work done for a complete cycle around a unit circle is

(a) 0 (b) 1 (c) 3 (d) $2\pi$

37. If $\bar{F}=\left(5xy-6x^2\right)\hat{i}+\left(2y-4x\right)\hat{j}$, then the value of the line integral $\int_C \bar{F}.\bar{dr}$ along the curve C in the x-y plane given by $y=x^3$, from the point (1,1) to (2,8) is equal to

[✓ (a) 35] (b) -35 (c) 47 (d) -47

38. If a force field $\bar{A}=(y+2x)\hat{i}+(3x+2y)\hat{j}$ is applied to a particle, then the work done by the force field in traversing the particle around a circle C in x-y plane, with center at the origin and radius 2 units is equal to ( C is traverse in the counter-clockwise direction )

(a) $2\pi$ (b) $4\pi$ [✓ (c) $8\pi$] (d) $16\pi$

39. When a force $\bar{F}=\left(2x+y^2\right)\hat{i}+\left(3y-4x\right)\hat{j}$ is applied on a particle, then it starts to move along a right angled triangle PQR having vertices P(0,0), Q(2,0), R(2,1) respectively. The amount of work done by the force is equal to

(a) 14/3 [✓ (b) -14/3] (c) 22/3 (d) -22/3

40. The flux of the vector field $\vec{V} = r^2\vec{r}$ over the region bounded by the surface S of a sphere of radius 'a', centered at the origin is equal to

(a) $\pi a^5$ (b) $2\pi a^5$ (c) $4\pi a^5$ (d) $8\pi a^5$

41. If $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, then the value of the integral $\int_S \vec{A}.\hat{n}dS$ (where S is the surface of a unit cube with two opposite corners at (0,0,0) and (1,1,1) respectively) is equal to

(a) 3/2 (b) 5/2 (c) 6 (d) 9/2

42. The value of the integral $\int_{(0,0)}^{(1,1)} \left[\left(3x^2 + 4xy + y^2\right)dx + \left(2x^2 + 2xy\right)dy\right]$ along the path $x^4 + y^4 = 2xy$ is

(a) 0 (b) 1 (c) 2 (d) 4 Green's theorem

43. The volume of the portion of the cyllinder $x^2 + y^2 = 4$ in the first octant between the planes z = 0 and 3x - z = 0 is

(a) 2 (b) 4 (c) 8 (d) 16

44. Let 'S' be the Boundary of the region consiting of the parabolic cyllinder $z = 1 - x^2$ and the planes y = 0, y = 2, z = 0. The flux of the vector field $\vec{F} = xy\hat{i} + \left(y^2 + e^{xz^2}\right)\hat{j} + \sin(xy)\hat{k}$ through the surface 'S' is

(a) 2 (b) 4 (c) 8 (d) 16

## CSIR Previous Year Questions

2011 45. Let $\bar{a}$ and $\bar{b}$ be two distinct three-dimensional vectors. Then the component of $\bar{b}$ that is perpendicular to $\bar{a}$ is given by **[CSIR June - 2011]**

(a) $\dfrac{\bar{a}\times\left(\bar{b}\times\bar{a}\right)}{a^2}$ (b) $\dfrac{\bar{b}\times\left(\bar{a}\times\bar{b}\right)}{b^2}$ (c) $\dfrac{\left(\bar{a}.\bar{b}\right)\bar{b}}{b^2}$ (d) $\dfrac{\left(\bar{b}.\bar{a}\right)\bar{a}}{a^2}$

46. The equation of the plane that is tangent to the surface xyz = 8 at the point (1, 2, 4) is

**[CSIR Dec - 2011]**

(a) x + 2y + 4z = 12 (b) 4x + 2y + z = 12
(c) x + 4y + 2z = 12 (d) x + y + z = 7

47. A vector perpendicular to any vector that lies on the plane defined by x + y + z = 5, is **[CSIR June - 2012]**

(a) $\bar{i} + \bar{j}$ (b) $\bar{j} + \bar{k}$ (c) $\bar{i} + \bar{j} + \bar{k}$ (d) $2\bar{i} + 3\bar{j} + 5\bar{k}$

48. The unit normal vector at the point $\left(\dfrac{a}{\sqrt{3}}, \dfrac{b}{\sqrt{3}}, \dfrac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$, is **[CSIR Dec - 2012]**

(a) $\dfrac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (b) $\dfrac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (c) $\dfrac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (d) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

49. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120° with respect to $\hat{i}$. The angle between the vectors $\vec{u} = a\hat{i} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{i}$ will be 60° if **[CSIR June - 2013]**

(a) $b = \sqrt{3}a/2$ (b) $b = 2a/\sqrt{3}$ (c) $b = a/2$ (d) $b = a$

50. If $\vec{A} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$, then the integral $\oint_C \vec{A}.\,d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by x = 0, x = a and y = 0, y = b) is **[CSIR Dec - 2013]**

(a) $\frac{1}{2}\left(a^3 + b^3\right)$ (b) $\pi\left(ab^2 + a^2b\right)$ (c) $\pi\left(a^3 + b^3\right)$ (d) 0

51. If $\vec{A} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.\,d\vec{l}$ is **[CSIR June - 2014]**

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{\pi}{4}$ (d) 0

## ANSWER KEY

**1.(c) 2.(d) 3.(c) 4.(b) 5.(a) 6.(c) 7.(d) 8.(b) 9.(c) 10.(a)**

**11.(a)(c) 12.(a) 13.(a) 14.(a) 15.(a) 16.(b) 17.(c) 18.(b) 19.(b) 20.(b)**

**21.(c) 22.(b) 23.(d) 24.(d) 25.(b) 26.(a) 27.(b) 28.(a) 29.(d) 30.(c)**

**31.(b) 32.(c) 33.(c) 34.(b) 35.(b) 36.(a) 37.(a) 38.(c) 39.(b) 40.(c)**

**41.(a) 42.(a) 43.(c) 44.(c) 45.(a) 46.(b) 47.(c) 48.(a) 49.(c) 50.(d)**

**51.(d)**

## Subject: PHYSICAL SCIENCES (NET-JRF/GATE)

## Section: MATHEMATICAL PHYSICS

## Assignment 2: VECTOR ALGEBRA & VECTOR CALCULUS

1. Find the unit vector normal to the plane of two vectors $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+4\hat{j}-\hat{k}$; and sin of the angle between the vectors. [**Ans.** $\left(-3\hat{i}+5\hat{j}+11\hat{k}\right)/\sqrt{155};\sqrt{155/156}$ ]

2. Show that $\vec{a}=\left[\frac{1}{3},\frac{-2}{3},\frac{-2}{3},\right]$ and $\vec{b}=\left[\frac{2}{3},\frac{-1}{3},\frac{2}{3}\right]$ are orthogonal and find a vector $\vec{C}$ orthogonal to both $\bar{a}$ and $\bar{b}$. $\left[\textbf{Ans.}\,\bar{C}=\left[\frac{-2}{3},\frac{-2}{3},\frac{1}{3}\right]\right]$

3. If the points P, Q, R are not lying on the same straight line having position vectors $\bar{x}$, $\bar{y}$, $\bar{z}$ relative to a given origin, show that $\bar{x}\times\bar{y}+\bar{y}\times\bar{z}+\bar{z}\times\bar{x}$ is perpendicular to the plane containing P, Q, R.

4. Find a set a vectors reciprocal to the following set $\left(2\hat{i}+3\hat{j}-\hat{k}\right),\left(\hat{i}-\hat{j}-2\hat{k}\right),\left(-\hat{i}+2\hat{j}+2\hat{k}\right)$, [**Ans.** $\left(2\hat{i}+\hat{k}\right)/3,\left(-8\hat{i}+3\hat{j}-7\hat{k}\right)/3,\left(-7\hat{i}+3\hat{j}-5\hat{k}\right)/3$]

5. If $\bar{r}=\bar{a}\cos\omega t+\bar{b}\sin\omega t$, show that

   (i) $\bar{r}\times\dfrac{d\bar{r}}{dt}=\omega\left(\bar{a}\times\bar{b}\right)$ (ii) $\dfrac{d^2\bar{r}}{dt^2}=-\omega^2\bar{r}$

   [$\bar{a}$, $\bar{b}$ are constant vectors and '$\omega$' is constant]

6. If $\bar{A}$ has constant magnitude show that $\bar{A}$ and $\dfrac{d\bar{A}}{dt}$ are perpendicular to each other.

7. A particle moves so that its position vector is given by, $\bar{r}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ where '$\omega$' is constant. Show that,

   (i) The velocity $\bar{v}$ of the particle is perpendicular to $\bar{r}$.

   (ii) The acceleration is directed towards the origin and has magntidue proportional to the distance from the origin

   (iii) $\bar{r}\times\bar{v}$ = constant vector.

8. If $\dfrac{d\bar{m}}{dt}=\bar{u}\times\bar{m}$ and $\dfrac{d\bar{n}}{dt}=\bar{u}\times\bar{n}$, then show that $\dfrac{d}{dt}\left(\bar{m}\times\bar{n}\right)=\bar{u}\times\left(\bar{m}\times\bar{n}\right)$

9. Find $\nabla\phi$ if (i) $\phi=\ln|\bar{r}|$, (ii) $\phi=\dfrac{1}{r}$ and (iii) $\phi=r^n$ [**Ans.** (i) $\dfrac{\bar{r}}{r^2}$, (ii) $\dfrac{-\bar{r}}{r^3}$ ]

10. If $f = x+y+z, g = x^2+y^2+z^2, h = xy+yz+zx$, then show that $\left[\nabla f \nabla g \nabla h\right] = 0$.

11. Find the direction derivative of the scalar fn. $\phi = 4e^{(2x-y+z)}$ at the point $(1,+1,-1)$ in a direction towards the point $(-3,5,6)$. What will be its maximum value? **[Ans.** $-20/9$, $4\sqrt{6}$ **]**

12. If $\vec{\nabla} U = 2r^4\vec{r}$, find U. **[Ans.** $U = \frac{r^6}{3} + \text{constant}$ **]**

13. Prove that, (i) $\vec{\nabla}.\left(\frac{\vec{r}}{r^3}\right) = 0\ (r \neq 0)$ (ii) $\vec{\nabla}.\left(r^n\vec{r}\right) = (3+n)r^n$ (iii) $\nabla\left[\frac{\vec{a}.\vec{r}}{r^n}\right] = \frac{\vec{a}}{r^n} - \frac{n(\vec{a}.\vec{r})\vec{r}}{r^{n+2}}$

14. Prove that, (i) $\vec{\nabla}\times\left(\vec{a}\times\vec{r}\right) = 2\vec{a}$ (ii) $\vec{\nabla}.\left(\hat{e}.\vec{r}\right)\hat{e} = 1$

(iii) $\vec{\nabla}\times\left(\frac{\hat{k}}{r}\right) = \frac{-\hat{i}y+\hat{j}x}{r^3}$ (iv) $\nabla.\left\{\vec{a}\times\left(\vec{r}\times\vec{a}\right)\right\} = 2a^2$

Given: $\hat{e}$ is an unit vector and $\vec{a}$ is a constant vector.

15. Find the values of a, b, c so that the function $\vec{F} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational **[Ans.** $a = 4, b = 2, c = -1$ **]**

16. Show that, $\vec{v} = (x+2y+4z)\hat{i} + (2x-3y-z)\hat{j} + (4x-y+2z)\hat{k}$ is irrotational. Find the corresponding scalar potential '$\phi$'. **[Ans.** $\phi = \frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz$ **]**

17. Prove that, if a rigid body is rotating with a constant angular velocity, then the curl of linear velocity at any point gives twice its angular velocity.

18. Show that $r^n\vec{r}$ is irrotational vector for any value of n, but it is solenoidal for $n = -3$ only.

19. Find an equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point $(1,-1,2)$. **[Ans.** $7x - 3y + 8z = 26$ **]**

20. Given: $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when $t = 2$ and $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ when $t = 3$. Show that, $\int_2^3 \vec{r}.\frac{d\vec{r}}{dt}dt = 10$

21. Evaluate $\int \vec{F}.d\vec{r}$ where $\vec{F} = C\left[-3a\sin\theta\cos\theta\hat{i} + a\left(2\sin\theta - 3\sin^2\theta\right)\hat{j} + b\sin 2\theta\hat{k}\right]$ and the curve is given by, $\vec{r} = a\cos\theta\hat{i} + a\sin\theta\hat{j} + b\theta\hat{k}$ ($\theta$ is varying from $\frac{\pi}{4}$ to $\frac{\pi}{2}$) **[Ans.** $\frac{1}{2}c\left(a^2+b^2\right)$ **]**

22. The acceleration of a particle at any time $t \geq 0$ is given by, $\vec{a} = 12\cos 2t\ \hat{i} - 8\sin 2t\ \hat{j} + 16t\ \hat{k}$

If velocity $\vec{v}$ and displacement $\vec{r}$ are zero, at $t = 0$; find $\vec{v}$ and $\vec{r}$ at any time.

**[Ans.** (i) $6\sin 2t\ \hat{i} + (4\cos 2t - 4)\hat{j} + 8t^2\hat{k}$, (ii) $(3 - 3\cos 2t)\hat{i} + (2\sin 2t - 4t)\hat{j} + \frac{8}{3}t^3\hat{k}$ **]**

23. If $\vec{A} = \left(3x^2+6y\right)\hat{i} - 14yz\ \hat{j} + 20\ xz^2\ \hat{k}$, evaluate $\int \vec{A}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the following paths:

(i) $x = t, y = t^2, z = t^3$.

(ii) The straight lines from $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$

(iii) The straight line joining (0, 0, 0) and (1, 1, 1) **[Ans.** (i) 5, (ii) 23/3, (iii) 13/3]

24. Find $I = \int (x\,dy - y\,dx)$ over the

(i) Straight line $y = x$ from (0, 0) to (1, 1) (ii) Parabola $y = x^2$ from (0, 0) to (1, 1)

(iii) Circle $x = a\cos t,\ y = 1 + \sin t;\ \frac{-\pi}{2} \le t \le 0$ **[Ans.** (i) 0, (ii) 1/3, (iii) $a\left(\frac{\pi}{2} - 1\right)$]

25. Show that, $\int_C \left[-\frac{y}{x^2+y^2}\hat{i} + \frac{x}{x^2+y^2}\hat{j}\right].d\bar{r} = 2\pi$, where 'C' is the circle $x^2 + y^2 = 1$ in the x-y plane, described in counter clockwise direction.

26. Find $\int \bar{F}.d\bar{r}$ along the curve defined by $x^2 = 4y$ and $3x^2 = 8z$ from $x = 0$ to $x = 2$, where $\bar{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ **[Ans.** 441/40]

27. Find the total work done in moving a particle in a force field given by $\bar{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the curve $x = t^2 + 1,\ y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$ **[Ans.** 303]

28. (i) Show that, $\bar{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force fields.

(ii) Find the scalar potential.

(iii) Find the work done in moving an object in this field from (1, –2, 1) to (3, 1, 4)

**[Ans.** (ii) $x^2y + xz^3 + C$, (iii) 202 units]

29. Find the circulation of the field $\bar{F} = -x^2y\hat{i} + xy^2\hat{j} + (y^3 - x^3)\hat{k}$, around the curve the curve C, where C is the intersection of the sphere $x^2 + y^2 + z^2 = 25$ and the plane $z = 3$. **[Ans.** $128\pi$]

30. If 'f' is a scalar function which is a solution of laplace equation $\nabla^2 f = 0$ in a volume 'V' bounded by a piecewise smooth surface 'S', then apply Gauss divergence theorem to $\bar{u} = \nabla\bar{f}$ to prove that, $\iint_S \hat{n}.\nabla f\ ds = 0$

31. Using Gauss divergence theorem, evaluate $\iint_S (x^3 dydz + y^3 dzdx + z^3 dxdy)$, where 'S' is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ **[Ans.** $\frac{12}{5}\pi a^5$]

32. Consider the following vector field: $\bar{V} = xz^2\hat{i} - yz^2\hat{j} + z(x^2 - y^2)\hat{k}$

(a) Calculate the volume integral of the divergence of $\bar{V}$ over the region defined by $-a \le x \le a$, $-b \le y \le b$ and $0 \le z \le c$.

(b) Calculate the flux of $\bar{V}$ out of the region through the surface at $z = c$.

(c) Hence deduce the net flux through the rest of the boundary of the region.

[Ans. (a) $\frac{4}{3}abc\left(a^2-b^2\right)$, (b) $\frac{4}{3}abc\left(a^2-b^2\right)$, (c) 0]

33. Calculate the flux of the vector field $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ through the surface bounded by the region $x^2+y^2=4, z=0, z=3$. [Ans. $84\pi$]

34. Evaluate $\iint_S \left(\nabla\times\vec{F}\right).\overline{ds}$ for $\vec{F}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$, where 'S' is upper half surface of the sphere $x^2+y^2+z^2=1$ and 'C' is its boundary. [Ans. $\pi$]

35. Evaluate $\iint_S \left(\nabla\times\vec{F}\right).\overline{ds}$, where $\vec{F}=(3x-2y)\hat{i}+x^2z\hat{j}+y^2(z+1)\hat{k}$ for a plane rectangular area with vertices at (0, 0), (1, 0), (1, 2), (0, 2) in the xy plane. [Ans. 4]

36. (a) Consider a constant vector field $\vec{v}=v_0\hat{k}$. Find any one of the many possible vectors $\vec{u}$ for which $\vec{\nabla}\times\vec{u}=\vec{v}$.

(b) Calculate the flux associated with the field $\vec{v}$ through the hemispherical surface defined by

$x^2+y^2+z^2=r^2, z>0$ [Ans. (a) $v_0x\hat{j}$, (b) $\pi r^2 v_0$]

37. Evaluate $\int_C \left(x^2y\,dx+x^2dy\right)$, where 'C' is boundary described counter-clockwise direction of the triangle with vertices (0, 0), (1, 0), (1, 1) [Ans. 5/12]

38. Evaluate $\oint_C \left(xy+y^2\right)dx+x^2dy$, where 'C' is the closed curve of the region bounded by $y=x^2$ and $y=x$

[Ans. -1/12]

39. Evaluate $\oint_C \left[\left(2x^2-y^2\right)dx+\left(x^2+y^2\right)dy\right]$, where 'C' is the boundary of the surface enclosed by x-axis and the semicircle $y=\left(1-x^2\right)^{1/2}$. [Ans. 4/3]

40. Find the mass of a metallic body of varying density $\rho(x,y,z)=\rho_0+xyz$, bounded by the planes $x=0, y=0$, $z=0$ and the sphere $x^2+y^2+z^2=a^2$ lying in the the first octant. [Ans. $\frac{1}{6}\rho_0\pi a^3+\frac{a^6}{48}$]

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# CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
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## Subject: PHYSICAL SCIENCES (NET-JRF/GATE)

## Section: MATHEMATICAL PHYSICS

## Assignment-3: MATRIX ALGEBRA

1. Let $a_{ij}$ denote the element of the $i^{th}$ row and $j^{th}$ column in a 3 × 3 determinant $(1 \le i \le 3, 1 \le j \le 3)$ and let $a_{ij} = -a_{ji}$ of every $i$ and $j$. Then the determinant has all the principal diagonal elements as   Skew matrix

   (a) 1   (b) – 1   [✓ (c) 0]   (d) none

2. Which of the following statements is wrong?   H K doo

   (a) Trace of a unit matrix is always a fixed natural number
   (b) Trace of the sum of two matrices is equal to sum of their traces
   (c) Trace of the scalar matrix is some multiple of its order
   (d) $\text{Tr}(AB) = \text{Tr}(BA)$

3. If A and B are matrices of order $3 \times 2$ and C is a matrix of order $2 \times 3$, then which of the following matrices is not defined?

   (a) $A^T + B$   (b) $A^T$ and $B^T$   (c) $A^T + C$   (d) $B + C^T$

4. If $A = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$, then $A^{10}$ equals

   [✓ (a)] $\begin{bmatrix} 1 & 10\alpha \\ 0 & 1 \end{bmatrix}$   (b) $\begin{bmatrix} 1 & 10\alpha \\ 0 & 10 \end{bmatrix}$   (c) $\begin{bmatrix} 1 & \alpha^{10} \\ 0 & 1 \end{bmatrix}$   (d) $\begin{bmatrix} 1 & \alpha^{10} \\ 0 & 10 \end{bmatrix}$

5. Consider the matrix $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $B^{1023}$ equals

   (a) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$   [✓ (b)] $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$   (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   (d) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

α 6. Which of the following is true for the matrix $\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$?

   (a) It is an idempotent matrix   (b) It is nilpotent matrix of order 3
   (c) It is an involutory matrix   (d) None of the above

7. $(x\ y)\begin{pmatrix}5 & -7\\ 7 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=15$. This matrix equation represents

(a) a circle of radius $\sqrt{15}$ units
(b) an ellipse with semi-minor axis $\sqrt{5}$ units
[✓ (c) an ellipse with semi-major axis $\sqrt{5}$ units]
(d) a hyperbola

8. Which of the following statements is true for the matrix $\begin{pmatrix}ab & b^2\\ -a^2 & -ab\end{pmatrix}$?

[✓ (a) It is a idempotent matrix]
(b) It is a nilpotent matrix of order 2
(c) It is an involutary matrix
(d) It is a periodic matrix of order 2

9. The product of the two matrices $\begin{pmatrix}\cos^2\theta & \cos\theta.\sin\theta\\ \cos\theta.\sin\theta & \sin^2\theta\end{pmatrix}$ and $\begin{pmatrix}\cos^2\phi & \cos\phi.\sin\phi\\ \cos\phi.\sin\phi & \sin^2\phi\end{pmatrix}$ is zero. Which of the following statements is true ?

[✓ (a) $\theta-\phi$ is odd multiple of $(\pi/2)$]
(b) $\theta-\phi$ is even multiple of $(\pi/2)$ A
(c) $\theta-\phi$ is odd multiple of $\pi$
(d) $\theta-\phi$ is even multiple of $\pi$

10. If each element of '$k\times k$' square matrix A can be chosen as a Boolean Variable, then one can construct 'M' number of different combinations of matrix 'A'. Therefore, 'M' is equal to

(a) $2k$
[✓ (b) $2^{k^2}$]
(c) $k^2$
(d) $2^k$

11. If A and B $n\times n$ square symmetric matrices, then AB will be symmetric in nature if

[✓ (a) A and B both are triangular]
(b) A and B both are non-singular
(c) A and B both commute
(d) A and B both anti-commute

12. The symmetric part of $P=\begin{pmatrix}a\\ b\end{pmatrix}\begin{pmatrix}a-2 & b\end{pmatrix}$ is

(a) $\begin{pmatrix}a^2-2 & ab-1\\ ab-1 & b^2-2\end{pmatrix}$
(b) $\begin{pmatrix}a(a-2) & b\\ b & b^2\end{pmatrix}$
(c) $\begin{pmatrix}a(a-1) & b(a-1)\\ b(a-1) & b^2\end{pmatrix}$
[✓ (d) $\begin{pmatrix}a(a-2) & b(a-1)\\ b(a-1) & b^2\end{pmatrix}$]

13. To convert a Hermitian matrix into skew-Hermitian one, the Hermitian matrix must be multiplied by

(a) – 1
[✓ (b) i]
(c) – i
(d) None of the above

14. Suppose the matrix $B^{\dagger}AB$ is Hermitian in nature. Which of the following statement is true ?

[✓ (a) A is a hermitian matrix]
(b) B is a hermitian matrix
(c) A is a skew hermitian matrix
(d) B is a skew hermitian matrix

15. Given two (n x n) matrices $\hat{p}$ and $\hat{Q}$ such that $\hat{p}$ is hermitian and $\hat{Q}$ is skew hermitian. Which one of the following combiantions of $\hat{p}$ and $\hat{Q}$ is necessarily a hermitian matrix?

(a) $\hat{P}\hat{Q}$ [✓ (b) $i\hat{P}\hat{Q}$] [✓ (c) $\hat{P}+i\hat{Q}$] (d) $\hat{P}+\hat{Q}$

16. Which of the following statements is true?

[✓ (a) Product of two orthogonal matrices A and B will be orthogonal if A and B commutes]

(b) Product of two orthogonal matrices A and B will be always orthogonal

(c) Product of two orthogonal matrices A and B will be never orthogonal

(d) None of these

17. The matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

(a) Symmetric (b) Skew Hermitian (c) Orthogonal (d) Unitary

18. If 'I' is an identity matrix of order 'k x k' and adj (2I) = $2^k$ I, then 'k' is equal to

(a) 1 (b) 2 [✓ (c) n-1] (d) n

19. Suppose A is n x n square invertible matrix. Which of the following statements is true for A (adj A)?

(a) It is a null matrix of order 'n' (b) It is an unit matrix of order 'n'

[✓ (c) It is a scalar matrix of order 'n'] (d) None of the these

20. If 'A' is $4 \times 4$ matrix with determinant 5, then the deteminant of adj.(A) is equal to

[✓ (a) 5] (b) 25 (c) 125 (d) 625

21. If 'A' is a $3 \times 3$ matrix with determinant 2, then the determinant of $adj.[adj.[adj.(A^{-1})]]$ is equal to

(a) $\frac{1}{512}$ (b) $\frac{1}{1024}$ (c) $\frac{1}{128}$ [✓ (d) $\frac{1}{256}$]

22. For which value of $\lambda$, the following matrix $A = \begin{bmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{bmatrix}$ will be invertible in nature ?

(a) $\lambda = 4$ (b) $\lambda \neq 8$ (c) $\lambda \neq 4$ (d) none

23. If 'U' is unit matrix and the matrix 'A' is defined as $A = 1 - 2UU^T$, then $A^{-1}$ is equal to

(a) $1-2UU^T$ (b) $1+2UU^T$ (c) $2UU^T - 1$ [✓ (d) $4UU^T$]

24. If a square matrix A has an eigenvalue $\lambda$, then the eigenvalue of the matrix $(kA)^T$ ( where $k \neq 0$) is

(a) $\lambda/k$ [✓ (b) $k/\lambda$] (c) $\lambda k$ (d) None of these

25. The eignvalues of the matrix. $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are

(a) 1, 3, 1 [✓ (b) 2, 3, 2] (c) 5, 1, 1 (d) 3, 5, 2

26. Two eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 & -\frac{a}{2} \\ 0 & 1 & 0 & -\frac{a}{2} \\ 0 & 0 & 1 & -\frac{a}{2} \\ 0 & 0 & 0 & a \end{bmatrix}$ are

(a) 1 and a [✓ (b) 1 and – a] (c) $\frac{1}{2}$ and $-\frac{a}{2}$ (d) $\frac{1}{2}$ and $\frac{a}{2}$

27. A matrix is given by $M = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of M are

6

[✓ (a) real and positive] (b) purely imaginary with modulus 1

(c) real and negative (d) complex number with modulus 1

28. Consider a matrix $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$. A 3-dimensional basis formed by eigenvectors of M is

(a) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ [✓ (b) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$] (c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

29. The eigenvalues of $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ are

(a) (1,i,1,i) (b) (1,-1,0,0) (c) (1,1,1,1) (d) (1,1,i,-i)

30. The trace of the matrix of a 2 x 2 matrix is 1 and determinant 1. Which of the following has to be true?

(a) one of the eigenvalues is 0 (b) one of the eigenvalue is 1

(c) both of the eigenvalues are 1 (d) neither of the eigenvalues are 1

$\lambda_1\lambda_2 = 1$

$\lambda_1 = 1$

$\lambda_2 = \cancel{0}$

31. For which of the following matrices both eigenvalues are positive?

(a) $\begin{pmatrix} -7 & 2 \\ 2 & -7 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & -3 \\ -3 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix}$

32. For an N X N matrix consisting of all ones,

(a) all eigenvalues are 1 [✓ (b) all eigenvalues are 0]

(c) eigenvalues are 1,2,3,...... N (d) one eigenvalue is N and others are 0

33. If trace of matrix M is 11 and its determinant is 36, then the largest eigenvalue of M is equal to

(a) 4 [✓ (b) 6] (c) 9 (d) 12

34. A 2×2 matrix 'A' has eigenvalues $e^{i\pi/4}$ and $e^{i\pi/5}$ such that $A^n = I$. The smallest possible value of 'n' is

[✓ (a) 20] (b) 30 (c) 40 (d) 80

35. The degenerate eigenvalues of the matrix $M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ is:

(a) 2 (b) 3 [✓ (c) 4] (d) 5

36. Let A be a 3 x 3 hermitian matrix which satisfies the matrix equation $M^2 - 7M + 12I = 0$, where I refers to the identity matrix. What is the determinant of the matrix M given that the trace is 10?

(a) 27 (b) 36 (c) 48 (d) 64

37. Consider a 3 x 3 matrix of the following form:

$$\begin{pmatrix} a_1^2 & a_1a_2 & a_1a_3 \\ a_1a_2 & a_2^2 & a_2a_3 \\ a_1a_3 & a_2a_3 & a_3^2 \end{pmatrix}$$

The number of zero eigenvalues for this matrix is

(a) 0 [✓ (b) 1] (c) 2 [✓ (d) 3]

38. The eigenvalues of the matrix $A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ are

$Tr = 0$

$\lambda^2 = 1$

$= \lambda = \pm 1$

$= \lambda = -1, 1, -1\ -1$

(a) {1,1,1,1} [✓ (b) {1,1,-1,-1}] (c) $\{0, 0, \sqrt{2}, -\sqrt{2}\}$ (d) $\{\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}$

39. The eigen values of $(A^4+3A-2I)$, where $A=\begin{bmatrix}1&1&1\\0&2&1\\0&0&3\end{bmatrix}$, are

(a) 2, 20, 88 [✓ (b) 1, 2, 3] (c) 2, 20, 3 (d) 1, 20, 88

40. If a matrix A is such that $3A^3+2A^2+5A+I=0$, then $A^{-1}$ is equal to

(a) $-(3A^2+2A+5)$ (b) $3A^2+2A+5$ (c) $3A^2-5A-5$ [✓ (d) None of these]

41. Let; 'M' be a n x n diogonalizable matrix which satisfies the equations $M^2=M$, then Det (M) is equal to

(a) 0 (b) 1 (c) n (d) n-1

42. Let; 'P' be a 3 x 3 Hermitian matrix which satisfies the matrix equation $M^2-5M+6I=0$. Which of the following are the possible eigenvalues of M?

(a) (0,1,2) (b) (0,2,3) (c) (1,3,4) (d) (2,3,4)

43. The matrix $\begin{pmatrix}1&1&0\\1&1&1\\0&1&1\end{pmatrix}$ can be related by a similarity transformation to a matrix

(a) $\begin{pmatrix}1&1&1\\1&-1&1\\1&1&0\end{pmatrix}$ (b) $\begin{pmatrix}2&1&0\\1&-1&-1\\0&1&2\end{pmatrix}$ (c) $\begin{pmatrix}1&-1&0\\-1&1&1\\0&1&-1\end{pmatrix}$ (d) $\begin{pmatrix}1&-1&0\\-1&1&1\\0&1&1\end{pmatrix}$

44. Two different 2 x 2 matrices A and B found to be have same eigenvalues. It is then correct to state that $A=SBS^{-1}$ where S can be a

(a) arbitary 2 x 2 matrix (b) hermitian 2 x 2 matrix

(c) traceless 2 x 2 matrix (d) unitary 2 x 2 matrix

45. The trace of a real 4 x 4 matrix $U=\exp(A)$, where $A=\begin{pmatrix}0&0&0&\frac{\pi}{4}\\0&0&-\frac{\pi}{4}&0\\0&\frac{\pi}{4}&0&0\\-\frac{\pi}{4}&0&0&0\end{pmatrix}$ is equal to

(a) $2\sqrt{2}$ (b) $\frac{\pi}{4}$ (c) 0 (d) 2

46. The rank of the matrix $\begin{pmatrix} \Omega & -1 & 0 \\ 0 & \Omega & -1 \\ -1 & 0 & \Omega \end{pmatrix}$ is 2, for $\Omega$ equal to

(a) 0 (b) 1 (c) 2 (d) 4

47. The rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}$ (a, b, c, being all real), is 3 if

(a) $a = b = c$
(b) a, b, c are all different but $a + b + c = 0$
(c) Two of the numbers a, b, c are equal but are different from the third
(d) a, b, c are all different and $a + b + c \neq 0$

48. The values of a and b for which the following systems of linear equation.

$$ax + y + 3z = a$$
$$2x + by - z = 3$$
$$5x + 7y + z = 7$$

has an infinite number of solutions, are

(a) $a = 1, b = 1$ (b) $a = 1, b = 3$ (c) $a = 2, b = 3$ (d) $a = 2, b = 1$

49. The value of 'a' for which the system of equations

$$a^3x + (a+1)^3 y + (a+2)^3 z = 0$$
$$ax + (a+1)y + (a+2)z = 0$$
$$x + y + z = 0$$

has a non-zero solution, is equal to

(a) 1 (b) 0 (c) -1 (d) None of these

## CSIR Previous Year Questions

Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ **[CSIR June - 2011]**

50. The eigenvalues of M are

(a) 0,1,2 (b) 0,0,3 (c) 1,1,1 (d) -1,1,3

51. The exponential of M simplifies ( I is the 3 x 3 identity matrix)

(a) $e^M = I + \left(\frac{e^3 - 1}{3}\right) M$ (b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$ (d) $e^M = (e - 1) M$

52. A 3 x 3 matrix M has $Tr[M] = 6, Tr[M^2] = 26, Tr[M^3] = 90$. Which of the following can be possible set of eigenvalues of M? **[CSIR Dec - 2011]**

(a) {1,1,4} (b) {-1,0,7} (c) {-1,3,4} (d) {2,2,2}

53. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ **[CSIR June - 2012]**

(a) (1,4,9) (b) (0,7,7) (c) (0,1,13) (d) (0,0,14)

54. The eiegenvalues of the anti-symmetric matrix $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where $n_1, n_2, n_3$ are components of an unit vector, are **[CSIR June - 2012]**

(a) 0, -i, i (b) 0, 1, -1 (c) 0, 1+i, -1-i (d) 0, 0, 0

55. A 2 x 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = I$ **[CSIR Dec - 2012]**

(a) 20 (b) 30 (c) 60 (d) 120

56. Given a 2 x 2 unitary matrix satisfying $U'U = UU' = I$ with $\det U = e^{i\varphi}$, one can construct a unitary matrix V $(V'V = VV' = I)$ with det V = 1 from it by **[CSIR Dec - 2012]**

(a) Multiplying U by $e^{-i\varphi/2}$ (b) Multiplying a single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column by $e^{-i\varphi/2}$ (d) Multiplying U by $e^{-i\varphi}$

57. Consider a n x n (n > 1) matrix A, in which $A_{ij}$ is the product of the indices i and j (namely $A_{ij} = ij$) The matrix A **[CSIR Dec - 2013]**

(a) has one degenerate eigenvalue with degeneracy (n-1)

(b) has two degenerate eigenvalues with degeneracies 2 and (n-2)

(c) has one degenerate eigenvalues with degeneracy n

(d) does not have any degenerate eigenvalues

58. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are **[CSIR June - 2014]**

(a) -5,-2,7 (b) -7,0,7 (c) -4i,2i,2i (d) 2,3,6

59. The matrices

$$A=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}, B=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, C=\begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}$$

satisfy the commutation relation **[CSIR June - 2014]**

(a) $[A,B]=B+C,[B,C]=0,[C,A]=B+C$

(b) $[A,B]=C,[B,C]=A,[C,A]=B$

(c) $[A,B]=B,[B,C]=0,[C,A]=A$

(d) $[A,B]=C,[B,C]=0,[C,A]=B$

## ANSWER KEY

**1.(c) 2.(a) 3.(a) 4.(a) 5.(b) 6.(c) 7.(c) 8.(b) 9.(a) 10.(b)**
**11.(c) 12.(d) 13.(b)(c) 14.(a) 15.(c) 16.(b) 17.(d) 18.(c) 19.(c) 20.(c)**
**21.(d) 22.(b) 23.(a) 24.(c) 25.(c) 26.(a) 27.(d) 28.(b) 29.(b) 30.(d)**
**31.(c) 32.(d) 33.(b) 34.(c) 35.(d) 36.(b) 37.(c) 38.(b) 39.(a) 40.(d)**
**41.(a) 42.(b)(d) 43.(d) 44.(d) 45.(a) 46.(b) 47.(d) 48.(b) 49.(c) 50.(b)**
**51.(a) 52.(c) 53.(d) 54.(a) 55.(c) 56.(a) 57.(a) 58.(b) 59.(d)**

60. Determine the values of a, b, c such that $\begin{bmatrix}0&2b&c\\a&b&-c\\a&-b&c\end{bmatrix}$ is orthogonal.

61. Find the eigenvalues and normalized eigen vectors of the following matrices:

(i) $\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$ (ii) $\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$ (iii) $\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$

62. Find the characteristic equation, of the following matrix and verify Cayley-Hamilton theorem.

$$\begin{bmatrix}1&2&3\\2&-1&4\\3&1&1\end{bmatrix}$$

63. Show that, $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ satisfies his own eigen equation and hence find $A^{-1}$.

64. Evaluate $M^{50}$ for the matrix, $M = \begin{bmatrix} 4/3 & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{bmatrix}$

65. Diagonalise the following matrices:

(i) $\begin{bmatrix} 4/3 & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{bmatrix}$ (ii) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

66. Evaluate $e^A$ for: (i) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, (ii) $\begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$

67. If $\alpha$ is the characteristic root of a non-singular matrix, then show that, $\frac{|A|}{\alpha}$ is a characteristic root of (adj. A).

68. Represent the following transformations

$$x_1 = 3y_1 + 2y_2, x_2 = -y_1 + 4y_2, y_1 = z_1 + 2z_2, y_2 = 3z_1$$

by the use of matrices and findthe composite transformation which expresses $x_1, x_2$ in terms of $z_1, z_2$.

69. Consider the following transformation in three dimensions:

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$
$$z' = z$$

(i) Write down the transformation matrix [A].

(ii) Show that, $[A(\theta_1)] \times [A(\theta_2)] = [A(\theta_1 + \theta_2)]$

(iii) Is $A(\theta)$ is unitary?

(iv) What are the eigenvalues of $A(\theta)$?

70. Find the Characteristic equation of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and hence find

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

71. The matrix $A = \begin{pmatrix} p & r \\ r & q \end{pmatrix}$ is transformed to the diagonal form $D = M^{-1}AM$, where

$M = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$. Find the value of $\phi$ which gives the diagonal transformation.

**CAREER ENDEAVOUR**
ACADEMY PRIVATE LIMITED
Institute for NET-JRF/GATE & IIT-JAM Exams

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment-4: COMPLEX ANALYSIS

1. If $x + iy = \left(1 - i\sqrt{3}\right)^{100}$, then the co-ordinate of point $P(x, y) =$
(a) $\left(2^{99}, 2^{99}\sqrt{3}\right)$ (b) $\left(2^{99}, -2^{99}\sqrt{3}\right)$ [✓ (c) $\left(-2^{99}, 2^{99}\sqrt{3}\right)$] (d) none of these

2. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n$ is real
(a) 3 [✓ (b) 2] (c) 4 (d) none ofr these

3. The sum of the following series:
$$i^2 + i^4 + i^6 \ldots \text{ up to } (2n+1) \text{ terms}$$
will be
(a) $i$ (b) $-i$ (c) 1 [✓ (d) $-1$]

4. If 1, $\omega$, $\omega^2$ are the cube roots of unity, then $\left(1 - \omega + \omega^2\right)^5 + \left(1 + \omega - \omega^2\right)^5$ will be
(a) 4 (b) 8 (c) 16 [✓ (d) 32]

5. Multiplying a complex number $z$ by $1 + i$ rotates the radius vector of $z$ by an angle of
(a) 90° clockwise [✓ (b) 45° anticlockwise]
(c) 45° clockwise (d) 90° anticlockwise

6. If 1, $\omega$, $\omega^2$ be cube roots of unity, then the sum of the following series:
$$\left(1 - \omega + \omega^2\right)\left(1 - \omega^2 + \omega^4\right)\left(1 - \omega^4 + \omega^8\right) \ldots \text{ to } 2n \text{ factors}$$
will be
(a) $2n$ [✓ (b) $2^{2n}$] (c) 0 (d) 1

7. If $z = (\lambda + 3) + i\sqrt{5 - \lambda^2}$ ($\lambda$ is a real parameter and $i = \sqrt{-1}$), then the locus of $z$ will be
[✓ (a) circle] (b) ellipse (c) parabola (d) hyperbola

8. If sum and products of two complex numbers are real, then which of the following statements is/are true?
[✓ (a) both complex numbers are real] (b) both complex numbers are purely imaginary
(c) one of the complex numbers are real (d) both are complex conjugate of each other

9. The expression $\sinh^{-1} x$ can be simplified to
[✓ (a) $\ln\left(x + \sqrt{x^2 + 1}\right)$] (b) $\ln\left(x + \sqrt{x^2 - 1}\right)$ (c) $\ln\left(x - \sqrt{x^2 - 1}\right)$ (d) $\ln\left(x - \sqrt{x^2 + 1}\right)$

10. The value of the following limit:

$$\underset{z\to 0}{Lt}\frac{\operatorname{Re}\left(z^2\right)+\operatorname{Im}\left(z^2\right)}{z^2}$$

(a) 0 (b) 1 (c) -1 [✓ (d) does not exist]

11. The value of the following limit:

$$\underset{z\to \infty}{Lt}\frac{iz^3+iz-1}{(2z+3i)(z-i)^2}$$

[✓ (a) $\frac{i}{2}$] (b) $-\frac{i}{2}$ (c) $\frac{1}{2}$ (d) does not exist

12. Which of the following statement is true for the compex function $f(z)=\frac{1}{z}$ ?

[✓ (a) $f(z)$ is analytic everywhere] (b) $f(z)$ is analytic everywhere except z = 0

(c) $f(z)$ is analytic nowhere (d) $f(z)$ is analytic everywhere except z = 1

13. Which of the following are not a complex analytic function of z ?

(a) $f(z)=\left(x^2-y^2+2ixy\right)^7(x+iy)^{17}$ (b) $f(z)=\left(x^2-y^2+2ixy\right)^{12}(x-iy)^4$

(c) $f(z)=(x+iy-5)^{13}$ (d) $f(z)=(2x+iy-5)^{19}$

14. Given: $f(z)=x^2+Py^2-2xy+i\left(Qx^2-y^2+2xy\right)$ is analytic in nature. The value of P and Q will be

(a) P = 1, Q = 1, (b) P = -1, Q = 1, (c) P = 1, Q = -1, (d) P = -1, Q = -1

15. The value of m, n such that $f(x,y)=x^2+mxy+ny^2$ will be a harmonic function, will be

(a) n = -1, m = any value (b) n = 1, m = any value

(c) n = 0 , m = -1 (d) n = 0, m = 1

16. The harmonic conjugate function of $u(x,y)=2x(1-y)$ corresponding to a complex analytic function $\omega=u(x,y)+iv(x,y)$ is given $v(x,y)=\alpha x^2+\beta y+\gamma y^2$ (Taking the integration constant to be zero). Which of the following statement is true ?

[✓ (a) $\alpha-\gamma=\beta$] (b) $\alpha+\gamma+\beta=0$ (c) $\alpha+\gamma=\beta$ (d) $\alpha\gamma\beta=1$

17. An electtrostatic field in the x-y plane is given by the potential function $\phi(x,y)=3x^2y-y^3$. The corresponding stream function will be

(a) $-x^3-3x^2y$ (b) $-x^3+3xy^2$ (c) $-x^3-3xy^2$ (d) $x^3-3xy^2$

18. If the real part of a complex analytic function f(z) is given as, $u(x,y)=e^{-2xy}\sin\left(x^2-y^2\right)$, then f(z) can be written as

(a) $ie^{iz^2}+C$ (b) $-ie^{iz^2}+C$ (c) $-ie^{-iz^2}+C$ (d) $ie^{-iz^2}+C$

19. The value of the integral $\int_0^{1+i}\left(x^2-iy\right)dz$ along the path $y=x^2$ is

(a) $\frac{5}{6}+\frac{i}{6}$ (b) $\frac{5}{6}-\frac{i}{6}$ (c) $\frac{1}{6}-\frac{5i}{6}$ (d) $\frac{1}{6}+\frac{5i}{6}$

20. The taylor series expansion of $f(z)=\cos z$ about $z=\frac{\pi}{6}$ will be

(a) $f(z)=\frac{\sqrt{3}}{2}+\frac{1}{2}\left(z-\frac{\pi}{6}\right)+\frac{\sqrt{3}}{4}\left(z-\frac{\pi}{6}\right)^2+\dots\dots\dots\dots$

(b) $f(z)=\frac{\sqrt{3}}{2}-\frac{1}{2}\left(z-\frac{\pi}{6}\right)-\frac{\sqrt{3}}{4}\left(z-\frac{\pi}{6}\right)^2+\dots\dots\dots\dots$

(c) $f(z)=\frac{\sqrt{3}}{2}-\frac{1}{2}\left(z-\frac{\pi}{6}\right)-\frac{\sqrt{3}}{2}\left(z-\frac{\pi}{6}\right)^2+\dots\dots\dots\dots$

(d) $f(z)=\frac{\sqrt{3}}{2}+\frac{1}{2}\left(z-\frac{\pi}{6}\right)-\frac{\sqrt{3}}{2}\left(z-\frac{\pi}{6}\right)^2-\dots\dots\dots\dots$

21. If $\sin^{-1}z$ is expanded in terms of z, it can be written as

(a) $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+\dots\dots\dots\dots$ (b) $z-\frac{z^3}{6}+\frac{3z^5}{40}-\frac{5z^7}{112}+\dots\dots\dots\dots$

(c) $z+\frac{z^3}{3}+\frac{z^5}{5}+\frac{z^7}{7}+\dots\dots\dots\dots$ (d) $z+\frac{z^3}{6}+\frac{3z^5}{40}+\frac{5z^7}{112}+\dots\dots\dots\dots$

22. Suppose a complex function f(z) such that $f(1)=1, f'(1)=1, f''(1)=1$ and all other higher derivatives of f(z) at z =1. The value of f(z) at z = 1/2 will be

(a) 1/4 (b) 1/2 (c) 5/8 (d) 7/8

23. In the Laurent series expansion of $f(z)=\frac{\left(z^2+3z+2\right)}{z(z+2)^3}$ about $z=-2$, the highest negative power (<0) terms of (z + 2) present in the expansion, will be

(a) 0 (b) 1 [✓ (c) 2] (d) 3

24. The Laurent series expansion of the complex function

$$f(z)=\frac{z}{(z-1)(z-2)}$$

in the region $|z-1|>1$ can be expressed as

(a) $\frac{1}{z-1}+\frac{2}{(z-1)^2}+\frac{2}{(z-1)^3}+\frac{2}{(z-1)^4}+\dots\dots\dots\dots\dots$

(b) $\frac{1}{z-1}-\frac{2}{(z-1)^2}+\frac{2}{(z-1)^3}-\frac{2}{(z-1)^4}+$...............

(c) $\frac{1}{z-1}-\frac{2}{(z-1)^2}+\frac{3}{(z-1)^3}-\frac{4}{(z-1)^4}+$...............

(d) $\frac{1}{z-1}-\frac{2}{(z-1)^2}+\frac{1}{(z-1)^3}-\frac{2}{(z-1)^4}+$...............

25. The function $f(z)=(z-3)\sin\left(\frac{1}{z+2}\right)$ has singularity at $z=-2$ of the following nature :

(a) pole of order 1
(b) removable isolated singularity
(c) isolated essential singularity
(d) non-isolated essential singularity

26. The function $f(z)=\frac{1}{1-e^z}$ has singularity at $z=2\pi i$ of the following nature:

[✓ (a) pole of order 1]
(b) removable isolated singularity
(c) isolated essential singularity
(d) non-isolated essential singularity

27. The function $f(z)=(z-3)^n\sin\left(\frac{1}{z-3}\right)$ has a residue of $\frac{1}{120}$ at the point $z=3$. The value of 'n' is equal to

(a) 2
(b) 4
(c) 5
(d) 6

28. The contribution of the point $z=\pi/2$ in evaluation of $\oint_C \frac{\tan z}{z}dz$ (where C is a circle $|z|=2$) is equal to

(a) 0
(b) $-4e^{i\pi/2}$
(c) $4e^{i\pi/2}$
[✓ (d) $-2/\pi$]

29. The value of the integral $\oint_C \frac{z\,dz}{(9-z^2)(z+i)}$, where C is a circle $|z|=2$ in the argand plane, described in the positive sense is equal to

(a) $\pi/2$
(b) $\pi/4$
(c) $\pi/3$
(d) $\pi/5$

30. The value of the integral $\oint_C \frac{3z^2+z+1}{(z^2-1)(z+3)}dz$ around the curve $C:|z|=2$ (where, 'C' is traversed in the clockwise direction) is equal to

(a) $\frac{3\pi i}{4}$
(b) $-\frac{3\pi i}{4}$
(c) $\frac{\pi i}{4}$
(d) $-\frac{\pi i}{4}$

31. The value of the integral $\oint_C \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}dz$, where 'C' is the circle $|z|=3$ traversed in the anticlockwise direction, will be

(a) $2\pi i$
(b) $-2\pi i$
[✓ (c) $4\pi i$]
(d) $-4\pi i$

32. The value of the integral $\int_0^{2\pi}\frac{d\theta}{5+4\cos\theta}$ will be

(a) $\frac{2\pi}{3}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$
(d) $\frac{5\pi}{6}$

33. The value of the integral $\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta - n\theta)\,d\theta$ will be

(a) $\frac{2\pi}{(n+1)!}$ (b) $\frac{2\pi}{n!}$ (c) $\frac{\pi}{n!}$ (d) $\frac{\pi}{(n+1)!}$

34. The value of the integral $\int_{-\infty}^{\infty}\frac{x^2-x+2}{x^4+10x^2+9}dx$ will be

(a) $\frac{5\pi}{12}$ (b) $\frac{5\pi}{24}$ (c) $\frac{7\pi}{12}$ (d) $\frac{7\pi}{24}$

35. The value of the integral $\int_0^{\infty}\frac{\sin mx}{x\left(x^2+a^2\right)}dx$ will be

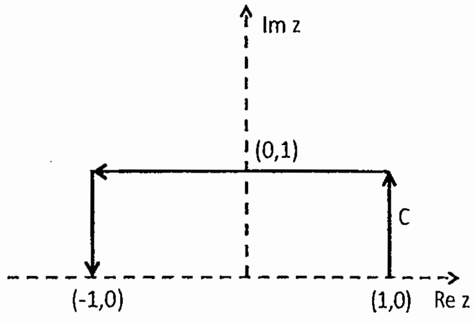
(a) $\frac{\pi}{2a^2}\left(1+e^{-ma}\right)$ (b) $\frac{\pi}{2a^2}\left(1-e^{-ma}\right)$ (c) $\frac{\pi}{2a^2}\left(1-e^{ma}\right)$ (d) $\frac{\pi}{2a^2}\left(1+e^{ma}\right)$

36. The following integral $\int_0^{\infty}\frac{\cos 3x}{\left(x^2+1\right)\left(x^2+4\right)}dx$ has a value.

(a) $\frac{\pi}{2}\left[\frac{e^3}{3}-\frac{e^6}{6}\right]$ (b) $\frac{\pi}{2}\left[\frac{e^{-3}}{3}-\frac{e^{-6}}{6}\right]$ (c) $\pi\left[\frac{e^{-3}}{3}-\frac{e^{-6}}{6}\right]$ (d) $\pi\left[\frac{e^3}{3}-\frac{e^6}{6}\right]$

## CSIR Previous Year Questions

37. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in figure below: **[CSIR June 2011]**



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$ (c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

38. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$? **[CSIR June 2011]**

(a) $(3+x-iy)^7$ [✓ (b) $(1+x+iy)^4(7-x-iy)^3$]

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

39. The first few terms in the Taylor series expansion of the function f(x) = sinx around $x=\frac{\pi}{4}$ are **[CSIR Dec 2011]**

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\ldots\ldots\ldots\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\ldots\ldots\ldots\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\ldots\ldots\ldots\right]$

(d) $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\ldots\right]$

40. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1\le|z|\le2$ and around z = 1is **[CSIR June 2012]**

(a) $\frac{1}{2}\left[1+z+z^2+\ldots\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\ldots\ldots\right]$

(b) $\frac{1}{1-z}+z+(1-z)^2+(1-z)^3+\ldots\ldots$

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+\ldots\ldots\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+\ldots\ldots\right]$

(d) $2(z-1)+5(z-1)^2+7(z-1)^3+\ldots\ldots$

41. The value of the integral $\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}\cos\left(\frac{\pi t}{2R}\right)dt$ **[CSIR June 2012]**

(a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

42. The taylor series expansion of the function $\ln(\cosh x)$, where x is real, about point x = 0 starts with the following terms: **[CSIR Dec 2012]**

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+\ldots\ldots$

(b) $\frac{1}{2}x^2-\frac{1}{12}x^4+\ldots\ldots$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+\ldots\ldots$

(d) $\frac{1}{2}x^2+\frac{1}{6}x^4+\ldots\ldots$

43. The value of the integral $\oint_C\frac{z^3}{z^2-5z+6}dz$, where C is closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is **[CSIR Dec 2012]**

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

44. With z = x+ iy, which of the following functions f (x,y) is NOT a complex analytic function of z? **[CSIR June 2013]**

(a) $(x+iy-8)^3\left(4+x^2-y^2+2ixy\right)^7$

(b) $(x+iy)^7(1-x-iy)^3$

(c) $\left(x^2-y^2+2ixy-3\right)^5$

(d) $(1-x+iy)^4(2+x+iy)^6$

45. Which of the following functions cannot be the real part of a complex analytic function of z = x + iy?

**[CSIR Dec 2013]**

[✓ (a) $x^2y$] (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2y - y - y^3$

46. Given that the integral $\int_0^\infty \frac{dx}{y^2 + x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{\left(y^2 + x^2\right)^2}$ is

**[CSIR Dec 2013]**

(a) $\frac{\pi}{y^3}$ [✓ (b) $\frac{\pi}{4y^3}$] (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

47. The function $\phi(x, y, z, t) = \cos(z - vt) + \text{Re}\left[\sin(x + iy)\right]$ satisfies the equation

**[CSIR June 2014]**

(a) $\frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi$

(b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\phi$

(d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\phi$

48. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral $\oint_C \frac{dz}{\sin^2 z}$ is

(a) $\infty$ (b) $2\pi i$ (c) 0 (d) $\pi i$

## ANSWER KEY

**1.(c) 2.(b) 3.(d) 4.(d) 5.(b) 6.(b) 7.(a) 8.(a)(d) 9.(a) 10.(d)**

**11.(a) 12.(b) 13.(b)(d) 14.(b) 15.(a) 16.(a) 17.(b) 18.(b) 19.(a) 20.(b)**

**21.(d) 22.(c) 23.(c) 24.(a) 25.(c) 26.(a) 27.(b) 28.(a) 29.(d) 30.(c)**

**31.(c) 32.(a) 33.(b) 34.(a) 35.(b) 36.(b) 37.(c) 38.(b) 39.(b) 40.(b)**

**41.( ) 42.( ) 43.(a) 44.(d) 45.(a) 46.(b) 47.(a) 48.(c)**

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment-5: COMPLEX ANALYSIS

1. If $m=\cos\theta+i\sin\theta$, then prove that $1+m+m^2=e^{i\theta}(1+2\cos\theta)$

2. If $m=e^{i\theta}$, $n=e^{i\phi}$, then show that $\frac{m-n}{m+n}=i\tan\left(\frac{\theta-\phi}{2}\right)$

3. Find the complex numbers 'z' such that $\arg(z+1)=\frac{\pi}{6}$ and $\arg(z-1)=2\frac{\pi}{3}$ [**Ans.** $z=\frac{1}{2}+i\frac{\sqrt{3}}{2}$]

4. Find the area of the triangle enclosed by the vectors z, iz, z − iz. [**Ans.** $\frac{1}{2}|z|^2$]

5. If $2\cos\theta=m+\frac{1}{m}$ and $2\cos\phi=n+\frac{1}{n}$, then prove that $m^pn^q+\frac{1}{m^pn^q}=2\cos(p\theta+q\phi)$

6. If 'n' be a positive integer, prove that $(1+i)^n+(1-i)^n=2^{\frac{n}{2}+1}.\cos\frac{n\pi}{4}$

7. Check whether the following complex functions are analytic or not:

   (i) $e^x(\cos y+i\sin y)$ (ii) $\sin z$ (iii) $e^{\sin z}$ (iv) $r^ne^{in\theta}$

8. Determine the imaginary or real part of the complex analytic function whose real or imaginary part are the following:

   (i) $u(x,y)=x^3-3xy^2+3x^2-3y^2+1$ (ii) $v(x,y)=6xy-5x+3$

   (iii) $v(r,\theta)=r^2\cos2\theta-r\cos\theta+2$ (iv) $u(r,\theta)=-r^3\sin3\theta$

   [Ans. (*i*) $3x^2y+6xy-y^3+C$, (*ii*) $3x^2-3y^2+5y+C$, (*iii*) $-r^2\sin2\theta+r\sin\theta+C$, (*iv*) $r^3\cos3\theta+C$]

9. Use Milne Thomson Method, find the complex analytic function $f(z)$ for which either the real part or the imaginary part is given as following:

   (i) $v=e^x(x\cos y-y\sin y)$ (ii) $u-v=(x-y)(x^2+4xy+y^2)$ (iii) $v=\tan^{-1}\left(\frac{y}{x}\right)$

   [Ans. (i) $ize^Z+C$, (ii) $-iz^3+C$, (iii) $\ln z+C$]

10. Show that, the function $u=\sin x.\cosh y+2\cos x.\sinh y+x^2-y^2+4xy$ is harmonic in nature and if the above function is the real part of a complex analytic function, then determine the corresponding analytic function. [**Ans.** $(1-2i)(\sin z+z^2)+C$]

11. If $f(z)=u+iv$ is an analytic fuinction of $z=x+iy$ and $u-v=\dfrac{e^{y}-\cos x+\sin x}{\cosh y-\cos x}$, find f(z) subject to the condition $f\left(\dfrac{\pi}{2}\right)=\dfrac{3-i}{2}$. **[Ans.** $f(z)=\cot\left(\dfrac{z}{2}\right)+\dfrac{1-i}{2}$**]**

12. Find the first three terms of the Taylor's series expansion of $f(z)=\dfrac{z}{(z-3)(z-4)}$ about $z=2$. Find the region of convergence.

13. Find the first three terms of the Taylor's series expansion of $f(z)=\dfrac{2z^3+1}{z^2+z}$ about $z=1$.

14. Expand $f(z)=\tan^{-1} z$ in powers of z. $(|z|<1)$.

15. Expand the following function in Laurent series:

(i) $f(z)=\dfrac{1}{z(z+2)^3}$ about the pole $z=-2$.

(ii) $f(z)=\dfrac{1}{z^2-3z+2}$ in the region (i) $|z|<1$, (ii) $|z|>3$

16. Discuss the nature of singularity regarding the following complex functions:

(i) $f(z)=\dfrac{1}{\cos z-\sin z}$ (ii) $f(z)=\dfrac{\cot \pi z}{(z-a)^2}$ at $z=a$, $z=\infty$ (iii) $f(z)=\tan\left(\frac{1}{z}\right)$ at $z=0$

(iv) $f(z)=\sin\left(\dfrac{1}{1-z}\right)$ (v) $f(z)=\dfrac{1-e^{z}}{1+e^{z}}$ at $z=\infty$ (vi) $f(z)=\dfrac{z}{1+z^4}$

(vii) $f(z)=\dfrac{\sin z}{(z-\pi)^2}$

17. Determine the poles and corresponding residues of following complex functions:

(i) $f(z)=\dfrac{1-2z}{z(z-1)(z-2)}$ (ii) $f(z)=\dfrac{z^2}{z^2+a^2}$ (iii) $f(z)=\dfrac{ze^{iz}}{z^4+a^4}$

(iv) $f(z)=\dfrac{z^2}{(z+1)^2(z-2)}$ (v) $f(z)=\dfrac{z}{\sin z}$ (vi) $f(z)=\dfrac{e^z}{z^2(z^2+9)}$ at (0,-3)

(vii) $f(z)=\dfrac{2z+1}{z^2-z-2}$ (viii) $f(z)=z^2e^{1/z}$ (ix) $f(z)=z^2\sin\left(\dfrac{1}{z}\right)$

18. Evaluate the following integral:

(i) $\displaystyle\oint_C \frac{z}{z^2-3z+2}\,dz$; C: circle of $|z-2|=\dfrac{1}{2}$ (ii) $\displaystyle\oint_C \frac{dz}{z(z+\pi i)}$; C: circle of $|z+3i|=1$

(iii) $\displaystyle\oint_C \frac{\cosh(\pi z)}{z(z^2+1)}\,dz$; C: circle of $|z|=2$ (iv) $\displaystyle\oint_C \frac{4-3z}{z(z-1)(z-2)}\,dz$; C: circle of $|z|=\dfrac{3}{2}$

(v) $\oint_C \frac{1}{\sinh z} dz$; C : circle of $|z| = 4$  (vi) $\oint_C \frac{e^{2z} + z^2}{(z-1)^5} dz$; C : circle of $|z| = 2$

(vii) $\oint_C z^4 e^{1/z} dz$; C : circle of $|z| = 1$  (viii) $\oint_C \frac{\sin z}{z^6} dz$; C: circle of $|z| = 2$

[*Ans.* (*i*) $4\pi i$, (*ii*) $-2$, (*iii*) $4\pi i$, (*iv*) $2\pi i$, (*v*) $-2\pi i$, (*vi*) $\frac{4\pi i e^2}{3}$, (*vii*) $\frac{\pi i}{60}$, (*viii*) $\frac{\pi i}{60}$]

19. Evaluate the following integral:

(i) $\int_0^{2\pi} \frac{\sin^2\theta}{5 - 4\cos\theta} d\theta$  (ii) $\int_0^{2\pi} \frac{1 + 2\cos\theta}{5 + 4\cos\theta} d\theta$

(iii) $\int_0^{2\pi} \frac{d\theta}{1 - 2m\cos\theta + m^2}, (m^2 < 1)$  (iv) $\int_0^{2\pi} \frac{\sin^2\theta - 2\cos\theta}{2 + \cos\theta} d\theta$

[*Ans.* (*i*) $\frac{\pi}{4}$, (*ii*) 0, (*iii*) $\frac{2\pi}{1 - m^2}$, (*iv*) $\frac{2\pi}{\sqrt{3}}$]

20. Evaluate the following integral:

(i) $\int_{-\infty}^{\infty} \frac{1}{(1 + x^2)^3} dx$  (ii) $\int_0^{\infty} \frac{dx}{x^4 + a^4}$

(iii) $\int_{-\infty}^{\infty} \frac{x^2}{(1 + x^2)(x^2 + 4)} dx$  (iv) $\int_0^{\infty} \frac{\cos mx}{(x^2 + a^2)} dx$; $(a > 0, m > 0)$

(v) $\int_0^{\infty} \frac{\cos 2x}{(x^2 + a^2)(x^2 + b^2)} dx (a > b > 0)$  (vi) $\int_0^{\infty} \frac{x^3 \sin mx}{(x^4 + a^4)} dx$; $(a > 0, m > 0)$

[*Ans.* (*i*) $\frac{3\pi}{8}$, (*ii*) $\frac{\pi}{\sqrt{2a^3}}$, (*iii*) $\frac{\pi}{3}$, (*iv*) $\frac{\pi}{2a} e^{-ma}$, (*v*) $\frac{\pi}{2(a^2 - b^2)}\left(\frac{e^{-2b}}{b} - \frac{e^{-2a}}{a}\right)$,

(*vi*) $\frac{\pi}{2} \exp\left(-\frac{ma}{\sqrt{2}}\right) \cos\left(\frac{ma}{\sqrt{2}}\right)$]

# CSIR-UGC-NET/JRF|GATE-PHYSICAL SCIENCES

## SECTION: MATHEMATICAL PHYSICS

### Assignment-8: PROBABILITY

1. A triangle is formed by joining three points of a regular hexagon. What is the probability that the triangle will be equilateral?

(a) 1/6 (b) 1/10 (c) 1/12 (d) 1/20

2. Two dice are thrown once the probability of getting a sum 9 is given by:

(a) $\frac{1}{12}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) None of these

3. Let $P\begin{bmatrix}1 & a\\ b & c\end{bmatrix}$, where a, b, c are chosen randomly from the set $\{1, 2, 3, 4, 5\}$. The probability that P is singular is

(a) $\frac{2}{25}$ (b) $\frac{1}{11}$ (c) $\frac{2}{21}$ (d) $\frac{1}{5}$

4. What is the probability that a leap year, selected at random, will contain 53 Mondays?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

5. What is the probability that the numbers formed between 60000 and 70000 using the digits 1,2,3,4,5,6,7,8,9 should end with '7'? (no digit is repeated)

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{11}$

6. What is probability that the words formed by the letters of the word "STRANGE" can be made so that the vowels should be in even places?

(a) $\frac{1}{7}$ (b) $\frac{2}{9}$ (c) $\frac{3}{11}$ (d) $\frac{4}{7}$

7. What is the probability that a 6-digit telephone number formed using digits 0 to 9, should start with 2 and end with 8? (no digits is repeated)

(a) $\frac{1}{32}$ (b) $\frac{1}{45}$ (c) $\frac{1}{90}$ (d) $\frac{2}{35}$

8. A bag contains 10 black and 15 red balls. Two balls are drawn from the bag at random. The probability that two balls drawn from the bag will be of same colour?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{5}$

9. Consider the experiment of throwing two fair dice. What is the probability that the sum of the numbers obtained in these dice is even ?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

10. A bag contains 6 red and 4 green balls A fair dice is rolled and a number of balls equal to that appearing on the dice is chosen from the urn at random. The probability that all the balls selected are red is.

(a) 1/3 (b) 3/10 (c) 1/8 (d) 68/315

11. A six faced die is a biased one. it is thrice more likely to show an odd number than to show an even number. It is thrown twice. The probability that the sum of the numbers in the two throws is even, is.
(a) 4/8 (b) 5/8 (c) 6/8 (d) 7/8

12. Two balls are drawn at random from a bag containing 6 white, 4 red and 5 black balls. The probability that both these balls are black, is:

(a) $\frac{1}{21}$ (b) $\frac{2}{15}$ (c) $\frac{2}{21}$ (d) $\frac{2}{35}$

13. A problem is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that this problem will be solved is:

(a) $\frac{1}{24}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

14. 6 boys and 6 girls sit in a row randomly. The probability that all the girls sit together is:

(a) $\frac{1}{132}$ (b) $\frac{1}{44}$ (c) $\frac{5}{132}$ (d) $\frac{7}{132}$

15. Different words are written with the letters of PEACE. The probability that both E's come together is:

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

16. The probability of randomly choosing 3 defectless bulbs from 15 electric bulbs of which 5 bulbs are defective is:

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{24}{91}$ (d) $\frac{67}{91}$

17. The choice of throwing 12 in a single throw with three dice is
(a) 12/216 (b) 21/216 (c) 15/216 (d) 25/216

18. The probability that a number in $\{1, 2, ..., 1001\}$ is divisible by 7 or 11 or both, is

(a) 143/1001 (b) 221/1001 (c) 247/1001 (d) 91/1001

19. A point P is chosen at random on a line AB of length $2l$. The probability $P\left[\left(AP \times BP\right) > l^2/2\right]$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\left(1-\frac{1}{\sqrt{2}}\right)l$ (d) $\frac{1}{\sqrt{2}}l$

20. A number is chosen from each of the two sets {1, 2, 3, 4, 5, 6, 7, 8, 9} and {1, 2, 3, 4, 5, 6, 7, 8, 9}. If $P_1$ denotes the probability that the sum of the two numbers be 10 and $P_2$ the probability that their sum be 8, then $(P_1 + P_2)$ is

(a) 7/729 (b) 137/729 (c) 16/81 (d) 137/81

21. The probability that three men hit a target are $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ respectively. Each shoots once the target. What is the probability that exactly one them hits the target ?

(a) $\frac{23}{96}$ (b) $\frac{42}{127}$ (c) $\frac{19}{48}$ (d) $\frac{31}{72}$

22. From a pack of 52 cards two are drawn at random. What is the probability that one is a king and the other a queen ?

(a) $\frac{5}{629}$ (b) $\frac{8}{663}$ (c) $\frac{10}{687}$ (d) None of the above

23. The probability that the sum of two numbers x and y randomly chosen in the interval (0, 1) greater than 1 while the sum of the squares less than 1 is equal to

(a) $\frac{2}{\pi}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}-\frac{1}{2}$ [✓ (d) $\frac{\pi}{4}-\frac{1}{2}$]

24. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 3 is

(a) 16/25 (b) 9/10 (c) $(9/10)^2$ (d) 18/25

25. At a railway station a passenger leaves his luggage in a locker which is opened by dialling a three-digit code. The passenger chooses the code, closes the locker and leaves for the town. A stranger who does not know the code tries to open the locker by dialling three digits at random without repeating the unsuccessful codes. The probability that the stranger does not have to perform more than 100 trials for a success is

(a) $\frac{9}{10}$ (b) $\frac{999}{1000}$ (c) $\frac{1}{1000}$ [✓ (d) $\frac{1}{10}$]

26. A cow is tied with a pole by a 100 meter long rope. What is the probability that at some point of time the cow is at least 60 meters away from the pole ?

(a) $\frac{9}{25}$ (b) $\frac{13}{25}$ [✓ (c) $\frac{16}{25}$] (d) $\frac{18}{25}$

27. A and B play a game of dice. A throws the die first. The person who first gets a 6 is the winner. What is the probability that A wins ?

(a) $\frac{6}{11}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

28. Two teams A and B play a series of four matches. If the probability that team A wins a match is $\frac{2}{3}$, then the probability that team A wins three matches, looses one and the third win occurs in the fourth match is

(a) $\frac{8}{27}$ (b) $\frac{16}{27}$ (c) $\frac{8}{81}$ (d) $\frac{32}{81}$

29. The probabilities that a husband and wife will be alive 20 years from now are give by 0.8 and 0.9 respectively. What is the probability that in 20 years at least one, will be alive ?
(a) 0.98 (b) 0.02 (c) 0.72 (d) 0.28

30. What is the probability that an urn containing 5 balls contains only white balls if the first two balls drawn from it was found to be white
(a) 1/20 (b) 1/10 (c) 1/2 (d) 1/4

31. If the probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is
(a) pq (b) p(1 – q) (c) q(1 – p) (d) p + q – 2pq

32. If A and B are two events such that P(A) = p, P(B) = q, $P(A \cap B) = r$ then the probability that A occurs but B does not occur is
(a) p – r (b) p – q (c) p – qr (d) p – pq

33. A box contains n enumerated articles. All the articles are taken out one by one at random. The probability that the numbers of the selected articles are in the sequence 1, 2, .... n is
(a) $\frac{1}{n!}$ (b) $\frac{n}{n!}$ (c) $\frac{1}{n(n!)}$ (d) none of the above

34. An anti-aircraft gun can take a maximum of four shots at a plane moving away from it. The probability of hitting the plane in the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probabilities that the gun hits the plane ?
(a) 0.7325 (b) 0.7286 [✓ (c) 0.6976] (d) 0.6574

35. For n independent events $A_1, A_2, A_3, .......A_n$, let $P(A_i) = \frac{1}{i+1}$, i = 1, 2, ...., n. Then the probability that none of the events will occur is
(a) n/(n + 1) (b) n – 1/(n + 1) (c) 1/(n + 1) (d) 1/n

36. Let A and B be two events with P(A) = 1/2, P(B) = 1/3 and $P(A \cap B) = 1/4$. What is $P(A \cup B)$ ?
(a) 3/7 (b) 4/7 (c) 7/12 (d) 9/122

37. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that A and B occur is 0.14. The probability that neither A nor B occur will be
(a) 0.61 (b) 0.39 (c) 0.86 (d) 0.75

38. If $P(A^c \cap B^c) = \frac{1}{3}, P(A \cap B) = \frac{1}{2}$ and $P(B) = 2P(A)$, the $P(A)$ will be
(a) 4/7 (b) 11/18 [✓ (c) 7/18] (d) 13/18

39. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled out at random from one of the two purses, then the probability that it will be silver coin will be
(a) 2/7 (b) 4/7 [✓ (c) 19/42] (d) 23/42

40. Let A, B and C be independent and mutually exclusive events with probability of occurrences $\frac{1-3p}{2}$, $\frac{1+4p}{3}$, $\frac{1+p}{6}$, respectively, then p lies in

(a) [– 1/4, 5/6] [✓ (b) [– 1/4, 1/3]] (c) [– 1/4, 1/2] (d) [– 1/2, 1/3]

41. Three persons play a game by tossing a fair coin each independently. The game ends in a trial if all of them get the same outcome in that trial, otherwise they continue to the next trial. What is the probability that the game ends in an even number of trials ?

(a) 2/7 [✓ (b) 3/7] (c) 1/2 (d) 4/7

42. In a certain office, 1/3 of the workers are women, 1/2 of the women are married and 1/3 of the married women have children. If 3/4 of the men are married and 2/3 of the married men have children, what part of workers are without children ?

(a) 5/18 (b) 4/9 (c) 11/18 (d) 17/36

43. A and B are independent witnesses in a case. The probability that A speaks the truth is 'x' and that B speaks the truth is 'y'. If A and B agree on a certain statement, the probability that the statement is true is

(a) $\dfrac{xy}{xy+(1-x)(1-y)}$ (b) $\dfrac{xy}{(1-x)(1-y)}$

(c) $\dfrac{(1-x)(1-y)}{xy+(1-x)(1-y)}$ (d) $\dfrac{(1-x)(1-y)}{xy}$

44. A bag contains 4 white and 3 black balls and a second bag contains 3 white and 3 black balls. If a ball is drawn from each of the bags, then the probability that both are of same colour is :

(a) $\dfrac{3}{14}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{3}{13}$ (d) $\dfrac{1}{3}$

45. Suppose there are two bags with first bag contaions 3 white and 2 black balls, second bag contains 2 white and 4 black balls. one ball is transferred fron bag I to bag II. and then a ball is drawn from the latter. It happens to be white. What is the probability that the tranferred ball is white?

(a) $\dfrac{9}{13}$ (b) $\dfrac{7}{13}$ (c) $\dfrac{6}{13}$ (d) $\dfrac{11}{13}$

46. Three urns are given each cotaining red and white balls. Urn 1 contains 6 red and 4 white balls, Urn II contains 2 red and 6 white balls and Urn III 1 red and 4 white balls. An urn is selected at random and a ball is drawn. If the selected ball is red, what is the probability that it is from the first urn?

(a) $\dfrac{2}{5}$ (b) $\dfrac{2}{7}$ [✓ (c) $\dfrac{4}{7}$] (d) $\dfrac{4}{9}$

47. The probability of getting a defective floppy in three boxes A, B and C are 1/3, 1/6, and 3/4, respectively. A box is selected randomly and a floppy is drawn from it. The probability that the floppy is defective and is drawn from box A is

(a) 4/15 (b) 12/15 (c) 2/15 (d) 3/5

48. A box contains 99 fair coins and 1 double-headed coin. One coin is chosen from the box at random. The coin is tossed 3 times and we get heads all the time. What is the probability that the chosen coin will be double-headed?

(a) 0.075 (b) 0.18 (c) 0.26 (d) 0.31

Suppose, 'x' is a continuos random varible with probability density function given by,

$$
\begin{aligned}
f(x) &= kx\,;(0 \le x < 2) \\
&= 2k\,;(2 \le x < 4) \\
&= -kx + 6k\,;(4 \le x < 6)
\end{aligned}
$$

49. The value of 'k' will be
(a) 1/8 (b) 1/6 (c) 1/2 (d) 4/3

50. The mean value of the random variable 'x' will be
(a) 3/2 (b) 3 (c) 5/3 (d) 7/4

51. The probability density function of a continuous random variable is given as,

$$f(x) = Ke^{-|x|} \quad (-\infty < x < \infty)$$

(a) Find the value of 'K'.

(b) Find the mean and variance of the distribution.

(c) Find the probability that the variate lies between 0 and 4.

**[Ans.** (a) 1/2, (b) 0, 2, (c) $\frac{1}{2}\left(1-e^{-4}\right)$ ]

52. A continuous random variable has the following probability density function:

$$f(x) = a + bx \quad \text{for } 0 \le x \le 1$$

If the mean of the distribution is 1/2, the value of a and b will be

(a) $a=1, b=1$ (b) $a=0, b=1$ (c) $a=1, b=0$ (d) $a=1, b=-1$

**Common Data Questions: Q.48 and Q.49**

The probability density function of the random variable 'x' is given as

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P(x)$: | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

53. The probability that the value of 'x' will be less than 4, will be
(a) 9/49 (b) 16/49 (c) 24/49 (d) 25/49

54. The probability that the value of 'x' will be atleast '5', will be
(a) 9/49 (b) 16/49 (c) 24/49 (d) 25/49

**Common Data Questions: Q.50 and Q.51**

Suppose an probability distribution is defined as

$$f(x) = \frac{1}{n} \quad \text{for } x = 1, 2, 3.....n$$

55. The mean of the distribution will be

(a) $\frac{n+1}{2}$ (b) $\frac{n}{2}$ (c) $\frac{n-1}{2}$ (d) $\frac{n}{2}-1$

56. The variance of the distribution will be

(a) $\frac{n^2+1}{12}$ (b) $\frac{n^2-1}{12}$ (c) $\frac{n^2-1}{6}$ (d) $\frac{n^2+1}{6}$

57. A die is tossed once. If the random variable X is defined as

$$x = \begin{cases} 1, \text{ if the die results in an even number} \\ 0, \text{ if the die results in an odd number} \end{cases}$$

The mean of the probability distribution will be

(a) 1/3 (b) 1/4 (c) 1/2 (d) 2/3

58. An unbiased coin is tossed 6 times. What is the probability of obtaining 4 heads?

(a) 3/16 (b) 15/64 (c) 15/32 (d) 21/64

59. The overall percentage of failure in an examination is 20 percent. If 7 candidates appear in the examination, what is the probability that atleast 6 pass the examination?

(a) $\frac{4^6.11}{5^7}$ (b) $\frac{4^7.11}{5^7}$ (c) $\frac{4^7.11}{5^6}$ (d) $\frac{4^6.11}{5^6}$

60. A die is thrown 9 times. What is the probability of getting '5' exactly 3 times?

(a) $\frac{5^6.14}{6^9}$ (b) $\frac{5^6.14}{6^8}$ (c) $\frac{5^6.84}{6^8}$ (d) $\frac{5^3.84}{6^8}$

61. The chance that out of 10 telephone lines, one is busy at a time is 0.8. What will be the most probable number of busy lines?

(a) 2 (b) 5 (c) 8 (d) 10

62. Three unbiased coins are tossed together and this is repeated five times. What is the probabilty of getting atleast one head each time?

(a) $\frac{1}{8}$ (b) $\left(\frac{1}{8}\right)^5$ (c) $\frac{7}{8}$ [✓ (d) $\left(\frac{7}{8}\right)^5$]

63. Which of the following is NOT an assumption of the Binomial distribution?
(a) All trials must be identical.
(b) All trials must be independent.
(c) Each trial must be classified as a success or a failure.
(d) The probability of success is equal to 0.5 in all trials.

64. In a triangle test, a tester is presented with three food samples, two of which are alike, and is asked to pick out the odd one by testing. If a tester has no well developed sense and can pick the odd one only, by chance, what is the probability that in five trials he will make four or more correct decisions?
[✓ (a) 11/243] (b) 1/243 (c) 10/243 (d) 233/243

65. The probability that a certain machine will produce a defective item is 1/4. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?
(a) 3/4096 (b) 4/4096 (c) 18/4096 (d) 19/4096

66. Suppose 60% of a herd of cattle is infected with a particular disease. Let Y = the number of non-diseased cattle in a sample of size 5. The distribution of Y is
(a) binomial with n = 5 and p = 0.6
(b) binomial with n = 5 and p = 0.4
(c) binomial with n = 5 and p = 0.5
(d) none of these

67. If the probability of unfavorable reaction from a certain injection is 0.001, what is the probability that out of 2000 individuals, more than 2 will suffer from unfavorable reaction?
(a) $\frac{5}{e^2}$ (b) $1-\frac{5}{e^2}$ (c) $1-\frac{3}{e^2}$ (d) $\frac{3}{e^2}$

68. The chance of a coal miner being killed in a mine accident is 1/1400 during a calendar year. What is probability that a mine with 3500 employers will have at least one accident in a year?
(Given: $e^{-5/2} = 0.082$)
(a) 0.164 (b) 0.492 (c) 0. 618 (d) 0.918

69. There are 20 buses passing a bus-stop randomly within an hour. What is the probability that there will be no buses within 5 minutes? (Given: $e^{-5/3} = 0.19$)
(a) 0.19 (b) 0.23 (c) 0.38 (d) 0.52

70. A card is drawn from a well-shuffled pack of 52 cards. A sequence of 156 trials is made. What is the probability of getting a queen of clubs at least once?
(a) $\frac{1}{e^3}$ (b) $1-\frac{1}{e^3}$ (c) $1-\frac{1}{e^{52}}$ (d) $\frac{1}{e^{52}}$

71. Suppose flaws (cracks, chips, specks, etc.) occur on the surface of glass with density of 3 per square metre. What is the probability of there being exactly 4 flaws on a sheet of glass of area 0.5 square metre? (Given: $e^{-3/2} = 0.223$)
(a) 0.047 (b) 0.168 (c) 0.815 (d) 0.647

72. In a certain communications system, there is an average of 1 transmission error per 10 seconds. What is the probability of more than 1 error in a communication one-half minute in duration? (Given: $e^{-3} = 0.05$)
(a) 0.2 (b) 0.32 (c) 0.56 (d) 0.8

73. Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored more than 60 marks?
(a) 0.25 (b) 0.5 (c) 0.6 (d) 0.75

74. A manufacturer produces airmail envelopes whose weight is normally distributed with a mean 1.95 gm and standard deviation 0.05 gm. The envelopes are sold in lots of 1000. How many envelopes in a lot will heavier than 2 gm?
(a) 78 (b) 142 (c) 155 (d) 221

75. In a test on 2000 electric bulbs, it was found that the life of a particular bulb was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. How many bulbs likely to burn for more than 2160 hours?

(a) 45 (b) 65 (c) 84 (d) 105

## CSIR Previous Year Questions

76. An unbiased dice is thrown three times successively. The probability that the number of dots on the uppermost surface add up to 16 is **[CSIR Dec 2011]**

(a) 1/16 (b) 1/36 (c) 1/108 (d) 1/216

77. A ball is picked at random from one of the two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white? **[CSIR June 2012]**

(a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

78. In a series of five cricket matches, one of the captains calls ‘heads’ every time when the toss is taken. The probability that he will win 3 times and lose 2 times is **[CSIR Dec 2012]**

(a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

79. Consider three particles A, B and C, each with attribute S that can take two values $\pm 1$. Let $S_A = 1, S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to -S with probability 1/3. The probability that $S_A + S_B + S_C$ remains unchanged is

**[CSIR June 2013]**

(a) 2/3 (b) 1/3 (c) 2/9 (d) 4/9

80. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}$ of turning up 1,2,3,4,5 and 6 respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?

**[CSIR Dec 2013]**

(a) 144/441 (b) 225/441 (c) 221/441 (d) 220/441

81. A child makes a random walk on a square lattice of lattice constant ‘a’, taking a step in the north, east, south or west directions with probabilities 0.255, 0.255, 0.245, 0.245 respectively. After a large number of steps N, the expected position of the child w.r.t the strating point is ata distance

**[CSIR Dec 2013]**

(a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction (b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction

(c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction (d) 0

82. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps? **[CSIR June 2014]**

(a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

## CSIR-UGC-NET/JRF|GATE-PHYSICAL SCIENCES

## SECTION: MATHEMATICAL PHYSICS

### Assignment-8: PROBABILITY

### ANSWER KEY

1.(b) 2.(d) 3.(a) 4.(b) 5.(c) 6.(a)
7.(c) 8.(b) 9.(a) 10.(d) 11.(b) 12.(c)
13.(d) 14.(a) 15.(b) 16.(c) 17.(d) 18.(b)
19.(b) 20.(c) 21.(d) 22.(b) 23.(d) 24.(d)
25.(d) 26.(c) 27.(a) 28.(a) 29.(a) 30.(c)
31.(d) 32.(a) 33.(a) 34.(c) 35.(c) 36.(c)
37.(b) 38.(c) 39.(c) 40.(b) 41.(b) 42.(c)
43.(a) 44.(b) 45.(a) 46.(c) 47.(a) 48.(a)
49.(a) 50.(b) 52.(c) 53.(b) 54.(c) 55.(a)
56.(b) 57.(c) 58.(b) 59.(a) 60.(b) 61.(c)
62.(d) 63.(d) 64.(a) 65.(d) 66.(b) 67.(b)
68.(d) 69.(a) 70.(b) 71.( ) 72.( ) 73.(b)
74.(c) 75.(a) 76.(b) 77.(b) 78.(d) 79.(d)
80.(b) 81.(a) 82.(a)

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment-9: FOURIER SERIES

1. Which of the following is NOT a periodic function of $\theta$?

(a) $\sin\theta + \pi\cos\theta$ (b) $\sin(\pi\theta) + \cos(\pi\theta)$

(c) $\sin\theta + \cos(\pi\theta)$ (d) $\sin\theta + \cos(\theta + \pi)$

2. In the fourier series expansion of $y = (x^2 - 1)^{1/2}\ (-\pi \le x \le \pi)$ the coefficient $a_1$, $b_1$ will be

(a) $\frac{1}{\pi}, 1$ (b) $1, -\frac{1}{\pi}$ (c) $2, \frac{\pi^2}{2}$ (d) Unable to find

3. In the Fourier series expansion of $y = \frac{1}{x^2(x-2)}(-2 \le x \le 2)$ the co-efficient $a_0$ will be

(a) Unable to find (b) 0 (c) $\frac{\pi^2}{3}$ (d) $\frac{2}{3}$

4. Suppose f(x) is defined as following:

$$\begin{aligned} f(x) &= 1 \qquad -1 \le x \le 0 \\ &= -2 \qquad 0 \le x \le 1 \end{aligned}$$

then $a_0, a_3, b_3$ in the fourier series exapansion of f(x) are

(a) $\left(0, 0, -\frac{2}{\pi}\right)$ (b) $\left(-1, 0, -\frac{2}{\pi}\right)$ (c) $\left(1, 0, \frac{2}{\pi}\right)$ (d) $\left(-1, 0, \frac{2}{\pi}\right)$

5. Suppose f(x) is defined as following:

$$\begin{aligned} f(x) &= -k; \qquad -\pi < x < 0 \\ &= k; \qquad 0 < x < \pi \end{aligned}$$

then co-efficient $b_2$, $b_3$ in the fourier series exapnasion will be

(a) $\frac{4k}{2\pi}, \frac{4k}{3\pi}$ (b) $-\frac{4k}{2\pi}, -\frac{4k}{3\pi}$ (c) $0, \frac{4k}{3\pi}$ (d) $\frac{4k}{2\pi}, 0$

6. Suppose f(x) is defined as following:

$$\begin{aligned} f(x) &= 0; \qquad -\pi < x < 0 \\ &= 2\pi; \qquad 0 < x < \pi \end{aligned}$$

In the fourier series expansion, the coefficient of $\cos 2x$ is:

(a) $\frac{2}{3\pi}$ (b) $\frac{1}{\pi}$ (c) 0 (d) $-\frac{2}{3\pi}$

7. Suppose f(x) is defined as following:

$$f(x) = 0 \qquad -\pi < x \le 0$$
$$= \frac{\pi x}{4} \qquad 0 \le x < \pi$$

then the co-efficient $a_n$ will be

(a) $\frac{(-1)^n}{4n^2}$ (b) $\frac{(-1)^n - 1}{4n^2}$ (c) $\frac{(-1)^n - 1}{4n}$ (d) $\frac{(-1)^n}{4n}$

8. In the fourier expansion for $f(x) = x\cos x\,(-\pi < x < \pi)$. Which of the following statement is true?

(a) $b_n = 0$ for all n (b) $b_n = \frac{2n}{n^2 - 1}$ for all even n

(c) $b_n = -\frac{2n}{n^2 - 1}$ for all even n (d) $b_n = -\frac{2n}{n^2 - 1}$

9. In the fourier series expansion of $f(x) = |\sin x|$, which of the following co-efficients are non-zero? $(-\pi \le x \le \pi)$

(a) $a_n$ for odd n (b) $a_n$ for even n. (c) $b_n$ for odd n (d) $b_n$ for even n.

10. If $f(x) = x - x^2$ from $x = -\pi$ to $\pi$ is expanded into

$$f(x) = -\frac{\pi^2}{3} + \sum \frac{4}{n^2}(-1)^{n+1}\cos nx + \sum \frac{2}{n}(-1)^{n+1}\sin nx$$

then $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots$ is equal to

(a) $\frac{\pi^2}{12}$ (b) $\frac{-\pi^2}{12}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{-\pi^2}{4}$

11. The function $f(x) = x^2, 0 < x < 2\pi$ has a Fourier series representation

$$x^2 = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos nx - \frac{4\pi}{n}\sin nx\right)$$

At $x = 0$ and $2\pi$, the series converges to $2\pi^2$. Then the value of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is:

(a) $2\pi^2$ (b) $\pi^2$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi^2}{6}$

12. If f(x) be an even, period function of x which is continuous. Then
(a) It can be represented by a cosine series.
(b) It can be represented by a sine series
(c) It cannot be represented by a Fourier series.
(d) It needs both sine and cosine terms in its Fourier expansion.

13. $f_1(x) = \sin^2 x$ and $f_2(x) = \cos^2 x$ are expanded in their corresponding fourier series. Which of the following is correct?

(a) $a_2^{(1)} = \frac{1}{2}, b_2^{(2)} = -\frac{1}{2}$ (b) $b_2^{(1)} = \frac{1}{2}, a_2^{(2)} = -\frac{1}{2}$

(c) $a_2^{(1)} = \frac{1}{2}, a_2^{(2)} = -\frac{1}{2}$ (d) $b_2^{(1)} = \frac{1}{2}, b_2^{(2)} = -\frac{1}{2}$

14. Consider the Fourier series corresponding to the function

$$f(x) = \begin{cases} 3, & 0 < x < 5 \\ -3, & -5 < x < 0 \end{cases}$$

Then the fourier series contains.

(a) Cosine terms only (b) Both cosine and sine terms

(c) Sine terms only (d) Sine terms and non-zero constant terms.

15. The Fourier series expansion of the function $f(x) = x(\pi - x), 0 \le x \le 2\pi$, is:

$$\frac{\pi^2}{6} - \left( \frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \ldots\ldots\ldots \right)$$

The value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is deduced as

(a) $\frac{\pi^2}{6}$ (b) $\frac{2\pi^2}{6}$ (c) $-\frac{\pi^2}{6}$ (d) Zero

16. The Fourier coefficients $a_n$ in the Fourier series expansion.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx, \text{for } f(x) = \begin{cases} -1, & -\pi \le x \le 0 \\ +1, & 0 \le x \le \pi \end{cases}, \text{are}$$

(a) $\frac{2}{n\pi}\left[1 - (-1)^n\right]$ (b) $\frac{4}{n\pi}$ (c) $\frac{2}{n\pi}\left[1 + (-1)^n\right]$ (d) 0

17. Find the fourier series expansion for

(i) Rectangular Wave: $f(x) = \begin{Bmatrix} -k & \text{for } -\pi < x < 0 \\ k & \text{for } \ 0 < x < \pi \end{Bmatrix}$

(ii) Half Wave Rectifier: $f(x) = \begin{Bmatrix} 0 & \text{for } -\frac{\pi}{\omega} < t < 0 \\ E \sin \omega t & \text{for } \ 0 < t < \frac{\pi}{\omega} \end{Bmatrix}$

(120)

(iii) Saw Tooth Wave: $f(x) = x + \pi$ for $-\pi < x < \pi$

**Ans.** (i) $\frac{4k}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \dots\dots\dots\right]$

(ii) $\frac{E}{\pi} + \frac{E}{2}\sin \omega t - \frac{2E}{\pi}\left[\frac{1}{1.3}\cos 2\omega t + \frac{1}{3.5}\cos 4\omega t + \dots\dots\dots\right]$

(iii) $\pi + 2\left[\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \dots\dots\dots\right]$

18. Obtain the complex form of the fourier series of the expansion

$$f(x) = 0; \quad -\pi \le x \le 0$$
$$= 1; \quad 0 \le x \le \pi$$

**Ans.** $\frac{1}{2} + \frac{1}{i\pi}\left[\frac{e^{ix}}{1} + \frac{e^{i3x}}{3} + \frac{e^{i5x}}{5} + \dots\dots\right] + \frac{1}{i\pi}\left[-\frac{e^{-ix}}{1} - \frac{e^{-3ix}}{3} - \frac{e^{-5ix}}{5} + \dots\dots\right]$

## ANSWER KEY

**1.(c)** **2.(d)** **3.(a)** **4.(b)** **5.(c)** **6.(c)** **7.(b)**

**8.(b)** **9.(b)** **10.(a)** **11.(d)** **12.(a)** **13.(c)** **14.(c)**

**15.(a)** **16.(d)**

upto 3

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment-9: DIFFERENTIAL EQUATIONS

1. The degree and order of differential equation $\sqrt{2\left(\frac{dy}{dx}\right)^3+4} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$ are respectively

(a) order 2, degree 3 (b) order 1, degree 3 (c) order 3, degree 2 (d) order 3, degree1

2. What is the degree of the differential equation $3y = x\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)$?

(a) 1 (b) 2 (c) 8 (d) cannot be determined

3. The half-life of a radioactive substance is the time required for one-half of that substance to decay. The amount of $^{11}C$, an isotope of carbon present at a future time t (in months) is given by A(t) = 100 exp [– 0.0338t]. The half-life of the material in months is

(a) In 2 (b) 0.0338 [✓ (c) In 2/0.0338] (d) 2 In 2

4. A cup of cofee has a temperature 95°C and is in a room where the temperature is 20°C. Let, T = temperature of the coffee after 't' mintues. Assuming Newton's law of cooling, which of the following options describes T? (k is a positive constant)

(a) $\frac{dT}{dt} = k(T-20)$ (b) $\frac{dT}{dt} = -k(T-20)$ (c) $\frac{dT}{dt} = -k(T-95)$ (d) $\frac{dT}{dt} = k(T-95)$

5. Suppose that, a tumour in a rat is approximately spherical and its rate of growth is proportional to its diameter. If the tumour has diameter 5 mm. When detected and 8 mm, after three months later, what will the diameter be after another three months?

[✓ (a) 10.1 mm] (b) 12.8 mm (c) 11.6 mm (d) 13.2 mm

6. A tank holds 1000 liters of water, in which 15kg of salt is dissolved. Pure water enter the tank at the rate of 10 liters/minutes. The solution is kept throughout mixed and drained from the tank at the same rate. If 'm' is the mass of the salt in the tank at anty time 't', which of the following options describes the rate of change of the mass of the salt in the tank?

(a) $\frac{dm}{dt} = 15-\frac{m}{100}$ [✓ (b) $\frac{dm}{dt} = -\frac{m}{100}$] (c) $\frac{dm}{dt} = -\frac{15}{100}$ (d) $\frac{dm}{dt} = -\frac{15-m}{100}$

7. The size of a popoulation 'P' is modeled by the differential equation $\frac{dP}{dt} = 1.2P\left(1-\frac{P}{4200}\right)$

For what values of P, population is increasing?

(a) $P > 0$ [✓ (b) $0 < P < 4200$]
(c) $P < 2100$ (d) Not enough information provided.

8. The differential equation of all circles passing through the origin and having their centres on the y-axis

(a) $\left(x^2+y^2\right)dy/dx=2xy$ [✓ (b) $\left(x^2-y^2\right)dy/dx=2xy$]

(c) $dy/dx=2xy\left(x^2+y^2\right)$ (d) $dy/dx=2xy\left(x^2-y^2\right)$

9. Classify the following differential equations:

$$e^x\frac{dy}{dx}+3y=x^2y$$

(a) Separable but not linear (b) Linear but not separable

(c) Both separable and linear (d) Neither separable nor linear.

10. Classify the following differential equation:

$$\frac{dx}{dt}=\frac{x+2xt+\cos t}{1+t^2}$$

(a) Separable but not linear (b) Linear but not separable.

(c) Both separable and linear (d) Neither separable nor linear.

11. Which of the following differential equations is with separated variables ?

(a) $\log\left(\frac{dy}{dx}\right)=ax+by$ (b) $dy=\left(e^{x-y}+x^2e^{-y}\right)dx$

(c) $\tan y\,dy=\left[\sin(x+y)+\sin(x-y)\right]dx$ [✓ (d) All of the above equations]

12. The geometrical interpretation of the graph of the differential equation: $\frac{dy}{dx}=-\frac{x}{y}$ is a family of

(a) Hyperbola (b) Ellipse (c) Parabola (d) Circle

13. The integrating factor of $\frac{dy}{dx}+\frac{x}{1+x}y=1+x$ is:

(a) $e^x$ (b) $\frac{e^x}{1+x}$ (c) $e^x(1+x)$ (d) $e^{x+x^2/2}$

14. Identify the functions p(t) and q(t) if the differential equation $\frac{dx}{dt}=\frac{x+t^2-2x\sqrt{t}}{t}$ is written in the form

$$\frac{dx}{dt}+p(t)x=q(t)$$

(a) It is not possible to write (b) $p(t)=2\sqrt{t}-1,\ q(t)=t^2$

(c) $p(t)=1-2\sqrt{t},\ q(t)=t$ (d) $p(t)=\frac{2\sqrt{t}-1}{t},\ q(t)=t$

15. An integrating factor I(x) is found for the differential equation $\left(1+x^2\right)\frac{dy}{dx}+xy=0$ and the equation is rewritten as $\frac{d}{dx}\left[I(x)y\right]=0$. Which of the following option is correct.

(a) $I(x)=e^{x^2/2}$ (b) $I(x)=\sqrt{1+x^2}$ (c) $I(x)=1+x^2$ (d) $I(x)=0$

16. Find the general solution of the differential equation $\frac{dy}{dx}+\frac{x}{1+x}y=1+x$

(a) $\left(1+c\,e^{-x}\right)(1+x)$ (b) $1+x+c$

(c) $c(1+x)$ (d) $e^{-x}\left(x+\frac{x^2}{2}+c\right)(1+x)$

17. $\tan^{-1}x+\tan^{-1}y=c$ is the general solution of the differential equation

(a) $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ (b) $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

(c) $\left(1+x^2\right)dy+\left(1+y^2\right)dx=0$ (d) $\left(1-x^2\right)dx+\left(1-y^2\right)dy=0$

18. Find the particular solution of differential equation $\frac{dy}{dx}=x\sin\left(3x^2\right)$ given $y(0)=0$

(a) $y=-\frac{\cos\left(3x^2\right)}{6}$ (b) $y=\frac{\cos\left(3x^2\right)-1}{6}$ (c) $y=-\frac{\cos\left(3x^2\right)}{6}+c$ (d) $y=\frac{1-\cos\left(3x^2\right)}{6}$

19. If $f_1$ and $f_2$ be the integrating factors of the differential equations $x\frac{dy}{dx}+2y=1$ and $\frac{dy}{dx}-\frac{2}{x}y=\frac{1}{x}$ respectively, then

(a) $f_1=x^4f_2$ (b) $f_1=x^3f_2$ (c) $f_1=x^2f_2$ (d) $f_1=f_2$

20. Consider the following differential equation: $\frac{dy}{dx}-2x=f(x);(x\in R)$, satisfying $y(0)=0$;

where $f(x)=\begin{cases}0 & \text{for } x\le 0\\ 1 & \text{for } x>0\end{cases}$. This initial value problem

[✓ (a)] has a continuous solution which is not differentiable at $x=0$

(b) has a continuous solution which is differentiable at $x=0$

(c) has a continuous solution which is differentiable on 'R'

(d) does not have a continuous solution on 'R'

21. Which of the following transformations reduces the differential equation $\frac{dy}{dx}-\frac{\tan y}{1-x}=(1+x)e^x\sec y$

into the form $\frac{dz}{dx}+P(x)z=Q(x)$, by the substitution

(a) $z=\sin y$ (b) $z=\cos y$ (c) $z=\operatorname{cosec} y$ (d) $z=\tan y$

22. If y (x) satisfies the differential equation $\frac{dy}{dx}+2y=2+e^{-x^2}$ with $y(0)=0$, then $\underset{n\to\infty}{Lt}\,y(x)$ is equal to

(a) 0 (b) 1 (c) 2 (d) -1

23. Consider the system of differential equations: $\frac{dx}{dt}=-y, \frac{dy}{dt}=x$

The trajectory of the particle 'P' located at point (x,y) will be

(a) hyperbola (b) parabola [✓ (c) circle] (d) Straight line

24. From the solution of the differential equation: $\frac{dy}{dx}=\frac{1}{1+x^2}$

What can you say regarding the following series?

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\ldots\ldots\ldots$$

(a) The series is divergent

(b) The series converges to 1

[✓ (c) The series converges to $\frac{\pi}{4}$]

(d) The series converges to 0

25. Which of the following is general solution to $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-10y=0$?

(a) $y=A\,e^{2x}+B\,e^{-5x}$

(b) $y=A\,e^{-2x}+B\,e^{5x}$

(c) $y=A\,e^{2x}+B\,e^{5x}$

(d) $y=A\,e^{-2x}+B\,e^{-5x}$

26. If $y=e^{2t}$ is a solution to $\frac{d^2y}{dt^2}-5\frac{dy}{dt}+ky=0$, what is the value of k?

[✓ (a) 6] (b) 4 (c) $-6$ (d) 0

27. Which of the following is general solution to $\frac{d^2x}{dt^2}-4\frac{dx}{dt}+13x=0$?

(a) $x=Ae^{5t}+B\,e^{-t}$

(b) $x=e^{2t}\left(A\cos 3t+B\sin 3t\right)$

(c) $x=e^{3t}\left(A\cos 3t+B\sin 3t\right)$

(d) $x=e^{-3t}\left(A\cos 2t+B\sin 2t\right)$

28. The general solution to $\frac{d^2z}{dt^2}+6\frac{dz}{dt}+9z=0$ is z. Which of the following options are correct?

(a) As $t\to\infty, z\to A$ for any value of B

(b) The behaviour of z as $t\to\infty$ depends on A, B

[✓ (c) As $t\to\infty, z\to 0$ for any value of A and B]

(d) As $t\to\infty, z\to\infty$ for any value of A and B.

29. The particular solution to $\frac{d^2y}{dt^2}-3\frac{dy}{dt}-4y=0$ satisfying $y(0)=0$ and $y'(0)=5$

(a) $y=e^{t}-e^{-4t}$ (b) $y=e^{-t}-e^{4t}$ (c) $y=e^{-4t}-e^{t}$ (d) $y=e^{4t}-e^{-t}$

30. Find a solution to $\frac{d^2x}{dt^2}+\frac{dx}{dt}-2x=0$, which satisfies $x(0)=3$ and does not tend to $\infty$( or $-\infty$) as $t\to\infty$

(a) $x=3e^{-t}$ (b) $4e^{-2t}-e^{t}=x$ (c) $x=3e^{-2t}$ (d) $x=2e^{-2t}+e^{-t}$

31. If $e^{2x}$ and $xe^{2x}$ are particular solutions of a 2nd order differential equation (homogeneous) with constant co-efficients, then the equation is:

(a) $y''-4y'+4y=0$ (b) $y''-5y'+6y=0$ (c) $y''-4y=0$ (d) $y''-3y'+2y=0$

32. The general solution of the differential equation $\frac{d^5y}{dx^5}-\frac{3d^4y}{dx^4}+\frac{3d^3y}{dx^3}-\frac{d^2y}{dx^2}=0$ is

(a) $c_1+c_2x+\left(c_3+c_4x+c_5x^2\right)e^x$ (b) $c_1x+c_2x^2+c_3e^x+c_4e^{2x}+c_5e^{3x}$

(c) $\left(c_1+c_2+c_3+c_4+c_5\right)e^x$ (d) None of these

33. The particular integral of the differential equation $y''+y'+3y=5\cos(2x+3)$ is:

(a) $2\cos(2x+3)-\sin(2x+3)$ (b) $2\sin(2x+3)+\cos(2x+3)$

(c) $\sin(2x+3)-2\cos(2x+3)$ (d) $2\sin(2x+3)-\cos(2x+3)$

34. The particular integral of $\frac{d^4y}{dx^4}-y=\cos x\cosh x$ is:

(a) $y=-\frac{1}{5}\sinh x\cos x$ (b) $y=-\frac{1}{5}\cosh x\cos x$

(c) $y=-\frac{1}{5}\sinh x\sin x$ (d) $y=-\frac{1}{5}\cosh x\sin x$

35. Which one of the following differential equations represents all circles with radius a ?

(a) $1+\left(\frac{dy}{dx}\right)^2+\sqrt{a^2-x^2}\,\frac{d^2y}{dx^2}=0$ (b) $1+\left(\frac{dy}{dx}\right)^2+\sqrt{a^2-y^2}\,\frac{d^2y}{dx^2}=0$

(c) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3+a^2\left(\frac{d^2y}{dx^2}\right)^2=0$ (d) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$

36. Find the general solution of differential equation $\frac{d^2y}{dx^2}=\frac{1+\cos^3x}{\cos^2x}$

(a) $y=\tan x+\sec x+C$ (b) $y=\ell n|\cos x|+\sin x+D$

(c) $y=\tan x\sec x+\cos x+Cx+D$ (d) $y=-\ell n|\cos x|-\cos x+Cx+D$

37. The general solution of differential equation $4x^2y''-8xy'+9y=0$ is:

(a) $C_1e^{5x/2}+C_2e^{-3x/2}$ (b) $C_1e^{3x/2}+C_2e^{-3x/2}$

(c) $\left(C_1+C_2\log x\right)x^{3/2}$ (d) $C_1x^{3/2}+C_2x^{-3/2}$

38. The solution of the differential equation $yy' + y^2 - x = 0$, where c is a constant, is

(a) $y^2 = x + ce^{-2x}$ (b) $y^2 = x + ce^{-2x} - \frac{1}{2}$ (c) $y^2 = x = ce^{-2x} + \frac{1}{2}$ (d) $y^2 = x + ce^{-2x} + 1$

39. The differential equation $\frac{d^2y}{dx^2} + y = 0$ satisfying the conditions $y(0) = 1, y(\pi) = 0$, has

(a) a unique solution (b) infinite no.of solutions

(c) no solution ✓ (d) none of these

40. Consider a forced harmonic oscillator which obeys the differential equation : $\frac{d^2y}{dt^2} + y = \sin t$

Which of thw following is the solution of the differential equation with the initial condition y (0) = 0?

(a) $y(t) = 6\sin t$ (b) $y(t) = 12\sin t + \frac{t}{2}\cos t$

(c) $y(t) = 12\sin t - \frac{t}{2}\cos t$ (d) $y(t) = 6\cos t - \frac{t}{2}\cos t$

41. The function $f(x) = x^2$ can be written in terms of Legendre Polynomials as

(a) $\frac{2P_2(x) + P_0(x)}{3}$ (b) $\frac{2P_1(x) - P_2(x)}{3}$ (c) $\frac{P_2(x) - P_1(x)}{3}$ (d) $\frac{P_2(x) - P_0(x)}{3}$

42. The function $f(x) = x^2 - x$ can be written in terms of Legendre Polynomials as

(a) $\frac{3P_2(x) - P_1(x) + P_0(x)}{2}$ (b) $\frac{2P_2(x) - 3P_1(x) + P_0(x)}{3}$

(c) $\frac{2P_2(x) - P_1(x) - P_0(x)}{3}$ (d) $\frac{2P_2(x) - P_1(x) + P_0(x)}{3}$

43. The solution of the differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$ is

(a) $P_n(x)$ ~~(b)~~ $P_1(x)$ ~~(c)~~ $P_2(x)$ (d) $P_3(x)$

44. The value of the integral $\int_{-1}^{1} [P_3(x)]^2 \, dx$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{2}{7}$ (d) $\frac{1}{5}$

45. The value of the integral $\int_{-1}^{1} P_n(x) \, dx \; (n \neq 0)$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{2n+1}$ (d) $\frac{2}{n^2 - 1}$

46. The value of the integral $\int_{0}^{1} P_{2n}(x) \, dx \, (n \neq 0)$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{2n+1}$ (d) $\frac{2}{2n+1}$

47. The value of the integral $\int_{-1}^{1} P_n(x)\left(1-2xz+z^2\right)^{-\frac{1}{2}} dx$ is equal to

(a) $\frac{2}{2n+1}$ (b) $\frac{2z^n}{2n+1}$ (c) $\frac{z^n}{2n+1}$ (d) 0

48. The expression $P'_{2n}(0)$ can be written as

(a) 0 (b) 1 (c) $\frac{1}{2n}$ (d) $\frac{(-1)^n.n!}{2^n}$

49. The bessen function $J_{-2}(x)$ can be written as

(a) $J_1(x)$ (b) $\frac{d}{dx}(J_1(x))$ (c) $J_2(x)$ (d) $\int J_1(x)dx$

50. The expression $\frac{d}{dx}\left(x^3 J_3(x)\right)$ can be simplified to

(a) $x^2 J_2(x)$ (b) $x^3 J_4(x)$ (c) $x^3 J_2(x)$ (d) $-x^3 J_2(x)$

51. The bessel function $J_0(1)$ can be best described by the series:

(a) $1-\frac{1}{2^2}+\frac{1}{2^2.4^2}-\frac{1}{2^2.4^2.6^2}+\dots\dots\dots$ (b) $1+\frac{1}{2^2}+\frac{1}{2^2.4^2}+\frac{1}{2^2.4^2.6^2}+\dots\dots\dots$

(c) $1+\frac{1}{2.2^2}+\frac{1}{3!.2^2.4^2}+\frac{1}{4!.2^2.4^2.6^2}+\dots\dots\dots$ (d) $1-\frac{1}{1^2}+\frac{1}{1^2.3^2}-\frac{1}{1^2.3^2.5^2}+\frac{1}{1^2.3^2.5^2.7^2}-\dots\dots\dots$

52. The value of the integral $\int_0^1 x\left[J_1(x)\right]^2 dx$ is equal to

(a) $J_2(1)^2$ (b) 0 (c) $\frac{1}{2}[J_2(1)]^2$ (d) $J_1(1)^2$

53. The bessel function $J_2(x)$ can be written as

(a) $J_1(x)-J_0(x)$ (b) $2J_1(x)-J_0(x)$ (c) $\frac{2}{x}J_1(x)-J_0(x)$ (d) $\frac{2}{x^2}J_1(x)-J_0(x)$

54. The expression $2J'_1(x)$ can be simpified to

(a) $J_0(x)-J_2(x)$ (b) $J_1(x)-J_0(x)$ (c) $J_0(x)-J_3(x)$ (d) $J_1(x)-J_3(x)$

55. The co-efficient of $z^{-3}$ in the expansion of $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)}$ is equal to

(a) $J_3(x)$ (b) $J_3(0)$ (c) $J_3(-x)$ (d) $-J_3(x)$

56. Let $p_n(x)$ (where n = 0,1,2, .....) be ploynomial of order 'n' wirh real co-efficients defined in the interval $2 \le n \le 4$. If $\int_2^4 p_m(x)p_n(x)dx = \delta_{mn}$, then **[CSIR June -2011]**

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

57. The genarating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x)t^n$ for Legendre polynomials $P_n(x)$ is $F(x,t) = (1-2xt+t^2)^{-1/2}$. The value of $P_3(-1)$ is **[CSIR Dec -2011]**

(a) 5/2 (b) 3/2 (c) 1 (d) -1

58. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$. Let, $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$. If $w(0) = 1$, then $w(1)$ is given by **[ CSIR Dec -2011]**

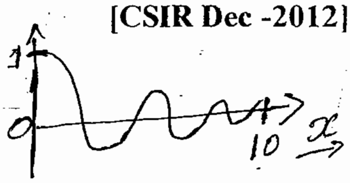
(a) 1 (b) $e^2$ (c) $\frac{1}{e}$ (d) $\frac{1}{e^2}$

59. Let y(x) be a continuous real function in the range of 0 to $2\pi$, satisfying the inhomogeneous differential equation: $\sin x\frac{d^2y}{dx^2} + \cos x\frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of $\frac{dy}{dx}$ at the point $x = \frac{\pi}{2}$ **[CSIR June -2012]**

(a) is continuous (b) has a discontinuity of 3

(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

60. The graph of the function f(x) shown below is best described by **[CSIR Dec -2012]**



(a) The bessel function $J_0(x)$ (b) $\cos x$

(c) $e^{-x}\cos x$ (d) $\frac{\cos x}{x}$

61. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2} - (3-4i)f = 0$ and satisfies the condition $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is **[CSIR Dec -2012]**

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi}$

62. Given that $\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$, the value of $H_4(0)$ is **[CSIR June -2013]**

(a) 12 (b) 6 (c) 24 (d) -6

63. The solution of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition x(0)=1 will blow up as t tend to **[CSIR June -2013]**

(a) 1 (b) 2 (c) 1/2 (d) ∞

64. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$ **[CSIR June -2014]**

with initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$. The solution $x(t)$ attains its maximum value when 't' is

(a) 1/2 ~~(b) 1~~ (c) 2 (d) ∞

65. Given, $\sum_{n=0}^{\infty} P_n(x)t^n = \left(1 - 2xt + t^2\right)^{-1/2}$ for $|t| < 1$, the value of $P_5(-1)$ is **[CSIR June -2014]**

(a) 0.26 (b) 1 (c) 0.5 (d) -1

## ANSWER KEY

**1.(a) 2.(d) 3.(c) 4.(b) 5.(a) 6.(b) 7.(b) 8.(b) 9.(c) 10.(b)**
**11.(c) 12.(d) 13.(b) 14.(d) 15.(b) 16.(a) 17.(c) 18.(d) 19.(a) 20.(a)**
**21.(a) 22.(b) 23.(c) 24.(c) 25.(a) 26.(a) 27.(b) 28.(c) 29.(d) 30.(c)**
**31.(a) 32.(a) 33.(d) 34.(b) 35.(d) 36.(d) 37.(c) 38.(b) 39.(c) 40.(c)**
**41.(a) 42.(b) 43.(d) 44.(c) 45.(a) 46.(a) 47.(b) 48.(a) 49.(c) 50.(c)**
**51.(a) 52.(c) 53.(c) 54.(a) 55.(d) 56.(d) 57.(d) 58.(d) 59.(d) 60.(a)**
**61.(c) 62.(a) 63.(a) 64.(b) 65.(d)**

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

upto =9

## SECTION: MATHEMATICAL PHYSICS

### Assignment-10: DIFFERENTIAL EQUATIONS

1. Solve the differential equations: (Separation of variables):

(i) $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$ given that $y(0)=\sqrt{3}$

(ii) $(e^y+1)\cos x dx + e^y \sin x dy = 0$ given that $y\left(\frac{\pi}{3}\right)=0$

(iii) $3\tan y . e^x dx + (1-e^x)\sec^2 y dy = 0$ given that $y(1)=\frac{\pi}{4}$

(iv) $\frac{dy}{dx} = (4x+y+1)^2$ given that $y(0)=1$

(v) $x^4\frac{dy}{dx} + x^3 y = -\sec(xy)$ given that $y(1)=\frac{\pi}{6}$

[*Ans.* (*i*) $\sqrt{1+x^2}+\sqrt{1+y^2}=3$; (*ii*) $\sin x\left(e^y+1\right)=\sqrt{3}$; (*iii*) $\left(1-e^x\right)^3=\left(1-e\right)^3\tan y$

(*iv*) $\tan^{-1}\left(\frac{4x+y+1}{2}\right)=2x+\frac{\pi}{4}$; (*v*) $\sin(xy)=\frac{1}{2x^2}$]

2. Solve the differential equations: (Homogeneous type)

(i) $(y^2-xy)dx + x^2 dy = 0$ given that $y(1)=1$

(ii) $\left(y\sin\frac{y}{x}-x\right)dx - x\sin\frac{y}{x}dy = 0$ given that $y(4)=\pi$

(iii) $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$ given that $y(0)=1$

(iv) $x\frac{dy}{dx} = y\left(\log\frac{y}{x}+1\right)$ given that $y(1)=2$

[*Ans.* (*i*) $\frac{x}{y}=\ln x+1$; (*ii*) $\cos\left(\frac{y}{x}\right)=\frac{1}{\sqrt{2}}-\ln 4+\ln x$; (*iii*) $e^{x/y}+\frac{x}{y}=\frac{1}{y}$; (*iv*) $\ln\frac{y}{x}=x\ln 2$]

3. Solve the differential equations: (Linear type)

(i) $\cos^2 x\frac{dy}{dx} + y = \tan x$ given that $y\left(\frac{\pi}{4}\right)=1$

(ii) $(1+y^2)dx = (\tan^{-1}y - x)dy$ given that $x(1)=-1$

(iii) $(x^3-x)\frac{dy}{dx} - (3x^2-1)y = x^5-2x^3+x$ given that $y(e)=e$

(iv) $(1-x^2)\dfrac{dy}{dx}+2xy=x(1-x^2)^{1/2}$ given that $y(0)=1$

(v) $3\dfrac{dy}{dx}+3\dfrac{y}{x}=2x^4y^4$ given that $y(1)=1$

(vi) $\cos x\dfrac{dy}{dx}+4y\sin x=4\sqrt{y}\sec x$ given that $y\left(\dfrac{\pi}{4}\right)=1$

(vii) $r\sin\theta-\dfrac{dr}{d\theta}\cos\theta=r^2$ given that $r\left(\dfrac{\pi}{4}\right)=1$

[*Ans.* $(i)\ y=(\tan x-1)+e^{(1-\tan x)}$; $(ii)\ x=\tan^{-1}y-1-\dfrac{\pi}{4}e^{\left(\frac{\pi}{4}-\tan^{-1}y\right)}$; $(iii)\ \dfrac{y}{x^3-x}=\ln x-\dfrac{e^2}{e^2-1}$;

$(iv)\ y=\sqrt{1-x^2}$; $(v)\ \dfrac{1}{y^3}=-x^5+2x^3$; $(vi)\ \sec^2 x.\sqrt{y}=2\left(\tan x+\dfrac{1}{3}\tan^3 x\right)-\dfrac{2}{3}$;

$(vii)\ \dfrac{1}{r}\sec\theta=\tan\theta+\left(\sqrt{2}-1\right)$]

4. Solve the following differential equations: (exact equation)

(i) $\left(x^4-2xy^2+y^4\right)dx-\left(2x^2y-4xy^3+\sin y\right)dy=0$ given that y (1) = 0

(ii) $\left[1+\ln(xy)\right]dx+\left[1+\dfrac{x}{y}\right]dy=0$ given that y (1) = 1

[*Ans.* $(i)\ \dfrac{x^5}{5}-x^2y^2+xy^4+\cos y=\dfrac{6}{5}$; $(ii)\ x\ln(xy)+y=1$ ]

5. A inductance 'L' and resistance 'R' are connected in seriers with an e.m.f 'E'. If the current is zero at time t = 0, find the current as a function of time. Also find the current at the end of 0.01 sec. If E = 100Volts, L=100 H, R = 20 Ohm.

6. A particle falls freely under gravity from a great height, there being a resistance proportional to the velocity of fall. Find the velocity and displacement as function of time.

7. The rate at which a hot body cools is proportional to the difference in temparature between it and surroundings (**Newton's law of Cooling**). A body is heated to $110^0$ C and placed in air at $10^0$. It's temparature becomes $60^0$ after 1 hour. After what additional time it cool down to $30^0$?

8. In one dimensional steady state heat coduction for a hollow cyllinder with thermal conductivity 'k' in the region $a\le r\le b$, the temparature $T_r$ at a distance r ($a\le r\le b$) is given by $\dfrac{d}{dr}\left[r\dfrac{dT_r}{dr}\right]=0$, subjected to the conditions $T_r=T_1$ at $r=a$ and $T_r=T_2$ at $r=b$. Determine the steaady state temaparature distributuion $T_r$.

[Ans. (5) $i=\dfrac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$; $i=5\left(1-e^{-0.002}\right)$, (6) $x=\dfrac{gm}{k}\left(t+\dfrac{m}{k}\left(e^{-(k/m)t}-1\right)\right)$;

(7) $\left(\dfrac{\ln 5}{\ln 2}-1\right)hrs.$, (8) $T_r=\dfrac{T_1-T_2}{\ln\left(\dfrac{r_1}{r_2}\right)}\ln r+\dfrac{T_2\ln r_1-T_1\ln r_2}{\ln\left(\dfrac{r_1}{r_2}\right)}$ ]

9. Solve the differential equations: (Homogeneous type):

(i) $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$ given that $y(0)=-1, y'(0)=0$

(ii) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=0$ given that $y(0)=2, y'(0)=1$

(iii) $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=0$ given that $y(0)=1, y'(0)=2$

(iv) $\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}+4\frac{dy}{dx}-8y=0$ given that $y(\infty)=0, y'(0)=2i$

[*Ans.* $(i)\ y=2e^{3x}-3e^{2x}$; $(ii)\ y=e^{-3x}\left(2\cos 2x+\frac{7}{2}\sin 2x\right)$; $(iii)\ y=(1-2x)e^{4x}$; $(iv)\ y=-e^{-2ix}$ ]

10. Solve the differential equations: (Non-homogeneous type):

(i) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=e^x\cosh 2x$ (ii) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=\sinh x$

(iii) $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}+4y=x^2-2x$ (iv) $(D^2-4D+3)y=x^3$

(v) $\frac{d^2y}{dx^2}+4y=2\cos x\cos 3x$ (vi) $\frac{d^3y}{dx^3}+y=\cos^2\left(\frac{x}{2}\right)+e^{-x}$

(vii) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^3e^{2x}$ (viii) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$

(ix) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x\sin x$ (x) $\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{3x}$

[*Ans.* $(i)\ y=c_1e^{3x}+c_2e^{-2x}+\frac{1}{10}xe^{3x}-\frac{1}{8}e^{-x}$; $(ii)\ y=e^{-x}(c_1\cos x+c_2\sin x)+\frac{1}{10}e^x+\frac{1}{2}e^{-x}$;

$(iii)\ y=e^{-\frac{3}{4}x}\left(c_1\cos\frac{\sqrt{23}}{4}x+c_2\sin\frac{\sqrt{23}}{4}x\right)+\frac{1}{32}(8x^2-28x+13)$; $(iv)\ y=c_1e^x+c_2e^{3x}+$

$\frac{1}{27}(9x^3+36x^2+78x+80)$; $(v)\ y=c_1\cos 2x+c_2\sin 2x-\frac{1}{12}\cos 4x+\frac{x}{4}\sin 4x$;

$(vi)\ y=c_1e^{-x}+e^{\frac{x}{2}}\left(c_2\cos\frac{\sqrt{3}}{2}x+c_3\sin\frac{\sqrt{3}}{2}x\right)+\frac{1}{2}+\frac{1}{4}(\cos x-\sin x+e^{-x})$;

$(vii)\ y=(c_1+c_2x)e^{2x}+e^{2x}\frac{x^5}{20}$; $(viii)\ y=(c_1+c_2x)e^x-e^x(x\sin x+2\cos x)$;

$(ix)\ y=(c_1+c_2x)e^x+\frac{1}{2}x\cos x+\frac{1}{2}\cos x-\frac{1}{2}\sin x$; $(x)\ y=c_1e^x+c_2e^{3x}+c_3e^{-2x}+\frac{x}{10}e^{3x}$]

11. Solve the following differential equations:

(i) $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}-4y=x^4$ (ii) $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=\sin\left(\ln x^2\right)$

(iii) $\dfrac{d^2y}{dx^2}+y=\sec x\tan x$ (iv) $\dfrac{d^2y}{dx^2}-y=\dfrac{2}{1+e^x}$

(v) $\dfrac{dx}{dt}+4x+3y=t,\ \dfrac{dy}{dt}+2x+5y=e^t$ (vi) $\dfrac{dx}{dt}+2y+\sin t=0,\ \dfrac{dy}{dt}-2x-\cos t=0$

[*Ans.* $(i)\ y=c_1x^4+\dfrac{c_2}{x}+\dfrac{(\ln x)}{5}x^4;\ (ii)\ y=c_1\cos(\ln x)+c_2\sin(\ln x)-\dfrac{1}{3}\sin\left(\ln x^2\right);$

$(iii)\ y=c_1\cos x+c_2\sin x+x\cos x+\sin x\ln(\sec x)-\sin x;$

$(iv)\ y=c_1e^x+c_2e^{-x}-1+e^x\ln\left(e^{-x}+1\right)-e^{-x}\ln\left(e^x+1\right);$

$(v)\ x=c_1e^{-2t}+c_2e^{-7t}+\dfrac{5}{14}t-\dfrac{31}{196}-\dfrac{1}{8}e^t,\ y=-\dfrac{2}{3}c_1e^{-2t}+c_2e^{-7t}-\dfrac{1}{7}t+\dfrac{9}{98}+\dfrac{5}{24}e^t;$

$(vi)\ x=c_1\cos 2t+c_2\sin 2t-\cos t,\ y=c_1\sin 2t-c_2\cos 2t-\sin t$]

12. The radial displacement ‘u’ in a rotating disc at a distance ‘r’ from the axis is given by,

$$r^2\frac{d^2u}{dr^2}+r\frac{du}{dr}-u+kr^3=0\quad \text{(k is constant)}$$

Find the displacement ‘u’ as a function of ‘r’ subject to the conditions $u(0)=0, u(a)=0$.

13. Check whether the following functions are linearly dependent or not?

(i) $e^{ax}, e^{bx}$ (ii) $e^{ax}\sin bx, e^{ax}\cos bx$

14. Find the singular points and discuss their nature for the following differential equation.

(i) $2x^2y''+3xy'+\left(x^2-4\right)y=0$ (ii) $x(x-2)^2y''+2(x-2)y'+(x+3)y=0$

15. Express the following polynomials into legendre polynomials:

(i) $4x^3+6x^2+7x+2$ **[Ans.** $\dfrac{8}{5}P_3(x)+4P_2(x)+\dfrac{47}{5}P_1(x)+4P_0(x)$**]**

(ii) $4x^3-2x^2-3x+8$ **[Ans.** $\dfrac{8}{5}P_3(x)-\dfrac{4}{3}P_2(x)-\dfrac{3}{5}P_1(x)+\dfrac{22}{3}P_0(x)$**]**

(iii) $x^4+2x^3-6x^2+5x-3$ **[Ans.** $\dfrac{8}{35}P_4(x)+\dfrac{4}{5}P_3(x)-\dfrac{24}{7}P_2(x)+\dfrac{31}{5}P_1(x)-\dfrac{24}{5}P_0(x)$**]**

(iv) $x^5$ **[Ans.** $\dfrac{8}{63}P_5(x)+\dfrac{7}{9}P_3(x)+\dfrac{3}{7}P_1(x)$**]**

16. Prove that:

(i) $\displaystyle\sum_{n=0}^{\infty}P_n(x)=\frac{1}{\sqrt{2-2x}}$ (ii) $\displaystyle\frac{1-z^2}{\left(1-2xz+z^2\right)^{3/2}}=\sum_{n=0}^{\infty}(2n+1)P_n(x)$

(iii) $\log \dfrac{1+\sin\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = 1+\frac{1}{2}P_1(\cos\alpha)+\frac{1}{3}P_2(\cos\alpha)+\frac{1}{4}P_3(\cos\alpha)+\ldots\ldots$

(iv) $P'_{n+1}+P'_n = P_0+3P_1+5P_2+\ldots\ldots+(2n+1)P_n$

(v) $\int_{-1}^{1} x^2 P_{n+1}(x)P_{n-1}(x)\,dx = \dfrac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$

(vi) $\int_{-1}^{1} (1-x^2)\left[P'_n(x)\right]^2\,dx = \dfrac{2n(n+1)}{(2n+1)}$

(vii) $\int_{-1}^{1} xP_n(x)P_{n-1}(x)\,dx = \dfrac{2n}{4n^2-1}$

17. Prove that,

(i) $\sqrt{\dfrac{\pi x}{2}}J_{3/2}(x) = \dfrac{\sin x}{x} - \cos x$

(iii) $\dfrac{d}{dx}\left[J_n^2(x)+J_{n+1}^2(x)\right] = 2\left[\dfrac{n}{x}J_n^2(x)-\dfrac{n+1}{x}J_{n+1}^2(x)\right]$

(iii) $\cos(x.\sin\theta) = J_0+2J_2\cos 2\theta+2J_4\cos 4\theta+\ldots\ldots\ldots\ldots\ldots\ldots$

$\sin(x.\sin\theta) = 2J_1\sin\theta+2J_3\sin 3\theta+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

(iv) $\dfrac{d}{dx}\left[x.J_n(x).J_{n+1}(x)\right] = x\left[J_n^2(x)-J_{n+1}^2(x)\right]$

18. Express the following polynomials as Hermite polynomials:

(i) $64x^4+8x^3-32x^2+40x+10$ **[Ans.** $4H_4(x)+H_3(x)+40H_2(x)+26H_1(x)+42H_0(x)$**]**

(ii) $8x^3+8x^2-6x+2$ **[Ans.** $H_3(x)+2H_2(x)+3H_1(x)+6H_0(x)$**]**

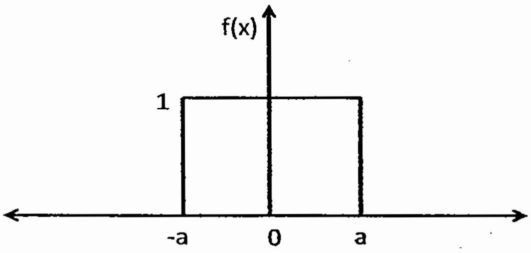
19. Find the value of: (i) $\int_{-\infty}^{\infty} e^{-x^2}H_{20}(x)H_{10}(x)dx$ (ii) $\int_{-\infty}^{\infty} e^{-x^2}\left[H_4(x)\right]^2 dx$

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment-10: FOURIER TRANSFORM

1. The fourier tranform of the following function will be



(a) $\sqrt{\frac{2}{\pi}}\frac{\sin sa}{s}$ (b) $\sqrt{\frac{2}{\pi}}\frac{\cos sa}{s}$ (c) $\frac{1}{\sqrt{2\pi}}\frac{\cos sa}{s}$ (d) $\frac{1}{\sqrt{2\pi}}\frac{\sin sa}{s}$

2. The fourier transform of the function: $f(x)=1-x^2$ for $|x|\le 1$
$=0$ for $|x|>1$

(a) $\frac{1}{\sqrt{2\pi}}\frac{4}{s^3}[\text{-s.cos(s)+sin(s)}]$ (b) $\frac{1}{\sqrt{2\pi}}\frac{4}{s^3}[\text{s.cos(s)+sin(s)}]$

(c) $\frac{1}{\sqrt{2\pi}}\frac{4}{s^3}[\text{s.sin(s)+cos(s)}]$ (d) $\frac{1}{\sqrt{2\pi}}\frac{4}{s^3}[\text{s.sin(s)-cos(s)}]$

3. The fourier transform of the Gaussian wave packet given as:

$$f(x)=e^{-x^2/2} \quad \text{for} \quad -\infty<x<\infty$$

will be

(a) $\frac{1}{\sqrt{2\pi}}e^{-s^2/2}$ (b) $e^{-s^2/2}$ (c) $\sqrt{\frac{2}{\pi}}e^{-s^2}$ (d) $\sqrt{\frac{2}{\pi}}e^{-s^2/4}$

4. The fourier tranform of the function: $f(x)=e^{-a|x|}$ for $-\infty<x<\infty\,(a>0)$

(a) $\sqrt{\frac{2}{\pi}}\frac{2}{a^2-s^2}$ (b) $\frac{1}{\sqrt{2\pi}}\frac{2}{a^2-s^2}$ (c) $\frac{1}{\sqrt{2\pi}}\frac{2a}{a^2+s^2}$ (d) $\frac{1}{\sqrt{2\pi}}\frac{2a}{a^2-s^2}$

5. The fourier transform of $f(x)=e^{-x^2+2x-1}(-\infty<x<\infty)$ will be

(a) $e^{-\left(is+\frac{s^2}{2}\right)}$ (b) $e^{-\left(is-\frac{s^2}{2}\right)}$ (c) $\frac{1}{\sqrt{2}}e^{\left(is-\frac{s^2}{4}\right)}$ (d) $\frac{1}{\sqrt{2}}e^{\left(is+\frac{s^2}{2}\right)}$

6. If the fourier transform of f (x) is f (s), then the fourier transform of $\frac{d}{dx}\left[f(x)\right]$ will be

(a) $sf(s)$ (b) $-isf(s)$ (c) $-2isf(s)$ (d) $2isf(s)$

7. The fourier tranform of the following function: $f(x)=e^{-s^2}\cos 2x\left(-\infty<x<\infty\right)$ will be

(a) $\frac{1}{\sqrt{2}}e^{-\left(\frac{s^2+4}{4}\right)}\cosh(4s)$ (b) $\frac{1}{\sqrt{2}}e^{-\left(\frac{s^2+4}{4}\right)}\cosh(s)$

(c) $\frac{1}{\sqrt{2}}e^{-\left(s^2-4\right)}\cosh(s)$ (d) $e^{-\left(s^2-4\right)}\sinh(2s)$

8. If the fourier transform of f (x) is f (s), then the fourier transform of $\frac{d^n}{dx^n}\left[f(x)\right]$ will be

(a) $isf(s)$ (b) $-isf(s)$ (c) $(-is)^n f(s)$ (d) $(-is)^{n-1}f(s)$

9. Use the information of Q.1. and then the value of the following integral will be :

$$\int_0^\infty\left(\frac{\sin t}{t}\right)^2 dt$$

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) 0 (d) 1

10. The value of the following integral:

$$\int_0^\infty\frac{dx}{\left(a^2+x^2\right)\left(b^2+x^2\right)}$$

(a) $\frac{\pi}{2ab(a-b)}$ (b) $\frac{\pi}{2ab(a+b)}$ (c) $\frac{\pi}{2ab}$ (d) $\frac{\pi}{2(a+b)}$

11. The fourier transform of the dirac delta function $f(x)=\delta(x-a)$ will be

(a) $\frac{1}{\sqrt{2\pi}}$ (b) $\frac{1}{\sqrt{\pi}}$ (c) $\frac{1}{\sqrt{2\pi}}e^{isa}$ (d) $\frac{1}{\sqrt{2\pi}}e^{sa}$

## CSIR Previous Year Questions

12. Fourier Transform of the derivative of the dirac $\delta$ -function, namely $\delta'(x)$ is proportional to

**[CSIR Dec 2013]**

(a) 0 (b) 1 (c) $\sin k$ (d) ik

## ANSWER KEY

**1. (a)** **2. (a)** **3. (b)** **4. (c)** **5. (c)** **6. (b)** **7. (b)**

**8. (c)** **9. (a)** **10. (b)** **11. (c)** **12. (d)**

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

## Assignment 11: LAPLACE TRANSFORM

1. Evaluate the Laplace transform of the following functions:

(i) $f(x) = (1+\cos 2x)$

(ii) $f(x) = (x-1) \quad \text{if } 1<t<2$
$\quad\; = (3-x) \quad \text{if } 2<t<3$

(iii) $f(x) = 3e^{3x} + 5x^4 - 4\cos(3x)$

(iv) $f(x) = 2\sin 2x \cos 4x$

(v) $f(x) = e^{-x}(3\sinh 2x - 5\cosh 2x)$

(vi) $f(x) = \left(1+xe^{-x}\right)^3$

(vii) $f(x) = \cos^2 ax$

(viii) $f(x) = \sinh(ax)\cos(ax)$

(ix) $f(x) = e^{-ax}\dfrac{x^{n-1}}{(n-1)!}$

(x) $f(x) = \left(x^2-3x+2\right)\sin 3x$

(xi) $f(x) = \sin \omega x \quad \text{if } 0 < x < \dfrac{\pi}{\omega}$
$\quad\; = 0 \quad \text{if } \dfrac{\pi}{\omega} < x < \dfrac{2\pi}{\omega}$

(xii) $f(x) = \dfrac{1}{\in} \quad \text{if } 0 \le x \le \in$
$\quad\; = 0 \quad \text{if } x > \in$

(xiii) $f(x) = \dfrac{1-e^{x}}{x}$

(xiv) $f(x) = \dfrac{\sin 2x}{x}$

$$\left[\begin{array}{l}
\text{Ans. }(i)\ \dfrac{\left(2s^2+4\right)}{s\left(s^2+4\right)},\ (ii)\ \dfrac{1}{s^2}\left(e^{-s}-2e^{-2s}+e^{-3s}\right),\ (iii)\ \dfrac{3}{s-3}+\dfrac{120}{s^5}-\dfrac{4s}{s^2+9},\ (iv)\ \dfrac{6}{s^2+36}-\dfrac{2}{s^2+4}, \\
(v)\ \dfrac{1-5s}{s^2+2s-3},\ (vi)\ \dfrac{1}{s}+\dfrac{3}{(s+1)^2}+\dfrac{6}{(s+2)^3}+\dfrac{6}{(s+3)^4},\ (vii)\ \dfrac{s^2+2a^2}{s\left(s^2+4a^2\right)},\ (viii)\ \dfrac{a\left(s^2-2a^2\right)}{s^4+4a^4}, \\
(ix)\ \dfrac{1}{(s+a)^n},\ (x)\ -\dfrac{18s^2+54}{\left(s^2+9\right)^3}-\dfrac{18s}{\left(s^2+9\right)^2}+\dfrac{6}{\left(s^2+9\right)}\ (xi)\ \dfrac{1}{1-e^{-\frac{2\pi s}{\omega}}}\left(\dfrac{\omega e^{-\frac{s\pi}{\omega}}+\omega}{s^2+\omega^2}\right), \\
(xii)\ \dfrac{1}{\in s}\left(1-e^{-\in s}\right),\ (xiii)\ \ln\dfrac{s-1}{s},\ (xiv)\ \cot^{-1}\dfrac{s}{2}
\end{array}\right]$$

2. Laplace transform of $f'(x)$ can be represented as

(a) $sL[f(x)]$ (b) $sL[f(x)]-f(0)$ (c) $L[f(x)]-f(0)$ (d) $L[f(x)]-f(1)$

3. Laplace transform of $f''(x)$ can be represented as

(a) $L^2[f(x)]$ (b) $s^2L^2[f(x)]-f(0)$

(c) $s^2L[f(x)]-f'(0)-sf(0)$ (d) $s^2L[f(x)]$

4. Laplace tranform of $J_0(x)$ can be represented as

(a) $\frac{1}{1+s^2}$ (b) $\frac{1}{\sqrt{1+s^2}}$ (c) $\frac{s}{\sqrt{1+s^2}}$ (d) $\frac{s}{1+s^2}$

5. Laplace transform of unit step function $u_a(x)=u(x-a)=0 \quad if\ x<a$ is given by,
$=1 \quad if\ x\ge a$

(a) $e^{-sa}$ (b) $\frac{e^{-sa}}{s}$ (c) $\frac{e^{-sa}}{s^2}$ (d) $e^{-2sa}$

6. The value of the integral $\int_0^\infty xe^{-2x}\sin x\,dx$ is equal to

(a) $\frac{4}{25}$ (b) $\frac{4}{50}$ (c) $\frac{2}{15}$ (d) $\frac{3}{35}$

7. The value of the integral $\int_0^\infty \frac{\sin x}{x}dx$ is equal to

(a) 0 (b) $\frac{\pi}{2}$ (c) $\pi$ (d) $2\pi$

8. If the error function is defined as: $erf(t)=\frac{2}{\sqrt{\pi}}\int_0^t e^{-x^2}dx$, then the laplace transform of $erf\left(\sqrt{t}\right)$ is equal to

(a) $\frac{1}{\sqrt{s(s+1)}}$ (b) $\frac{1}{\sqrt{(s+1)}}$ (c) $\frac{1}{s\sqrt{s(s+1)}}$ (d) $\frac{1}{s\sqrt{(s+1)}}$

9. The laplace transform of the following function:

$$f(x)=k\left[u_0(x)-2u_a(x)+2u_{2a}(x)-2u_{3a}(x)+\ldots\ldots\ldots\ldots\right]$$

(a) $\frac{k}{s}\tanh\left(\frac{as}{2}\right)$ (b) $\frac{k}{s}\tanh(as)$ (c) $\frac{k}{2s}\tanh(as)$ (d) $\frac{k}{2s}\tanh\left(\frac{as}{2}\right)$

10. The inverse laplace transform of $f(s)=\frac{s-2}{(s-2)^2+25}+\frac{s+4}{(s+4)^2+81}+\frac{1}{(s+2)^2+9}$ is

(a) $e^{-2x}\cos 5x+e^{-4x}\cos 9x+\frac{e^{-2x}}{3}\sin 3x$ (b) $e^{2x}\cos 5x+e^{-4x}\cos 9x+\frac{e^{-2x}}{3}\sin 3x$

(c) $e^{2x}\cos 5x+e^{4x}\cos 9x+\frac{e^{2x}}{3}\sin 3x$ (d) $e^{2x}\cos 5x+e^{4x}\cos 9x+\frac{e^{-2x}}{3}\sin 3x$

11. The inverse laplace transform of $f(s)=\frac{1}{(s-4)^5}+\frac{5}{(s-2)^2+25}$ is

(a) $e^{4x}\frac{x^4}{12}-e^{2x}\cos 5x$ (b) $e^{4x}\frac{x^4}{12}+e^{2x}\cos 5x$

(c) $e^{4x}\frac{x^4}{24}+e^{2x}\sin 5x$ (d) $e^{4x}\frac{x^4}{12}-e^{2x}\sin 5x$

12. The inverse Laplace transform of $f(s)=\frac{1}{s^2(s+1)^2}$ is

(a) $te^{-t}+2e^{-t}+t+2$ (b) $te^{-t}+2e^{-t}+t-2$

(c) $te^{-t}+e^{-t}+t-2$ (d) $te^{-t}-e^{-t}+t-2$

13. The inverse Laplace transform of $f(s)=\frac{3s+7}{s^2-2s-3}$ is

(a) $2e^{3x}-e^{-x}$ (b) $4e^{3x}+e^{-x}$ (c) $2e^{3x}+e^{-x}$ (d) $4e^{3x}-e^{-x}$

**CSIR Previous Year Questions**

14. The graph of the function $f(x)=\left\{\begin{matrix}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{matrix}\right\}$ (where n = 0,1,2........)..Its laplae transform $\tilde{f}(s)$ is **[CSIR Dec 2011]**

(a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s\left(1+e^{-s}\right)}$ (d) $\frac{1}{s\left(1-e^{-s}\right)}$

15. The inverse Laplace tranform of $\frac{1}{s^2(s+1)}$ is **[CSIR June 2013]**

(a) $\frac{1}{2}t^2e^{-t}$ (b) $\frac{1}{2}t^2+1-e^{-t}$ (c) $t-1+e^{-t}$ (d) $\frac{1}{2}t^2\left(1-e^{-t}\right)$

**ANSWER KEY**

**2.(b)** **3.(c)** **4.(b)** **5.(b)** **6.(a)** **7.(b)**

**8.(d)** **9.(a)** **10.(b)** **11.(c)** **12.(b)** **13.(d)**

**14.(c)** **15.(c)**

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## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)
## SECTION: MATHEMATICAL PHYSICS

### Assignment 12: PARTIAL DIFFERENTIAL EQUATION

1. The solution of the differential equation $2x\frac{\partial u}{\partial x}-3y\frac{\partial u}{\partial y}=0$ is

(a) $Ax^{k/2}y^{k/3}$ (b) $Ax^{k/3}y^{k/2}$ (c) $Ax^{k/3}y^{k/3}$ (d) $Ax^{k/3}y^{k/4}$

2. The solution of the differential equation $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ (Given that, $u(x,0)=6e^{-3x}$) is

(a) $u(x,t)=6e^{-3x+2t}$ (b) $u(x,t)=6e^{-3x-3t}$ (c) $u(x,t)=6e^{-3x-2t}$ (d) $u(x,t)=6e^{-2x+3t}$

3. The solution of the differential equation $4\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=3x$ (Given that, $u(x,0)=3e^{-x}-e^{-5x}$) is

(a) $u(x,t)=3e^{t-x}+e^{2t-5x}$ (b) $u(x,t)=3e^{t-x}-e^{2t-5x}$

(c) $u(x,t)=3e^{t-x}+e^{-2t+5x}$ (d) $u(x,t)=3e^{t-x}-e^{-2t-5x}$

4. The vibration of a string of length 'L', fixed at both ends is described by the equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

subjected to the conditions $u(0,t)=u(L,t)=0$, $u(x,0)=1$ *and* $\frac{\partial}{\partial t}u(x,0)=0$. The solution of the differential equation can be written as

(a) $\frac{2}{n\pi}\left[1-(-1)^n\right]\sin\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}$ (b) $\frac{2}{n\pi}\left[1-(-1)^n\right]\sin\frac{n\pi x}{L}\sin\frac{n\pi ct}{L}$

(c) $\frac{2}{n\pi}\left[(-1)^n-1\right]\sin\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}$ (d) $\frac{2}{n\pi}\left[1-(-1)^n\right]\cos\frac{n\pi x}{L}\sin\frac{n\pi ct}{L}$

5. The vibration of an elastic spring is governed by the partial differential equation

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$$

The length of the spring is $\pi$ and both ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0)=2(\sin x+\sin 3x)$. The deflection $u(x,t)$ of the vibrating string will be

(a) $u(x,t)=2[\cos 2t\sin x+\cos 3t\sin 3x]$ (b) $u(x,t)=2[\cos t\sin x+\cos 3t\sin 3x]$

(c) $u(x,t)=2[\sin t\sin x+\cos 3t\cos 3x]$ (d) $u(x,t)=2[\sin t\sin x-\cos 3t\cos 3x]$

6. A string is streched and fastened to two points distance '$l$' apart. Motion is started by displacing the string into the form $y = kx(l-x)$ from which it is released at time t = 0. The displacement $y(x,t)$ at any time is written as

$$y(x,t) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}$$

The value of $A_n$ is

(a) 0 for odd n (b) 0 for even n (c) $\frac{8kl^2}{n^3\pi^3}$ for odd n (d) $\frac{8kl^2}{n^3\pi^3}$ for even n

7. The temparature distribututíon of a bar of length 2 units, (ends are kept at zero temparature) is governed by the following dfferential equation

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$$

subjected to the conditions $u(x,0) = \sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right)$

The temparature of the rigid bar will be

(a) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2c^2t/4} + 3\sin\left(\frac{5\pi x}{2}\right)e^{-25\pi^2c^2t/4}$ (b) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2c^2t/4} + 3\sin\left(\frac{5\pi x}{2}\right)e^{-16\pi^2c^2t/4}$

(c) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2c^2t/4} + \cos\left(\frac{5\pi x}{2}\right)e^{-25\pi^2c^2t/4}$ (d) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2c^2t/4} + 3\cos\left(\frac{5\pi x}{2}\right)e^{-25\pi^2c^2t/4}$

8. A rectangular plate with insulated surface is 8 cm. wide and its length can be treated as infinte compared to its width so that it may be considered as infinite in length. If the temparature along one short edge y = 0 is given by

$$u(x,0) = 100\sin\left(\frac{\pi x}{8}\right); (0 < x < 8)$$

The steady state temparature any plate will be
(Assume that all the edges of the rectangular plate are kept at $0^0$ C)

(a) $100\sin\left(\frac{\pi x}{4}\right)e^{-\frac{\pi y}{4}}$ (b) $100\cos\left(\frac{\pi x}{4}\right)e^{-\frac{\pi y}{4}}$ (c) $100\sin\left(\frac{\pi x}{8}\right)e^{-\frac{\pi y}{8}}$ (d) $100\cos\left(\frac{\pi x}{8}\right)e^{-\frac{\pi y}{8}}$

9. The solution of the differential equation:

$$\frac{\partial^2}{\partial t^2}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = 0$$

satisfying the boundary conditions $u(0,t) = 0 = u(L,t)$ and initial conditions

$u(x,0) = \sin\left(\frac{\pi x}{L}\right)$ and $\frac{\partial}{\partial t}u(x,t)|_{t=0} = \sin\left(\frac{2\pi x}{L}\right)$ is **[CSIR June 2013]**

(a) $\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi t}{L}\right) + \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right)$

(b) $2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi t}{L}\right)-\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right)$

(c) $\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right)+\frac{L}{\pi}\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{\pi t}{L}\right)$

(d) $\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi t}{L}\right)+\frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi t}{L}\right)$

## ANSWER KEY

**1.(a)** **2.(c)** **3.(b)** **4.(a)** **5.(b)** **6.(b), (c)** **7.(a)**

**8.(c)** **9.(d)**

## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment-13: NUMERICAL TECHNIQUES

1. Each of the following numbers are rounded-off upto three decimal places:

$$3.14750,\ 10.54761,\ 8.23450,\ 3.46756$$

Which of the following is true?

(a) $3.14750 \approx 3.148,\ 10.54761 \approx 10.548,\ 8.23450 \approx 8.235,\ 3.46756 \approx 3.468$

(b) $3.14750 \approx 3.148,\ 10.54761 \approx 10.548,\ 8.23450 \approx 8.234,\ 3.46756 \approx 3.468$

(c) $3.14750 \approx 3.147,\ 10.54761 \approx 10.548,\ 8.23450 \approx 8.234,\ 3.46756 \approx 3.468$

(d) $3.14750 \approx 3.148,\ 10.54761 \approx 10.547,\ 8.23450 \approx 8.234,\ 3.46756 \approx 3.468$

2. If the number 9.0894 is rounded-off to three significant figures, then the round-off error is

(a) 0.0004 (b) -0.0004 (c) 0.001 (d) -0.001

3. Consider the following series:

$$\sqrt{1}+\sqrt{2}+\sqrt{3}+\dots\dots\dots+\sqrt{100}$$

If each of the roots are calculated upto rounding-off two decimal places, then the maximum possible error in the calculation of the sum of the series, is

(a) 0.05 (b) 0.5 (c) 5 (d) 50

4. The interval for the smallest positive root of the transcendental equation $x = e^{x}$, is

(a) [-1,0] (b) [-1,1] (c) [0,1] (d) [1,2]

5. If the equation $x^3 - 3x + K = 0$ has a real root in the interval [-1,1], the the range of K is

(a) (-2,2) (b) (0,2) (c) (-2,0) (d) (1,3)

6. The Newton raphson iterative formula to evaluate the square root of a natural number 'N' is

(a) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ (b) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$

(c) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{2x_n}\right)$ (d) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{2x_n}\right)$

7. The Newton raphson iterative formula to evaluate the cube root of a natural number 'N' is

(a) $x_{n+1} = \frac{3}{2}\left(x_n + \frac{N}{2x_n^2}\right)$ (b) $x_{n+1} = \frac{3}{2}\left(x_n - \frac{N}{2x_n^2}\right)$

(c) $x_{n+1} = \frac{2}{3}\left(x_n + \frac{N}{2x_n^2}\right)$ (d) $x_{n+1} = \frac{2}{3}\left(x_n - \frac{N}{2x_n^2}\right)$

8. Consider the following equation: $x^4 - x - 10 = 0$.
To solve the equation numerically using Newton-Raphson method, which one will be the best possible first guess value?
(a) 0 (b) 1 (c) 3/2 (d) 2

9. The first order forward difference of $\ln\left[f(x)\right]$ is

(a) $\ln\left[1+\frac{\Delta f(x)}{f(x)}\right]$ (b) $\ln\left[1-\frac{\Delta f(x)}{f(x)}\right]$ (c) $\ln\left[1-\frac{f(x)}{\Delta f(x)}\right]$ (d) $\ln\left[1+\frac{f(x)}{\Delta f(x)}\right]$

10. The second order forward difference of $f(x) = \cos 2x$ is

(a) $4\sin^2 h.\cos(2x+2h)$ (b) $-4\sin^2 h.\cos(2x+2h)$

(c) $-4\cos^2 h.\sin(2x+2h)$ (d) $4\cos^2 h.\sin(2x+2h)$

Consider the following data:

| $x$: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$: | 1 | 7 | 17 | 31 |

11. The value of $f(x)$ at $x=1.5$ is
(a) 2.5 (b) 3 (c) 3.5 (d) 4

12. The functional relationship between x and $f(x)$ for the above mentioned data, is

(a) $f(x)=x^2-1$ (b) $f(x)=2x^2-1$ (c) $f(x)=2x^2+1$ (d) $f(x)=x^2+1$

13. Consider the following data:

| $x$: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$: | 0 | 1 | 3 | 10 |

The polynomial corresponding to the given data, is

(a) $\frac{2}{3}x^3-\frac{3}{2}x^2+\frac{11}{6}x$ (b) $\frac{2}{3}x^3-\frac{3}{2}x^2-\frac{11}{6}x$ (c) $\frac{2}{3}x^3+\frac{3}{2}x^2-\frac{11}{6}x$ (d) $\frac{2}{3}x^3+\frac{3}{2}x^2+\frac{11}{6}x$

14. Consider the following data:

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$: | 0 | 3 | 0 | 3 | 0 | ? | ? |

If the given data a describes fourth order polynomial, then the missing terms are
(a) 45,192 (b) 45,−192 (c) −45,192 (d) −45,−192

15. Consider the following operators:

$$E\left[f(x)\right]=f(x+h);\ D\left[f(x)\right]=\frac{df}{dx}$$

Which of the following relation is true?

(a) $E=e^{hD}$ (b) $E=e^{-hD}$ (c) $E=1+e^{hD}$ (d) $E=1-e^{hD}$

16. Consider the following opeators:

$$\delta[f(x)]=f\left(x+\frac{h}{2}\right)-f\left(x-\frac{h}{2}\right);\Delta[f(x)]=f(x+h)-f(x)$$

Which of the following relations is true?

(a) $\Delta=\frac{1}{2}\delta^2-\delta\sqrt{1+\frac{1}{4}\delta^2}$ (b) $\Delta=\frac{1}{2}\delta^2+\delta\sqrt{1+\frac{1}{4}\delta^2}$

(c) $\Delta=\frac{1}{2}\delta^2+\delta\sqrt{1-\frac{1}{4}\delta^2}$ (d) $\Delta=\frac{1}{2}\delta^2-\delta\sqrt{1-\frac{1}{4}\delta^2}$

17. The missing term in the following table:

| | | | | | |
|---|---|---|---|---|---|
| $x$: | 2 | 3 | 4 | 5 | 6 |
| $f(x)$: | 45.0 | 49.2 | 54.1 | ? | 67.4 |

(a) 12.5 (b) 24.75 (c) 45.8 (d) 60.05

18. The missing values of the following table:

| | | | | | |
|---|---|---|---|---|---|
| $x$: | 45 | 50 | 55 | 60 | 65 |
| $f(x)$: | 3.0 | ? | 2.0 | ? | –2.4 |

(a) 2.925, 0.225 (b) 2.525, 0.225 (c) 2.925, 0.775 (d) 2.525, 0.775

19. Consider the following data:

| | | | | | |
|---|---|---|---|---|---|
| $x$: | –1 | 0 | 1 | 2 | 3 |
| $f(x)$: | 1 | 5 | 3 | 1 | 5 |

The value of the integral $\int_{-1}^{3} f(x)\,dx$ is

(a) 8 (b) 10 (c) 12 (d) 16

20. Consider the following data:

| | | | | |
|---|---|---|---|---|
| $x$: | 3 | 6 | 9 | 12 |
| $f(x)$: | –1 | 1 | 2 | 3 |

The value of the integral $\int_{3}^{12} f(x)\,dx$ is

(a) 15 (b) 10 (c) 12 (d) 5

21. To evaluate $\int_{x_0}^{x_n} y\,dx$ by Simpson's 1/3 rd rule as well as Simpson's 3/8 th rule, the number of intervals should be

(a) 2 (b) 3 (c) 6 (d) 10

22. To evaluate $\int_{x_0}^{x_n} y\,dx$ by Simpson's 1/3 rd rule as well as Trapezoidal rule, the number of intervals should be

(a) 2 (b) 3 (c) 6 (d) 10

(110)

A second degree polynomial f(x) has the following data:

| | | | |
|---|---|---|---|
| $x$: | 0 | 1 | 2 |
| $f(x)$: | 1 | 4 | 15 |

23. The value of the integral $\int_0^2 f(x)\,dx$ (using Trapezoidal rule) will be

(a) 8 (b) 12 (c) 16 (d) 20

24. The error in the estimation by trapezoidal rule, will be

(a) -4/3 (b) 4/3 (c) 5/3 (d) -5/3

25. Consider the following differential equation: $\frac{dy}{dx} = -y$ given that, at x = 0, y=1

The value of y at x = 0.1 (using Runge-kutta method of third order) is

(a) 0.24 (b) 0.46 (c) 0.91 (d) 0.99

26. Consider the following differential equation: $\frac{dy}{dx} = x + y$ given that, at x = 0, y=1

The value of y at x = 0.2 (using Runge-kutta method of fourth order) is

(a) 0.94 (b) 1.11 (c) 1.24 (d) 1.67

## ANSWER KEY

**1.(b)** **2.(d)** **3.(c)** **4.(c)** **5.(a)** **6.(a)**
**7.(c)** **8.(d)** **9.(a)** **10.(b)** **11.(c)** **12.(b)**
**13.(a)** **14.(d)** **15.(a)** **16.(b)** **17.(d)** **18.(a)**
**19.(c)** **20.(c)** **21.(c)** **22.(a)** **23.(b)** **24.(a)**
**25.(c)** **26.(c)**

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## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment 14: TENSOR ANALYSIS

1. Which of the following is a tensor of order '0'?

(a) $\vec{A}+\vec{B}$ (b) $\vec{A}-\vec{B}$ (c) $\vec{A}.\vec{B}$ (d) $\vec{A}\times\vec{B}$

2. Which of the following statements is/are true?

(a) Gradient of scalar field is a contravariant tensor of rank 1
(b) Gradient of scalar field is a covariant tensor of rank 1
(c) Velocity is contravariant tensor of rank 1
(d) Velocity is contravariant tensor of rank 2

3. If A is a scalar field, show that the second order derivatives of A i.e $\frac{\partial^2 A}{\partial x^i \partial x^j}$ are not the components of a tensor.

4. A covariant tensor has components $2x-z,\ x^2y,\ yz$ in cartesian co-ordinate system. Find its components in cyllindrical co-ordinate system.

[Ans. $2\rho\cos^2\varphi - z\cos\varphi + \rho^3\cos^2\varphi\sin^2\varphi$; $-\rho^2\sin 2\varphi + \rho z\sin\varphi + \rho^4\cos^3\varphi\sin\varphi$; $\rho z\sin\varphi$]

5. The number of components for a tensor of rank 3 is

(a) 9 (b) 27 (c) 81 (d) 3

6. The Krocnker delta tensor $\delta_i^j$ can be expressed as

(a) $\frac{\partial x^j}{\partial \bar{x}^k}\frac{\partial \bar{x}_k}{\partial x^i}$ (b) $\frac{\partial x^j}{\partial \bar{x}^k}\frac{\partial \bar{x}^k}{\partial x^i}$ (c) $\frac{\partial x^j}{\partial \bar{x}_k}\frac{\partial \bar{x}^k}{\partial x^i}$ (d) $\frac{\partial x^i}{\partial \bar{x}^k}\frac{\partial \bar{x}^k}{\partial x^j}$

7. In contraction process,

(a) the rank of the tensor is reduced by 2.
(b) the rank of the tensor is reduced by 1.
(c) the rank of the tensor can be reduced by any number.
(d) the rank of the tensor is increased by 2.

8. $S_{ij}$ and $A_{ij}$ represent a symmetric and anti-symmetric real valued tensor respectively in three dimension. The number of independent components of $S_{ij}$ and $A_{ij}$ are

(a) 3 and 6 (b) 6 and 3 (c) 6 and 6 (d) 9 and 6

9. Let, $T_{ij}=\varepsilon_{ijk}a_k$ and $\beta_k=\varepsilon_{ijk}T_{ij}$ where $\varepsilon_{ijk}$ is the Levi-Civita tensor. Then $\beta_3$ is equal to

[✓ (a)] $2a_3$ (b) $-2a_3$ (c) $a_3$ (d) $-a_3$

10. Which of the following is/are correct for Kroncker delta tensor $\delta_{ij}$ and Alternate tensor $\varepsilon_{ijk}$?

(a) $\varepsilon_{ijk}\delta_{ij}=1$ (b) $\delta_{ij}\varepsilon_{ijk}=1$ (c) $\varepsilon_{ijm}\varepsilon_{klm}=\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}$ (d) $\varepsilon_{ijl}\varepsilon_{kjl}=2\delta_{ik}$

11. Find the metric tensor for the following:
(a) a two dimensional plane in cartesian co-ordinate system.
(b) a two dimensional plane in plane polar co-ordinate system.
(c) three dimensional cartesian co-ordinate system.
(d) three dimensional spherical polar co-ordinate system.
(e) three dimensional cyllindrical co-ordinate system.

12. If the line element 'ds' in a certain co-ordinate system is given by,

$$(ds)^2=(dr)^2+r^2\sin\theta(d\theta)^2+r\sin^2\varphi(d\varphi)^2+r\sin 2\varphi(drd\theta)$$

then, the metric tensor will be

(a) $\begin{bmatrix}1 & 0 & 0\\ 0 & r^2\sin\theta & 0\\ 0 & 0 & r\sin^2\varphi\end{bmatrix}$ (b) $\begin{bmatrix}1 & r\sin 2\varphi & 0\\ r\sin 2\varphi & r^2\sin\theta & 0\\ 0 & 0 & r\sin^2\varphi\end{bmatrix}$

(c) $\begin{bmatrix}1 & r\sin\varphi\cos\varphi & 0\\ r\sin\varphi\cos\varphi & r^2\sin\theta & 0\\ 0 & 0 & r\sin^2\varphi\end{bmatrix}$ [✓ (d)] $\begin{bmatrix}1 & \frac{r}{2}\sin\varphi\cos\varphi & 0\\ \frac{r}{2}\sin\varphi\cos\varphi & r^2\sin\theta & 0\\ 0 & 0 & r\sin^2\varphi\end{bmatrix}$

13. Consider a co-ordinate system $(u,v,w)$ which is related to the cartesian co-ordinates by

$$x=vw,\ y=uw,\ z=uv$$

The metric tensor in terems of $u,v,w$ is

[✓ (a)] $\begin{bmatrix}v^2+w^2 & uv & uw\\ uv & u^2+w^2 & vw\\ uw & vw & u^2+v^2\end{bmatrix}$ (b) $\begin{bmatrix}0 & uv & uw\\ uv & 0 & vw\\ uw & vw & 0\end{bmatrix}$

(c) $\begin{bmatrix}v^2+w^2 & 0 & 0\\ 0 & u^2+w^2 & 0\\ 0 & 0 & u^2+v^2\end{bmatrix}$ (d) None of these

14. If the line element 'ds' in a certain co-ordinate system is given by, $(ds)^2=(dr)^2+r^2(d\theta)^2+r^2\sin\theta(d\varphi)^2$

then find the christoffel symbols: (i) [2 2, 3], (ii) [2 2, 1], (iii) $\begin{Bmatrix}1\\ 2\,2\end{Bmatrix}$ [Ans. (i) 0, (ii) -r, (iii) -r]

15. Show that, $[i\,k,j]+[j\,k,i]=\dfrac{\partial g_{ij}}{\partial x^k}$

## ANSWER KEY

**1.(c)** **2.(a), (c)** **5.(b)** **6.(b)** **7.(a)** **8.(b)**
**9.(a)** **10.(c),(d)** **12.(c)** **13.(a)**

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## SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

## SECTION: MATHEMATICAL PHYSICS

### Assignment 15: GROUP THEORY

Consider the following set $\{1,-1,i,-i\}$

1. Which of the following statements is/are CORRECT?
   (a) The set is a finite non-abelian group under multiplication.
   (b) The set is a finite abelian group under multiplication.
   (c) The set is a finite abelian group under addition.
   (d) The set is a finite non-abelian group under addition.

Consider an element of a group as: $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ where x is a non-zero real number

2. The identity element of the group is

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{3}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix}$

3. The inverse element of the group is

   (a) $\begin{bmatrix} \frac{1}{x} & \frac{1}{x} \\ \frac{1}{x} & \frac{1}{x} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{2x} & \frac{1}{2x} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{2}{x} & \frac{2}{x} \\ \frac{2}{x} & \frac{2}{x} \end{bmatrix}$

4. If a group is defined by $a*b=a+b-1$, then the inverse of the group is
   (a) $2-a$ (b) $a$ (c) $a^{-1}$ (d) 0

5. In a group G, the elements are satisfying the relation $x^2=E$ (identity element). Show that G is abelian in nature.

6. If G is an abelian group and $a,b\in G$ such that $O(a)=5$ and $O(b)=7$, then $(ab)^{14}$ is

   (a) $a$ (b) $a^{-1}$ (c) $a^2$ (d) $a^3$

7. The generators of the following group:

   $$G=\{a,a^2,a^3,a^4,a^5,a^6,a^7,a^8=E\}$$

   (a) $a,a^2,a^4$ (b) $a^2,a^4,a^6$ [✓ (c) $a,a^3,a^5,a^7$] (d) $a^2,a^4,a^6,a^8$

8. (i) Show that, the set of all $n^{th}$ roots of unity forms a finite abelian group under multiplication.
(ii) Check that if it is cyclic or not. If yes, find its one generator.

9. The special unitary group of matrices i.e. $SU(2)$ can be represented by

(a) $\begin{bmatrix} a & b \\ b^* & a^* \end{bmatrix}$ (b) $\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ (c) $\begin{bmatrix} a & b \\ -b^* & -a^* \end{bmatrix}$ (d) $\begin{bmatrix} a & b \\ b^* & -a^* \end{bmatrix}$

10. Which of the following matrices is an element of the group SU(2)?

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{bmatrix}$ (c) $\begin{bmatrix} 2+i & i \\ 3 & 1+i \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

## ANSWER KEY

**1.(b)** **2.(c)** **3.(b)** **4.(a)** **6.(b)** **7.(c)**

**9.(b)** **10.(b)**

# PERMUTATIONS AND COMBINATIONS

- **Fundamental Principle of Counting:**

  If an event can occur in 'm' different ways, following which another event will occur in 'n' different ways, then total number of ways in two events will occur in succession is 'm x n'.

  Example:

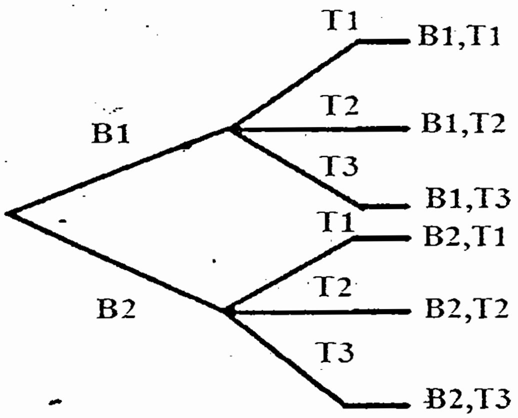
  Q. Rahul has 2 school bags and 3 tiffin boxes. In how many ways can he carry these items?

  Ans. Two events are following:

  (A)The event of choosing a bag : It can chosen in 2 ways.

  (B)The event of choosing a tiffin box : It can be chosen in 3 ways.

  So, Rahul can carry these items in (2 x 3) = 6 ways which are following:



  Q. Find the number of 4 letter words, which can be formed out of letters of the word 'ROSE' ( Repetition is not allowed )

  Ans. There are 4 vacant places. First place can be filled in 4 ways, second place can be filled in 3 ways, third place can be filled in 2 ways and fourth place can be filled in 1 way. (Since repetition is not allowed).

  The total number of ways in which 4 places can be filled is (4 x 3 x 2 x 1) = 24.

  If repetition is allowed, then each of the 4 places can be filled in 4 ways. Then, total number of ways in which 4 places can be filled is (4 x 4x 4 x 4) = 256.

  Q. How many 2 digit even numbers can be formed from digits 1,2,3,4,5 if digits can be repeated?

Ans. There are 2 vacant places. Since, the required number is even, then, the unit's place can be filled in 2 ways (either 2 or 4). The ten's place can be filled in 5 ways. Therefore, the required number of two digit numbers is (2 x 5) = 10.

**Exercise:**

Q. A coin is tossed 3 times. How many possible outcomes are there? (Ans. 8)

Q. There are 11 different buses between Place 'A' and Place 'B'. In how many different ways a person can go from 'A' to 'B', if he travels with different buses in going and returning? (Ans. 110)

Q. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each telephone number starts with 67 and no digit appears more than once? (Ans: 336)

- **Permutations:**

A permutation is an arrangement of objects (taking some or all at a time) in a definite order.

Example: We consider the number of ways in which we can arrange the A,B,C in a row. There are 6 different ways: ABC, ACB, BAC, BCA, CAB, CBA. Each arrangement is called permutation of 3 different letters taken all at a time.

The code of a locker is '472'. If someone presses '742' it will not open. In this case order is important. That is 'Permutation'.

Theorem 1: The number of permutations of 'n' different objects taken 'r' $\left(0 \le r \le n\right)$ at a time (the objects do not repeat) is denoted by ${}^{n}P_{r}$.

Proof : The possible number of permutations of 'n' things taken 'r' at a time is equivalent the number ways of filling 'r' things in 'r' positions. The 1st place can be filled in 'n' ways, the 2nd place can be placed in 'n-1' ways, .................. the rth place can be filled in [n-(r-1)] ways. Therefore,

$${}^{n}P_{r} = n\left(n-1\right)\left(n-2\right)...........\left(n-r+1\right)$$

$$= \frac{n\left(n-1\right)\left(n-2\right)...........\left(n-r+1\right)\left(n-r\right)\left(n-r-1\right)...............3.2.1}{\left(n-r\right)\left(n-r-1\right)...............3.2.1}$$

$$= \frac{n!}{\left(n-r\right)!}$$

✓ The number of permutations of 'n' different objects taken 'n' at a time (the object do not repeat) is $^{n}P_{n} = n!$.

Q. In how many ways can the letters of the word "FORMULA" be arranged?

Ans. "FORMULA" contains 7 different letters. Total number of arrangements is $7! = 5040$.

Q. How many numbers (no digit is repeated) between 1000 and 10000 can be formed with 7,6,5,4,3,2.

Ans. Each number contains 4 digits. The required number = no. of permutations of 6 digits, taking 4 digits at a time = $^{6}P_{4} = \frac{6!}{(6-4)!} = 6 \times 5 \times 4 \times 3 = 360$.

Q. If the letters of the word "MEANS" are arranged taken all at a time, how many numbers do not start with 'AS'?

Ans. Total number of arrangements = 5! = 120

The numbers which starts with 'AS', the first two places can be filled in 1 way. Then, the remaining 3 letters can be arranged in 3! = 6 ways

∴ The numbers which start with 'AS' = 1 x 6 = 6

The numbers which does not start with 'AS' = 120 – 6 = 114

**Exercise:**

Q. How many numbers can be formed between 5000 and 6000 (no digit is repeated) using the digits 1,2,3,4,5,6,7,8? (Ans. 210)

Q. How many arrangements of the letters of the word "COINS" can be made, without changing the places of the vowels in the word? (Ans. 6)

Q. Find the number of arrangements of the letters of the word "STRANGE" can be made so that the vowels may be only in odd places. (Ans. 1440)

Q. If all the letters A,B,C,D,E,F,G can be arranged, find the number of the arrangements in which (i) A,B,C are all together, (ii) A,B,C are not all together. (Ans. (i) 720, (ii) 4320)

Theorem 2: The number of permutations of 'n' different objects taken 'r' at a time, (where repetition is allowed) is $n^{r}$.

Theorem 3: The number of permutations of 'n' things taken all at a time, where 'r' are of first kind, 's' are of second kind, ............. and the rest are different will be

$$\frac{n!}{r!s!\ldots\ldots}$$

Q. Find the number of ways in which the letters of the word "INFINITESIMAL" can be arranged.

Ans. There are 13 letters among which 4 'I' and 2 'N' are present and the rest are different. Therefore the required number of permutations = $\frac{13!}{4!2!}$ =129729600

Q. If the letters of the word "COSONANT" are arranged all at a time, find the number of arrangements in which (i) the two 'O' s are together, (ii) three 'N' s will be first.

Ans . (i) If the two 'O' s are regarded as one thing, then there are 8 letters. Required number of arrangements = $\frac{8!}{3!} = 6720$

(ii) If three 'N' s are in first three places, other 6 letters can be arranged in $\frac{6!}{2!} = 360$ ways.

- **Combinations:**

A combination is an arrangement of objects (taking some or all at a time) where order does not matter.

Example: Let us assume there is group of 3 Cricketers X,Y,Z. A team of 2 cricketers is to be formed. There are 3 possible teams that can be constructed i.e XY, YZ, XZ. Here, the team of X and Y is not different from Y and X i.e order is not important. Here each selection is called a Combination of 3 different objects taken 2 at a time.

Theorem 4: The number of combinations of 'n' different objects taken 'r' at a time is $^nC_r = \frac{n!}{r!(n-r)!}$

Q. There are 5 teachers and 10 students. In how many ways can a committee of 3 teachers and 2 students be formed?

Ans. 3 teachers can be selected from 5 teachers in $^5C_3$ ways.

2 students can be selected from 10 students in $^{10}C_2$ ways.

The required number of ways = $^5C_3 \times ^{10}C_2 = \frac{5!}{3!2!} \times \frac{10!}{2!8!} = 450$

Q. If a word consists of 5 vowels and 15 consonants, how many words of 3 different vowels and 2 different consonants can be made?

Ans. 3 vowels can be selected from 5 vowels in ${}^{5}C_{3}$ ways.

2 consonants can be selected from 15 consonants in ${}^{15}C_{2}$ ways.

3 vowels and 2 consonants can be selected in ${}^{5}C_{3} \times {}^{15}C_{2} = 1050$ ways.

5 letters can be arranged among themselves in 5! ways.
Hence, number of words = 1050 x 120 = 126000

Q. What is the number of ways of choosing 4 cards from a pack of 52 cards? In how many ways of these

(i) Four cards are of same suit?
(ii) Four cards are of different suits?
(iii) Two cards are ace and two cards are queen?
(iv) Two cards are red and two cards are black?
(v) Four cards are of same colour?

Ans. There are four suits: Diamond, club, Spade, Heart (13 each). The number of ways in which 4 cards can chosen from 52 cards = ${}^{52}C_{4}$.

(i) Four cards will be of same suit i.e either 4 diamonds or 4 clubs or 4 spades or 4 hearts.

Required number of ways = ${}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4} = 2890$

(ii) Four cards will be of different suits i.e. 1 diamond, 1 club, 1 spade and 1 heart.

Required number of ways = ${}^{13}C_{1} \times {}^{13}C_{1} \times {}^{13}C_{1} \times {}^{13}C_{1} = 13^{4}$

(iii) Required number of ways = ${}^{4}C_{2} \times {}^{4}C_{2} = 36$

(iv) Required number of ways = ${}^{26}C_{2} \times {}^{26}C_{2} = 105625$

(v) Required number of ways = ${}^{26}C_{4} + {}^{26}C_{4} = 29900$

**Exercise:**

Q. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls? (Ans. (i) 21, (ii) 441, (iii) 91)

Q. In how many ways can 5 girls and 3 boys be seated in arrow so that no two boys are together? (Ans. 14400)

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## Physical Sciences (NET-JRF/GATE)
## Test-II: Mathematical Physics

**Time : 01:00 Hour** **Date : 07-10-2014**
**M.M. : 70**

### INSTRUCTIONS

- **This question booklet contains 10 questions in Part-A + 7 questions Part-B only. You have to answer all questions from Part-A and Part-B.**
- **In Part-A, 3.5 marks will be awarded for a right answer and (–1) marks will be awarded for a wrong answer.**
- **In Part-B, 5 marks will be awarded for a right answer and (–1.5) marks will be awarded for a wrong answer.**

### Part-A

1. Consider the following complex function:

$$f(z)=\frac{z-\sin z}{z^3}$$

At z = 0, the function will have the following type of singularity?
(a) Isolated pole of order 3 [✓ (b) Isolated essential singularity]
(c) Isolated removable singularity (d) Non-isolated singularity

2. Consider the following power series expansion:

$$\frac{1}{2}+\frac{1}{3}z+\frac{1}{5}z^2+\frac{1}{9}z^3+\frac{1}{17}z^4+\ldots\ldots\ldots\ldots\ldots$$

The radius of convergence for the above power series will be
(a) 1 (b) 1.5 [✓ (c) 1.8] (d) 2

3. Consider the following functions:

$$w(\rho,\varphi)=\rho(\cos\varphi+i\sin\varphi) \text{ and } z(\rho,\varphi)=\ln\rho+i\varphi$$

The value of $z$ for which the function $w$ ceases to be analytic, is
[✓ (a) 0] (b) 1 (c) -1 (d) $\infty$

4. The co-efficient of $(z-2)^3$ in the taylor series expansion $e^z$ about z = 2, is

[✓ (a) $\frac{1}{3!}$] (b) $\frac{2^3}{3!}$ (c) $\frac{e^{-2}}{3!}$ [✓ (d) $\frac{e^2}{3!}$]

5. Consider the following differential equation:

$$x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+2y=0$$

The solution of the differential equation will be

(a) $x^2-4x$ (b) $\frac{1}{2}(x^2-4x+2)$ (c) $\frac{1}{2}(x^2-x+2)$ (d) None of these

6. For any complex number z in complex argand plane, the operator $\hat{A}=\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}$ is equivalent to

(a) $\frac{\partial}{\partial z}$ [✓ (b) $2\frac{\partial}{\partial z}$] (c) $2\frac{\partial}{\partial z^*}$ (d) $\frac{\partial}{\partial z}+\frac{\partial}{\partial z^*}$

7. The particular integral of the differential equation

$$\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}-3y=e^x$$

will be

(a) $\frac{xe^x}{4}$ (b) $\frac{xe^x}{8}$ (c) $\frac{e^x}{8}$ (d) $\frac{x^2e^x}{8}$

8. If the polynomial $4x^3+6x^2+7x+2$ can be wriiten as $\sum_n a_nP_n(x)$, [where $P_n(x)$ is Legendre Polynomial of order 'n' and $a_n$ are constant co-effiecients] then which of the following statement is true?

(a) $a_2=a_1$ (b) $a_2=-a_1$ (c) $a_2=-a_0$ (d) $a_2=a_0$

9. For the differential equation $z^3(z-4)\frac{d^2y}{dz^2}+z\frac{dy}{dz}+\frac{1}{(z-4)}y=0$, which of the following statement is true?

[✓ (a) z = 0 is an ordinary point]
(b) z = 0 is regular singular point
(c) z =0 and z = 4 are regular singular points
[✓ (d) z = 0 is an irregular singular point and z = 4 is a regular singular point]

10. Given the recurrence relation: $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$

(where $P_n(x)$ is the Legendre polynomial of order 'n')

The value of the integral $\int_{-1}^{1}x^2P_n^2(x)\,dx$ will be

(a) $\frac{2(n+1)^2}{(2n+1)^2(2n-3)}+\frac{2n^2}{(2n+1)^2(2n-1)}$ (b) $\frac{2(n+1)^2}{(2n+1)^2(2n+3)}+\frac{2n^2}{(2n+1)^2(2n-1)}$

(c) $\frac{2n^2}{(2n+1)^2(2n+3)}+\frac{2(n+1)^2}{(2n+1)^2(2n-1)}$ (d) $\frac{2n^2}{(2n+1)^2(2n-3)}+\frac{2(n+1)^2}{(2n+1)^2(2n-1)}$

## PART-B

11. The value of the integral $\int_0^\infty \frac{x^2}{(1+x^2)^3}dx$ is equal to

(a) $\frac{\pi}{2}$ [✓ (b) $\frac{\pi}{4}$] (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{16}$

12. The point which is making a significant contribution in evaluating the integral $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta$ is

(a) $z=-\frac{1}{2}$ (b) $z=-\frac{3}{5}$ (c) $z=\frac{1}{2}$ (d) $z=\frac{3}{5}$

13. The laurent series expansion $f(z)=\frac{1}{(z+1)(z+3)}$ about z = -1 is equal to

[✓ (a)] $\frac{1}{2z}-\frac{1}{2}+\frac{z}{6}-\frac{z^2}{10}+$........................

(b) $\frac{1}{2(1+z)}-\frac{1}{4}+\frac{(z+1)}{8}-\frac{(z+1)^2}{16}+$........................

(c) $\frac{1}{3(1+z)^2}-\frac{1}{2(1+z)}-\frac{1}{4}+\frac{(z+1)}{6}-\frac{(z+1)^2}{9}+$........................

(d) $\frac{1}{4(1+z)^2}-\frac{1}{(1+z)}-\frac{1}{2}+\frac{(z+1)}{9}-\frac{(z+1)^2}{16}+$........................

14. If the real part of a complex analytic function $f(z)$ is

$$e^{2x}(x\cos 2y - y\sin 2y)$$

then the complex function $f(z)$ will be of the following form: (Take the constant to be zero)

(a) $e^{2z}$ (b) $ze^{2z}$ (c) $e^{-2z}$ (d) $ze^{-2z}$

15. The equation of the curve passing through (1/2,0) and having slope at (x,y) given by the differential equation

$$2(1+y^2)dx+(2x-\tan^{-1}y)dy=0$$

will be

(a) $x=\frac{1}{2}(\tan^{-1}y-1)+e^{-\tan^{-1}y}$ (b) $x=(\tan^{-1}y-1)+e^{-\tan^{-1}y}$

(c) $x=\frac{1}{2}(\tan^{-1}y-1)-e^{-\tan^{-1}y}$ (d) $x=(\tan^{-1}y-1)-e^{-\tan^{-1}y}$

16. The differential equation of all circles passing through the origin and having their centres on the x-axis is

[✓ (a)] $(y^2-x^2)dx/dy=2xy$ (b) $(x^2-y^2)dy/dx=2xy$

(c) $dx/dy=2xy(x^2+y^2)$ (d) $dy/dx=2xy(x^2-y^2)$

17. Suppose a function is defined as follows:

$$f(x)=\frac{1}{8\sqrt{6}\pi^{1/4}}e^{-x^2/2}(16x^4-48x^2+12)$$

The value of the integral $\int_0^{\infty}[f(x)]^2 dx$ will be

(a) 0 (b) 1 (c) 1/2 (d) 1/4

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## Physical Sciences (NET-JRF/GATE)

## Test-I: Mathematical Physics

**Time : 01:00 Hour**
**Date : 16-08-2014**
**M.M. : 70**

### INSTRUCTIONS

- **This question booklet contains 10 questions in Part-A + 7 questions Part-B only. You have to answer all questions from Part-A and Part-B.**
- **In Part-A, 3.5 marks will be awarded for a right answer and (–1) marks will be awarded for a wrong answer.**
- **In Part-B, 5 marks will be awarded for a right answer and (–1.5) marks will be awarded for a wrong answer.**

### Part-A

1. Consider a set of three vectors $\vec{P}, \vec{Q}, \vec{R}$ such that sum of them is equal to zero. Which of the following can be the magnitudes of $\vec{P}, \vec{Q}, \vec{R}$?
(a) (2, 1/2, 2) [✓ (b) (1, 3/2, 1/2)] (c) (1, 2, 1/2) (d) (1/2, 1/2, 3/2)

2. Suppose $f(r)$ is a twice differentiable function of $r$ and

$$\nabla^2 f(r) = pf''(r) + \frac{m}{r^n} f'(r), \text{ (where 'm', 'n', 'p' are scalar)}$$

Which of the following relation is true?
[✓ (a) $m = n = p$] (b) $m + n = p$ (c) $m = p + n$ (d) none of these

3. Suppose $M$ is a periodic matrix of order '1', then which of the following statements is true about $M$?
(a) $(I - M)$ is a periodic matrix of order '2' (b) $(I - M)$ is an idempotent matrix
(c) $(I - 2M)$ is a periodic matrix of order '2' (d) $(I - 2M)$ is an idempotent matrix

4. Suppose $R(\theta)$ denotes transformation matrix for the rotation of Y-Z plane in the clockwise direction. Which of the following statement is wrong for matrix $R\left(\frac{\pi}{2}\right)$?
[✓ (a) Matrix is traceless] (b) Eigenvalues of the matrix will be purely imaginary
(c) Eigenvectors of the matrix are orthogonal (d) Matrix is singular

5. Consider a matrix $P = \begin{bmatrix} 2 & -3 \\ 5 & -6 \end{bmatrix}$. Then the value of

$$P^6 + 4P^5 + 3P^4 + 2P^3 + 6P^2 - 4P + 7I_2$$

will be
[✓ (a) $P^2 - 2P + 4I_2$] (b) $P^2 - P + 4I_2$ (c) $P^3 + 2P^2 - 7P + 7I_2$ (d) $P^3 + 2P^2 - P + 7I_2$

6. Suppose $\vec{a}$ and $\vec{b}$ are two three dimensional vectors and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector w.r.t. the origin of 3-D rectangular right handed cartesian coordinate system. Then the direction of $\vec{\nabla}\left[\left\{\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right\} . \vec{r}\right]$ (where $\vec{a}$ and $\vec{b}$ are constant vectors) is

(a) perpendicular to $\vec{a}$ (b) perpendicular to $\vec{c}$

[✓ (c) perpendicular to $\vec{a}$ and $\vec{b}$] (d) parellel to $\vec{a}$ and $\vec{b}$

7. Consider a n x n matrix having elements $a_{ij} = m\det(I_n)[\text{for all } i \text{ and } j, m \text{ is scalar}]$. The eigenvalues of matrix A are

(a) $0,0,0$ [✓ (b) $n^m,0,0$] (c) $mn,0,0$ [✓ (d) $m^n,0,0$]

8. Suppose $M$ is hermitian and $N$ is a skew hermitian operator. Then the product $MN$ will be

(a) hermitian if $[M,N]=0$ and skew hermitian if $\{M,N\}=0$

[✓ (b) hermitian if $\{M,N\}=0$ and skew hermitian if $[M,N]=0$]

(c) hermitian if $[M,MN]=0$ and skew hermitian if $[M,N]=0$

(d) none of the above

9. For vectors $\vec{a}=\hat{j}+\hat{k}, \vec{b}=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{c}=\hat{j}-\hat{k}$, then the vector product $\vec{a}\times(\vec{b}\times\vec{c})$ is

(a) in the same direction as $\vec{c}$ (b) in the direction opposite to $\vec{c}$

(c) in the same direction as $\vec{b}$ [✓ (d) in the direction opposite to $\vec{b}$]

10. The direc tion along which the rate of change in the value of the function $\psi(x,y,z)=xyz^2$ from the point (1,0,3) is maximum, will be

(a) $\frac{(\hat{x}+\hat{y})}{\sqrt{2}}$ (b) $\frac{(\hat{x}-\hat{y})}{\sqrt{2}}$ (c) $\hat{x}$ [✓ (d) $\hat{y}$]

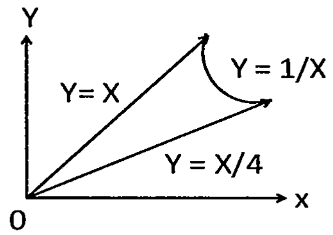
## PART-B

11. If the net flux of the vector field $\vec{F}(x,y,z)=2px\hat{i}+qy\hat{j}-3rz\hat{k}$ passing through the closed surface of an arbitary unit sphere is to be zero, then which of the following relations is true?

(a) $6p+q-6r=0$ [✓ (b) $2p+q-3r=0$] (c) $3p-6q+2r=0$ [✓ (d) $\frac{2}{p}+\frac{1}{q}-\frac{3}{r}=0$]

12. The area of the region in the first quadrant bounded by the curves (shown below)

$$y=x,\ y=\frac{x}{4},\ y=\frac{1}{x}$$



will be

(a) 2 ln 2 [✓ (b) ln 2] (c) 4 ln 2 [✓ (d) 1]

13. The eigenvalues of the anti-symmetric matrix $A=\begin{pmatrix} 0 & -p & q \\ p & 0 & -r \\ -q & r & 0 \end{pmatrix}$, (where $p,q,r$ are the roots of the equation $x^3=64$) are

[✓ (a) 0, i, -i] (b) 0, 2i, -2i (c) 0, 4i, -4i (d) 0, 0, 0

14. If $\begin{bmatrix} \frac{1}{2} & \text{m} \\ 0 & \text{n} \end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ then 'mn' is equal to

(a) $\frac{1}{60}$ [✓ (b) $\frac{1}{180}$] (c) $\frac{1}{3}$ (d) None of these

15. If an electrostatic field $\vec{E}$ can be expressed as $(x+2y+4z)\,\hat{i}+(2x-3y-z)\,\hat{j}+(4x-y+2z)\,\hat{k}$, then the corresponding electrostatic potential at point (1,1,1) is equal to

(a) -5 units [✓ (b) 5 units] (c) -10 units (d) 10 units

16. Let $\vec{F}=ay\hat{i}+z\hat{j}+x\hat{k}$ and C be the negatively oriented closed curve (in a plane that is parrallel to x-y plane) given by $x^2+y^2=1, z=5$. If $\oint_C \vec{F}.d\vec{r}=\pi$, then the value of 'a' will be

(a) -1 [✓ (b) 0] (c) 1/2 (d) 1

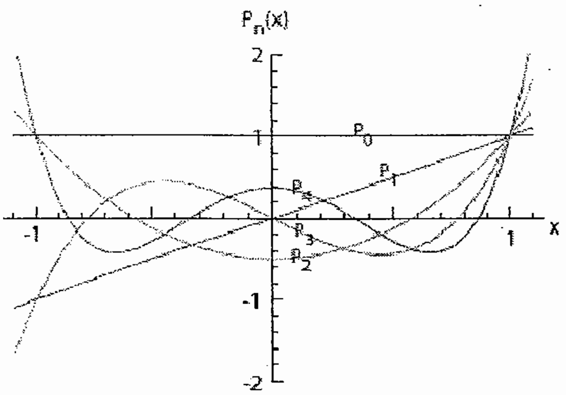
17. The line integral $\oint \vec{A}.d\vec{l}$ of a vector field $\vec{A}(x,y)=\frac{1}{r^2}\left(-y\hat{i}+x\hat{j}\right)$, where $r^2=x^2+y^2$, is taken around a circle of radius 'R' units and centered at $(x_0,y_0)$ with $|x_0|>R$ and $|y_0|>R$. If the value of the line integral

(a) depends on $(x_0,y_0)$

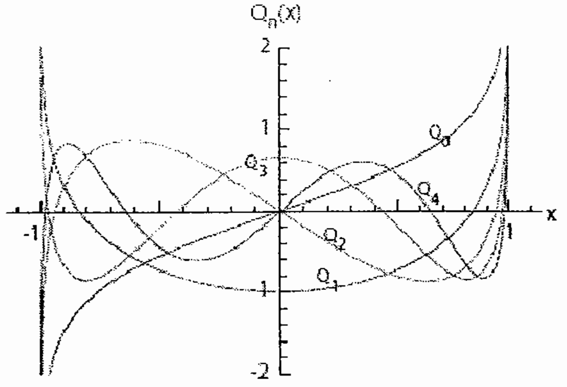
(b) is independent of $(x_0,y_0)$ and its value is -1

(c) is independent of $(x_0,y_0)$ and its value is 0

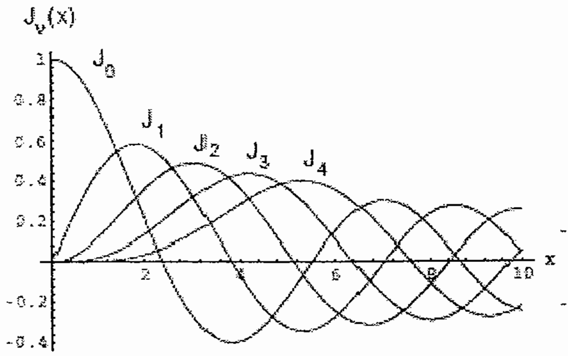
[✓ (d) is independent of $(x_0,y_0)$ and its value is 2]



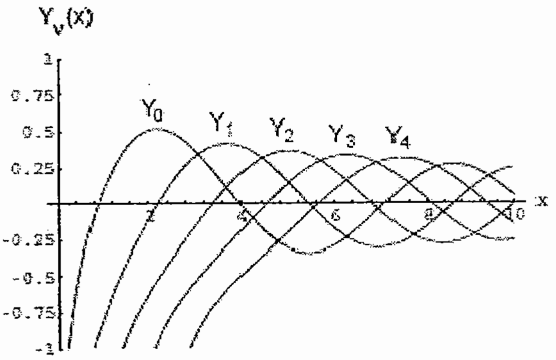
Legendre Polynomial



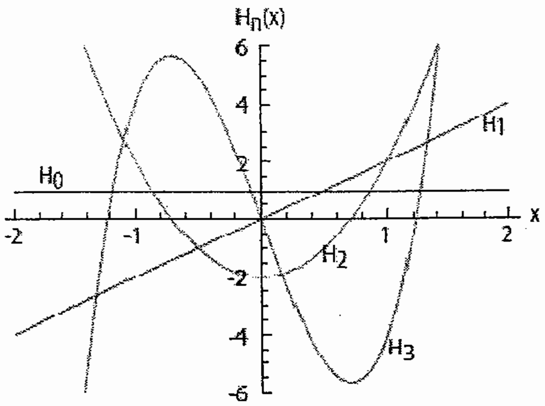
Legendre Function of 2nd Kind



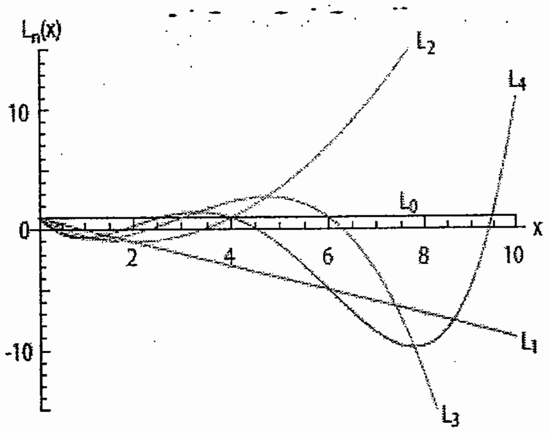
Bessel Function



Neumann Function



Hermite Polynomial



Laguarre Polynomial

क्रमांक 3128

**H**

# 2014 (II)
# भौतिक विज्ञान
# प्रश्न पत्र

विषय कोड **5** पुस्तिका कोड **B**

समय : 3:00 घंटे

पूर्णांक : 200 अंक

## अनुदेश

1. आपने हिन्दी को माध्यम चुना है । इस परीक्षा पुस्तिका में पचहत्तर (20 भाग 'A' में + 25 भाग 'B' + 30 भाग 'C' में ) बहुल विकल्प प्रश्न (MCQ) दिए गए हैं । आपको भाग 'A' में से अधिकतम 15 और भाग 'B' में 20 तथा भाग 'C' में से 20 प्रश्नों के उत्तर देने हैं । यदि निर्धारित से अधिक प्रश्नों के उत्तर दिए गए तब केवल पहले भाग 'A' से 15, भाग 'B' से 20 तथा भाग 'C' से 20 उत्तरों की जांच की जाएगी ।
2. ओ.एम.आर. उत्तर पत्रक अलग से दिया गया है । अपना रोल नम्बर और केन्द्र का नाम लिखने से पहले यह जांच लीजिए कि पुस्तिका में पृष्ठ पूरे और सही हैं तथा कहीं से कटे–फटे नहीं हैं । यदि ऐसा है तो आप इन्विजीलेटर से उसी कोड की पुस्तिका बदलने का निवेदन कर सकते हैं । इसी तरह से ओ.एम.आर. उत्तर पत्रक को भी जांच लें । इस पुस्तिका में रफ काम करने के लिए अतिरिक्त पन्ने संलग्न हैं ।
3. ओ.एम.आर. उत्तर पत्रक के पृष्ठ 1 में दिए गए स्थान पर अपना रोल नम्बर, नाम तथा इस परीक्षा पुस्तिका का क्रमांक लिखिए, साथ ही अपना हस्ताक्षर भी अवश्य करें ।
4. आप अपनी ओ.एम.आर. उत्तर पत्रक में रोल नंबर, विषय कोड, पुस्तिका कोड और केन्द्र कोड से संबंधित समुचित वृतों को काले बॉल पेन से अवश्य काला करें। यह एक मात्र परीक्षार्थी की जिम्मेदारी है कि वह ओ.एम.आर. उत्तर पत्रक में दिए गए निर्देशों का पूरी सावधानी से पालन करें, ऐसा न करने पर कम्प्यूटर विवरणों का सही तरीके से अकूटित नहीं कर पाएगा, जिससे अंततः आपको हानि, जिसमें आपकी ओ.एम.आर. उत्तर पत्रक की अस्वीकृति भी शामिल है, हो सकती है।
5. भाग 'A' में प्रत्येक प्रश्न के 2 अंक, भाग 'B' में प्रत्येक प्रश्न के 3.5 अंक तथा 'C' में प्रत्येक प्रश्न 5 अंक का है । प्रत्येक गलत उत्तर का ऋणात्मक मूल्यांक 25 % की दर से किया जाएगा ।
6. प्रत्येक प्रश्न के नीचे चार विकल्प दिए गए हैं । इनमें से केवल एक विकल्प ही "सही" अथवा "सर्वोत्तम हल" है । आपको प्रत्येक प्रश्न का सही अथवा सर्वोत्तम हल ढूंढना है ।
7. नकल करते हुए या अनुचित तरीकों का प्रयोग करते हुए पाए जाने वाले परीक्षार्थियों का इस और अन्य भावी परीक्षाओं के लिए अयोग्य ठहराया जा सकता है ।
8. परीक्षार्थी को उत्तर या रफ पन्नों के अतिरिक्त कहीं और कुछ भी नहीं लिखना चाहिए ।
9. केलकूलेटर का उपयोग करने की अनुमति नहीं है ।
10. परीक्षा समाप्ति पर छिद्र बिन्दु चिन्हित स्थान से ओ.एम.आर. उत्तर पत्रक को विभाजित करें। इन्विजीलेटर को मूल ओ.एम.आर. उत्तर पत्रक सौंपने के पश्चात आप इसकी कॉर्बनलैस प्रतिलिपि ले जा सकते हैं।
11. हिन्दी माध्यम/संस्करण के प्रश्न में विसंगति होने/पाये जाने पर अंग्रेजी संस्करण प्रमाणिक होगा ।
12. केवल परीक्षा की पूरी अवधि तक बैठने वाले परीक्षार्थी को ही परीक्षा पुस्तिका साथ ले जाने की अनुमति दी जाएगी ।

रोल नंबर : 515236
नाम MOHAN KHUBCHANDANI
OMR उत्तर पत्रक नंबर : 515236

परीक्षार्थी द्वारा भरी गई जानकारी को मैं सत्यापित करता हूँ ।

..............................
इन्विजीलेटर के हस्ताक्षर

H

$$-\frac{\sin\frac{\pi x}{2a}}{\frac{\pi}{2a}} \qquad -\frac{\cos\frac{\pi x}{2a}}{\left(\frac{\pi}{2a}\right)^2}$$

$$\int_{-a}^{+a} A\cos\frac{\pi x}{2a}\,H\,.\,A\cos\frac{\pi x}{2a}\,dx$$

$$|A|^2\int_{-a}^{+a}\cos\frac{\pi x}{2a}\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-b\,\delta(x)\right)\cos\left(\frac{\pi x}{2a}\right)dx$$

---

$$|A|\int_{-a}^{+a}\cos^2\frac{\pi x}{2a} \qquad \sin x\frac{\pi x}{2a}\ \sin\pi x \qquad \frac{\sin\frac{\pi x}{a}}{\pi/a}$$

$$|A|\left(\frac{-\hbar^2}{2m}\right)\int_{-a}^{+a}\cos\frac{\pi x}{2a}\cdot\left(-\cos\frac{\pi x}{2a}\right)\left(\frac{2a}{\pi}\right)^2-\int_{-a}^{+a}b\cos^2\frac{\pi x}{a}\,\delta(x)\,dx$$

$$\Rightarrow \frac{4a^2}{\pi^2}\frac{\hbar^2}{2m}\int_{-a}^{+a}\frac{1+\cos\frac{2\pi x}{2a}}{2}\,dx-b\int_{-a}^{+a}\left(1+\cos\frac{2\pi x}{a}\right)\delta(x=0)\,dx$$

$$\Rightarrow \frac{4a^2}{\pi^2}\cdot\frac{\hbar^2}{2m}\cdot\int_{-a}^{a}\left(\frac{1}{2}\,2a\right)+\frac{1}{2}(-1)\ -\ b\int_{-a}^{a}(1+1)$$

$$\Rightarrow -\frac{\hbar^2 4a^3}{\pi^2 m} \qquad -2ab$$

---

$$\Rightarrow \frac{\hbar^2\left(\frac{1}{\frac{2a}{\hbar^2}}\right)+\cos 2\pi x}{\pi^2 m} \quad -2abm^2 \qquad 2b+0$$

$$\frac{\frac{2\hbar^2a^2}{2m}\frac{1}{2}(2a)}{a}-2ab^2\hbar^2\ \frac{2b^2m}{m\hbar^2}\Rightarrow 2bm.$$

## उपयोगी मूलभूत नियतांक

| | | |
|---|---|---|
| m | इलेक्ट्रान का द्रव्यमान | $9.11 \times 10^{-31}$kg |
| h | प्लांक नियतांक | $6.63 \times 10^{-34} J\ sec$ |
| e | इलेक्ट्रान का आवेश | $1.6 \times 10^{-19} C$ |
| k | बोल्ट्समान नियतांक | $1.38 \times 10^{-23} J/K$ |
| c | प्रकाश का वेग | $3.0 \times 10^{8} m/sec$ |
| IeV | $1.6 \times 10^{-19}$J | |
| amu | $1.67 \times 10^{-27}$kg | |
| G | $6.67 \times 10^{-11} Nm^{2} kg^{-2}$ | |
| $R_y$ | रिडबर्ग नियतांक | $1.097 \times 10^{7} m^{-1}$ |
| $N_A$ | आवोगाद्रो संख्या | $6.023 \times 10^{23} mole^{-1}$ |
| $\varepsilon_o$ | $8.854 \times 10^{-12} Fm^{-1}$ | |
| $\mu_o$ | $4\pi \times 10^{-7} Hm^{-1}$ | |
| R | मोलर गैस नियतांक | $8.314 JK^{-1} mole^{-1}$ |

## USEFUL FUNDAMAENTAL CONSTANTS

| | | |
|---|---|---|
| m | Mass of electron | $9.11 \times 10^{-31}$kg |
| h | Planck's constant | $6.63 \times 10^{-34} J\ sec$ |
| e | Charge of electron | $1.6 \times 10^{-19} C$ |
| k | Boltzmann constant | $1.38 \times 10^{-23} J/K$ |
| c | Velocity of Light | $3.0 \times 10^{8} m/sec$ |
| IeV | $1.6 \times 10^{-19}$J | |
| amu | $1.67 \times 10^{-27}$kg | |
| G | $6.67 \times 10^{-11} Nm^{2} kg^{-3}$ | |
| $R_y$ | Rydberg constant | $1.097 \times 10^{7} m^{-1}$ |
| $N_A$ | Avogadro's number | $6.022 \times 10^{23} mole^{-1}$ |
| $\varepsilon_o$ | $8.854 \times 10^{-12} Fm^{-1}$ | |
| $\mu_o$ | $4\pi \times 10^{-7} Hm^{-1}$ | |
| R | Molar Gas constant | $8.314 JK^{-1} mole^{-1}$ |

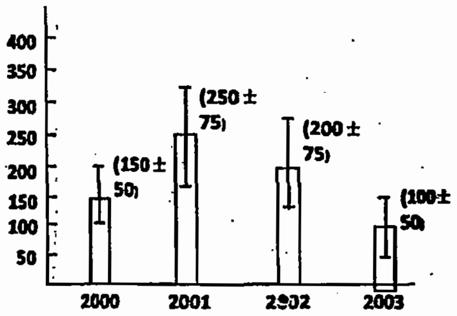
## *LIST OF THE ATOMIC WEIGHTS OF THE ELEMENTS*

| Element | Symbol | Atomic Number | Atomic Weight |
|---|---|---|---|
| Actinium | Ac | 89 | (227) |
| Aluminium | Al | 13 | 26.98 |
| Americium | Am | 95 | (243) |
| Antimony | Sb | 51 | 121.75 |
| Argon | Ar | 18 | 39.948 |
| Arsenic | As | 33 | 74.92 |
| Astatine | At | 85 | (210) |
| Barium | Ba | 56 | 137.34 |
| Berkelium | Bk | 97 | (249) |
| Beryllium | Be | 4 | 9.012 |
| Bismuth | Bi | 83 | 208.98 |
| Boron | B | 5 | 10.81 |
| Bromine | Br | 35 | 79.909 |
| Cadmium | Cd | 48 | 112.40 |
| Calcium | Ca | 20 | 40.08 |
| Californium | Cf | 98 | (251) |
| Carbon | C | 6 | 12.011 |
| Cerium | Ce | 58 | 140.12 |
| Cesium | Cs | 55 | 132.91 |
| Chlorine | Cl | 17 | 35.453 |
| Chromium | Cr | 24 | 52.00 |
| Cobalt | Co | 27 | 58.93 |
| Copper | Cu | 29 | 63.54 |
| Curium | Cm | 96 | (247) |
| Dysprosium | Dy | 66 | 162.50 |
| Einsteinium | Es | 99 | (254) |
| Erbium | Er | 68 | 167.26 |
| Europium | Eu | 63 | 151.96 |
| Fermium | Fm | 100 | (253) |
| Fluorine | F | 9 | 19.00 |
| Francium | Fr | 87 | (223) |
| Gadolinium | Gd | 64 | 157.25 |
| Gallium | Ga | 31 | 69.72 |
| Germanium | Ge | 32 | 72.59 |
| Gold | Au | 79 | 196.97 |
| Hafnium | Hf | 72 | 178.49 |
| Helium | He | 2 | 4.003 |
| Holmium | Ho | 67 | 164.93 |
| Hydrogen | H | 1 | 1.0080 |
| Indium | In | 49 | 114.82 |
| Iodine | I | 53 | 126.90 |
| Iridium | Ir | 77 | 192.2 |
| Iron | Fe | 26 | 55.85 |
| Krypton | Kr | 36 | 83.80 |
| Lanthanum | La | 57 | 138.91 |
| Lawrencium | Lr | 103 | (257) |
| Lead | Pb | 82 | 207.19 |
| Lithium | Li | 3 | 6.939 |
| Lutetium | Lu | 71 | 174.97 |
| Magnesium | Mg | 12 | 24.312 |
| Manganese | Mn | 25 | 54.94 |
| Mendelevium | Md | 101 | (256) |
| Mercury | Hg | 80 | 200.59 |
| Molybdenum | Mo | 42 | 95.94 |
| Neodymium | Nd | 60 | 144.24 |
| Neon | Ne | 10 | 20.183 |
| Neptunium | Np | 93 | (237) |
| Nickel | Ni | 28 | 58.71 |
| Niobium | Nb | 41 | 92.91 |
| Nitrogen | N | 7 | 14.007 |
| Nobelium | No | 102 | (253) |
| Osmium | Os | 76 | 190.2 |
| Oxygen | O | 8 | 15.9994 |
| Palladium | Pd | 46 | 106.4 |
| Phosphorus | P | 15 | 30.974 |
| Platinum | Pt | 78 | 195.09 |
| Plutonium | Pu | 94 | (242) |
| Polonium | Po | 84 | (210) |
| Potassium | K | 19 | 39.102 |
| Praseodymium | Pr | 59 | 140.91 |
| Promethium | Pm | 61 | (147) |
| Protactinium | Pa | 91 | (231) |
| Radium | Ra | 88 | (226) |
| Radon | Rn | 86 | (222) |
| Rhenium | Re | 75 | 186.23 |
| Rhodium | Rh | 45 | 102.91 |
| Rubidium | Rb | 37 | 85.47 |
| Ruthenium | Ru | 44 | 101.1 |
| Samarium | Sm | 62 | 150.35 |
| Scandium | Sc | 21 | 44.96 |
| Selenium | Se | 34 | 78.96 |
| Silicon | Si | 14 | 28.09 |
| Silver | Ag | 47 | 107.870 |
| Sodium | Na | 11 | 22.9898 |
| Strontium | Sr | 38 | 87.62 |
| Sulfur | S | 16 | 32.064 |
| Tantalum | Ta | 73 | 180.95 |
| Technetium | Tc | 43 | (99) |
| Tellurium | Te | 52 | 127.60 |
| Terbium | Tb | 65 | 158.92 |
| Thallium | Tl | 81 | 204.37 |
| Thorium | Th | 90 | 232.04 |
| Thulium | Tm | 69 | 168.93 |
| Tin | Sn | 50 | 118.69 |
| Titanium | Ti | 22 | 47.90 |
| Tungsten | W | 74 | 183.85 |
| Uranium | U | 92 | 238.03 |
| Vanadium | V | 23 | 50,94 |
| Xenon | Xe | 54 | 131.30 |
| Ytterbium | Yb | 70 | 173.04 |
| Yttrium | Y | 39 | 88.91 |
| Zinc | Zn | 30 | 65.37 |
| Zirconium | Zr | 40 | 91.22 |

*Based on mass of $C^{12}$ at 12.000... . The ratio of these weights of those on the order chemical scale (in which oxygen of natural isotopic composition was assigned a mass of 16.0000...) is 1.000050. (Values in parentheses represent the most stable known isotopes.)

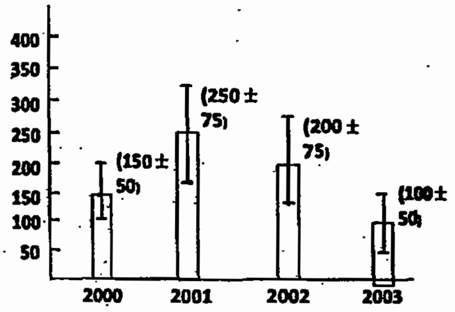
# भाग 'क' / PART 'A'

1. निम्न आयतचित्र में किसी उत्पाद का विभिन्न वर्षों में माध्य उपज दर्शाया गया है। यदि ऊर्ध्वाधर त्रुटि रेखायें वर्षों में प्रसरण को इंगित करती हैं, तो किस वर्ष में, उस वर्ष के माध्य पर प्रसरण प्रतिशत न्यूनतम था?



1. 2000 [✓ 2. 2001]
3. 2002 4. 2003

1. Average yield of a product in different years is shown in the histogram. If the vertical bars indicate variability during the year, then during which year was the percent variability over the average of that year the least?

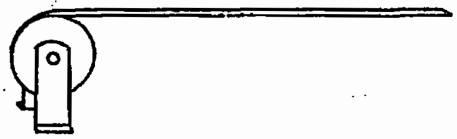


1. 2000 [✓ 2. 2001]
3. 2002 4. 2003

2. लम्बाई *d* तथा चौड़ाई *d*/2 के एक आयत को एक बार अपनी लंबाई के गिर्द एवं एक बार अपनी चौड़ाई के गिर्द पूरी तरह घुमाया जाता है । दोनों बार में उत्पन्न आयतनों का अनुपात है
1. 1:1 [✓ 2. 1:2]
3. 1:3 4. 1:4

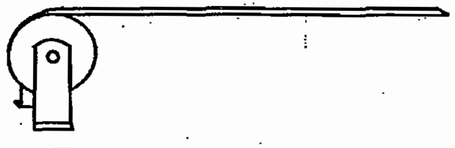
2. A rectangle of length *d* and breadth *d*/2 is revolved once completely around its length and once around its breadth. The ratio of volumes swept in the two cases is
1. 1:1 [✓ 2. 1:2]
3. 1:3 4. 1:4

3. एक लंबी पट्टी एक चरखी पर त्रिज्या *R* तक लपेटी गयी है । पट्टी की छोर को पकड़ते हुये, तथा खोली गयी पट्टी के भाग को क्षैतिज रखते हुये, एक लड़का चरखी से दूर, एक स्थिर गति के साथ भागता है । चार मिनटों में लपेटी हुई पट्टी की त्रिज्या $\frac{R}{\sqrt{2}}$ हो जाती है । त्रिज्या को *R*/2 हो जाने में और कितना समय लगेगा?



1. $\sqrt{2}$ मिनट 2. 2 मिनट
[✓ 3. $2\sqrt{2}$ मिनट] 4. 4 मिनट

3. A long ribbon is wound around a spool up to a radius *R*. Holding the tip of the ribbon, a boy runs away from the spool with a constant speed maintaining the unwound portion of the ribbon horizontal. In 4 minutes, the radius of the wound portion becomes $\frac{R}{\sqrt{2}}$. In what further time, it will become *R*/2?



1. $\sqrt{2}$ min 2. 2 min
[✓ 3. $2\sqrt{2}$ min] 4. 4 min

4. यदि *n* एक धन पूर्णांक है, तो $n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)$ विभाजनीय है:
1. 3 से लेकिन 7 से नहीं
2. 3 और 7 से
3. 7 से लेकिन 3 से नहीं
4. न तो 3 से, न 7 से

4. If $n$ is a positive integer, then $n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)$ is divisible by

1. 3 but not 7
[✓ 2. 3 and 7]
3. 7 but not 3
4. neither 3 nor 7

5. आयाम 50 मी., 120 मी. एवं 130 मी. के एक त्रिकोणीय उद्यान का क्षेत्रफल (वर्ग मीटर में) है

1. 3000 2. 3250
3. 5550 4. 7800

5. The area (in $m^2$) of a triangular park of dimensions 50 m, 120 m and 130 m is

[✓ 1. 3000] 2. 3250
3. 5550 4. 7800

6. किसी व्यक्ति का मध्याह्न भोजन एवं रात्रि-भोजन का प्रतिरूप *m* दिनों के लिये नीचे दिया गया है। वह अपने मध्याह्न भोजन/रात्रि भोजन के लिये शाकाहारी या मांसाहारी भोजन को चुन सकता है।

(a) यदि वह मांसाहारी मध्याह्न भोजन लेता है तो रात्रि-भोजन के लिये मात्र शाकाहारी लेगा ।
(b) ठीक 9 दिनों में वह मांसाहारी रात्रि भोजन लेता है ।
(c) ठीक 15दिनों में वह शाकाहारी मध्याह्न भोजन लेता है ।
(d) वह कुल मिलाकर 14 मांसाहारी भोजन लेता है।

*m* का क्या मान है?

1. 18 2. 24
3. 20 4. 38

6. Lunch-dinner pattern of a person for *m* days is given below. He has a choice of a VEG or a NON-VEG meal for his lunch/dinner

(a) If he takes a NON-VEG lunch, he will have only VEG for dinner
(b) He takes NON-VEG dinner for exactly 9 days
(c) He takes VEG lunch for exactly 15 days
(d) He takes a total of 14 NON-VEG meals

What is *m*?

1. 18 2. 24
[✓ 3. 20] 4. 38

7. एक बैंक एक योजना प्रदान करती है जिसमें 1600 दिनों के लिए रखा गया निवेश अपने मूल्य का दुगना होता है, दैनिक ब्याज जोड़ते हुये । रूपये 1000/- के निवेश के पहले 400 दिनों में प्राप्त ब्याज होगा रूपये

1. 250 2. 183
3. 148 4. 190

7. A bank offers a scheme wherein deposits made for 1600 days are doubled in value, the interest being compounded daily. The interest accrued on a deposit of Rs.1000/- over the first 400 days would be Rs.

1. 250 2. 183
3. 148 [✓ 4. 190]

8. निम्न अनुक्रम में अगली संख्या क्या है?
2, 3, 4, 7, 6, 11, 8, 15, 10 ...

1. 12 2. 13
3. 17 4. 19

8. What is the next number of the following sequence?
2, 3, 4, 7, 6, 11, 8, 15, 10 ...

1. 12 2. 13
3. 17 [✓ 4. 19]

9. 60 तथा 40 कि.मी. प्रति घंटे की गतियों के साथ दो इंजन एक दूसरे की ओर दौड़ रहे हैं । एक वस्तु एक इंजन के अग्रभाग से दूसरे इंजन के अग्रभाग तक, आगे -पीछे दोनों तरफ 70 कि.मी. प्रति घंटे की गति से उड़ती रहती हैं । 30 मिनट बाद दोनों इंजन टकराते हैं तथा वस्तु चूर-चूर हो जाती है । चूर-चूर होने के पहले वस्तु से पारित कुल दूरी क्या है?

1. 50 कि.मी. 2. 45 कि.मी.
3. 35 कि.मी. 4. 10 कि.मी.

9. Two locomotives are running towards each other with speeds of 60 and 40 km/h. An object keeps on flying to and fro from the front tip of one locomotive to the front tip of the other with a speed of 70 km/h. After 30 minutes, the two locomotives collide and the object is crushed. What distance did the object cover before being crushed?

1. 50 km
2. 45 km
[✓ 3. 35 km]
4. 10 km

10. 13 व्यक्तियों के वज़न (कि.ग्रा.) में नीचे दिये गये हैं।

70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94

दो और व्यक्ति, जिनके वज़न 100 कि.ग्रा. तथा 79 कि.ग्रा. हैं, समूह में शामिल होते हैं । तो समूह का माध्य वज़न इतने से बढ़ता है:

1. 0 कि.ग्रा.
2. 1 कि.ग्रा.
3. 1.6 कि.ग्रा.
4. 1.8 कि.ग्रा.

10. Weights (in kg) of 13 persons are given below:

70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94

Two new persons having weights 100 kg and 79 kg join the group. The average weight of the group increases by

1. 0 kg
[✓ 2. 1 kg]
3. 1.6 kg
4. 1.8 kg

11. एक कूटलेख में अधिकतम दो सर्वसम (अंग्रेजी) अक्षर, तथा उसके बाद अधिकतम चार सर्वसम अंक अंतर्विष्ट हैं । कूटलेख में कम से कम एक अक्षर एवं एक अंक का होना जरूरी है । अक्षरों A से Z तक, एवं 1 से 9 तक के उपयोग से कितने पृथक कूटलेख बनाये जा सकते हैं?

1. 936
2. 1148
3. 1872
4. 2574

11. A code consists of at most two identical letters followed by at most four identical digits. The code must have at least one letter and one digit. How many distinct codes can be generated using letters A to Z and digits 1 to 9?

1. 936
2. 1148
[✓ 3. 1872]
4. 2574

12. दो लोहे के गोलों को 100°से. तक गरम करके ठंडे होने के लिये छोड़ दिया जाता है । उनमें से एक फुटबाल के आकार में तथा दूसरा मटर के आकार में है, तो कौन-सा गोला सबसे पहले कमरा-ताप (अचर) तक पहुँचेगा?

1. बड़ा गोला
2. छोटा गोला
3. दोनों गोले एक साथ
4. यह कमरा-ताप पर निर्भर होगा ।

12. Two solid iron spheres are heated to 100°C and then allowed to cool. One has the size of a football; the other has the size of a pea. Which sphere will attain the room temperature (constant) first?

[✓ 1. The bigger sphere]
2. The smaller sphere
3. Both spheres will take the same time
4. It will depend on the room temperature

13. लापता अक्षर को खोजें:

| | | | |
|---|---|---|---|
| A | ? | Q | E |
| C | M | S | C |
| E | K | U | A |
| G | I | W | Y |

1. L
2. Q
3. N
[✓ 4. O]

13. Find the missing letter:

| | | | |
|---|---|---|---|
| A | ? | Q | E |
| C | M | S | C |
| E | K | U | A |
| G | I | W | Y |

1. L
2. Q
3. N
[✓ 4. O]

14. एक 8-द्वयंक द्वि-आधारी संख्या का न्यूनतम महत्वपूर्ण द्वयंक शून्य है । इस संख्या से 8 गुनी मान रखती संख्या के
1. 12 द्वयंक हैं, अंत में तीन शून्यों के साथ।
2. 11 द्वयंक हैं, अंत में चार शून्यों के साथ।
3. 11 द्वयंक हैं, अंत में तीन शून्यों के साथ।
4. 12 द्वयंक हैं, अंत में चार शून्यों के साथ।

14. The least significant bit of an 8-bit binary number is zero. A binary number whose value is 8 times the previous number has
1. 12 bits ending with three zeros
[✓ 2.] 11 bits ending with four zeros
3. 11 bits ending with three zeros
4. 12 bits ending with four zeroes

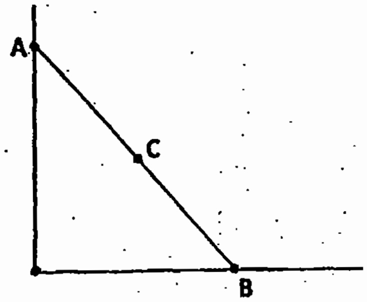
15. एक व्यक्ति दो वस्तुओं में हर एक को रूपये1035/- में बेचता है। पहली वस्तु में वह 10% की हानि भोगता है तथा दूसरी में 15% का लाभ । उसके कुल लाभ/हानि का प्रतिशत क्या है?
1. 5% लाभ
2. < 1% लाभ
3. < 1% हानि
4. कोई हानि या लाभ नहीं

15. A person sells two objects at Rs.1035/- each. On the first object he suffers a loss of 10% while on the second he gains 15%. What is his net loss/gain percentage?
1. 5% gain
[✓ 2.] < 1% gain
3. < 1% loss
4. no loss, no gain

16. अनुक्रम को जारी रखें:
2, 5, 10, 17, 28, 41, –, –, –
1. 58, 77, 100
2. 64, 81, 100
3. 43, 47, 53
4. 55, 89, 113

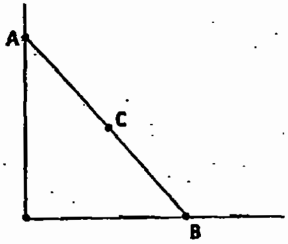
16. Continue the sequence
2, 5, 10, 17, 28, 41, –, –, –
[✓ 1.] 58, 77, 100
2. 64, 81, 100
3. 43, 47, 53
4. 55, 89, 113

17. जैसे चित्र में दर्शाया गया है, एक सीढ़ी एक दीवार पर रखी हुई है । सीढ़ी के ऊपरी तथा निचली छोर A एवं B से अंकित हैं । आधार B सरकता है, तो सीढ़ी का मध्यबिंदु C गिरते हुये बनाता है



1. एक परवलय
[✓ 2.] एक वर्तुल का चाप
3. एक सीधी रेखा
4. एक अतिपरवलय

17. A ladder rests against a wall as shown. The top and the bottom ends of the ladder are marked A and B. The base B slips. The central point C of the ladder falls along



1. a parabola
[✓ 2.] the arc of a circle
3. a straight line
4. a hyperbola

18. किसी खास पाठ्यक्रम के विधार्थियों में से 20% उत्तीर्ण होने के एक साल के अंदर नौकरी पाते हैं । उत्तीर्ण होने के दूसरे साल के पूरे होने से पूर्व बाकी विधार्थियों में से 20% नौकरी पाते हैं । इसके बाद यदि 16 विधार्थी बेरोज़गार हैं, तो पाठ्यक्रम में कुल कितने लोग उत्तीर्ण हुये?
1. 32
2. 64
3. 25
4. 100

**18.** 20% of students of a particular course get jobs within one year of passing. 20% of the remaining students get jobs by the end of second year of passing. If 16 students are still jobless, how many students had passed the course?

1. 32
2. 64
3. 25
4. 100

**19.** बीजगणित में द्विपद-प्रमेय देती है $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, जहां $a_0, a_1, \dots, a_n$ $n$ पर निर्भर स्थिरांक हैं । योगफल $a_0 + a_1 + a_2 + \dots + a_n$ क्या है?

1. $2^n$
2. $n$
3. $n^2$
4. $n^2+n$

**19.** Binomial theorem in algebra gives $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where $a_0, a_1, \dots, a_n$ are constants depending on $n$. What is the sum $a_0 + a_1 + a_2 + \dots + a_n$?

1. $2^n$
2. $n$
3. $n^2$
4. $n^2+n$

**20.** बढ़ती त्रिज्याओं के संकेंद्री, बहुत पतले कोषों से, बिना कोई अंतर छोड़े एक गोला बना है। किसी यादृच्छिक रूप से चुने गये कोष का द्रव्यमान

1. उसके तुरंत पूर्व कोष के बराबर है।
2. उसके आयतन के अनुपात में है।
3. उसकी त्रिज्या के अनुपात में है।
4. उसके सतही क्षेत्रफल के अनुपात में है।

**20.** A sphere is made up of very thin concentric shells of increasing radii (leaving no gaps). The mass of an arbitrarily chosen shell is

1. equal to the mass of the preceding shell
2. proportional to its volume
3. proportional to its radius
4. proportional to its surface area

# भाग 'ख' / PART 'B'

**21.** द्रव्यमान $m$ का एक कण विभव $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ में गतिशील है, जहां $a$ तथा $b$ धन अचर हैं। स्थायी साम्यावस्था के आर-पार अल्प दोलन की आवृत्ति है

1. $\sqrt{a/m}$
2. $\sqrt{2a/m}$
3. $\sqrt{3a/m}$
4. $\sqrt{6a/m}$

**21.** A particle of mass $m$ is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ where $a, b$ are positive constants. The frequency of small oscillations about a point of stable equilibrium is

1. $\sqrt{a/m}$
2. $\sqrt{2a/m}$
3. $\sqrt{3a/m}$
4. $\sqrt{6a/m}$

**22.** पृथ्वी कि त्रिज्या है लगभग 6400 कि.मी.। पृथ्वी की सतह पर पृथ्वी के गुरूत्व के कारण जो त्वरण $g$ है उससे लगभग 1% अंतर रखता त्वरण इस ऊँचाई $h$ पर पाया जायेगा।

1. 64 कि.मी.
2. 48 कि.मी.
3. 32 कि.मी.
4. 16 कि.मी.

**22.** The radius of Earth is approximately 6400 km. The height $h$ at which the acceleration due to Earth's gravity differs from $g$ at the Earth's surface by approximately 1% is

1. 64 km
2. 48 km
3. 32 km
4. 16 km

**23.** आपेक्षिकता के विशिष्ट सिद्धान्त के अनुसार द्रव्यमान $m$ तथा कुल ऊर्जा $E$ युक्त एक मुक्त कण की गति $v$ है:

1. $v = c\sqrt{1 - \frac{mc^2}{E}}$
2. $v = \sqrt{\frac{2E}{m}}\left(1 + \frac{mc^2}{E}\right)$
3. $v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$
4. $v = c\left(1 + \frac{mc^2}{E}\right)$

**23.** According to the special theory of relativity, the speed $v$ of a free particle of mass $m$ and total energy $E$ is:

1. $v = c\sqrt{1 - \frac{mc^2}{E}}$
2. $v = \sqrt{\frac{2E}{m}\left(1 + \frac{mc^2}{E}\right)}$
3. [✓ 3.] $v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$
4. $v = c\left(1 + \frac{mc^2}{E}\right)$

**24.** मानें कि $\vec{r}$ त्रिविम आकाश में किसी बिंदु के स्थिति सदिश को निर्दिष्ट करता है, तथा $r = |\vec{r}|$. तो

1. $\vec{\nabla}\cdot\vec{r} = 0$ and $\vec{\nabla}\times\vec{r} = \vec{r}/r$
2. $\vec{\nabla}\cdot\vec{r} = 0$ and $\nabla^2 r = 0$
3. $\vec{\nabla}\cdot\vec{r} = 3$ and $\nabla^2\vec{r} = \vec{r}/r^2$
4. $\vec{\nabla}\cdot\vec{r} = 3$ and $\vec{\nabla}\times\vec{r} = 0$

**24.** Let $\vec{r}$ denote the position vector of any point in three-dimensional space, and $r = |\vec{r}|$. Then

1. $\vec{\nabla}\cdot\vec{r} = 0$ and $\vec{\nabla}\times\vec{r} = \vec{r}/r$
2. $\vec{\nabla}\cdot\vec{r} = 0$ and $\nabla^2 r = 0$
3. $\vec{\nabla}\cdot\vec{r} = 3$ and $\nabla^2\vec{r} = \vec{r}/r^2$
4. [✓ 4.] $\vec{\nabla}\cdot\vec{r} = 3$ and $\vec{\nabla}\times\vec{r} = 0$

**25.** स्तंभ सदिश $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$, $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ तथा $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ का युगपत् अभिलक्षणिक सदिश है यदि

1. $b = 0$ या $a = 0$
2. $b = a$ या $b = -2a$
3. $b = 2a$ या $b = -a$
4. $b = a/2$ या $b = -a/2$

**25.** The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

1. $b = 0$ or $a = 0$
2. [✓ 2.] $b = a$ or $b = -2a$
3. $b = 2a$ or $b = -a$
4. $b = a/2$ or $b = -a/2$

**26.** समाकल $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3}\, dx$ का मुख्य मान है

1. $-2\pi$
2. $-\pi$
3. $\pi$
4. $2\pi$

**26.** The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3}\, dx$ is

1. [✓ 1.] $-2\pi$
2. $-\pi$
3. $\pi$
4. $2\pi$

**27.** दो स्वतंत्र यादृच्छिक चर $m$ तथा $n$, जो पूर्णांक मान $0, 1, 2, \cdots, \infty$ ले सकते हैं, प्वासों बंटन का अनुकरण करते हैं, स्पष्ट माध्य मान $\mu$ तथा $\nu$ के साथ। तो

1. यादृच्छिक चर $l = m + n$ का प्रायिकता बंटन एक द्विपद बंटन है
2. यादृच्छिक चर $r = m - n$ का प्रायिकता बंटन भी एक प्वासों बंटन है
3. यादृच्छिक चर $l = m + n$ का प्रसरण $\mu + \nu$ के समान है
4. यादृच्छिक चर $r = m - n$ का माध्य मान शून्य के बराबर है

**27.** Two independent random variables $m$ and $n$, which can take the integer values $0, 1, 2, \cdots, \infty$, follow the Poisson distribution, with distinct mean values $\mu$ and $\nu$ respectively. Then

1. the probability distribution of the random variable $l = m + n$ is a binomial distribution.

2. the probability distribution of the random variable $r = m - n$ is also a Poisson distribution.
3. the variance of the random variable $l = m + n$ is equal to $\mu + \nu$.
4. the mean value of the random variable $r = m - n$ is equal to 0.

**28.** $z = 0$ के आर-पार फलन $f(z) = e^z + e^{1/z}$ का लौराँ श्रेणी विस्तार इससे दिया जाता है:

1. सभी $|z| < \infty$ के लिए $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$
2. केवल यदि $0 < |z| < 1$ है तो ही $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right) \frac{1}{n!}$
3. सभी $0 < |z| < \infty$ के लिए $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right) \frac{1}{n!}$
4. केवल यदि $|z| < 1$ तो ही $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$

**28.** The Laurent series expansion of the function $f(z) = e^z + e^{1/z}$ about $z = 0$ is given by

1. $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ for all $|z| < \infty$
2. $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right) \frac{1}{n!}$ only if $0 < |z| < 1$
3. $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right) \frac{1}{n!}$ for all $0 < |z| < \infty$
4. $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$, only if $|z| < 1$

**29.** समय निर्भर लग्रांजी $L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$ से वर्णित एक तंत्र का गति समीकरण है:

1. $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$
2. $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$
3. $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$
4. $m\ddot{x} + \frac{dV}{dx} = 0$

**29.** The equation of motion of a system described by the time-dependent Lagrangian $L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$ is

~~1.~~ $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$
2. $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$
3. $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$
4. $m\ddot{x} + \frac{dV}{dx} = 0$

$= 1$

**30.** त्रिज्या $R$ के एक ठोस गोले का आवेश घनत्व $\rho(r) = \rho_0\left(1 - \frac{ar}{R}\right)$ से दी जाती है, जहाँ $r$ त्रिज्यी निर्देशांक है तथा $\rho_0$, $a$ एवं $R$ धन अचर हैं। $r = R$ की तुलना में यदि $r = R/2$ पर विद्युत क्षेत्र का परिमाण 1.25 गुना है, तो $a$ का मान है

1. 2 — 2. 1
3. $1/2$ — 4. $1/4$

**30.** A solid sphere of radius $R$ has a charge density, given by

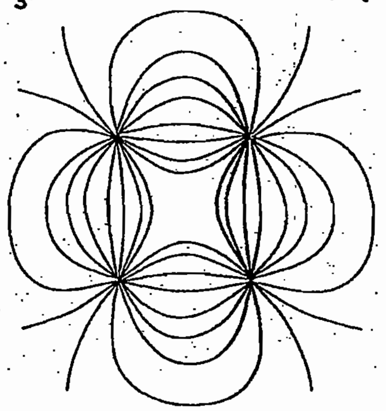
$1 \frac{a}{2} \cdot \frac{1}{2} \rho$

$$\rho(r) = \rho_0\left(1 - \frac{ar}{R}\right),$$

where $r$ is the radial coordinate and $\rho_0$, $a$ and $R$ are positive constants. If the magnitude of the electric field at $r = R/2$ is 1.25 times that at $r = R$, then the value of $a$ is

1. 2 — 2. 1
3. $1/2$ — 4. $1/4$

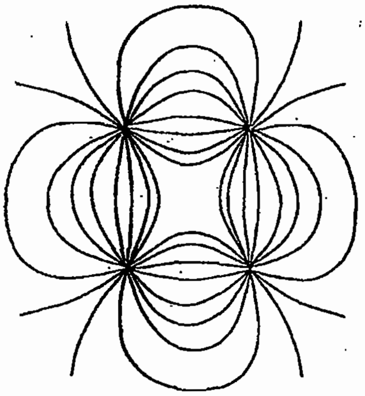
**31.** चार बिंदु आवेशों के एक तंत्र से जनित विद्युतस्थैतिक बल रेखायें निम्नवत हैं।



बड़ी दूरी $r$ पर विद्युतस्थैतिक विभव का अग्रग उपगामी आचरण इस अनुपात में है:

1. $r$ — 2. $r^{-1}$
3. $r^{-2}$ — 4. $r^{-3}$

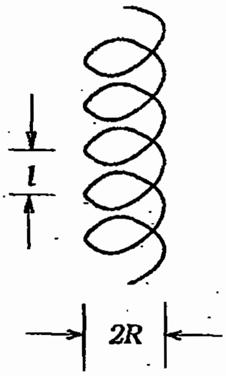
**31.** The electrostatic lines of force due to a system of four point charges is sketched below.



At a large distance $r$, the leading asymptotic behaviour of the electrostatic potential is proportional to

1. $r$  2. $r^{-1}$
3. $r^{-2}$  [✓ 4.] $r^{-3}$

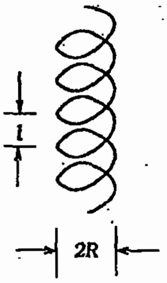
**32.** एक अचर चुंबकीय क्षेत्र के प्रभाव के अंदर एक आवेशित कण एक कुंडलिनी पथ पर गतिशील है। प्रारंभिक गति ऐसी है कि चुंबकीय क्षेत्र के समांतर वाला घटक, चुंबकीय क्षेत्र से लंब समतल में घटक की तुलना में दुगुना है।



कुंडलिनी पथ के कुंडलिनी-अंतराल $\ell$ तथा $R$ त्रिज्या का अनुपात $\ell/R$ है

1. $\pi/2$  2. $4\pi$
3. $2\pi$  4. $\pi$

**32.** A charged particle moves in a helical path under the influence of a constant magnetic field. The initial velocity is such that the component along the magnetic field is twice the component in the plane normal to the magnetic field.



The ratio $\ell/R$ of the pitch $\ell$ to the radius $R$ of the helical path is

1. $\pi/2$  [✓ 2.] $4\pi$
3. $2\pi$  4. $\pi$

**33.** तरंग-दैर्घ्य $\lambda$ युक्त एक समांतर प्रकाश पुंज, दोनों तरफ हवा से घिरी एक पतली पॉलिमर फिल्म पर लंब आपतित है। यदि फिल्म का अपवर्तन गुणांक $n > 1$ है, तो द्वितीय कोटि दीप्त धारियाँ प्रतिवर्तन में तब देखे जा सकती हैं जब फिल्म की मोटाई होगी

1. $\lambda/4n$  2. $\lambda/2n$
3. $3\lambda/4n$  4. $\lambda/n$

**33.** A parallel beam of light of wavelength $\lambda$ is incident normally on a thin polymer film with air on both sides. If the film has a refractive index $n > 1$, then second-order bright fringes can be observed in reflection when the thickness of the film is

1. $\lambda/4n$  2. $\lambda/2n$
[✓ 3.] $3\lambda/4n$  4. $\lambda/n$

**34.** प्रसामान्यीकृत तरंगफलन $\phi = a_1 \psi_{11} + a_2 \psi_{10} + a_3 \psi_{1-1}$ पर विचारें, जहाँ $\psi_{lm}$, कोणीय संवेग संकारकों $L^2$ तथा $L_z$ के युगपत् प्रसामान्यीकृत अभिलक्षणिक फलन है, क्रमश: अभिलक्षणिक मानों $l(l+1)\hbar^2$ तथा $m\hbar$ के साथ। अभिलक्षणिक मान $\hbar$ युक्त संकारक $L_x$ का यदि अभिलक्षणिक फलन $\phi$ है, तो

1. $a_1 = -a_3 = \frac{1}{2}$, $a_2 = \frac{1}{\sqrt{2}}$
2. $a_1 = a_3 = \frac{1}{2}$, $a_2 = \frac{1}{\sqrt{2}}$
3. $a_1 = a_3 = \frac{1}{2}$, $a_2 = -\frac{1}{\sqrt{2}}$
4. $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

**34.** Consider the normalized wavefunction

$$\phi = a_1 \psi_{11} + a_2 \psi_{10} + a_3 \psi_{1-1}$$

where $\psi_{lm}$ is a simultaneous normalized eigenfunction of the angular momentum operators $L^2$ and $L_z$, with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. If $\phi$ is an eigenfunction of the operator $L_x$ with eigenvalue $\hbar$, then

1. $a_1 = -a_3 = \frac{1}{2}, \quad a_2 = \frac{1}{\sqrt{2}}$
2. $a_1 = a_3 = \frac{1}{2}, \quad a_2 = \frac{1}{\sqrt{2}}$
3. $a_1 = a_3 = \frac{1}{2}, \quad a_2 = -\frac{1}{\sqrt{2}}$
4. $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

**35.** मानें कि $x$ तथा $p$ क्रमश: निर्देशांक तथा संवेग संकारक जिसका विहित क्रमविनिमय संबंध प्राकृतिक युनिट ($\hbar = 1$) में $[x,p] = i$ हैं। तो क्रमविनिमेयक $[x, pe^{-p}]$ का मान है

1. $i(1-p)e^{-p}$ 2. $i(1-p^2)e^{-p}$
3. $i(1-e^{-p})$ 4. $ipe^{-p}$

**35.** Let $x$ and $p$ denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x,p] = i$ in natural units ($\hbar = 1$). Then the commutator $[x, pe^{-p}]$ is

1. $i(1-p)e^{-p}$ 2. $i(1-p^2)e^{-p}$
3. $i(1-e^{-p})$ 4. $ipe^{-p}$

**36.** चिरप्रतिष्ठित् यांत्रिकी में एक संरक्षी निकाय की हैमिल्टनी है $H = \omega xp$, जहाँ $\omega$ एक अचर है तथा $x$ एवं $p$ क्रमश: स्थान एवं संवेग हैं। क्वांटम् यांत्रिकी में, निर्देशांक निरूपण में संगत हैमिल्टनी, है

1. $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$
2. $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$
3. $-i\hbar\omega x\frac{\partial}{\partial x}$
4. $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

**36.** Suppose the Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where $\omega$ is a constant and $x$ and $p$ are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is

1. $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$
2. $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$
3. $-i\hbar\omega x\frac{\partial}{\partial x}$
4. $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

**37.** मानें कि $\psi_1$ तथा $\psi_2$ क्रमश: ऊर्जा अभिलक्षणिक मान $E_1$ तथा $E_2$ ($E_2 > E_1$) युक्त एक कण के प्रसामान्यीकृत अभिलक्षणिक स्थितियां हैं। समय $t = 0$ पर कण स्थिति $\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$ में बनाया जाता है। लघुत्तम समय $T$, जब $\Psi(t=T)$, $\Psi(t=0)$ से लांबिक होगा, वह है

1. $\frac{2\hbar\pi}{(E_2-E_1)}$ 2. $\frac{\hbar\pi}{(E_2-E_1)}$
3. $\frac{\hbar\pi}{2(E_2-E_1)}$ 4. $\frac{\hbar\pi}{4(E_2-E_1)}$

**37.** Let $\psi_1$ and $\psi_2$ denote the normalized eigenstates of a particle with energy eigenvalues $E_1$ and $E_2$ respectively, with $E_2 > E_1$. At time $t = 0$ the particle is prepared in a state

$$\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2).$$

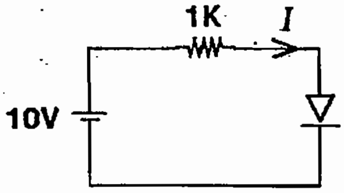
The shortest time $T$ at which $\Psi(t=T)$ will be orthogonal to $\Psi(t=0)$ is

1. $\frac{2\hbar\pi}{(E_2-E_1)}$ 2. $\frac{\hbar\pi}{(E_2-E_1)}$
3. $\frac{\hbar\pi}{2(E_2-E_1)}$ 4. $\frac{\hbar\pi}{4(E_2-E_1)}$

**38.** निम्न परिपथ में डॉयोड के $I$-$V$ अभिलक्षण इससे दिये जाते है:

$$I = \begin{cases} (V-0.7)/500 & \text{for } V \geq 0.7 \\ 0 & \text{for } V < 0.7 \end{cases}$$

जहाँ $V$ वोल्ट्स् में तथा $I$ ऐम्पियर में मापे जाते हैं ।



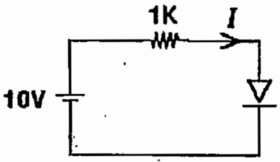
परिपथ में विद्युतधारा $I$ का मान है

1. 10.0 mA
2. 9.3 mA
3. 6.2 mA
4. 6.7 mA

38. The $I$-$V$ characteristics of the diode in the circuit below is given by

$$I = \begin{cases} (V - 0.7)/500 & \text{for } V \geq 0.7 \\ 0 & \text{for } V < 0.7 \end{cases}$$

where $V$ is measured in volts and $I$ is measured in amperes.



The current $I$ in the circuit is

1. 10.0 mA
2. 9.3 mA
3. 6.2 mA
4. 6.7 mA

39. कार्य-फलन $W_M$ के एक धातु तथा कार्य-फलन $W_s$, (जहाँ $W_M > W_s$), के एक डोपित अर्धचालक, दोनों का एक संगम स्थान बनाया जाता है। यदि अंतरापृष्ठ में विद्युत क्षेत्र को तीन गुना बढ़ाना है तो अर्धचालक में डोपंट सांद्रता को

1. 9 गुना बढ़ाना चाहिये
2. 3 गुना कम करना चाहिये
3. 3 गुना बढ़ाना चाहिये
4. $\sqrt{3}$ गुना कम करना चाहिये

39. A junction is made between a metal of work function $W_M$, and a doped semiconductor of work function $W_S$ with $W_M > W_S$. If the electric field at the interface has to be increased by a factor of 3, then the dopant concentration in the semiconductor would have to be

1. increased by a factor of 9
2. decreased by a factor of 3
3. increased by a factor of 3
4. decreased by a factor of $\sqrt{3}$

40. द्रव में गोलीय कणों से अनुभवित श्यानता कर्षण बल के एक मापन में, यह पाया गया कि बल $V^{1/3}$ के अनुपात में है, जहां $V$ हर कण का मापित आयतन है। यदि $V$ अनिश्चितता $2.7\ \text{mm}^3$ के साथ, $30\ \text{mm}^3$ मापा जाता है, तो मापित बल में सापेक्ष प्रतिशत अनिश्चितता है

1. 2.08
2. 0.09
3. 6
4. 3

40. In a measurement of the viscous drag force experienced by spherical particles in a liquid, the force is found to be proportional to $V^{1/3}$ where $V$ is the measured volume of each particle. If $V$ is measured to be $30\ \text{mm}^3$, with an uncertainty of $2.7\ \text{mm}^3$, the resulting relative percentage uncertainty in the measured force is

1. 2.08
2. 0.09
3. 6
4. 3

41. किसी द्रव का दाब $P$ उसके संख्या घनत्व $\rho$ से, अवस्था समीकरण $P = a\rho + b\rho^2$ द्वारा संबंधित है, जहाँ $a$ तथा $b$ अचर हैं। यदि द्रव का प्रांरभिक आयतन $V_0$ है, तो प्रांरभिक मान $\rho_0$ से $2\rho_0$ तक संख्या घनत्व को बढ़ाने के लिए निकाय को संपीडित किये जाने पर किया गया कार्य है

1. $a\rho_0 V_0$
2. $(a + b\rho_0)\rho_0 V_0$
3. $\left(\frac{3a}{2} + \frac{7\rho_0 b}{3}\right)\rho_0 V_0$
4. $(a \ln 2 + b\rho_0)\rho_0 V_0$

41. The pressure $P$ of a fluid is related to its number density $\rho$ by the equation of state

$$P = a\rho + b\rho^2$$

where $a$ and $b$ are constants. If the initial volume of the fluid is $V_0$, the work done on the system when it is compressed so as to

increase the number density from an initial value of $\rho_0$ to $2\rho_0$ is

1. $a\rho_0 V_0$
2. $(a+b\rho_0)\rho_0 V_0$
3. $\left(\frac{3a}{2}+\frac{7\rho_0 b}{3}\right)\rho_0 V_0$
[✓ 4.] $(a\ln 2+b\rho_0)\rho_0 V_0$

42. एक विम में गतिशील एक चिरप्रतिष्ठित् कण की हैमिल्टनी है $H=\frac{p^2}{2m}+aq^4$, जहां $a$ एक धन अचर हैं एवं $p$ तथा $q$ क्रमशः उसके संवेग एवं स्थान हैं। यदि यह दिया गया है कि उसकी कुल ऊर्जा $E \le E_0$ है, तो प्रावस्था समष्टि में पाये जाने वाला आयतन $E_0$ पर इस प्रकार निर्भर है:

1. $E_0^{3/4}$
2. $E_0$
3. $\sqrt{E_0}$
4. $E_0$ पर आश्रित नहीं है

42. The Hamiltonian of a classical particle moving in one dimension is

$$H=\frac{p^2}{2m}+\alpha q^4$$

where $\alpha$ is a positive constant and $p$ and $q$ are its momentum and position respectively. Given that its total energy $E \le E_0$ the available volume of phase space depends on $E_0$ as

[✓ 1.] $E_0^{3/4}$
2. $E_0$
3. $\sqrt{E_0}$
4. is independent of $E_0$

43. एक आदर्श बोस गैस, एक कण-भंडार से जुड़े एक पात्र के अंदर सीमित है। हर कण, एकल-कण क्वांटम अवस्थाओं के एक वियुक्त समुच्चय में रह सकता है। यदि किसी क्वांटम अवस्था के रिक्त रहने की प्रयिकता 0.1 है, तो उस अवस्था में बोसानों की माध्य संख्या है:

1. 8
2. 9
3. 10
4. 11

43. An ideal Bose gas is confined inside a container that is connected to a particle reservoir. Each particle can occupy a discrete set of single-particle quantum states. If the probability that a particular quantum state is unoccupied is 0.1, then the average number of bosons in that state is

1. 8
[✓ 2.] 9
3. 10
4. 11

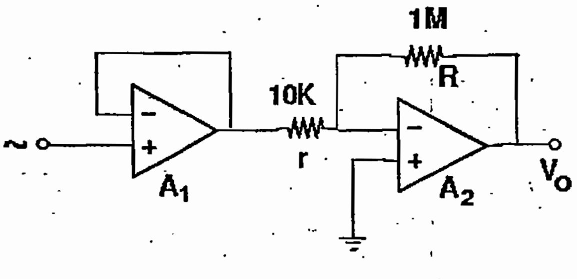
44. अल्प ताप में अल्प घनत्व के ऑक्सीजन गैस में, अणुओं के केवल स्थानांतरीय एवं घूर्णन मोड उत्तेजित होते हैं। इस गैस के प्रति अणु विशिष्ट ऊष्मा है

1. $\frac{1}{2}k_B$
2. $k_B$
3. $\frac{3}{2}k_B$
4. $\frac{5}{2}k_B$

44. In low density oxygen gas at low temperature, only the translational and rotational modes of the molecules are excited. The specific heat per molecule of the gas is

1. $\frac{1}{2}k_B$
2. $k_B$
3. $\frac{3}{2}k_B$
[✓ 4.] $\frac{5}{2}k_B$

45. दो ऑप-एम्पों $A_1$ तथा $A_2$ युक्त एक संवर्धक परिपथ, जो चित्र में दर्शाया गया है, के बारे में विचारें।

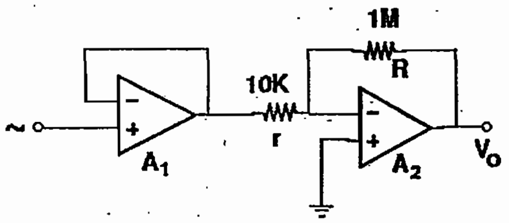


यदि निविष्ट ac संकेत स्रोत की प्रतिबाधा 50 kΩ है, तो निम्न कथनों में कौन-सा सही है?

1. परिपथ में $A_1$ की आवश्यकता इसलिए हैं कि स्रोत प्रतिबाधा $r$ से बहुत अधिक है।
2. परिपथ में $A_1$ की आवश्यकता इसलिए है कि स्रोत प्रतिबाधा $R$ से बहुत कम है।

3. कुल लब्धि को प्रभावित किये बिना परिपथ से $A_1$ को हटाया जा सकता है।
4. यदि निविष्ट संकेत की प्रावस्था को निर्गम को अनुकरण करना है, तो परिपथ में $A_1$ का होना आवश्यक है।

**45.** Consider the amplifier circuit comprising of the two op-amps $A_1$ and $A_2$ as shown in the figure.



If the input ac signal source has an impedance of 50 k$\Omega$, which of the following statements is true?

1. $A_1$ is required in the circuit because the source impedance is much greater than $r$
2. $A_1$ is required in the circuit because the source impedance is much less than $R$
3. $A_1$ can be eliminated from the circuit without affecting the overall gain
4. $A_1$ is required in the circuit if the output has to follow the phase of the input signal

## भाग 'ग' / PART 'C'

**46.** किसी द्रव्य के सतह पर लंबवत आपतित समतल विध्युतचुम्बकीय तरंग आंशिकत: प्रतिवर्तित होती है। अंतरापृष्ठ के सामने के क्षेत्र में बनी अचल तरंग पर किये गये मापन से पता चलता है कि उच्चतम एवं लघुत्तम पर विध्युत क्षेत्र के आयाम 5 के अनुपात में हैं। प्रतिवर्तित एवं आपतित तीव्रताओं का अनुपात है:

1. 4/9 2. 2/3
3. 2/5 4. 1/5

**46.** A plane electromagnetic wave incident normally on the surface of a material is partially reflected. Measurements on the standing wave in the region in front of the interface show that the ratio of the electric field amplitude at the maxima and the minima is 5. The ratio of the reflected intensity to the incident intensity is

1. 4/9 2. 2/3
3. 2/5 4. 1/5

**47.** अदिश एवं सदिश विभव $\varphi(\vec{x},t)$ तथा $\vec{A}(\vec{x},t)$ एक प्रमाप रूपांतरण $\varphi \to \varphi' = \varphi - \frac{\partial \xi}{\partial t}$ तथा $\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\xi$, जहां $\xi$, $\vec{x}$ तथा $t$ का एक स्वेच्छ, संतत एवं अवकलनीय फलन है, तक निर्धारित किये जाते हैं। यदि इसके ऊपर हम लॉरेण्ट्स् प्रमाप प्रतिबंध

$$\vec{\nabla}.\vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0$$

को भी लागू करें तो प्रमाप फलन $\xi(\vec{x},t)$ के लिए एक संभाव्य वरण है: (जहां $\omega, \vec{k}$ शून्येतर अचर हैं, $\omega = c|\vec{k}|$ के साथ)

1. $\cos\omega t \, \cosh \vec{k}.\vec{x}$
2. $\sinh\omega t \, \cos \vec{k}.\vec{x}$
3. $\cosh\omega t \, \cos \vec{k}.\vec{x}$
4. $\cosh\omega t \, \cosh \vec{k}.\vec{x}$

**47.** The scalar and vector potentials $\varphi(\vec{x},t)$ and $\vec{A}(\vec{x},t)$ are determined up to a gauge transformation $\varphi \to \varphi' = \varphi - \frac{\partial \xi}{\partial t}$ and $\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\xi$ where $\xi$ is an arbitrary continuous and differentiable function of $\vec{x}$ and $t$. If we further impose the Lorenz gauge condition

$$\vec{\nabla}.\vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0$$

then a possible choice for the gauge function $\xi(\vec{x},t)$ is (where $\omega, \vec{k}$ are nonzero constants with $\omega = c|\vec{k}|$)

1. $\cos\omega t \, \cosh \vec{k}.\vec{x}$
2. $\sinh\omega t \, \cos \vec{k}.\vec{x}$
3. $\cosh\omega t \, \cos \vec{k}.\vec{x}$
4. $\cosh\omega t \, \cosh \vec{k}.\vec{x}$

**48.** द्रव्यमान $m$ तथा आवेश $e$ युक्त एक अनापेक्षकीय कण, जो गति $\vec{v}$ तथा त्वरण $\vec{a}$ के साथ गतिशील है, तीव्रता $I$ वाली विकिरण छोड़ता है। द्रव्यमान $m/2$, आवेश $2e$, गति $\vec{v}/2$

एवं त्वरण $2\vec{a}$ युक्त कण से छूटती विकिरण की तीव्रता क्या है?

1. $16I$
2. $8I$
3. $4I$
4. $2I$

**48.** A non-relativistic particle of mass $m$ and charge $e$, moving with a velocity $\vec{v}$ and acceleration $\vec{a}$, emits radiation of intensity $I$. What is the intensity of the radiation emitted by a particle of mass $m/2$, charge $2e$, velocity $\vec{v}/2$ and acceleration $2\vec{a}$?

1. $16I$
2. $8I$
3. $4I$
4. $2I$

**49.** मानें कि $\alpha$ तथा $\beta$ सम्मिश्र संख्याएं हैं। निम्न आव्यूहों की समुच्चयों में कौन-सी आव्यूह गुणन पर एक समूह का निर्माण करतीं है?

1. $\begin{pmatrix}\alpha & \beta\\ 0 & 0\end{pmatrix}$
2. $\begin{pmatrix}1 & \alpha\\ \beta & 1\end{pmatrix}$, जहां $\alpha\beta \neq 1$
3. $\begin{pmatrix}\alpha & \alpha^*\\ \beta & \beta^*\end{pmatrix}$, जहां $\alpha\beta^*$ वास्तविक है।
4. $\begin{pmatrix}\alpha & \beta\\ -\beta^* & \alpha^*\end{pmatrix}$, जहां $|\alpha|^2 + |\beta|^2 = 1$ है।

**49.** Let $\alpha$ and $\beta$ be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication?

1. $\begin{pmatrix}\alpha & \beta\\ 0 & 0\end{pmatrix}$
2. $\begin{pmatrix}1 & \alpha\\ \beta & 1\end{pmatrix}$, where $\alpha\beta \neq 1$
3. $\begin{pmatrix}\alpha & \alpha^*\\ \beta & \beta^*\end{pmatrix}$, where $\alpha\beta^*$ is real
4. $\begin{pmatrix}\alpha & \beta\\ -\beta^* & \alpha^*\end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

**50.** व्यंजक

$$\sum_{i,j,k=1}^{3} \epsilon_{ijk}\{x_i, \{p_j, L_k\}\}$$

(जहां $\epsilon_{ijk}$ लेवी-चिविता प्रतीक है, $\vec{x}, \vec{p}, \vec{L}$ क्रमश: स्थान, संवेग एवं कोणीय संवेग हैं, एवं $\{A, B\}$, $A$ तथा $B$ के प्वासों कोष्ठक को निर्दिष्ट करता है) इस पर सरलीकृत होता है:

1. 0
2. 6
3. $\vec{x}.(\vec{p}\times\vec{L})$
4. $\vec{x}\times\vec{p}$

**50.** The expression

$$\sum_{i,j,k=1}^{3} \epsilon_{ijk}\{x_i, \{p_j, L_k\}\}$$

(where $\epsilon_{ijk}$ is the Levi-Civita symbol, $\vec{x}$, $\vec{p}$ $\vec{L}$ are the position, momentum and angular momentum respectively, and $\{A, B\}$ represents the Poisson bracket of $A$ and $B$) simplifies to

1. 0
2. 6
3. $\vec{x}.(\vec{p}\times\vec{L})$
4. $\vec{x}\times\vec{p}$

**51.** एक यांत्रिकी निकाय हैमिल्टनी $H(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2q^2$ से वर्णित है। $F(q,Q) = -\frac{Q}{q}$ से जनित विहित रूपांतरण के परिणाम, नये निर्देशांक $Q$ तथा संवेग $P$ में हैमिल्टनी बन जाती है:

1. $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}Q^2$
2. $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}P^2$
3. $\frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$
4. $\frac{1}{2m}Q^2P^4 + \frac{m\omega^2}{2}P^{-2}$

$\frac{a}{b} = \frac{a}{b}$ a b a

**51.** A mechanical system is described by the Hamiltonian $H(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2q^2$. As a result of the canonical transformation generated by $F(q,Q) = -\frac{Q}{q}$, the Hamiltonian in the new coordinate $Q$ and momentum $P$ becomes

1. $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}Q^2$
2. $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}P^2$
3. $\frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$
4. $\frac{1}{2m}Q^2P^4 + \frac{m\omega^2}{2}P^{-2}$

$-\frac{\partial F}{\partial p}$ $q$ $p = \frac{\partial F}{\partial}$ $p = +\frac{Q}{q}$ $2\frac{Q}{P}$

$-1$ , $p = +\frac{Q}{}$

**52.** मानें कि $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, जहां $\sigma_1, \sigma_2, \sigma_3$ पाऊली आव्यूह हैं। यदि $\vec{a}$ तथा $\vec{b}$ त्रिविम में दो स्वेच्छ अचर सदिश हैं, तो क्रमविनिमेयक $[\vec{a}.\vec{\sigma}, \vec{b}.\vec{\sigma}]$ इसके समान है: (निम्न में $I$ है तत्समक आव्यूह)

1. $(\vec{a}.\vec{b})(\sigma_1 + \sigma_2 + \sigma_3)$
2. $2i(\vec{a} \times \vec{b}).\vec{\sigma}$
3. $(\vec{a}.\vec{b})I$
4. $|\vec{a}||\vec{b}|I$

**52.** Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{a}$ and $\vec{b}$ are two arbitrary constant vectors in three dimensions, the commutator $[\vec{a}.\vec{\sigma}, \vec{b}.\vec{\sigma}]$ is equal to (in the following $I$ is the identity matrix)

1. $(\vec{a}.\vec{b})(\sigma_1 + \sigma_2 + \sigma_3)$
2. $2i(\vec{a} \times \vec{b}).\vec{\sigma}$
3. $(\vec{a}.\vec{b})I$
4. $|\vec{a}||\vec{b}|I$

$\bar{a}\bar{b}[\sigma, \sigma]$

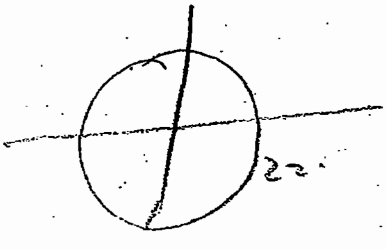
$\sigma\sigma - \sigma\sigma$

**53.** सम्मिश्र चर $z = re^{i\theta}$ $(r \geq 0, -\infty < \theta < \infty)$ के फलन $f(z) = \frac{1}{z}\ln(1-z)$ के बारे विचारें। $f(z)$ की विचित्रतायें निम्नवत हैं:

1. $z = 1$ तथा $z = \infty$ पर शाखा बिंदु, तथा केवल $0 \leq \theta < 2\pi$ के लिए $z = 0$ पर एक अनंतक
2. $z = 1$ तथा $z = \infty$ पर शाखा बिंदु, तथा $0 \leq \theta < 2\pi$ के अलावा सभी $\theta$ के लिए $z = 0$ पर एक अनंतक
3. $z = 1$ तथा $z = \infty$ पर शाखा बिंदु, तथा सभी $\theta$ के लिए $z = 0$ पर एक अनंतक
4. $z = 0$, $z = 1$ तथा $z = \infty$ पर शाखा बिंदु

**53.** Consider the function $f(z) = \frac{1}{z}\ln(1-z)$ of a complex variable $z = re^{i\theta}$ $(r \geq 0, -\infty < \theta < \infty)$. The singularities of $f(z)$ are as follows:

1. branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ only for $0 \leq \theta < 2\pi$
2. branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all $\theta$ other than $0 \leq \theta < 2\pi$
3. branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all $\theta$
4. branch points at $z = 0$, $z = 1$ and $z = \infty$.



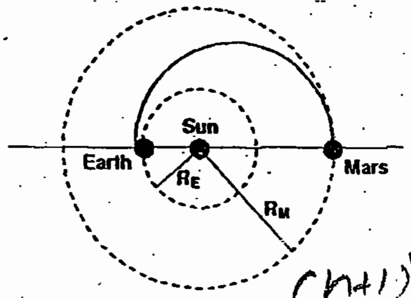
**54.** फलन $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$ निम्न अवकल समीकरण का समाधान करता है।

1. $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2+1)f = 0$
2. $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2-1)f = 0$
3. $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2-1)f = 0$
4. $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2-1)f = 0$

**54.** The function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$ satisfies the differential equation

1. $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2+1)f = 0$
2. $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2-1)f = 0$
3. $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2-1)f = 0$
4. $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2-1)f = 0$

**55.** हाल में मंगलयान गवेषणार्थ मंगल ग्रह के तरफ भेजा गया । प्रपथ का अंतर्ग्रहीय भाग, पृथ्वी (अवतरण के समय), सूर्य तथा मंगल (जब यान गंतव्य स्थान पर पहुँचती है) से बनाई गयी दीर्घ अक्ष पर एक दीर्घवृत्त है। मानें कि पृथ्वी एवं मंगल की कक्षायें क्रमश: त्रिज्यायों $R_E$ तथा $R_M$, के साथ लगभग वृत्ताकार हैं। पृथ्वी एवं मंगल के प्रभाव को उदासीन करते हुये, सूर्य के सापेक्ष यान की गति, जब वह सूर्य से $r$ $(R_E \ll r \ll R_M)$ की दूरी पर है, का मान है



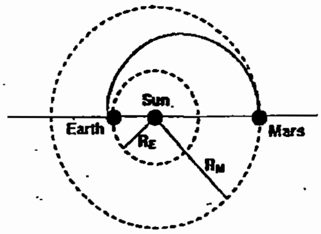
$(n+1)^2 \frac{a^2}{2}$

$-\frac{1}{2!}\left(\frac{x}{2}\right)^3$

$F^{1/3}$

1. $\sqrt{2GM\frac{(R_E+R_M)}{r(R_E+R_M-r)}}$
2. $\sqrt{2GM\frac{(R_E+R_M-r)}{r(R_E+R_M)}}$
3. $\sqrt{2GM\frac{R_E}{rR_M}}$
4. $\sqrt{\frac{2GM}{r}}$

**55.** The probe *Mangalyaan* was sent recently to explore the planet Mars. The inter-planetary part of the trajectory is approximately a half-ellipse with the Earth (at the time of launch), Sun and Mars (at the time the probe reaches the destination) forming the major axis. Assuming that the orbits of Earth and Mars are approximately circular with radii $R_E$ and $R_M$, respectively, the velocity (with respect to the Sun) of the probe during its voyage when it is at a distance $r$ ( $R_E \ll r \ll R_M$) from the Sun, neglecting the effect of Earth and Mars, is



1. $\sqrt{2GM\frac{(R_E+R_M)}{r(R_E+R_M-r)}}$
2. ✓ $\sqrt{2GM\frac{(R_E+R_M-r)}{r(R_E+R_M)}}$
3. $\sqrt{2GM\frac{R_E}{rR_M}}$ $\sqrt{\frac{2M(E-ax)}{\hbar}}$ $=\left(\frac{(n+1)\pi\hbar}{\sqrt{2M}}\right)$
4. $\sqrt{\frac{2GM}{r}}$ $2M(E - \frac{ax^2}{2})$

**56.** एक बड़े धा.ऑ.अ. ट्रांसिस्टर में समांतर संबंधित $N$ पृथक ट्रांसिस्टर अंतर्विष्टित हैं। यदि हर ट्रांसिस्टर में रव का एकमात्र रूप $1/f$ रव है, तो धा.ऑ.अ. ट्रांसिस्टर के लिए तुल्य वोल्टेज रव स्पैक्ट्रल घनत्व एकल ट्रांसिस्टर की तुलना में

1. $1/N$ गुना है
2. $1/N^2$ गुना है
3. $N$ गुना है
4. $N^2$ गुना है

**56.** A large MOS transistor consists of $N$ individual transistors connected in parallel. If the only form of noise in each transistor is $1/f$ noise, then the equivalent voltage noise spectral density for the MOS transistor is

1. $1/N$ times that of a single transistor
2. ~~2. $1/N$~~$^2$ times that of a single transistor
3. $N$ times that of a single transistor
4. $N^2$ times that of a single transistor

**57.** द्रव्यमान $m$ के एक कण विभव $V(x) = a|x|$, $(a > 0)$ के प्रभाव में है। WKB सन्निकटन में ऊर्जा अभिलक्षणिक मान $E_n$ $(n = 0, 1, 2, \cdots)$ हैं

1. $\left[\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{1/3}$ $\int_0^a 2mE - Vx\, dx$
2. ✓ $\left[\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$ $-\frac{ax^2}{2}$
3. $\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)$ $\frac{\hbar^2}{2M}$ $(n+1)\frac{\hbar^2}{2M}\pi$
4. $\left[\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{4/3}$ $\frac{2M}{\hbar^2}$

**57.** Consider a particle of mass $m$ in the potential $V(x) = a|x|$, $a > 0$. The energy eigen-values $E_n$ $(n = 0, 1, 2, \cdots)$, in the WKB approximation, are

1. $\left[\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{1/3}$
2. ✓ $\left[\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$
3. $\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)$
4. $\left[\frac{3ah\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{4/3}$

**58.** तरंग दैर्घ्य $\lambda$ का लेसर प्रकाश जब 1 mm उत्कीर्णन युक्त धातवी मापनी पर पृष्ठसर्पी कोण पर आपतित होता है, विवर्तित होकर मापनी से लंब रखे गये पर्दे पर विवर्तन चिह्नों का एक ऊर्ध्वाधर श्रृंखला निर्मित करता है। यदि लेसर का तरंगदैर्घ्य और 200 nm बढ़ाया जाता है, प्रथम कोटि विवर्तन चिह्न का कोण 5° से किस मान तक बदलता है?

1. 6.60°
2. 5.14°
3. 5.018°
4. 5.21°

$\frac{x}{2}0 + \frac{x1}{2} + (x^2+1)\left(\frac{x}{2}\right) \to 0$ $\frac{x^2-1}{x^2}$ $1 = \frac{1}{x^2} \to 0$ $x^2 = \pm 1$ $x^2 - 1$

**58.** When laser light of wavelength $\lambda$ falls on a metal scale with 1 mm engravings at a grazing angle of incidence, it is diffracted to form a vertical chain of diffraction spots on a screen kept perpendicular to the scale. If the wavelength of the laser is increased by 200 nm, the angle of the first-order diffraction spot changes from 5° to

1. 6.60°
2. 5.14°
3. 5.018°
4. 5.21°

**59.** मानें कि एक-विमी आवर्त दोलनी की आध्यावस्था तथा प्रथम उत्तेजित अवस्था के तरंग फलनों का एकघात संचय $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ है (जहां $c_0$ तथा $c_1$ अचर हैं, और $c_0^2 + c_1^2 = 1$)। $c_0$ के किस मूल्य के लिए $\langle x\rangle$ अपने अधिकतम मूल्य प्राप्त करता है?

1. $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}},\ c_0 = \frac{1}{\sqrt{2}}$
2. $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}},\ c_0 = \frac{1}{2}$
3. $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}},\ c_0 = \frac{1}{\sqrt{2}}$
4. $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}},\ c_0 = \frac{1}{2}$

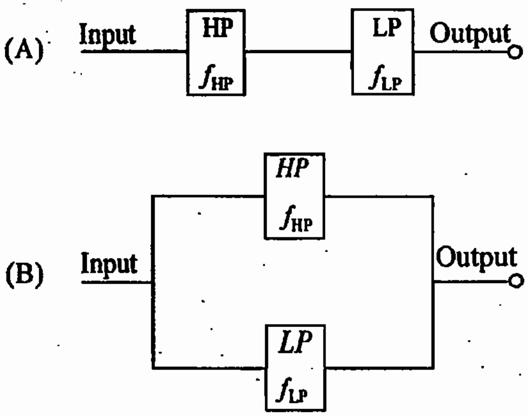
**59.** Let $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ (where $c_0$ and $c_1$ are constants with $c_0^2 + c_1^2 = 1$) be a linear combination of the wavefunctions of the ground and first excited states of the one-dimensional harmonic oscillator. For what value of $c_0$ is the expectation value $\langle x\rangle$ a maximum?

1. $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}},\ c_0 = \frac{1}{\sqrt{2}}$
2. $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}},\ c_0 = \frac{1}{2}$
3. $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}},\ c_0 = \frac{1}{\sqrt{2}}$ ✓
4. $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}},\ c_0 = \frac{1}{2}$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

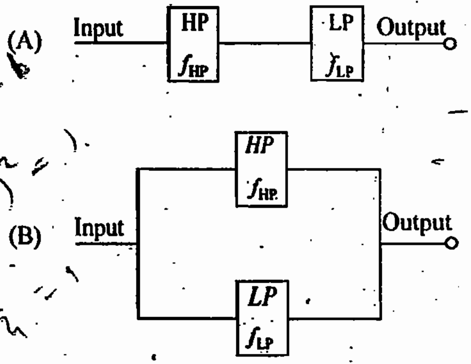
**60.** जैसे निम्न चित्रों A तथा B में दर्शाया गया है, एक निम्न आवृत्ति पारक फिल्टर (LP) एवं उच्च आवृत्ति पारक फिल्टर (HP) जिनकी अंतकीय बिंदु आवृत्तियां क्रमशः $f_{LP}$ एवं $f_{HP}$ है, श्रेणी या समांतर विन्यासों में संबंधित हैं।



निम्न कथनों में कौन-सा सही है?

1. $f_{HP} < f_{LP}$ के लिए, A बैंड पारक फिल्टर एवं B बैंड निराकरण फिल्टर के रूप में काम करते हैं
2. $f_{HP} > f_{LP}$ के लिए, A संकेत को पारित होने से रोकता है तथा B संकेत को फिल्टर किये बिना पारित करता है
3. $f_{HP} < f_{LP}$ के लिए, A बैंड पारक फिल्टर के रूप में कार्य करता है, तथा B संकेत को फिल्टर किये बिना पारित करता है
4. $f_{HP} > f_{LP}$ के लिए, A संकेत को फिल्टर किये बिना पारित करता है; तथा B बैंड-निराकरण फिल्टर के रूप में काम करता है

**60.** Consider a Low Pass (LP) and a High Pass (HP) filter with cut-off frequencies $f_{LP}$ and $f_{HP}$, respectively, connected in series or in parallel configurations as shown in the Figures A and B below.



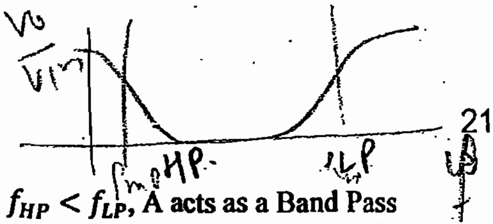
Which of the following statements is correct?

$H_6 = \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix} \Rightarrow$

$R_0 = \sum$

$(-2\ 0\ 0\ \frac{1}{3}\ \frac{2}{3}\ \frac{1}{n}$

$= 0;\ \frac{1}{3}\ \frac{2}{3}\ \frac{1}{3}\ \frac{2}{3}$



1. For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B acts as a Band Reject filter
2. For $f_{HP} > f_{LP}$, A stops the signal from passing through and B passes the signal without filtering
3. For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B passes the signal without filtering
4. For $f_{HP} > f_{LP}$, A passes the signal without filtering and B acts as a Band Reject filter

61. $N$ अन्योन्यक्रियाहीन प्रचक्रणों $S_i$, $i = 1,2,\cdots,N$ ($S_i = \pm1$) का एक संग्रह ताप $T$ में एक बाह्य चुंबकीय क्षेत्र $B$ में रखा गया है। निकाय की हैमिल्टनी है $H = -\mu B\sum_i S_i$ । माध्य $\langle S_i\rangle \geq \frac{1}{3}$ के लिए $\frac{\mu B}{k_BT}$ का न्यूनतम मान क्या होना चाहिए?

1. $\frac{1}{2}N\ln 2$ 2. $2\ln 2$
[✓ 3.] $\frac{1}{2}\ln 2$ 4. $N\ln 2$

61. A collection $N$ of non-interacting spins $S_i$, $i = 1,2,\cdots,N$, ($S_i = \pm1$) is kept in an external magnetic field $B$ at a temperature $T$. The Hamiltonian of the system is $H = -\mu B\sum_i S_i$. What should be the minimum value of $\frac{\mu B}{k_BT}$ for which the mean value $\langle S_i\rangle \geq \frac{1}{3}$?

1. $\frac{1}{2}N\ln 2$ 2. $2\ln 2$
[✓ 3.] $\frac{1}{2}\ln 2$ 4. $N\ln 2$

62. एक स्थैतिककल्प उत्क्रमणीय प्रक्रिया द्वारा जब एक गैस आयतन $V_1$ से $V_2$ तक रूद्धोष्मत: विस्तृत होती है, वह ताप $T_1$ से $T_2$ तक शीतित होती है। यदि यही प्रक्रिया रूद्धोष्मत: एवं अनुत्क्रमणीयत: की जाती है, तथा जब वह साम्यावस्थित है तब ताप $T_2'$ है, तो

1. $T_2' = T_2$ 2. $T_2' > T_2$
3. $T_2' = T_2\left(\frac{V_2-V_1}{V_2}\right)$ 4. $T_2' = \frac{T_2V_1}{V_2}$

62. When a gas expands adiabatically from volume $V_1$ to $V_2$ by a quasi-static reversible process, it cools from temperature $T_1$ to $T_2$. If now the same process is carried out adiabatically and irreversibly, and $T_2'$ is the temperature of the gas when it has equilibrated, then

1. $T_2' = T_2$ 2. $T_2' > T_2$
3. $T_2' = T_2\left(\frac{V_2-V_1}{V_2}\right)$ 4. $T_2' = \frac{T_2V_1}{V_2}$

63. एक यादृच्छिक पादचारी धनात्मक दिशा में प्रायिकता 2/3 के साथ एकक लंबाई का एक कदम तथा प्रायिकता 1/3 के साथ ऋणात्मक दिशा में एकक लंबाई का एक कदम लेता है। $n$ कदमों के बाद पादचारी का माध्य विस्थापन है

1. $n/3$ 2. $n/8$
3. $2n/3$ 4. 0

63. A random walker takes a step of unit length in the positive direction with probability $2/3$ and a step of unit length in the negative direction with probability $1/3$. The mean displacement of the walker after $n$ steps is

[✓ 1.] $n/3$ 2. $n/8$
3. $2n/3$ 4. 0

64. एक त्रि-अवस्था क्वांटम निकाय के लिए हैमिल्टनी आव्यूह $H_0 = \begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}$ है। जब यह $H' = \epsilon\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$ (जहां $\epsilon \ll 1$) से क्षोभित होती है, परिणाम स्वरूप ऊर्जा अभिलक्षणिक मान $E_0 = 2$ में अंतरण है:

1. $\epsilon, -2\epsilon$ 2. $-\epsilon, 2\epsilon$
3. $\pm\epsilon$ 4. $\pm 2\epsilon$

$\frac{2}{3} + \frac{1}{3}$

64. The Hamiltonian $H_0$ for a three-state quantum system is given by the matrix $H_0 = \begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}$.

When perturbed by $H' = \epsilon\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$ where $\epsilon \ll 1$, the resulting shift in the energy eigenvalue $E_0 = 2$ is

1. $\epsilon, -2\epsilon$ 2. $-\epsilon, 2\epsilon$
[✓ 3.] $\pm\epsilon$ 4. $\pm 2\epsilon$

$PV = nRT$

$\frac{3}{3}\ \frac{1}{2}$ $\frac{2}{3}\ \frac{1}{3}$ $\frac{1}{3}\begin{pmatrix}1\\0\\1\end{pmatrix} = |\phi_1\rangle$

**65.** यदि आकर्षण डेल्टा फलन $V(x) = -b\,\delta(x)$ जहां $b > 0$ की आध्यावस्था ऊर्जा विचरण परीक्षण फलन

$$\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & -a < x < a \text{ के लिये} \\ 0 \;, & \text{अन्यथा,} \end{cases}$$

के साथ परिकलित की जाती है, तो उसका मान है:

1. $-\frac{mb^2}{\pi^2\hbar^2}$
[✓ 2. $-\frac{2mb^2}{\pi^2\hbar^2}$]
3. $-\frac{mb^2}{2\pi^2\hbar^2}$
4. $-\frac{mb^2}{4\pi^2\hbar^2}$

**65.** The ground state energy of the attractive delta function potential

$$V(x) = -b\,\delta(x),$$

where $b > 0$, calculated with the variational trial function

$$\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for } -a < x < a, \\ 0 \;, & \text{otherwise,} \end{cases}$$

is

1. $-\frac{mb^2}{\pi^2\hbar^2}$
[✓ 2. $-\frac{2mb^2}{\pi^2\hbar^2}$]
3. $-\frac{mb^2}{2\pi^2\hbar^2}$
4. $-\frac{mb^2}{4\pi^2\hbar^2}$

**66.** किसी नाभिक (द्रव्यमान संख्या $A$ तथा आवेश $Z$) की बंधन ऊर्जा

$$B = a_V A - a_S A^{2/3} - a_{\text{sym}}\frac{(2Z-A)^2}{A} - \frac{a_c Z^2}{A^{1/3}}$$

से दी जाती है, जहां $a_V$ = 16 MeV, $a_S$ = 16 MeV, $a_{\text{sym}}$ = 24 MeV तथा $a_c$ = 0.75 MeV हैं। $A = 216$ युक्त एक नाभिक की अत्यंत स्थिर समभारिक परमाणु का $Z$ है:

1. 68
2. 72
3. 84
4. 92

**66.** If the binding energy $B$ of a nucleus (mass number $A$ and charge $Z$) is given by

$$B = a_V A - a_S A^{2/3} - a_{\text{sym}}\frac{(2Z-A)^2}{A} - \frac{a_c Z^2}{A^{1/3}}$$

where $a_V$ = 16 MeV, $a_S$ = 16 MeV, $a_{\text{sym}}$ = 24 MeV and $a_c$ = 0.75 MeV, then the $Z$ for the most stable isobar for a nucleus with $A = 216$ is

2 108

1. 68
2. 72
[✓ 3. 84]
4. 92

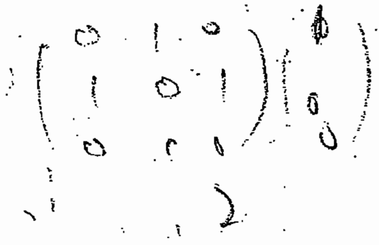
**67.** सोडियम क्लोराइड की स्फटिक संरचना स्पर्श करते गोलों के समुच्चय के रूप में प्रतिमानित है। हर सोडियम परमाणु की त्रिज्या $r_1$ तथा हर क्लोरिन परमाणु की त्रिज्या $r_2$ है। गोलों के केंद्र बिंदु एक सरल घन जालक निर्मित करते हैं। इस निकाय का संकुलन गुणांक है:

1. $\pi\left[\left(\frac{r_1}{r_1+r_2}\right)^3 + \left(\frac{r_2}{r_1+r_2}\right)^3\right]$
2. $\frac{2\pi}{3}\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$
3. $\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$
4. $\pi\frac{r_1^3+r_2^3}{2(r_1+r_2)^3}$

**67.** Consider the crystal structure of sodium chloride which is modeled as a set of touching spheres. Each sodium atom has a radius $r_1$ and each chlorine atom has a radius $r_2$. The centres of the spheres form a simple cubic lattice. The packing fraction of this system is

1. $\pi\left[\left(\frac{r_1}{r_1+r_2}\right)^3 + \left(\frac{r_2}{r_1+r_2}\right)^3\right]$
[✓ 2. $\frac{2\pi}{3}\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$]
3. $\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$
4. $\pi\frac{r_1^3+r_2^3}{2(r_1+r_2)^3}$

**68.** दो स्फटिकी ठोस पदार्थों के बारे में विचारें जिनमें से एक का सरल घन संरचना है तथा दूसरे का द्विसमलंबाक्ष संरचना है। $a$ तथा $b$ दिशाओं में परमाणुओं के बीच प्रभावी कमानी स्थिरांक की तुलना में $c$-दिशा में परमाणुओं के बीच प्रभावी कमानी स्थिरांक आधा है। निम्न तापों पर विशिष्ट ऊष्मा को जालक योगदान का व्यवहार ताप $T$ के फलन में इस प्रकार आश्रित होगा।



1. द्विसमलंबाक्ष ठोस पदार्थ के लिए $T^2$, परन्तु सरल घन ठोस पदार्थ के लिए $T^3$ के रूप में
2. द्विसमलंबाक्ष ठोस पदार्थ के लिए $T$, तथा सरल घन ठोस पदार्थ के लिए $T^3$ के रूप में
3. दोनों ठोस पदार्थों के लिए $T$ के रूप में
4. दोनों ठोस पदार्थों के लिए के $T^3$ रूप में

**68.** Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the $c$-direction is half the effective spring constant between atoms in the $a$ and $b$ directions. At low temperatures, the behavior of the lattice contribution to the specific heat will depend as a function of temperature $T$ as

1. $T^2$ for the tetragonal solid, but as $T^3$ for the simple cubic solid
2. $T$ for the tetragonal solid and as $T^3$ for the simple cubic solid
3. $T$ for both solids
4. $T^3$ for both solids

**69.** 1 टैस्ला के एक चुंबकीय क्षेत्र में एक परमाणविक संक्रमण $^1P \rightarrow {}^1S$ जेमानी विपाटन दर्शाता है । बोर मैग्नेटान $\mu_B = 9.27x10^{-24}$ J/T, तथा संक्रमण से संगत तरंगदैर्घ्य 250 nm है, तो जेमानी स्पैक्ट्रल रेखाओं के बीच पृथक्न है लगभग

1. 0.01 nm    2. 0.1 nm
3. 1.0 nm    4. 10 nm

**69.** An atomic transition $^1P \rightarrow {}^1S$ in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton $\mu_B = 9.27x10^{-24}$ J/T, and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately

1. 0.01 nm    2. 0.1 nm
3. 1.0 nm    4. 10 nm

**70.** यदि एक द्विपरमाणुक अणु की $n$-वीं कंपन स्तर की ऊर्जा की अग्रग अप्रसंवादी संशुद्धि $-x_e\left(n+\frac{1}{2}\right)^2 \hbar\omega$, जहां $x_e = 0.001$ है, तो संभाव्य ऊर्जा स्तरों की कुल संख्या लगभग है:

1. 500    2. 1000
3. 250    4. 750

**70.** If the leading anharmonic correction to the energy of the $n$-th vibrational level of a diatomic molecule is $-x_e\left(n+\frac{1}{2}\right)^2 \hbar\omega$ with $x_e = 0.001$, the total number of energy levels possible is approximately

1. 500    2. 1000
3. 250    4. 750

**71.** अपने समतल से लंब एक चुंबकीय क्षेत्र $\vec{B}$ की उपस्थिति में एक अतिचालक वलय एक अचर विद्युतधारा वहित करता है। **गलत** कथन को पहचाने:

1. अतिचालक के अंदर से पारित अभिवाह $hc/e$ की एकलों में क्वांटमित है।
2. अतिचालक में विद्युतधारा एवं चुंबकीय क्षेत्र दोनों समय पर निर्भर नहीं हैं।
3. विद्युतधारा घनत्व $\vec{j}$ तथा $\vec{B}$ समीकरण $\vec{\nabla}\times\vec{j}+\Lambda^2\vec{B}=0$ से संबंधित हैं, जहां $\Lambda$ एक अचर है।
4. अतिचालक एक ऊर्जा अंतराल दर्शाता है, जो अतिचालक के संक्रमण ताप के अनुपात में है।

**71.** A superconducting ring carries a steady current in the presence of a magnetic field $\vec{B}$ normal to the plane of the ring. Identify the **incorrect** statement.

1. The flux passing through the superconductor is quantized in units of $hc/e$
2. The current and the magnetic field in the superconductor are time independent.

3. The current density $\vec{j}$ and $\vec{B}$ are related by the equation $\vec{\nabla}\times\vec{j}+\Lambda^2\vec{B}=0$, where $\Lambda$ is a constant
4. The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

**72.** हाईड्रोजन परमाणु की आध्यावस्था में इलैक्ट्रोन प्रचक्रण $\vec{s}_e$ तथा प्रोटॉन प्रचक्रण $\vec{s}_p$ के बीच प्रभावी प्रचक्रण-प्रचक्रण अन्योन्यक्रिया $H' = a\vec{s}_e \cdot \vec{s}_p$ से दी जाती है । इस अन्योन्यक्रिया के परिणाम स्वरूप ऊर्जा स्तरें इस मात्रा में विपाटित होती हैं:

1. $\frac{1}{2}a\hbar^2$
2. $2a\hbar^2$
3. $a\hbar^2$
4. $\frac{3}{2}a\hbar^2$

**72.** The effective spin-spin interaction between the electron spin $\vec{s}_e$ and the proton spin $\vec{s}_p$ in the ground state of the Hydrogen atom is given by $H' = a\vec{s}_e \cdot \vec{s}_p$. As a result of this interaction, the energy levels split by an amount

1. $\frac{1}{2}a\hbar^2$
2. $2a\hbar^2$
3. [✓ $a\hbar^2$]
4. $\frac{3}{2}a\hbar^2$

**73.** यह निर्धारित करने के लिए कि क्या प्रोटॉनों की कुछ आंतरिक संरचना है या नहीं, गभीर अप्रत्यास्थ विकीर्णन में इलेक्ट्रोन प्रोटॉनों से प्रकीर्णित किये जाते हैं। इसके लिए इलेक्ट्रोन की ऊर्जा को कम से कम इतनी होनी चाहिये:

1. $1.25\times 10^9$ eV
2. $1.25\times 10^{12}$ eV
3. $1.25\times 10^6$ eV
4. $1.25\times 10^8$ eV

**73.** In deep inelastic scattering electrons are scattered off protons to determine if a proton has any internal structure. The energy of the electron for this must be at least

1. [✓ $1.25\times 10^9$ eV]
2. $1.25\times 10^{12}$ eV
3. $1.25\times 10^6$ eV
4. $1.25\times 10^8$ eV

**74.** एक सौर (सौलर) सेल पर आपतित सूर्यप्रकाश की शक्ति घनत्व 100 mW/cm$^2$ है। लघु पथ में विद्युत धारा 30 mA/cm$^2$ है तथा खुला पथ वोल्टेज 0.7 V है। यदि सौर सेल का भराव गुणांक 0.8 से 0.5 तक घटता है, तो प्रतिशत क्षमता घटेगी

1. 42.0 से 26.2 तक
2. 24.0 से 16.8 तक
3. 21.0 से 10.5 तक
4. 16.8 से 10.5 तक

**74.** The power density of sunlight incident on a solar cell is 100 mW/cm$^2$. Its short circuit current density is 30 mA/cm$^2$ and the open circuit voltage is 0.7 V. If the fill factor of the solar cell decreases from 0.8 to 0.5 then the percentage efficiency will decrease from

1. 42.0 to 26.2
2. 24.0 to 16.8
3. 21.0 to 10.5
4. [✓ 16.8 to 10.5]

**75.** निम्न चार प्रक्रियाओं पर विचारें:

(i) $p^+ \to n + e^+ + \nu_e$
(ii) $\Lambda^0 \to p^+ + e^+ + \nu_e$
(iii) $\pi^+ \to e^+ + \nu_e$
(iv) $\pi^0 \to \gamma + \gamma$

मुक्त कणों के लिए उपरोक्त में कौन-सा/से वर्जित है/हैं?

1. केवल (ii)
2. (ii) तथा (iv)
3. (i) तथा (iv)
4. [✓ (i) तथा (ii)]

**75.** Consider the four processes

(i) $p^+ \to n + e^+ + \nu_e$
(ii) $\Lambda^0 \to p^+ + e^+ + \nu_e$
(iii) $\pi^+ \to e^+ + \nu_e$
(iv) $\pi^0 \to \gamma + \gamma$

Which of the above is/are forbidden for free particles?

1. only (ii)
2. (ii) and (iv)
3. (i) and (iv)
4. (i) and (ii)

# GATE 2014 Examination

## PH: Physics

*Duration*: **180 minutes** *Maximum Marks*: **100**

**Read the following instructions carefully.**

1. To login, enter your Registration Number and password provided to you. Kindly go through the various symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **View All Questions** button in the screen.
3. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory.
   The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.
   The subject specific GATE paper section consists of **55** questions, out of which question numbers 1 to 25 are of 1-mark each, while question numbers 26 to 55 are of 2-mark each.
4. Depending upon the GATE paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Common Data** button that appears at the top, right hand side of the screen.
5. The computer allotted to you at the examination center runs specialized software that permits only one answer to be selected for multiple-choice questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual keyboard and mouse.
6. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
7. In each paper a candidate can answer a total of 65 questions carrying 100 marks.
8. The question paper may consist of questions of **multiple choice type** (MCQ) and **numerical answer type**.
9. Multiple choice type questions will have four choices against A, B, C, D, out of which only **ONE** is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
10. For numerical answer type questions, each question will have a numerical answer and there will not be any choices. **For these questions, the answer should be entered**by using the virtual keyboard that appears on the monitor and the mouse.
11. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of⅓ marks for a 1-mark question and ⅔ marks for a 2-mark question.
12. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
13. Non-programmable type Calculator is allowed. Charts, graph sheets, and mathematical tables are **NOT** allowed in the Examination Hall. You must use the Scribble pad provided to you at the examination centre for all your rough work. The Scribble Pad has to be returned at the end of the examination.

**Declaration by the candidate:**

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

only be able to see questions in this Section, but you will not be able to answer questions in the Section.

16. After clicking the **Save & Next** button for the last question in a Section, you will automatically be taken to the first question of the next Section in sequence.
17. You can move the mouse cursor over the name of a Section to view the answering status for that Section.

## Changing the Optional Section :

18. After answering the chosen optional Section, partially or completely, you can change the optional Section by selecting the checkbox for a new Section that you want to attempt. A warning message will appear along with a table showing the number of questions answered in each of the previously chosen optional Sections and a checkbox against each of these Sections. Click on a checkbox against a Section that you want to reset and then click on the **RESET** button. Note that RESETTING a Section will DELETE all the answers for questions in that Section. Hence, if you think that you may want to select this Section again later, you will have to note down your answers for questions in that Section. If you do not want to reset the Section and want to continue answering the previously chosen optional Section, then click on the **BACK** button.

19. If you deselect the checkbox for an optional Section in the top bar, the following warning message will appear: "Deselecting the checkbox will DELETE all the answers for questions in this Section. Do you want to deselect this Section?" If you want to deselect, click on the **RESET** button. If you do not want to deselect, click on the **BACK** button.

20. You can shuffle between different Sections or change the optional Sections any number of times.

**Caution:** Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

## Answering a Question :

8. Procedure for answering a multiple choice (MCQ) type question:
   a. Choose one answer from the 4 options (A,B,C,D) given below the question, click on the bubble placed before the chosen option.
   b. To deselect your chosen answer, click on the bubble of the chosen option again or click on the **Clear Response** button.
   c. To change your chosen answer, click on the bubble of another option.
   d. To save your answer, you MUST click on the **Save & Next** button.

9. Procedure for answering a numerical answer type question:
   a. To enter a number as your answer, use the virtual numerical keypad.
   b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.
   c. To clear your answer, click on the **Clear Response** button.
   d. To save your answer, you MUST click on the **Save & Next** button
10. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ) or entered (for numerical answer type) for a question that is* **Marked for Review**, *that answer will be considered in the evaluation unless the status is modified by the candidate.*

11. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.
12. Note that ONLY Questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

## Choosing a Section :

13. Sections in this question paper are displayed on the top bar of the screen. Questions in a Section can be viewed by clicking on the name of that Section. The Section you are currently viewing will be highlighted.
14. A checkbox is displayed for every optional Section, if any, in the Question Paper. To select the optional Section for answering, click on the checkbox for that Section.
15. If the checkbox for an optional Section is not selected, the **Save & Next** button and the **Mark for Review & Next** button will NOT be enabled for that Section. You will

# GATE 2014: General Instructions during Examination

1. Total duration of the GATE examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Common Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use a non-programmable type calculator, however, sharing of calculators is not allowed.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

1 You have not visited the question yet.

3 You have not answered the question.

5 You have answered the question.

7 You have NOT answered the question, but have marked the question for review.

9 ✓ You have answered the question, but marked it for review.

The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is answered, but marked for review, then the answer will be considered for evaluation unless the status is modified by the candidate.*

## Navigating to a Question :

7. To answer a question, do the following:
    a. Click on the question number in the Question Palette to go to that question directly.
    b. Select an answer for a multiple choice type question by clicking on the bubble placed before the 4 choices, namely A, B, C and D. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
    c. Click on **Save & Next** to save your answer for the current question and then go to the next question.
    d. Click on **Mark for Review & Next** to save your answer for the current question and also mark it for review, and then go to the next question.

## Q. 1 – Q. 5 carry one mark each.

Q.1 A student is required to demonstrate a high level of comprehension of the subject, especially in the social sciences.

The word closest in meaning to comprehension is

(A) understanding (B) meaning (C) concentration (D) stability

Q.2 Choose the most appropriate word from the options given below to complete the following sentence.

One of his biggest _______ was his ability to forgive.

(A) vice (B) virtues (C) choices (D) strength

Q.3 Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(A) Rajan has decided to work only in a group.
(B) Rajan and Sajan were formed into a group against their wishes.
(C) Sajan had decided to give in to Rajan's request to work with him.
(D) Rajan had believed that Sajan and he would be working together.

Q.4 If $y = 5x^2 + 3$, then the tangent at $x = 0, y = 3$

(A) passes through $x = 0, y = 0$ (B) has a slope of $+1$
(C) is parallel to the $x$-axis (D) has a slope of $-1$

Q.5 A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

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## Q. 6 – Q. 10 carry two marks each.

Q.6 Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

(A) ALRVX (B) EPVZB (C) ITZDF (D) OYEIK

Q.7 Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

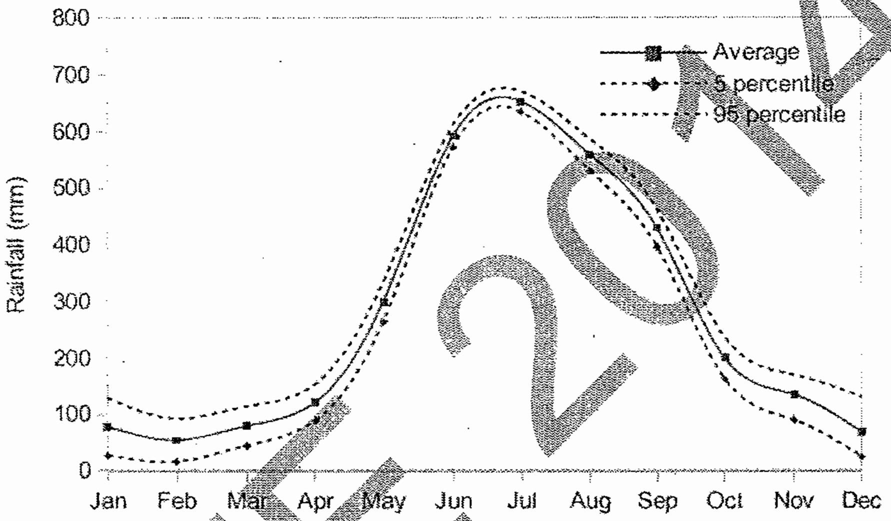
| | Anuj | Bhola | Chandan | Dilip | Eswar | Faisal |
|---|---|---|---|---|---|---|
| (A) | 6 | 2 | 5 | 1 | 3 | 4 |
| (B) | 2 | 6 | 5 | 1 | 3 | 4 |
| (C) | 4 | 2 | 6 | 3 | 1 | 5 |
| (D) | 2 | 4 | 6 | 1 | 3 | 5 |

Q.8 The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

Q.9 One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

(A) 3.0 (B) 2.5 (C) 1.5 (D) 1.25

Q.10 The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (*k* percentile is the value such that *k* percent of the data fall below that value)



(i) On average, it rains more in July than in December
(ii) Every year, the amount of rainfall in August is more than that in January
(iii) July rainfall can be estimated with better confidence than February rainfall
(iv) In August, there is at least 500 mm of rainfall

(A) (i) and (ii) (B) (i) and (iii)
(C) (ii) and (iii) (D) (iii) and (iv)

## END OF THE QUESTION PAPER

GATE 2014

## Some Useful Constants

| | |
|---|---|
| Speed of light in free space | $c = 3 \times 10^{8}\, m/s$ |
| Boltzmann constant | $k_B = 1.380 \times 10^{-23}\, J/K$ |
| Planck's constant | $h = 6.626 \times 10^{-34}\, J.s$ |
| Electron charge | $e = 1.602 \times 10^{-19}\, C$ |
| Permittivity of free space | $\varepsilon_0 = 8.854 \times 10^{-12}\, C^2/N.m^2$ |
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7}\, H/m$ |
| Mass of electron | $m_e = 9.31 \times 10^{-31}\, kg$ |
| Mass of proton | $m_p = 1.67 \times 10^{-27}\, kg$ |
| Mass of neutron | $m_n = 1.675 \times 10^{-27}\, kg$ |

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## Q. 1 – Q. 25 carry one mark each.

Q.1 The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point $(1,1,1)$ is

(A) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (B) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (C) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (D) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

Q.2 Which one of the following quantities is invariant under Lorentz transformation?

(A) Charge density (B) Charge (C) Current (D) Electric field

Q.3 The number of normal Zeeman splitting components of ${}^1P \to {}^1D$ transition is

(A) 3 (B) 4 (C) 8 (D) 9

Q.4 If the half-life of an elementary particle moving with speed $0.9c$ in the laboratory frame is $5\times10^{-8}\,s$, then the proper half-life is __________ $\times\,10^{-8}\,s$. ($c = 3 \times 10^8\,m/s$)

Q.5 An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

(A) $0^0$ (B) $45^0$ (C) $90^0$ (D) $120^0$

Q.6 Two masses $m$ and $3m$ are attached to the two ends of a massless spring with force constant $K$. If $m = 100\,g$ and $K = 0.3\,N/m$, then the natural angular frequency of oscillation is ___________ $Hz$.

Q.7 The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E} = \hat{x}10\cos\left(6\pi\times10^7 t - 0.4\pi z\right)\ V/m.$$

The phase velocity of the wave is ____________ $\times\,10^8\,m/s$.

Q.8 The matrix

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \text{ is}$$

(A) orthogonal (B) symmetric (C) anti-symmetric (D) unitary

Q.9 The recoil momentum of an atom is $p_A$ when it emits an infrared photon of wavelength 1500 $nm$, and it is $p_B$ when it emits a photon of visible wavelength 500 $nm$. The ratio $\frac{p_A}{p_B}$ is

(A) $1:1$ (B) $1:\sqrt{3}$ (C) $1:3$ (D) $3:2$

Q.10 For a gas under isothermal conditions, its pressure $P$ varies with volume $V$ as $P \propto V^{-5/3}$. The bulk modulus $B$ is proportional to

(A) $V^{-1/2}$ (B) $V^{-2/3}$ (C) $V^{-3/5}$ (D) $V^{-5/3}$

Q.11 Which one of the following high energy processes is allowed by conservation laws?

(A) $p + \bar{p} \to \Lambda^0 + \Lambda^0$ (B) $\pi^- + p \to \pi^0 + n$

(C) $n \to p + e^- + \nu_e$ (D) $\mu^+ \to e^+ + \gamma$

PH (GATE 2014)

Q.12 The length element $ds$ of an arc is given by, $(ds)^2 = 2(dx^1)^2 + (dx^2)^2 + \sqrt{3}\, dx^1 dx^2$. The metric tensor $g_{ij}$ is

(A) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$

(D) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

Q.13 The ground state and the first excited state wave functions of a one dimensional infinite potential well are $\psi_1$ and $\psi_2$, respectively. When two spin-up electrons are placed in this potential, which one of the following, with $x_1$ and $x_2$ denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(A) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)]$

(B) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$

(C) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$

(D) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

Q.14 If the vector potential

$$\vec{A} = \alpha x\,\hat{x} + 2y\,\hat{y} - 3z\,\hat{z},$$

satisfies the Coulomb gauge, the value of the constant $\alpha$ is _______________.

Q.15 At a given temperature, $T$, the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillators is _______________ $k_BT$ ($k_B$ is the Boltzmann constant).

Q.16 Which one of the following is a fermion?

(A) $\alpha$ particle (B) ${}_4Be^7$ nucleus (C) Hydrogen atom (D) Deuteron

Q.17 Which one of the following three-quark states ($qqq$), denoted by $X$, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript.

(A) $X^{++}$ (B) $X^{+}$ (C) $X^{-}$ (D) $X^{--}$

Q.18 The Hamilton's canonical equations of motion in terms of Poisson Brackets are

(A) $\dot{q} = \{q, H\}$; $\dot{p} = \{p, H\}$
(B) $\dot{q} = \{H, q\}$; $\dot{p} = \{H, p\}$
(C) $\dot{q} = \{H, p\}$; $\dot{p} = \{H, q\}$
(D) $\dot{q} = \{p, H\}$; $\dot{p} = \{q, H\}$

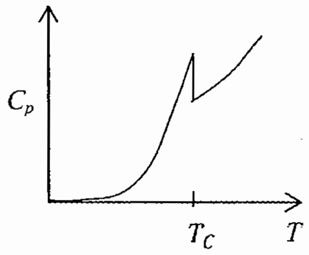
Q.19 The Miller indices of a plane passing through the three points having coordinates (0,0,1), (1,0,0), $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}\right)$ are

(A) (212) (B) (111) (C) (121) (D) (211)

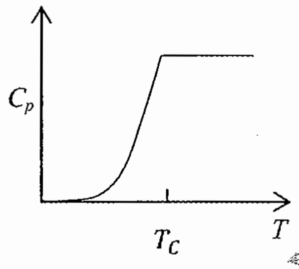
PH (GATE 2014)

Q.20 The plot of specific heat versus temperature across the superconducting transition temperature $(T_C)$ is most appropriately represented by

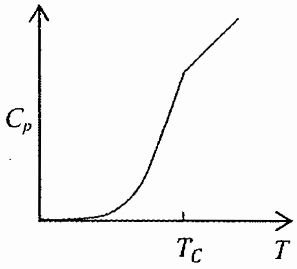
(A)



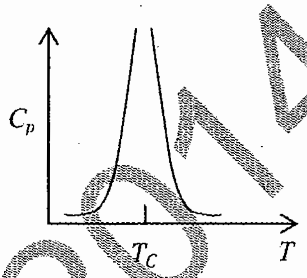
(B)



(C)



(D)



Q.21 If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does NOT commute with

(A) $S_z$ (B) $L^2$ (C) $S^2$ (D) $(\vec{L}+\vec{S})^2$

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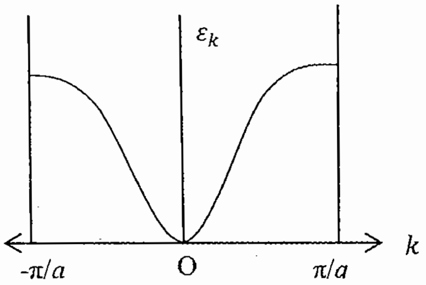
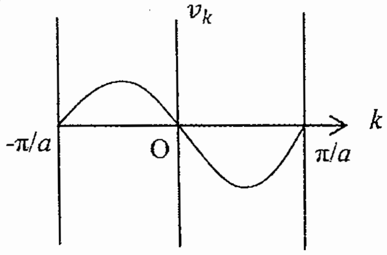
Q.22 The energy, $\varepsilon_k$ for band electrons as a function of the wave vector, $k$ in the first Brillouin zone $(-\pi/a \le k \le \pi/a)$ of a one dimensional monatomic lattice is shown as ($a$ is lattice constant)

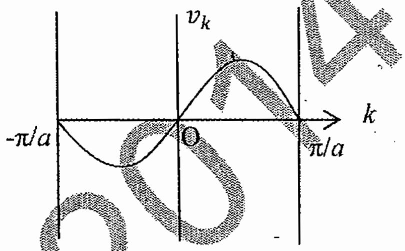


The variation of the group velocity $v_k$ is most appropriately represented by

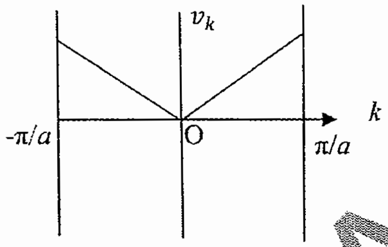
(A)



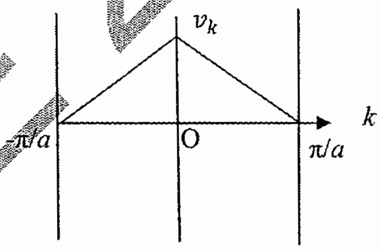
(B)



(C)

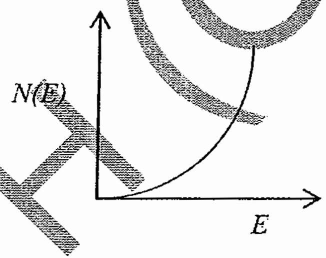


(D)

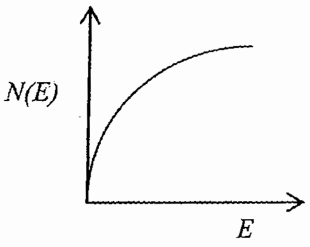


Q.23 For a free electron gas in two dimensions, the variation of the density of states, $N(E)$ as a function of energy $E$, is best represented by

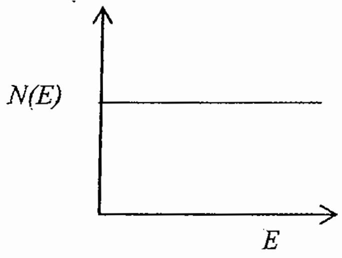
(A)



(B)



(C)



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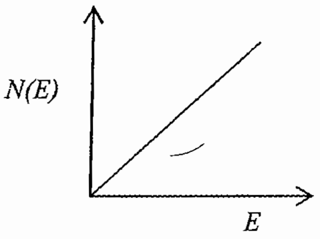

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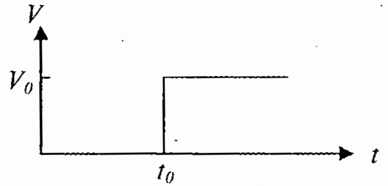
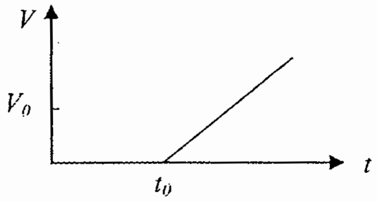
Q.24 The input given to an ideal OP-AMP integrator circuit is

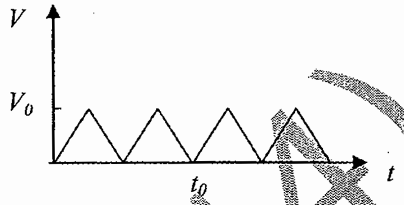


The correct output of the integrator circuit is

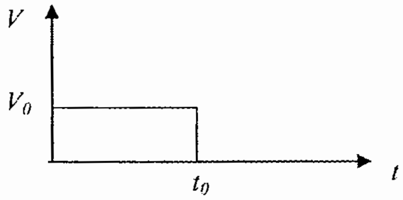
(A)



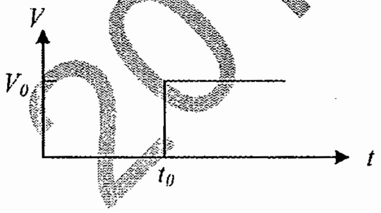
(B)



(C)



(D)



Q.25 The minimum number of flip-flops required to construct a mod-75 counter is _______.

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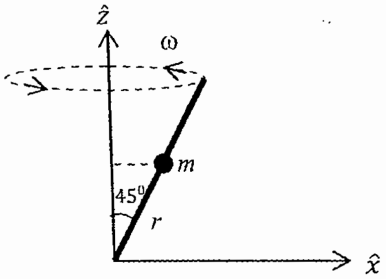
## Q. 26 – Q. 55 carry two marks each.

Q.26 A bead of mass $m$ can slide without friction along a massless rod kept at $45^0$ with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed $\omega$. At any instant, $r$ is the distance of the bead from the origin. The momentum conjugate to $r$ is



(A) $m\dot{r}$ (B) $\frac{1}{\sqrt{2}}m\dot{r}$ (C) $\frac{1}{2}m\dot{r}$ (D) $\sqrt{2}m\dot{r}$

Q.27 An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\left(r/a_0\right)}$$

where $a_0$ is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is

(Hint: $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$)

(A) $-a_0^2/2$ (B) $-a_0^2$ (C) $-3a_0^2/2$ (D) $-2a_0^2$

Q.28 For Nickel, the number density is $8 \times 10^{23}\ atoms/cm^3$ and electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic state is ________________ $\times 10^9\ A/m$.

(Given the value of Bohr magneton $\mu_B = 9.21 \times 10^{-21}\ Am^2$)

Q.29 A particle of mass $m$ is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{a r_0^2}{3r^3},$$

where $a$ and $r_0$ are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(A) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (B) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (C) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (D) $4\pi\sqrt{\frac{mr_0^3}{a}}$

Q.30 The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is __________ $meV$.

($k_B T = 25\ meV$ at $300\ K$)

Q.31 A particle of mass $m$ is subjected to a potential,

$$V(x,y) = \frac{1}{2}m\omega^2(x^2+y^2), -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is $g$-fold degenerate. The value of $g$ is ___________ .

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Q.32 A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}}\,\psi_{200} - \sqrt{\frac{3}{7}}\,\psi_{310} + \sqrt{\frac{4}{21}}\,\psi_{321},$$

where $n, l, m$ in $\psi_{nlm}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of $L^2$ is ______________ $\hbar^2$.

Q.33 A planet of mass $m$ moves in a circular orbit of radius $r_0$ in the gravitational potential $V(r) = -\frac{k}{r}$, where $k$ is a positive constant. The orbital angular momentum of the planet is

(A) $2r_0km$ (B) $\sqrt{2r_0km}$ (C) $r_0km$ (D) $\sqrt{r_0km}$

Q.34 The moment of inertia of a rigid diatomic molecule $A$ is 6 times that of another rigid diatomic molecule $B$. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers $J_A$ and $J_B$ are

(A) $J_A = 2, J_B = 1$ (B) $J_A = 3, J_B = 1$ (C) $J_A = 5, J_B = 0$ (D) $J_A = 6, J_B = 1$

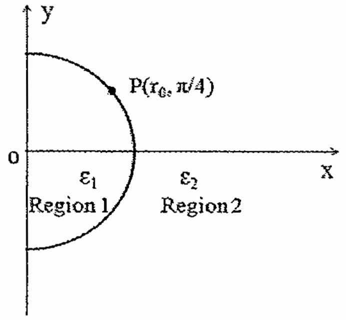
Q.35 The value of the integral

$$\oint_C \frac{z^2}{e^z + 1}\,dz,$$

where $C$ is the circle $|z| = 4$, is

(A) $2\pi i$ (B) $2\pi^2 i$ (C) $4\pi^3 i$ (D) $4\pi^2 i$

Q.36 A ray of light inside Region 1 in the $xy$- plane is incident at the semicircular boundary that carries no free charges. The electric field at the point $P(r_0, \pi/4)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$ , where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to $x$- axis. If $\varepsilon_1$ and $\varepsilon_2$ are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is ______________________



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Q.37 The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0,$$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is

(A) $\cos t + \sin t$ (B) $\cosh t + \sinh t$ (C) $\cos t - \sin t$ (D) $\cosh t - \sinh t$

Q.38 Given that the linear transformation of a generalized coordinate $q$ and the corresponding momentum $p$,

$$Q = q + 4ap$$
$$P = q + 2p$$

is canonical, the value of the constant $a$ is ___________ .

Q.39 The value of the magnetic field required to maintain non-relativistic protons of energy 1 $MeV$ in a circular orbit of radius 100 $mm$ is _____________ Tesla.
(Given: $m_p = 1.67 \times 10^{-27}\, kg$, $e = 1.6 \times 10^{-19}\, C$)

Q.40 For a system of two bosons, each of which can occupy any of the two energy levels 0 and $\varepsilon$, the mean energy of the system at a temperature $T$ with $\beta = \frac{1}{k_BT}$ is given by

(A) $\dfrac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (B) $\dfrac{1 + \varepsilon\, e^{-\beta\varepsilon}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(C) $\dfrac{2\varepsilon\, e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (D) $\dfrac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

Q.41 In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and $I_0$, respectively. The intensities of the individual waves are
(A) $3I_0$ and $I_0$ (B) $4I_0$ and $I_0$
(C) $5I_0$ and $4I_0$ (D) $9I_0$ and $I_0$

Q.42 $\psi_1$ and $\psi_2$ are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix},$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state $\psi_2$, its probability to be in the spin-up state is ______________ .

Q.43 Neutrons moving with speed $10^3$ m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is $30^0$, the interplanar spacing of the crystal is _________ Å.
(Given: $m_n = 1.675 \times 10^{-27}$ kg, $h = 6.626 \times 10^{-34}\, J.s$)

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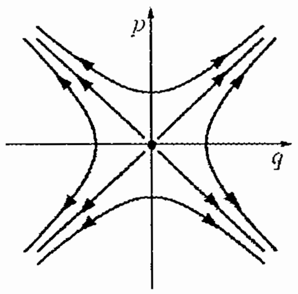
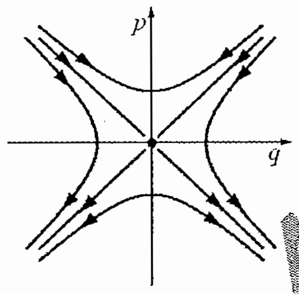
Q.44 The Hamiltonian of a particle of mass $m$ is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which one of the following figures describes the motion of the particle in phase space?

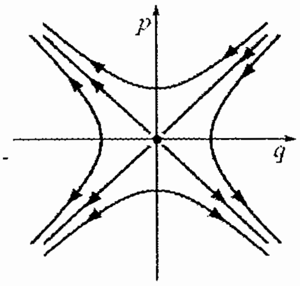
(A)



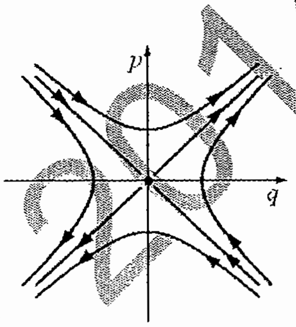
(B)



(C)



(D)



Q.45 The intensity of a laser in free space is 150 $mW/m^2$. The corresponding amplitude of the electric field of the laser is ________ $V/m$. $(\epsilon_0 = 8.854 \times 10^{-12}\, C^2/N.m^2)$

Q.46 The emission wavelength for the transition $^1D_2 \rightarrow {}^1F_3$ is 3122 $\AA$. The ratio of populations of the final to the initial states at a temperature 5000 $K$ is $(h = 6.626 \times 10^{-34}\, J.s,\ c = 3 \times 10^8\, m/s,\ k_B = 1.380 \times 10^{-23}\, J/K)$

(A) $2.03 \times 10^{-5}$ (B) $4.02 \times 10^{-5}$ (C) $7.02 \times 10^{-5}$ (D) $9.83 \times 10^{-5}$

Q.47 Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(A) $k_B \ln 2$ (B) $2k_B \ln 2$ (C) $2k_B \ln 4$ (D) $3k_B \ln 4$

Q.48 A particle is confined to a one dimensional potential box with the potential

$$V(x) = 0,\ 0 < x < a$$
$$= \infty, \text{ otherwise}$$

If the particle is subjected to a perturbation, within the box, $W = \beta x$, where $\beta$ is a small constant, the first order correction to the ground state energy is

(A) 0 (B) $a\beta/4$ (C) $a\beta/2$ (D) $a\beta$

Q.49 Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons ($\mu$) in the centre of mass frame required to produce the pion ($\pi$) pairs at rest is ________ $MeV$. (Given: $m_\mu = 105\, MeV/c^2$, $m_\pi = 140\, MeV/c^2$).

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Q.50 A one dimensional harmonic oscillator is in the superposition of number states, $|n\rangle$, given by

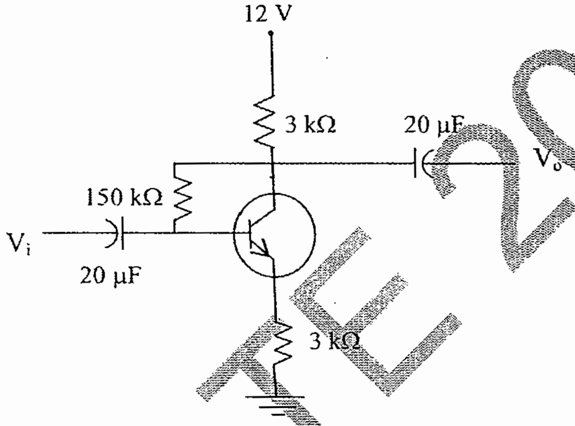
$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle .$$

The average energy of the oscillator in the given state is ______________ $\hbar\omega$.

Q.51 A nucleus $X$ undergoes a first forbidden $\beta$-decay to a nucleus $Y$. If the angular momentum ($I$) and parity ($P$), denoted by $I^P$ as $\frac{7}{2}^-$ for $X$, which of the following is a possible $I^P$ value for $Y$?

(A) $\frac{1}{2}^+$ (B) $\frac{1}{2}^-$ (C) $\frac{3}{2}^+$ (D) $\frac{3}{2}^-$

Q.52 The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current $I_c$ is ______________ $mA$.



Q.53 In order to measure a maximum of $1V$ with a resolution of $1mV$ using a $n$-bit $A/D$ converter, working under the principle of ladder network, the minimum value of $n$ is ______________ .

Q.54 If $L_+$ and $L_-$ are the angular momentum ladder operators, then, the expectation value of $(L_+L_- + L_-L_+)$, in the state $|l = 1, m = 1\rangle$ of an atom is ________ $\hbar^2$.

Q.55 A low pass filter is formed by a resistance $R$ and a capacitance $C$. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively, are

(A) 0.71 and $45^0$ (B) 0.71 and $-45^0$ (C) 0.5 and $-90^0$ (D) 0.5 and $90^0$

## END OF THE QUESTION PAPER

PH (GATE 2014)

# GATE 2014
## Answer Keys for PH - Physics

| Section | Q. No. | Key / Range | Marks |
|---|---|---|---|
| GA | 1 | A | 1 |
| GA | 2 | B | 1 |
| GA | 3 | D | 1 |
| GA | 4 | C | 1 |
| GA | 5 | 1300 to 1300 | 1 |
| GA | 6 | D | 2 |
| GA | 7 | B | 2 |
| GA | 8 | 180 to 180 | 2 |
| GA | 9 | D | 2 |
| GA | 10 | B | 2 |
| PH | 1 | D | 1 |
| PH | 2 | B | 1 |
| PH | 3 | A | 1 |
| PH | 4 | 2.1 to 2.3 | 1 |
| PH | 5 | C | 1 |
| PH | 6 | 1.99 to 2.01 | 1 |
| PH | 7 | 1.49 to 1.51 | 1 |
| PH | 8 | D | 1 |
| PH | 9 | C | 1 |
| PH | 10 | D | 1 |
| PH | 11 | B | 1 |
| PH | 12 | B | 1 |
| PH | 13 | D | 1 |
| PH | 14 | 0.99 to 1.01 | 1 |
| PH | 15 | 1.99 to 2.01 | 1 |
| PH | 16 | B | 1 |
| PH | 17 | D | 1 |
| PH | 18 | A | 1 |
| PH | 19 | C | 1 |
| PH | 20 | A | 1 |
| PH | 21 | A | 1 |
| PH | 22 | B | 1 |
| PH | 23 | C | 1 |
| PH | 24 | A | 1 |
| PH | 25 | 6.99 to 7.01 | 1 |
| PH | 26 | A | 2 |
| PH | 27 | D | 2 |
| PH | 28 | 40 to 43 | 2 |
| PH | 29 | A | 2 |
| PH | 30 | 114 to 117 | 2 |
| PH | 31 | 3.99 to 4.01 | 2 |
| PH | 32 | 1.99 to 2.01 | 2 |
| PH | 33 | D | 2 |
| PH | 34 | B | 2 |
| PH | 35 | C | 2 |
| PH | 36 | 2.3 to 2.4 | 2 |
| PH | 37 | D | 2 |
| PH | 38 | 0.24 to 0.26 | 2 |
| PH | 39 | 1.41 to 1.47 | 2 |
| PH | 40 | A | 2 |
| PH | 41 | B | 2 |
| PH | 42 | 0.66 to 0.68 | 2 |
| PH | 43 | 3.91 to 4.15 | 2 |
| PH | 44 | D | 2 |
| PH | 45 | 10.58 to 10.70 | 2 |
| PH | 46 | C | 2 |
| PH | 47 | B | 2 |
| PH | 48 | C | 2 |
| PH | 49 | 34.9 to 35.1 | 2 |
| PH | 50 | 3.2 to 3.3 | 2 |
| PH | 51 | C | 2 |
| PH | 52 | 1.4 to 1.7 | 2 |
| PH | 53 | 9.99 to 10.01 | 2 |
| PH | 54 | 1.99 to 2.01 | 2 |
| PH | 55 | B | 2 |

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## PHYSICAL SCIENCES

ANSWER KEY BOOKLET CODE : B

| Q. NO. | ANSWER | Q. NO. | ANSWER | Q. NO. | ANSWER |
|---|---|---|---|---|---|
| 1. | 2 | 26. | 1 | 51. | 4 |
| 2. | 2 | 27. | 3 | 52. | 2 |
| 3. | 3 | 28 | 3 | 53. | 2 |
| 4. | 2 | 29. | 1 | 54. | 3 |
| 5. | 1 | 30. | 2 | 55. | 2 |
| 6. | 3 | 31. | 4 | 56. | 2 |
| 7. | 4 | 32. | 2 | 57. | 2 |
| 8. | 4 | 33. | 3 | 58. | 3 |
| 9. | 3 | 34. | 2 | 59. | 3 |
| 10. | 2 | 35. | 1 | 60. | 3 |
| 11. | 3 | 36. | 2 | 61. | 3 |
| 12. | 1 | 37. | 2 | 62. | 2 |
| 13. | 4 | 38. | 3 | 63. | 1 |
| 14. | 2 | 39. | 1 | 64. | 3 |
| 15. | 2 | 40. | 4 | 65. | 2 |
| 16. | 1 | 41. | 4 | 66. | 3 |
| 17. | 2 | 42. | 1 | 67. | 2 |
| 18. | 3 | 43. | 2 | 68. | 4 |
| 19. | 1 | 44. | 4 | 69. | 1 |
| 20. | 4 | 45. | 1 | 70. | 1 |
| 21. | 2 | 46. | 1 | 71. | 1 |
| 22. | 3 | 47. | 4 | 72. | 3 |
| 23. | 3 | 48. | 1 | 73. | 1 |
| 24. | 4 | 49. | 4 | 74. | 4 |
| 25. | 2 | 50. | 2 | 75. | 4 |

Note: Career Endeavour Academy is not responsible for any inadvertent error that may have crept in the results being published.

$$\left\{\frac{\partial Q}{\partial \psi} = -\frac{Q}{\psi}\right\}$$

$$\left\{\frac{\partial P}{\partial p} = \frac{\partial F}{\partial \psi}\right\}$$

$$\frac{\partial F}{\partial \varphi}$$

# PH: PHYSICS

*Duration*: Three Hours *Maximum Marks*:100

Please read the following instructions carefully:

## General Instructions:

1. Total duration of examination is 180 minutes (3 hours).
2. The clock will be set at the server. The countdown timer in the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You will not be required to end or submit your examination.
3. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

| | |
|---|---|
| 1 | You have not visited the question yet. |
| 3 | You have not answered the question. |
| 5 | You have answered the question. |
| 7 | You have NOT answered the question, but have marked the question for review. |
| 9 | You have answered the question, but marked it for review. |

The Marked for Review status for a question simply indicates that you would like to look at that question again. ***If a question is answered and Marked for Review, your answer for that question will be considered in the evaluation.***

## Navigating to a Question

4. To answer a question, do the following:
   a. Click on the question number in the Question Palette to go to that question directly.
   b. Select an answer for a multiple choice type question. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
   c. Click on **Save and Next** to save your answer for the current question and then go to the next question.
   d. Click on **Mark for Review and Next** to save your answer for the current question, mark it for review, and then go to the next question.
   **e. Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on its question number.**
5. You can view all the questions by clicking on the **Question Paper** button. Note that the options for multiple choice type questions will not be shown.

## Answering a Question

6. Procedure for answering a multiple choice type question:
   a. To select your answer, click on the button of one of the options
   b. To deselect your chosen answer, click on the button of the chosen option again or click on the **Clear Response** button
   c. To change your chosen answer, click on the button of another option
   d. To save your answer, you MUST click on the **Save and Next** button
   e. To mark the question for review, click on the **Mark for Review and Next** button. ***If an answer is selected for a question that is Marked for Review, that answer will be considered in the evaluation.***

7. Procedure for answering a numerical answer type question:
   a. To enter a number as your answer, use the virtual numerical keypad
   b. A fraction (eg.,-0.3 or -.3) can be entered as an answer with or without '0' before the decimal point
   c. To clear your answer, click on the **Clear Response** button
   d. To save your answer, you MUST click on the **Save and Next** button
   e. To mark the question for review, click on the **Mark for Review and Next** button. ***If an answer is entered for a question that is Marked for Review, that answer will be considered in the evaluation.***

8. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.

9. Note that ONLY Questions for which answers are saved or marked for review after answering will be considered for evaluation.

## Paper specific instructions:

1. There are a total of 65 questions carrying 100 marks.Questions are of multiple choice type or numerical answer type. A multiple choice type question will have four choices for the answer with only **one** correct choice. For numerical answer type questions, the answer is a number and no choices will be given. **A number as the answer should be entered** using the virtual keyboard on the monitor.
2. Questions Q.1 – Q.25 carry 1mark each. Questions Q.26 – Q.55 carry 2marks each. The 2marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is not attempted, then the answer to the second question in the pair will not be evaluated.
3. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1mark each, and questions Q.61 – Q.65 carry 2marks each.
4. Questions not attempted will result in zero mark. Wrong answers for multiple choice type questions will result in **NEGATIVE** marks. For all 1 mark questions, ⅓ mark will be deducted for each wrong answer. For all 2 marks questions, ⅔ mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question. There is no negative marking for questions of numerical answer type.
5. Calculator is allowed. Charts, graph sheets or tables are **NOT** allowed in the examination hall.
6. Do the rough work in the Scribble Pad provided.

## Q. 1 – Q. 25 carry one mark each.

Q.1 $f(x)$ is a symmetric periodic function of $x$ i.e. $f(x) = f(-x)$. Then, in general, the Fourier series of the function $f(x)$ will be of the form

(A) $f(x) = \sum_{n=1}^{\infty}(a_n \cos(nkx) + b_n \sin(nkx))$
(B) $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nkx))$
(C) $f(x) = \sum_{n=1}^{\infty}(b_n \sin(nkx))$
(D) $f(x) = a_0 + \sum_{n=1}^{\infty}(b_n \sin(nkx))$

Q.2 In the most general case, which one of the following quantities is NOT a second order tensor?

(A) Stress (B) Strain
(C) Moment of inertia (D) Pressure

Q.3 An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is

(A) $c$ (B) $-c$ (C) $0.15c$ (D) $-0.15c$

Q.4 If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature $T$ would be ( $k$ is the Boltzmann constant and $T$ is nonzero)

(A) Zero (B) Infinite (C) $\frac{3}{2} kT$ (D) $kT$

Q.5 Across a first order phase transition, the free energy is

(A) proportional to the temperature
(B) a discontinuous function of the temperature
(C) a continuous function of the temperature but its first derivative is discontinuous
(D) such that the first derivative with respect to temperature is continuous

Q.6 Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same

(A) pressure and temperature
(B) volume and temperature
(C) pressure and volume
(D) volume and energy

Q.7 The entropy function of a system is given by $S(E) = a\,E(E_0 - E)$ where $a$ and $E_0$ are positive constants. The temperature of the system is

(A) negative for some energies
(B) increases monotonically with energy
(C) decreases monotonically with energy
(D) Zero

Q.8 Consider a linear collection of $N$ independent spin 1/2 particles, each at a fixed location. The entropy of this system is ($k$ is the Boltzmann constant)

(A) Zero (B) $Nk$ (C) $\frac{1}{2}Nk$ (D) $Nk\,ln(2)$

Q.9 The decay process $n \rightarrow p^+ + e^- + \bar{\nu}_e$ violates

(A) baryon number (B) lepton number (C) isospin (D) strangeness

Q.10 The isospin (I) and baryon number (B) of the up quark is

(A) I=1, B=1 (B) I=1, B=1/3 (C) I=1/2, B=1 (D) I=1/2, B=1/3

Q.11 Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range $r_0$. Given that,

$$\cot(kr_0 + \delta) \approx -\frac{\gamma}{k}$$

where δ is the phase shift, $k$ the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

(A) $\delta \approx -\frac{k}{\gamma} - kr_0$ (B) $\delta \approx -\frac{\gamma}{k} - kr_0$ (C) $\delta \approx \frac{\pi}{2} - kr_0$ (D) $\delta \approx -\frac{\pi}{2} - kr_0$

Q.12 In the β decay process, the transition $2^+ \rightarrow 3^+$, is

(A) allowed both by Fermi and Gamow-Teller selection rule
(B) allowed by Fermi and but not by Gamow-Teller selection rule
(C) not allowed by Fermi but allowed by Gamow-Teller selection rule
(D) not allowed both by Fermi and Gamow-Teller selection rule

Q.13 At a surface current, which one of the magnetostatic boundary condition is NOT CORRECT?

(A) Normal component of the magnetic field is continuous.
(B) Normal component of the magnetic vector potential is continuous.
(C) Tangential component of the magnetic vector potential is continuous.
(D) Tangential component of the magnetic vector potential is not continuous.

Q.14 Interference fringes are seen at an observation plane $z = 0$, by the superposition of two plane waves $A_1 exp[i(\vec{k}_1.\vec{r} - \omega t)]$ and $A_2 exp[i(\vec{k}_2.\vec{r} - \omega t)]$, where $A_1$ and $A_2$ are real amplitudes. The condition for interference maximum is

(A) $(\vec{k}_1 - \vec{k}_2).\vec{r} = (2m + 1)\pi$
(B) $(\vec{k}_1 - \vec{k}_2).\vec{r} = 2m\pi$
(C) $(\vec{k}_1 + \vec{k}_2).\vec{r} = (2m + 1)\pi$
(D) $(\vec{k}_1 + \vec{k}_2).\vec{r} = 2m\pi$

Q.15 For a scalar function $\varphi$ satisfying the Laplace equation, $\nabla\varphi$ has

(A) zero curl and non-zero divergence
(B) non-zero curl and zero divergence
(C) zero curl and zero divergence
(D) non-zero curl and non-zero divergence

Q.16 A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is CORRECT ?

(A) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.
(B) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.
(C) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.
(D) There will be no reflected light and the transmitted light is circularly polarized.

Q.17 Which one of the following commutation relations is NOT CORRECT ? Here, symbols have their usual meanings.

(A) $[L^2, L_z] = 0$
(B) $[L_x, L_y] = i\hbar L_z$
(C) $[L_z, L_+] = \hbar L_+$
(D) $[L_z, L_-] = \hbar L_-$

Q.18 The Lagrangian of a system with one degree of freedom $q$ is given by $L = \alpha\dot{q}^2 + \beta q^2$, where $\alpha$ and $\beta$ are non-zero constants. If $p_q$ denotes the canonical momentum conjugate to $q$ then which one of the following statements is CORRECT?

(A) $p_q = 2\beta q$ and it is a conserved quantity.
(B) $p_q = 2\beta q$ and it is not a conserved quantity.
(C) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity.
(D) $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

Q.19 What should be the clock frequency of a 6-bit A/D converter so that its maximum conversion time is $32\,\mu s$?

(A) 1 MHz (B) 2 MHz (C) 0.5 MHz (D) 4 MHz

Q.20 A phosphorous doped silicon semiconductor ( doping density: $10^{17}/\text{cm}^3$) is heated from 100°C to 200°C. Which one of the following statements is CORRECT ?

(A) Position of Fermi level moves towards conduction band
(B) Position of dopant level moves towards conduction band
(C) Position of Fermi level moves towards middle of energy gap
(D) Position of dopant level moves towards middle of energy gap

Q.21 Considering the BCS theory of superconductors, which one of the following statements is NOT CORRECT?
($h$ is the Planck's constant and $e$ is the electronic charge)

(A) Presence of energy gap at temperatures below the critical temperature
(B) Different critical temperatures for isotopes
(C) Quantization of magnetic flux in superconducting ring in the unit of $\left(\frac{h}{e}\right)$
(D) Presence of Meissner effect

Q.22 Group I contains elementary excitations in solids. Group II gives the associated fields with these excitations. MATCH the excitations with their associated field and select your answer as per codes given below.

| Group I | Group II |
|---|---|
| (P) phonon | (i) photon + lattice vibration |
| (Q) plasmon | (ii) electron + elastic deformation |
| (R) polaron | (iii) collective electron oscillations |
| (S) polariton | (iv) elastic wave |

Codes

(A) (P-iv), (Q-iii), (R-i), (S-ii)
(B) (P-iv), (Q-iii), (R-ii), (S-i)
(C) (P-i), (Q-iii), (R-ii), (S-iv)
(D) (P-iii), (Q-iv), (R-ii), (S-i)

Q.23 The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is ____________.

Q.24 A voltage regulator has ripple rejection of −50dB. If input ripple is 1 mV, what is the output ripple voltage in μV? The answer should be up to two decimal places. __________

Q.25 The number of spectral lines allowed in the spectrum for the $3\ {}^2D \rightarrow 3\ {}^2P$ transition in sodium is ____________.

## Q. 26 to Q. 55 carry two marks each.

Q.26 Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(s)$ is NOT CORRECT?

(A) $F(t) = \delta(t)$, $f(s) = 1$, (Singularity at +0)
(B) $F(t) = 1$, $f(s) = \frac{1}{s}$, $(s > 0)$
(C) $F(t) = \sin kt$, $f(s) = \frac{s}{s^2+k^2}$, $(s > 0)$
(D) $F(t) = te^{kt}$, $f(s) = \frac{1}{(s-k)^2}$, $(s > k, s > 0)$

Q.27 If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\nabla(\vec{A}\cdot\vec{B}\times\vec{r})$ is

(A) $\vec{A}\cdot\vec{B}$ (B) $\vec{A}\times\vec{B}$ (C) $\vec{r}$ (D) Zero

Q.28 $\Gamma\left(n+\frac{1}{2}\right)$ is equal to [Given $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma(1/2) = \sqrt{\pi}$ ]

(A) $\frac{n!}{2^n}\sqrt{\pi}$ (B) $\frac{2n!}{n!\,2^n}\sqrt{\pi}$ (C) $\frac{2n!}{n!\,2^{2n}}\sqrt{\pi}$ (D) $\frac{n!}{2^{2n}}\sqrt{\pi}$

Q.29 The relativistic form of Newton's second law of motion is

(A) $F = \frac{mc}{\sqrt{c^2-v^2}}\frac{dv}{dt}$
(B) $F = \frac{m\sqrt{c^2-v^2}}{c}\frac{dv}{dt}$
(C) $F = \frac{mc^2}{c^2-v^2}\frac{dv}{dt}$
(D) $F = m\frac{c^2-v^2}{c^2}\frac{dv}{dt}$

Q.30 Consider a gas of atoms obeying Maxwell-Boltzmann statistics. The average value of $e^{i\,\vec{a}\cdot\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and $a$ is its magnitude, $m$ is the mass of each atom, $T$ is temperature and $k$ is Boltzmann's constant) is,

(A) One (B) Zero (C) $e^{-\frac{1}{2}a^2\,m\,k\,T}$ (D) $e^{-\frac{3}{2}a^2\,m\,k\,T}$

Q.31 The electromagnetic form factor $F(q^2)$ of a nucleus is given by,

$$F(q^2) = exp\left[-\frac{q^2}{2Q^2}\right]$$

where $Q$ is a constant. Given that

$$F(q^2) = \frac{4\pi}{q}\int_0^\infty rdr\,\rho(r)\,\sin qr$$

$$\int d^3r\,\rho(r) = 1$$

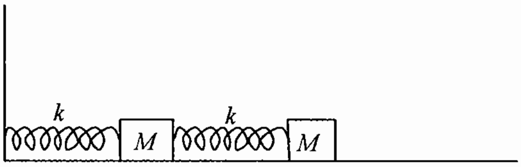
where ρ(r) is the charge density, the root mean square radius of the nucleus is given by,

(A) $1/Q$ (B) $\sqrt{2}/Q$ (C) $\sqrt{3}/Q$ (D) $\sqrt{6}/Q$

Q.32 A uniform circular disk of radius $R$ and mass $M$ is rotating with angular speed $\omega$ about an axis, passing through its center and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is,

(A) $\frac{\sqrt{3}}{4}\,\omega MR^2$ (B) $\frac{\sqrt{3}}{8}\,\omega MR^2$ (C) $\frac{\sqrt{7}}{8}\,\omega MR^2$ (D) $\frac{\sqrt{7}}{4}\,\omega MR^2$

Q.33 Consider two small blocks, each of mass $M$, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is $k$. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,



(A) $\sqrt{\frac{3+\sqrt{2}}{2}}\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{3+\sqrt{3}}{2}}\sqrt{\frac{k}{M}}$ (C) $\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{M}}$ (D) $\sqrt{\frac{3+\sqrt{6}}{2}}\sqrt{\frac{k}{M}}$

Q.34 A charge distribution has the charge density given by $\rho = Q\{\delta(x-x_0) - \delta(x+x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

(A) $\frac{2Q\hat{x}}{9\pi\epsilon_0 x_0^2}$ (B) $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^3}$ (C) $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^2}$ (D) $\frac{Q\hat{x}}{16\pi\epsilon_0 x_0^2}$

Q.35 A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If $\hat{k}_i$, $\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expressions is correct?

(A) $(\hat{k}_i - \hat{k}_r) \times \hat{n} \neq 0$ (B) $(\hat{k}_i - \hat{k}_r) \cdot \hat{n} = 0$
(C) $(\hat{k}_i \times \hat{n}) \cdot \hat{k}_r = 0$ (D) $(\hat{k}_i \times \hat{n}) \cdot \hat{k}_r \neq 0$

Q.36 In a normal Zeeman effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition $5\,^1D_2 \rightarrow 5\,^1P_1$ of cadmium atoms is to be observed. The spectrometer has a resolution of 0.01 nm. The minimum magnetic field needed to observe this is $(m_e = 9.1 \times 10^{-31}\,kg, e = 1.6 \times 10^{-19}\,C, c = 3 \times 10^8\,m/s)$

(A) 0.26 $T$ (B) 0.52 $T$ (C) 2.6 $T$ (D) 5.2 $T$

Q.37 The spacing between vibrational energy levels in CO molecule is found to be $8.44\text{x}10^{-2}$ eV. Given that the reduced mass of CO is $1.14\text{x}10^{-26}$ kg, Planck's constant is $6.626 \times 10^{-34}$ Js and 1eV = $1.6\text{x}10^{-19}$ J. The force constant of the bond in CO molecule is

(A) 1.87 N/m (B) 18.7 N/m (C) 187 N/m (D) 1870 N/m

Q.38 A lattice has the following primitive vectors (in Å): $\vec{a} = 2(\hat{j}+\hat{k})$, $\vec{b} = 2(\hat{k}+\hat{i})$, $\vec{c} = 2(\hat{i}+\hat{j})$. The reciprocal lattice corresponding to the above lattice is

(A) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)$ Å$^{-1}$
(B) BCC lattice with cube edge of $(2\pi)$ Å$^{-1}$
(C) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)$ Å$^{-1}$
(D) FCC lattice with cube edge of $(2\pi)$ Å$^{-1}$

Q.39 The total energy of an ionic solid is given by an expression $E = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where α is Madelung constant, $r$ is the distance between the nearest neighbours in the crystal and $B$ is a constant. If $r_0$ is the equilibrium separation between the nearest neighbours then the value of $B$ is

(A) $\frac{\alpha e^2 r_0^8}{36\pi\epsilon_0}$ (B) $\frac{\alpha e^2 r_0^8}{4\pi\epsilon_0}$ (C) $\frac{2\alpha e^2 r_0^{10}}{9\pi\epsilon_0}$ (D) $\frac{\alpha e^2 r_0^{10}}{36\pi\epsilon_0}$

Q.40 A proton is confined to a cubic box, whose sides have length $10^{-12}\ m$. What is the minimum kinetic energy of the proton? The mass of proton is $1.67 \times 10^{-27}\ kg$ and Planck's constant is $6.63 \times 10^{-34} Js$.

(A) $1.1 \times 10^{-17}\ J$ (B) $3.3 \times 10^{-17}\ J$ (C) $9.9 \times 10^{-17}\ J$ (D) $6.6 \times 10^{-17}\ J$

Q.41 For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z = 1$ is (your answer should be an integer) __________.

Q.42 The degenerate eigenvalue of the matrix

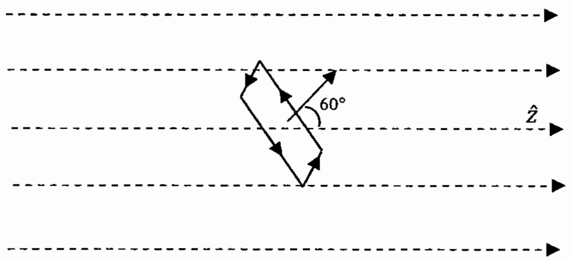
$$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$$ is (your answer should be an integer) ______________

Q.43 Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ in the pion rest frame.

$m_\pi = 139.6\ MeV/c^2,\ m_\mu = 105.7\ MeV/c^2,\ m_\nu \approx 0$

The energy (in MeV) of the emitted neutrino, to the nearest integer is ___________

Q.44 In a constant magnetic field of 0.6 Tesla along the z direction, find the value of the path integral $\oint \vec{A} \cdot \vec{dl}$ in the units of (Tesla m$^2$) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60° to the z-axis, as shown in the figure. The answer should be up to two decimal places. ___________



Q.45 A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of $\sigma_z$ then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$ ? Here, symbols have their usual meanings. ___________

Q.46 Consider the wave function $Ae^{ikr}(r_0/r)$, where $A$ is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k/m)$, is

_____________.

Q.47 An *n*-channel junction field effect transistor has 5*m*A source to drain current at shorted gate ($I_{DSS}$) and 5V pinch off voltage ($V_P$). Calculate the drain current in *m*A for a gate-source voltage ($V_{GS}$) of −2.5V. The answer should be up to two decimal places. ________________

## Common Data Questions

**Common Data for Questions 48 and 49:** There are four energy levels $E$, $2E$, $3E$ and $4E$ (where $E>0$). The canonical partition function of two particles is , if these particles are

Q.48 two identical fermions

(A) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$
(B) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$
(C) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$
(D) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

Q.49 two distinguishable particles

(A) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$
(B) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$
(C) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$
(D) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

**Common Data for Questions 50 and 51:** To the given unperturbed Hamiltonian

$$\begin{bmatrix}5 & 2 & 0\\ 2 & 5 & 0\\ 0 & 0 & 2\end{bmatrix},$$

we add a small perturbation given by

$$\varepsilon\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ 1 & -1 & 1\end{bmatrix},$$

where $\varepsilon$ is a small quantity.

Q.50 The ground state eigenvector of the unperturbed Hamiltonian is

(A) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (B) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$ (C) (0,0,1) (D) (1,0,0)

Q.51 A pair of eigenvalues of the perturbed Hamiltonian, using first order perturbation theory, is

(A) 3+2ε, 7+2ε (B) 3+2ε, 2+ε (C) 3, 7+2ε (D) 3, 2+2ε

## Linked Answer Questions

**Statement for Linked Answer Questions 52 and 53:** In the Schmidt model of nuclear magnetic moments, we have,

$$\vec{\mu} = \frac{e\hbar}{2Mc}\left(g_l \vec{l} + g_s \vec{S}\right)$$

where the symbols have their usual meaning

Q.52 For the case $J = l + 1/2$, where $J$ is the total angular momentum, the expectation value of $\vec{S} \cdot \vec{J}$ in the nuclear ground state is equal to,

(A) $(J-1)/2$ (B) $(J+1)/2$ (C) $J/2$ (D) $-J/2$

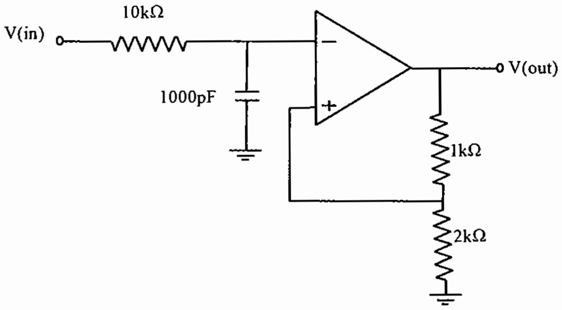
Q.53 For the $O^{17}$ nucleus (A=17, Z=8), the effective magnetic moment is given by,

$$\vec{\mu}_{eff} = \frac{e\hbar}{2Mc} g\vec{J},$$

where $g$ is equal to, ($g_s = 5.59$ for proton and $-3.83$ for neutron)

(A) 1.12 (B) −0.77 (C) −1.28 (D) 1.28

**Statement for Linked Answer Questions 54 and 55:** Consider the following circuit



Q.54 For this circuit the frequency above which the gain will decrease by 20 *d*B per decade is

(A) 15.9 kHz (B) 1.2 kHz (C) 5.6 kHz (D) 22.5 kHz

Q.55 At 1.2kHz the closed loop gain is

(A) 1 (B) 1.5 (C) 3 (D) 0.5

## General Aptitude (GA) Questions

## Q. 56 – Q. 60 carry one mark each.

Q.56 A number is as much greater than 75 as it is smaller than 117. The number is:

(A) 91 (B) 93 (C) 89 (D) 96

Q.57 The professor ordered to the students to go out of the class.
I II III IV
Which of the above underlined parts of the sentence is grammatically incorrect?

(A) I (B) II (C) III (D) IV

Q.58 Which of the following options is the closest in meaning to the word given below:

Primeval

(A) Modern (B) Historic
(C) Primitive (D) Antique

Q.59 Friendship, no matter how __________ it is, has its limitations.

(A) cordial

(B) intimate

(C) secret

(D) pleasant

Q.60 Select the pair that best expresses a relationship similar to that expressed in the pair:
**Medicine: Health**

(A) Science: Experiment (B) Wealth: Peace
(C) Education: Knowledge (D) Money: Happiness

## Q. 61 to Q. 65 carry two marks each.

Q.61 X and Y are two positive real numbers such that $2X + Y \leq 6$ and $X + 2Y \leq 8$. For which of the following values of $(X, Y)$ the function $f(X, Y) = 3X + 6Y$ will give maximum value?

(A) (4/3, 10/3)

(B) (8/3, 20/3)

(C) (8/3, 10/3)

(D) (4/3, 20/3)

Q.62 If $|4X - 7| = 5$ then the values of $2\,|X| - |-X|$ is:

(A) 2, 1/3 (B) 1/2, 3 (C) 3/2, 9 (D) 2/3, 9

Q.63 Following table provides figures (in rupees) on annual expenditure of a firm for two years - 2010 and 2011.

| **Category** | **2010** | **2011** |
|---|---|---|
| **Raw material** | 5200 | 6240 |
| **Power & fuel** | 7000 | 9450 |
| **Salary & wages** | 9000 | 12600 |
| **Plant & machinery** | 20000 | 25000 |
| **Advertising** | 15000 | 19500 |
| **Research & Development** | 22000 | 26400 |

In 2011, which of the following two categories have registered increase by same percentage?

(A) Raw material and Salary & wages

(B) Salary & wages and Advertising

(C) Power & fuel and Advertising

(D) Raw material and Research & Development

Q.64 A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

(A) 5 (B) 10 (C) 15 (D) 30

Q.65 Abhishek is elder to Savar.
Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

(A) Abhishek is elder to Anshul

(B) Anshul is elder to Abhishek

(C) Abhishek and Anshul are of the same age

(D) No conclusion follows

## END OF THE QUESTION PAPER

# GATE 2013 : Answer keys for PH - Physics

| Paper | Q.No | Key(s)/Value(s) |
|---|---|---|
| PH | 1 | B |
| PH | 2 | D |
| PH | 3 | B |
| PH | 4 | B |
| PH | 5 | B |
| PH | 6 | A |
| PH | 7 | A |
| PH | 8 | D |
| PH | 9 | C |
| PH | 10 | D |
| PH | 11 | A |
| PH | 12 | C |
| PH | 13 | D |
| PH | 14 | B |
| PH | 15 | C |
| PH | 16 | C |
| PH | 17 | D |
| PH | 18 | D |
| PH | 19 | B |
| PH | 20 | C |
| PH | 21 | C |
| PH | 22 | B |
| PH | 23 | 70 |
| PH | 24 | 3.16 |
| PH | 25 | 3 |
| PH | 26 | C |
| PH | 27 | B |
| PH | 28 | C |
| PH | 29 | C |
| PH | 30 | C |
| PH | 31 | C |
| PH | 32 | C |
| PH | 33 | C |
| PH | 34 | A |
| PH | 35 | C |

| Paper | Q.No | Key(s)/Value(s) |
|---|---|---|
| PH | 36 | B |
| PH | 37 | C |
| PH | 38 | A |
| PH | 39 | A |
| PH | 40 | C |
| PH | 41 | 3 |
| PH | 42 | 5 |
| PH | 43 | 30 |
| PH | 44 | 0.15 |
| PH | 45 | 7.6 |
| PH | 46 | 0.25 |
| PH | 47 | 1.25 |
| PH | 48 | B |
| PH | 49 | C |
| PH | 50 | C |
| PH | 51 | C |
| PH | 52 | B |
| PH | 53 | B |
| PH | 54 | A |
| PH | 55 | B |
| PH | 56 | D |
| PH | 57 | B |
| PH | 58 | C |
| PH | 59 | B |
| PH | 60 | C |
| PH | 61 | A |
| PH | 62 | B |
| PH | 63 | D |
| PH | 64 | A |
| PH | 65 | D |

# TEST SERIES UGC-CSIR-NET/JRF Dec. 2014

**BOOKLET SERIES** B

**Paper Code** 05

**Test Type:** TEST SERIES

## PHYSICAL SCIENCES

**Duration: 02:00 Hours**

**Date: 22-11-2014**

**Maximum Marks: 140**

**Read the following instructions carefully:**

- Single Paper Test is divided into **TWO** Parts.

1. Attempt all the questions.
2. **Part - A:** This part shall carry 20 questions. Each question shall be of **2 marks.**
3. **Part - B:** This part shall contain 50 questions. Each question shall be of **2 marks.**
4. For rough work, blank sheet is attached at the end of test booklet.
5. There will be negative marking @25% for each wrong answer.
6. Darken the appropriate bubbles with HB pencil/Ball Pen to write your answer.
7. The candidates shall be allowed to carry the Question Paper Booklet after completion of the exam.

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## PART-A (GENERAL APTITUDE)

1. Walking at $\frac{3}{4}$ of its usual speed a train is 8 min late. Find its usual point of time to cover the journey.
(a) 24 min (b) 30 min (c) 60 min (d) 48 min

2. A man can walk up a moving 'up' escalator in 30s. The same man can walk down this moving 'up' escalator in 90s. Assume that his walking speed is same upwards and downwards. How much time will he take to walk up escalator. When it is not moving.
(a) 30 sec (b) 45 sec (c) 60 sec (d) 90 sec

3. If $\log_5 y = x$ and $\log_2 z = x$, then value of $20^x$.
(a) $z^2y$ (b) $y^2z$ (c) $yz$ (d) $\sqrt{yz}$

4. If a, b, c are three consecutive integers, then $\log(ac+1)$ is equal to
(a) log 2b (b) $x = 2$ (c) $2\log b$ (d) none of these

5. The sum of n terms of an AP is $3n^2$. Find the n<sup></sup>th term
(a) 6n – 3 (b) 3n–3 (c) 3n + 3 (d) 6n + 3

6. The least value of n such that $1+3+3^2+......+3^n > 2007$ is
(a) 7 (b) 8 (c) 9 (d) 10

7. If 137 + 276 = 435, how much is 731 + 672
(a) 534 (b) 1403 (c) 1623 (d) 1513

8. The next term of series is 4, 3, 5, 9, 12, 17, ....
(a) 32 (b) 30 (c) 24 (d) 26

81 × 81 = 6448
9 × 9 × 9 × 9
3 × 3 × 3 × 3 × 3 × 3
137
276
719 × 3
1689 × 3
19387

9. Consider the set of numbers N = 1, 2, 3, 4, 5, 6, 7, 8. Take every distinct two element subset of N and write down the number that is smaller. For example, if you take the subset (2, 5), you will write down 2. What is sum of all the numbers that you write down
(a) 28 (b) 84 (c) 168 (d) none of these

10. A train without stoppages travels on an average 50 km/h and with stoppages, 40 km/h. For how many minute, on an average, does the train stop in an hour
(a) 6 (b) 12 (c) 20 (d) none of these

11. A ship is some distance away from the shore when it develops a leak. This leak admits water at the rate of 3000 L/sec while the ships two pumps can pump out water at the rate of 1200L/sec and 1800 L/sec, but the mechanics is such that only one of the pumps can be operated continuously while the other can be operated for half the same time. The ship's manufacturer has designed the ship to survive 36 k ton of water while its top speed is 12 km/h. If the ship travelling at top speed just made it to the shore before drawning, what was the maximum distance it could have been from teh shore when the leak developed.
(a) 400/3 km (b) 200 km (c) 100 km (d) the ship will not drown

12. In one hour the minute hand cover how much more angle than the hour hand
(a) 360° (b) 30° (c) 330° (d) none of these

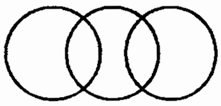
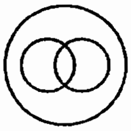
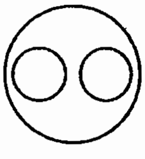
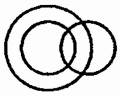
13. Under constant temperature, the pressure of a definite mass of a gas varies inversely as the volume. If the volume is reduced by 50% the pressure will be
(a) increase by 100%
(b) increase by 25%
(c) decrease by 25%
(d) decrease by 50%

14. Several identical cubes are arranged in a single layer. If area of layer is $A$ and volume $V$. Then number of cubes in arrangement
(a) $\frac{V}{A^2}$ (b) $\frac{A^3}{V}$ (c) $\frac{A^3}{V^2}$ (d) none of these

15. Fill in the blank : 0, 2, 6, 15, 23, 33, ......
(a) 42 (b) 45 (c) 60 (d) 48

16. **Statement :** (i) Some boys are clever, (ii) Mohan is a boy
**Conclusion :** (I) Mohan is clever, (II) Mohan is not clever
(a) Only I (b) Only II (c) Both I and II (d) Neither I nor II

17. The correct representation of Sea, Fish, Whale in Venn diagram

(a) 
(b) 
(c) 
(d) 

18. Find the day of 10th May, 1857
(a) Monday (b) Friday (c) Saturday (d) None of these

19. If area of any square $A_S$ and any rectangle $A_R$ is same. Then
(a) $P_S > P_R$ (b) $P_S = P_R$ (c) $P_S < P_R$ (d) none of these

20. Find the maximum volume of a cone which can be inscribed from a cube whose side is 7 cm
(a) 539/6 cm³ (b) 512 cm³ (c) 458 cm³ (d) none of these

## PART-B (QUANTUM, CLASSICAL MECHANICS, SOLID STATE)

21. The dynamics of a particle of mass 'm' moving under a one-dimensional potential $V(x)$ is governed by the Hamiltonian $H_0 = \frac{p^2}{2m} + V(x)$, where $p = -i\hbar\frac{d}{dx}$ is the momentum operator and $E_n^{(0)}$ $(n = 1,2,3.....)$ be the eigenvalues of $H_0$. Now, consider the following Hamiltonian:

$$H = H_0 + \frac{\lambda p}{m} \text{ (where } \lambda \text{ is real constant parameter)}$$

The eigenvalue of the hamiltonian $H$ is

(a) $E_n^{(0)} + \frac{\lambda}{m}$ (b) $E_n^{(0)} + \frac{\lambda^2}{2m}$ (c) $E_n^{(0)} - \frac{\lambda^2}{2m}$ (d) $E_n^{(0)} - \frac{\lambda}{m}$

22. Suppose a particle of mass 'm' is moving under a 1-D harmonic oscillator potential

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

$\frac{dV}{dt} = 0$

and at t = 0, the wave function of the particle is described as

$$\psi(x,0) = A\sum_{n=0}^{\infty} \frac{1}{2^{n/2}}\psi_n(x)$$

$E_0 = \frac{1}{2}\hbar\omega$

where $\psi_n(x)$ are the orthonormalized eigenstates of the energy with eigenvalues $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$. The expectation value of energy of the particle at t = 0, is

(a) $\sum_{n=0}^{\infty}\frac{1}{2^n}\left(n+\frac{1}{2}\right)\hbar\omega$ (b) $\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}\left(n+\frac{1}{2}\right)\hbar\omega$ (c) $\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}\left(n+\frac{3}{2}\right)\hbar\omega$ (d) $\sum_{n=0}^{\infty}\frac{1}{2^n}\left(n+\frac{3}{2}\right)\hbar\omega$

23. A free particle of mass 'm' moves in one dimension and at t = 0, the normalized wave function of the particle is given as:

$$\psi(x,0)=\left(\frac{1}{\sqrt{2\pi}\sigma_x}\right)^{1/2}\exp\left(-\frac{x^2}{4\sigma_x^2}\right)$$

$\sqrt{2h^2}$ $\begin{pmatrix} 0 & \phi \\ 1 & 0 \end{pmatrix}$

$\frac{1}{2}$ (x $\frac{1}{2}$

where $\sigma_x$ is the uncetanity in the position of the particle. The momentum spread associated with the particle will be

(a) $\frac{\hbar\sigma_x}{2}$ (b) $\frac{\hbar}{2\sigma_x}$ (c) $\frac{\hbar}{2\sigma_x^2}$ (d) $\frac{\hbar\sigma_x^2}{2}$

24. Consider a stream of particles of mass 'm' each moving in the positive x direction with kinetic energy $E$, toward a potential jump located at x = 0. The potential is zero for $x \le 0$ and $3E/4$ for $x > 0$. The fraction of the particles that are reflected at x = 0, is

(a) 1/9 (b) 1/3 (c) 1/4 (d) 2/3

25. Two identical non-interacting particles, each of mass 'm' and spin 1/2, are placed in an infinite potential well of width 'L'. Suppose the lowest possible value of energy for these system of two particles is $E_0$. If instead, four such particles are introduced into a similar 1-D infinte potential well of width '2L', then the lowest possible value of their total energy will be

(a) $2E_0$ (b) $1.25E_0$ (c) $1.5E_0$ (d) $E_0$

26. Suppose an operator in quantum mechanics is defined as

$$e^A = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots\ldots\ldots\ldots$$

Then, the operator $e^{\frac{d}{dx}}$ will corresponds to which of the physical phenomena?

(a) translation operator along x-axis corresponding to 1 unit of translation.

(b) translation operator along x-axis corresponding to 10 unit of translation.

(c) rotation operator about x-axis corresponding to 45° of rotation.

(d) rotation operator about x-axis corresponding to 90° of rotation.

27. Suppose a particle of mass 'm' is moving under a one-dimensional potential V(x) has the following form of wavefunction:

$$\psi(x)=\frac{A}{x^2+a^2}(a>0)$$

$\frac{d^2}{dx^2} + \frac{\hbar^2}{2m}(V(x))\psi(x) = 0$

where A is the normalization constant. The energy of the particle in the given state is 0. The one dimensional potential V(x) will be of the following form:

(a) $\frac{\hbar^2}{m}\frac{2x^2-a^2}{\left(x^2+a^2\right)^2}$ (b) $\frac{\hbar^2}{m}\frac{3x^2-a^2}{\left(x^2+a^2\right)^2}$ (c) $\frac{\hbar^2}{m}\frac{3x^2-a^2}{\left(x^2+a^2\right)}$ (d) $\frac{\hbar^2}{m}\frac{x^2-2a^2}{\left(x^2+a^2\right)}$

$2x^2 + 4xa^2$

28. At time t = 0, the wave function for the hydrogen atom is

$$\psi(\vec{r},0)=\frac{1}{\sqrt{10}}\left[2\psi_{100}+\psi_{210}+\sqrt{2}\psi_{211}+\sqrt{3}\psi_{21-1}\right]$$

$= \frac{13.6}{0.45}$ $\frac{2}{10} + \frac{3}{10}$ $\frac{20+30}{100}$

0.5

where the subscripts dènotes the principal, orbital and magnetic quantum numbers respectively.The expectation value for the energy of the system, will be

(a) -2.1 eV (b) -5.4 eV (c) -3.6 eV (d) -7.5 eV

29. The system of two spin-half particles are described by the following Hamiltonian:

$$\hat{H} = A\left(S_{1z} + S_{2z}\right) + B\left(\vec{S}_1 . \vec{S}_2\right)$$

$2\hbar A + B(1)$

where A and B are constants and $\vec{S}_1$ & $\vec{S}_2$ are the spin angular momentum of the two particles. The sum of the energies corresponding to triplet states will be

(a) $A\hbar + \frac{3}{4}B\hbar^2$ (b) $2A\hbar + \frac{3}{4}B\hbar^2$ (c) $2A\hbar - \frac{3}{4}B\hbar^2$ (d) $\frac{3}{4}B\hbar^2$

30. A particle of mass 'm' and charge 'e' is in the ground state of a one dimensional harmonic oscillator potential of the following form:

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

$x \neq \frac{\hbar}{\sqrt{2m\omega}}$

Consider the system is suddenly placed under a constant elctric field of the following form: $\vec{E} = E_0\hat{x}$. As a result, the particle will

(a) make a transtion to any harmonic oscillator state with $x = -\frac{eE}{m\omega^2}$ as origin without emiiting a photon.

(b) make a transtion to any harmonic oscillator state with $x = -\frac{eE}{m\omega^2}$ as origin and absorb a photon.

(c) make a transtion to any harmonic oscillator state with $x = \frac{eE}{m\omega^2}$ as origin and absorb a photon.

(d) settle into the ground state of the harmonic oscillator state with $x = \frac{eE}{m\omega^2}$ as origin and absorb a photon.

31. Consider a quantum mechanical system with three linear operator $\hat{P}, \hat{Q}, \hat{R}$ which are related by,

$$\hat{P}\hat{Q} - \hat{R} = \hat{I}$$

where $\hat{I}$ is the unitary operator. If $\hat{P} = \frac{d}{dx}$ and $\hat{Q} = x$, then $\hat{R}$ must be

(a) 0 (b) $\frac{d}{dx}$ (c) $x\frac{d}{dx}$ (d) $-x\frac{d}{dx}$

32. A ball bounces off earth. You are asked to solve this quantum mechanically assuming the earth is a infinitely hard sphere. Which one of the following wave functions for the ball that can represent a physically admissible solution with k > 0?

(a) $\psi = A\frac{e^{-kx}}{x}$ (b) $\psi = Axe^{-kx^2}$ (c) $\psi = -Axe^{kx}$ (d) $\psi = Ae^{-kx^2}$

33. Consider the following Hamiltonian:

$$\hat{H} = A\vec{S}.\hat{n}$$

where A is constant, $\vec{S}$ is the spin angular momentum of a spin-1/2 particle and $\hat{n}$ is a unit vector parametrized in terms of two angles as $\left(n_x, n_y, n_z\right) = \left(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\right)$

The energy eigenvalues are

$\frac{1}{2}A\cdot\hat{n}$ [illegible]

(a) $\pm\frac{1}{2}A\hbar\cos\theta$ (b) $\pm\frac{1}{2}A\hbar\sin\theta$ (c) $\pm\frac{1}{2}A\hbar$ (d) $\pm\frac{3}{2}A\hbar$

34. A crystal has some negative ion vacacies, each containing one electron. Treat this electrons as they are moving freely inside a volume whose dimensions are of the order of lattice constant (1 A$^0$). The longest wavelength of the electromagnetic radiation that can be absorbed by these electrons, will be approximately

(a) 7 nm (b) 9 nm. (c) 11 nm. (d) 13 nm.

35. Let, k be the wave number of the incident plane wave in a scattering experiment. If the scattering is a mixture of s-wave and p-wave with phase shifts $\frac{\pi}{6}$ and $\frac{\pi}{3}$ respectively, then the total scattering cross-section will be

(a) $\frac{2\pi}{k^2}$ (b) $\frac{4\pi}{k^2}$ (c) $\frac{6\pi}{k^2}$ (d) $\frac{10\pi}{k^2}$

36. The rotational part of the hamiltonian of a diatomic molecule is given as following:

$$H = \frac{1}{2I}\left(L_x^2 + L_y^2\right) + \frac{1}{I}L_z^2$$

$\hbar^2 m^2$

where I is the moment of inertia of the diatomic molecule. The energy of the diatomic molecule will be

(a) $\frac{\hbar^2}{2I}\left[l(l+1)+2m^2\right]$ (b) $\frac{\hbar^2}{2I}\left[l(l+1)-2m^2\right]$

(c) $\frac{\hbar^2}{2I}\left[l(l+1)+m^2\right]$ (d) $\frac{\hbar^2}{2I}\left[l(l+1)-m^2\right]$

37. A particle of mass 'm' is confined to a move under the following 1-D potential:

$$V(x) = \begin{cases} 0 & \text{for } 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

At time t = 0, the normalized wave function of the particle is given as follows:

$$\psi(x, t=0) = \sqrt{\frac{8}{5a}}\left[1+\cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$$

The probability of finding the particle in the left half of the box potential will be

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{16}{15\pi}\cos\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$

(c) $\frac{1}{2}-\frac{16}{15\pi}\cos\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$ (d) $\frac{1}{2}+\frac{16}{8\pi}\cos\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$

38. A thin circular disc of mass M and radius R has moment of inertia $\frac{3}{4}MR^2$ about one of chord. Perpendicular distance of chord from centre of disc is

$\frac{3}{4} mR^2$

(a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{4}$ (d) $\frac{R}{2\sqrt{2}}$

39. A thin rod of length '$l$' when suspended from one end it undergoes small oscillation about a horizontal axis with time period T$_1$. When it is suspended from some other point its time period becomes T$_2$ such that T$_1$ = T$_2$. Distance of second point from centre of the rod is

(a) $\frac{\ell}{6}$ (b) $\frac{\ell}{3}$ (c) $\frac{2\ell}{3}$ (d) $\frac{\ell}{4}$

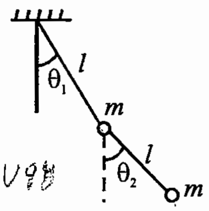
40. Equations of motion for small oscillation of double pendulum shown in figure are

(a) $2\ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0, \ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_2=0$

(b) $2\ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0, \ddot{\theta}_1+\ddot{\theta}_2+\frac{g}{\ell}\theta_2=0$

(c) $\ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0, \ddot{\theta}_1+\ddot{\theta}_2+\frac{4g}{\ell}\theta_2=0$

(d) $\ddot{\theta}_1+2\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0, \ddot{\theta}_1+\ddot{\theta}_2+\frac{4g}{3\ell}\theta_2=0$



$\frac{1}{2}m\dot{\theta}_1^2$ +V

41. A bead of mass $m$ slides on a smooth wire kept in $x$-$y$ plane. The equation of wire is described as $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ where '$y$' is in vertical direction. Lagrangian of the bead is

(a) $(ma^2\dot{\theta}^2-mga)(1-\cos\theta)$ (b) $(ma^2\dot{\theta}^2+mga)(1-\sin\theta)$

(c) $(ma^2\dot{\theta}^2-mga)(1-\sin\theta)$ (d) $ma^2\dot{\theta}^2(1-\sin\theta)-mga(1-\cos\theta)$

42. Lagrangian of a system is $L=\frac{\dot{x}^2}{2}-\frac{1}{2}\omega^2x^2-\alpha x^3+\beta x\dot{x}^2$ hamiltonian of the system is

(a) $\frac{p_x^2}{2(1+2\beta x)^2}+\frac{1}{2}\omega^2+\alpha x^3$ (b) $\frac{p_x^2}{2(1+2\beta x)}+\frac{1}{2}\omega^2x^2+\alpha x^3$

(c) $\frac{p_x^2}{2(1+2\beta x)}-\frac{1}{2}\omega^2x^2-\alpha x^3$ (d) $\frac{2p_x x-p_x^2}{2(1+2\beta x)}-\frac{1}{2}\omega^2x^2+\alpha x^3$

43. Lagrangian of a system is $L=\sum_k f_k(q_k)\dot{q}_k^2-\sum V_k(q_k)$ hamiltonian of the system is

(a) $\sum_k \frac{p_k^2}{2f_k(q_k)}+\sum_k V_k(q_k)$ (b) $\sum_k \frac{p_k^2}{4f_k^2(q_k)}+\sum_k V_k(q_k)$

(c) $\sum_k \frac{p_k^2}{4f_k(q_k)}+\sum_k V_k(q_k)$ (d) $\sum_k \frac{p_k^2}{2f_k^2(q_k)}+\sum_k V_k(q_k)$

44. If the transformation $P=\alpha(p^2+q^2)$, $Q=\beta\tan^{-1}\frac{q}{p}$ is canonical values of $\alpha$ and $\beta$ are respectively.

(a) 2, 1 (b) $\frac{1}{2},\frac{1}{2}$ (c) $\frac{1}{2},1$ (d) $\frac{1}{2},2$

45. A person standing at rear end of a train lights a torch and notices that light reaches ~~the~~ other the end after time $t$. If train is moving with speed v = 3c/5. What is time noticed by a person standing on the ground.

(a) $\frac{3t}{2}$ (b) $\frac{4t}{3}$ (c) $\frac{4t}{5}$ (d) $2t$

46. Generating function for a CT is $F_3=-Q\left[1+\log\left(\frac{p^2}{4Q}\right)\right]$ generating function $F_1(q,Q)$ is

(a) $Q-Q\log\left(\frac{Q}{q^2}\right)$ (b) $-Q-Q\log\left(\frac{Q}{q^2}\right)$ (c) $Q+Q\log\left(\frac{Q}{q^2}\right)$ (d) $-Q+Q\log\left(\frac{Q}{q^2}\right)$

47. A particle of mass $m$ is released at $x=\sqrt{\frac{a}{3b}}$. If the partile moves under potential $V(x)=ax-bx^3$. Maximum kinetic energy attained by the particle is

$\frac{a^2 a}{3b} - b^3\frac{a^4}{3b}$

$\frac{a^3}{3b} - \frac{b^2a^2}{3b}$

(a) $\sqrt{\frac{2a^3}{27b}}$ (b) $\sqrt{\frac{8a^3}{27b}}$ (c) $\sqrt{\frac{16a^3}{27b}}$ (d) $\sqrt{\frac{64a^3}{27b}}$

48. A particle of mass $m$ is moving on earth's surface at a point where latitude is $\lambda N$. If speed of particle is $\omega R$ and its direction is towards east. Net inertial forces (pseudo forces) on particle as seen from earth is

(a) $m\omega^2R(2+\cos\lambda)$ (b) $m\omega^2R(2-\sin\lambda)$

(c) $m\omega^2R(2-\cos\lambda)$ (d) $m\omega^2R(2+\sin\lambda)$

$v^2 = 6bx$

49. A sphere is moving on a plane surface degree of freedom of sphere is

(a) 2 (b) 3 (c) 5 (d) 6

50. A bead of mass $m$ is sliding on a wire whose equation is $y=\alpha x^2$, where $y$ is in vertically upward direction. Hamiltonian of the system is

(a) $\frac{p_x^2}{2m(1+2\alpha^2x^2)}+mg\alpha x^2$ (b) $\frac{p_x^2}{2m(1+4\alpha^2x^2)}+mg\alpha x^2$

(c) $\frac{p_x^2}{2m(1+2\alpha^2x^2)}-mg\alpha x^2$ (d) $\frac{p_x^2}{2m(1+4\alpha^2x^2)}-mg\alpha x^2$

51. A particle of mass m is projected with speed $\sqrt{\frac{GM}{2R}}$ from earth's surface due to which movs in elliptical path. Semi major axis of the ellipse is

(a) $\frac{R}{3}$ (b) $\frac{R}{2}$ (c) $2R$ (d) $\frac{2R}{3}$

52. An artificial satellite is to be launched in a circular orbit around the earth. If radius of circular orbit is $2R_e$, radius of earth $R_e$, acceleration due to gravity on earth's surface g and mass of satellite $m$, what is energy spent in the process.

(a) $\frac{mgR}{3}$ (b) $\frac{mgR}{2}$ (c) $\frac{3mgR}{4}$ (d) $\frac{3mgR}{2}$

53. Lagrangian of a system is, $L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)+k(x\dot{y}-y\dot{x})$

the conserved quantities are

(a) energy, $p_z$ only (b) energy, $p_z$ and $L_z$

(c) Energy and $L_z$ only (d) $L_z$ and $p_z$ only.

54. An object of mass $m$ is moving with speed $v_0$. If it is acted upon by a resistive force $\frac{B^2\ell^2 v}{R}$, speed of the object ~~particle~~ after it has travelled a distance $l$ is

(a) $v_0-\frac{B^2\ell^3}{2mR}$ (b) $v_0-\frac{B^2\ell^3}{mR}$ (c) $v_0-\frac{2B^2\ell^3}{3mR}$ (d) $v_0-\frac{B^2\ell^3}{3mR}$

55. A long thin and flat copper strip of rectangular cross-section (0.1 mm × 1.0 cm) carries a current of 5.0A along its length. If it is placed in a magnetic field of strength 1.0 Tesla perpendicular to the plane of the strip, what is the hall voltage generated? (Given Hall's coefficient $(R_H) = 11\ m^3/C$)

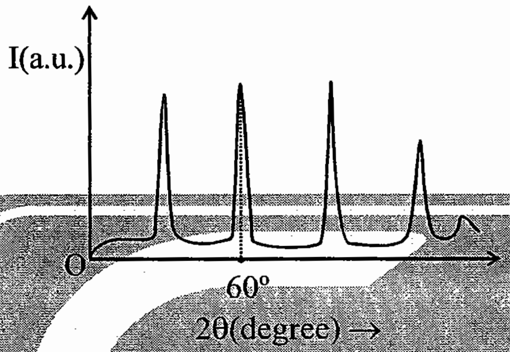
doubt

(a) 1.75 mV (b) 0.93 μV (c) 55 μV (d) 0.55 μV

56. If the transition temperature of superconductor is 30K, then the superconducting energy gap according to BCS theory is:
(a) 2.6 meV (b) 5.2 meV (c) 9.1 meV (d) 2.6 meV

**Common Data Questions for Q.57 and Q.58:**
A beam of X-rays of wavelength 2.5Å is reflected from sodium (Na) metal having bcc structure. A graph between intensity (I) $V_s$ diffraction angle $(2\theta)$ for this metal is shown in the figure below.

1



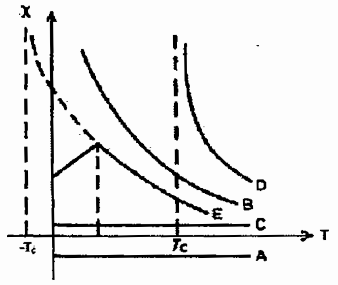
57. The lattice constant 'a' of Na is:
[✓ (a) 5Å] (b) 3.5Å (c) 2.5Å (d) 4.3Å

58. Volume of the primitive unit cell of this solid (in $Å^3$) is:
(a) 125 [✓ (b) 62.5] (c) 7.8 (d) 21.5

59. Fermi energy of a certain metal $M_1$ is 5eV. A second metal $M_2$ has an electron density which is 6% higher than that of $M_1$. Assuming that the free electron theory is valid for both the metals, the Fermi energy of $M_2$ is closest to
(a) 5.6 eV [✓ (b) 5.2 eV] (c) 4.8 eV (d) 4.4eV

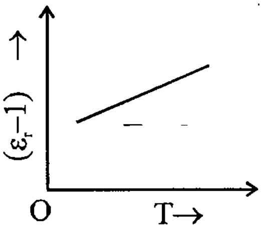
60. The ratio of the second-neighbour distance to the nearest neighbour distance in an fcc lattice is :
[✓ (a) $2\sqrt{2}$] (b) 2 (c) $\sqrt{3}$ [✓ (d) $\sqrt{2}$]

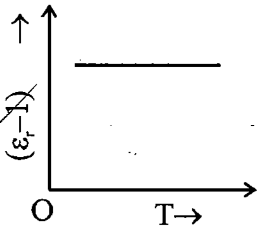
61. Susceptibility $(\chi)$ Vs Temperature (T) graph given below is for five solids A, B, C, D and E.

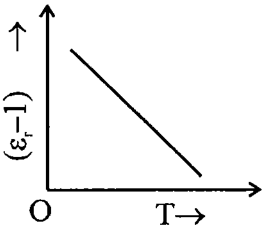


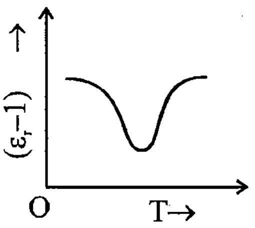
Choose correct answer:
(a) A is Diamagnetic, B is Ferromagnetic and C is Anti-Ferromagnetic.
[✓ (b) A is Diamagnetic, D is Ferromagnetic and C is Pauli-Paramagnetic.]
[✓ (c) C is Diamagnetic, D is Ferromagnetic and B is Pauli-Paramagnetic.]
(d) E is Anti-Ferromagnetic, D is Ferri-Magnetic and C is Pauli-Paramagnetic.

62. A dielectric is subject to a time dependent electric field (E) of frequency $f = 10^{14}$ Hz. At this frequency, choose the correct graph between $(\varepsilon_r - 1)$ and temperature (T). Where $\varepsilon_r$ is dielectric constant.

(a)


(b)


(c)


(d)


63. A Si pin photodiode has an active light receiving area of dimensions (0.4mm ×0.4 mm), when radiation of wavelength 700 nm and intensity $0.1\text{mw}/\text{cm}^2$ is incident on it, it generates a photocurrent of 10 nA. Quantum efficiency (Q.E.) of photodiode at 700 nm is:

(a) 44% (b) 22% (c) 11% (d) 88%

64. An X-ray diffraction (XRD) experiment is carried out on a crystalline solid having *FCC* structure at room temperature. The solid undergoes a phase transformation on cooling to –20°C and shows orthorhombic structure with small decrease in its unit cell lengths as compared to the *FCC* unit cell lengths. As a result, the (311) line of the XRD pattern corresponding to the *FCC* system

(a) will split into a doublet. (b) will split into a triplet

(c) will remain unchanged. (d) will split into two separate doublets.

65. In an experiment, the resistance of a rectangular slab of a semiconductor is measured as a function of temperature. The semiconductor shows a resistance of 300Ω at 200 K and 2 Ω at 250 K. Its energy band gap is [Given: ln(15) = 2.708, ln(10) = 2.303]

(a) 0.138 eV (b) 0.431 eV (c) 0.690 eV (d) 0.862 eV

66. Nickel is ferromagnetic, with a saturation magnetization per atom of $0.6\,\mu_B$, where $\mu_B$ is the Bohr magneton, equal to $9.27\times10^{-24}$ J/T. Given that the atomic weigth of nickel is 58.71 and that the density of nickel is 8.9 g/cm$^3$, the molar saturation magnetization of nickel is:

(a) $1.5\times10^{5}\ A-m^{-1}$ (b) $5.1\times10^{5}\ A-m^{-1}$

(c) $3.6\times10^{3}\ A-m^{-1}$ (d) $3.6\times10^{-3}\ A-m^{-1}$

67. A monovalent fcc metal has a lattice parameter of 0.41 nm at 273 K. Thermal conductivity and Lorentz number of same metal is 0.71 watt/am-k and $2.31\times10^{-5}$, respectively. What is the electrical resistivity of metal.

(a) $8.88\times10^{-8}\ \Omega-\text{m}$ (b) $2.82\times10^{-5}\ \Omega-\text{m}$

(c) $4.23\times10^{-5}\ \Omega-\text{m}$ (d) $8.46\times10^{-5}\ \Omega-\text{m}$

**Linked Answer Type Q.68 and 69.**

The tight binding energy dispersion (E–k) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is:

$$E = 2E_0\left[\sin^2\left(\frac{ka}{2}\right) - \frac{1}{6}\sin^2(ka)\right]$$

Where $E_0$ is constant and k is the wave-vector.

68. The density of states of electrons (including spin degeneracy) at $k = \frac{\pi}{2a}$ in the band is given by

[✓ (a) $\frac{L}{\pi E_0 a}$] (b) $\frac{3L}{\pi E_0 a}$ (c) $\frac{L}{2\pi E_0 a}$ (d) $\frac{2L}{\pi E_0 a}$

69. The effective mass (m*) at $k = \frac{\pi}{2a}$ is

(a) $-\frac{\hbar^2}{E_0 a^2}$ (b) $\frac{\hbar^2}{E_0 a^2}$ [✓ (c) $\frac{3\hbar^2}{2E_0 a^2}$] (d) $-\frac{3\hbar^2}{2E_0 a^2}$

70. The lattice parameter of a fcc solid is given as $a = 2$Å. If energy ($E_V$) of vacancy formation is 1 eV. The number of vacancies/m³ at 300K are: (Given : $\exp(-40) = 4.2\times10^{-18}$ )

[✓ (a) $2.1\times10^{12}$] (b) $1.70\times10^{12}$ (c) $4\times10^{19}$ (d) $1.70\times10^{20}$

# TEST SERIES UGC-CSIR-NET/JRF Dec. 2014

**BOOKLET SERIES** **A**

**Paper Code** **05**

**Test Type:** **TEST SERIES**

## PHYSICAL SCIENCES

**Duration: 2:00 Hours**

**Date: 15-11-2014**

**Maximum Marks: 140**

**Read the following instructions carefully:**

- Single Paper Test is divided into **TWO** Parts.

1. Attempt all the questions.
2. **Part - A:** This part shall carry 20 questions. Each question shall be of **2 marks.**
3. **Part - B:** This part shall contain 50 questions. Each question shall be of **2 marks.**
4. For rough work, blank sheet is attached at the end of test booklet.
5. There will be negative marking @25% for each wrong answer.
6. Darken the appropriate bubbles with HB pencil/Ball Pen to write your answer.
7. The candidates shall be allowed to carry the Question Paper Booklet after completion of the exam.

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## PART-A (GENERAL APTITUDE)

1. How many maximum intersection point can be formed of intersecting 4 circles and 4 lines.
(a) 50 (b) 54 (c) 56 (d) 60

2. There are 4 lines no two parallel and no three concurrent. How many new lines can be formed from intersection of these lines
(a) 4 (b) 5 (c) 3 (d) 6

3. A car has 2 front seat and 1 rear seat. In 6 person, there are two drivers. How many different ways these person can be seated.
(a) 40 (b) 36 (c) 60 (d) 72

4. In a survery in city, 25% of person drink tea and coffee and 35% drink coffee. How many maximum % of person who can drink tea
(a) 65% (b) 40% (c) 75% (d) 60%

5. A and B start to run at the same time with 60 km and 50 km per hour. If A is 15km ahead of B. After how many hours B will catch to A
(a) 10 hour (b) 15 hour (c) 12 hour (d) none of these

6. How much time after 8.00 pm, will the hour hand and the minute hand of a clock make an angle of 153 degrees for the first time?
(a) 6 minutes (b) 6 minutes 30 seconds (c) 5 minutes (d) 5 minutes 30 seconds

7. The revenue of ABC Ltd. Increased by 14.28% in the year 2008 over the previous year. The revenue further increased by 37.5% in the year 2009. If the revenue of ABC Ltd. In the year 2008 was Rs. 2, 400 crore, then what was the sum of its revenue in the year 2007 to 2009?
(a) Rs. 4600 crore (b) Rs. 4200 crore (c) Rs. 5400 crore (d) Rs. 4800

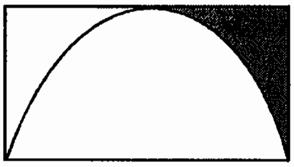
8. A class has a strength of 48 students in which 18 are boys. The students are to be seated in rows such that the number of students in all the rows is equal. If none of the rows contain both boys and girls, then what is the maximum number of students sitting in one row?
(a) 4 (b) 6 (c) 9 (d) 8

9. A man can complete as much work in 4 hours as a woman can in 6 hours. How long would 2 men and 3 women take in completing a work which 18 women can complete in 9 hours?
(a) 27 hours (b) 37 hours (c) 21 hours (d) 33 hours

10. The ages of A and B are in the ratio 2 : 3, the ages of B and C are in the ratio 4 : 5 and the ages of C and D are in the ratio $\frac{1}{6}:\frac{1}{5}$. If D is 30 years, older than A, then what is the age of B?
(a) 48 years (b) 24 years (c) 36 years (d) 60 years

11. The ratio of the incomes of Shyam and Ghanshyam is 4 : 5 and the ratio of their expenditures is 3 : 4. Who among the two saves more?
(a) Shyam (b) Ghanshyam
(c) both save equal money (d) can not be determined.

12. A semi-circle of a radius 5 cm is drawn inside a rectangle as shown in the figure given below. What is the area of the shaded region?



(a) $100-\frac{25\pi}{2}\text{cm}^2$ (b) $25-\frac{25\pi}{2}\text{cm}^2$ (c) $50-\frac{75\pi}{2}\text{cm}^2$ (d) $25-\frac{25\pi}{4}\text{cm}^2$

13. Which of the following is the largest?

(a) $333^3$ (b) $33^{33}$ (c) $3^{3^{3^3}}$ (d) $3^{33^3}$

14. A boat travels a certain distance down stream but while coming back it stops one km before the point from where it had started. it takes a total of 3 hours to complete this journey. Find the distance traveled by the boat while going upstream if it is given that speed of the stream is 2 km/hr and that of the boat is 10 km/hr in still water.

(a) 14 km (b) 16 km (c) 12 km (d) 10 km

15. In how many ways can we select a pair of prime numbers from the first 50 natural numbers such that the sum of the two numbers is a composite number?

(a) 8 (b) 14 (c) 99 (d) 210

**Directions for Q.16 and Q.17**

These are three movies namely Anaconda, true lies and eraser. In a survey it was found that 85% watches eraser, 20% watched true lies, and 30% watched Anaconda. 20% watched exactly two movies and 5% watched one.

16. What percentage of the people watched all three?

(a) 10% (b) 12% (c) 14% (d) none of these

17. Assuming that 20% of the people watched Anaconda and Eraser and 16% of the people watched Eraser and True Lies, what percentage of people watched only True Lies?

(a) 20 (b) 0 (c) 10 (d) 16

18. If today is Sunday, then what will be the day on 1757th day after today? (**Note :** Next day is considered as first day)

(a) Sunday (b) Monday (c) Tuesday (d) Wednesday

**Directions for Q.19 and Q.20.**

Amit is the son of Rahul. Sarika, who is Rahul's sister has a son Sonu and a daughter Rita. Raja is the maternal uncle of Sonu.

19. How is Amit related to Sonu?

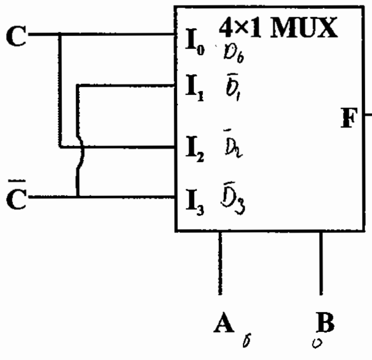
(a) Nephew (b) Cousin (brother) (c) Uncle (d) Brother

20. How is Rita related to Raja?

(a) Sister (b) Daughter (c) Niece (d) Aunt

**PART-B (ELECTRONICS, MATHEMATICAL PHYSICS, EMT)**

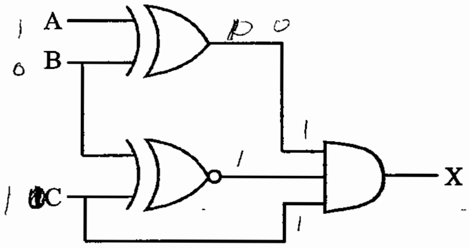
21. The logic realized by the circuit shown in figure is



$A\bar{D}_0 + B\bar{D}_1 +$

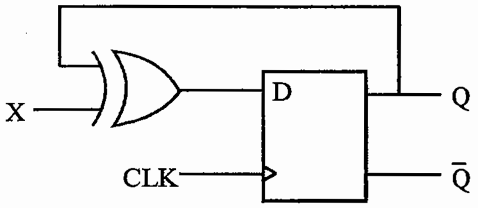
(a) $F = A.C$ (b) $F = A \oplus C$ (c) $F = B.C$ (d) $F = B \oplus C$

22. For the logic circuit shown in figure. The required input condition (A, B, C) to make the output ($X = 1$) is



(a) 1, 0, 1 (b) 0, 0, 1 (c) 1, 1, 1 (d) 0, 1, 1

23. The digital circuit shown in the figure works as



(a) 3K flip-flop (b) Clocked RS flip-flop

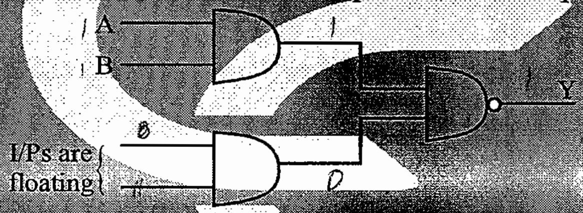
(c) T flip-flop (d) Ring counter

24. A 5-bit DAC produces $V_{out} = 0.2V$ for a digital input of 00001. Find the value of $V_{out}$ for an input of 11110.

(a) 3V (b) 6V (c) 9V (d) 12V

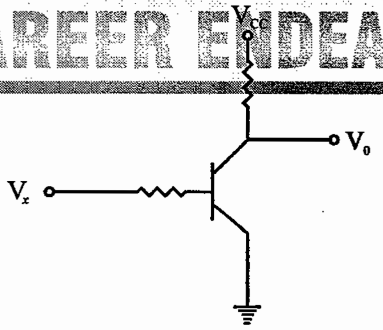
00001
11110
1
11111

25. Figure shows the internal schematic of a TTL IC. For the input shown the output Y is



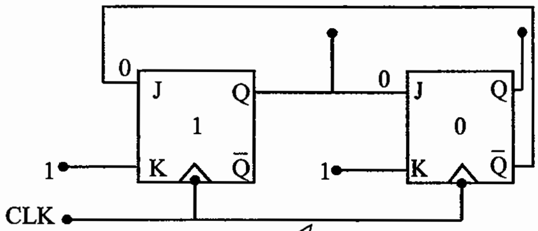
(a) 0 (b) 1 (c) AB (d) $\overline{AB}$

26. The GATE shown in figure is



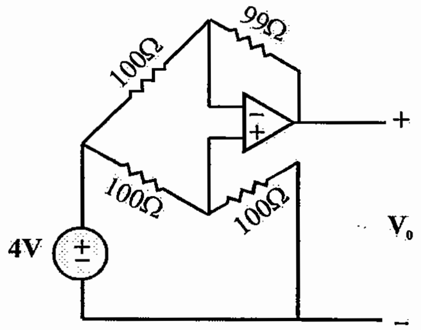
(a) AND gate (b) NAND gate (c) NOT gate (d) OR gate

27. Figure shown a mod-k counter, here k is equal to



(a) 1 (b) 2 (c) 3 (d) 4

28. For the ideal OP-AMP circuit of figure shown. What is the value of output voltage

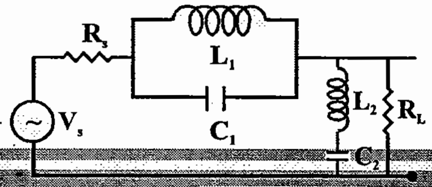

99Ω
100Ω
100Ω
100Ω
4V
+
−
+
−
V₀

199

200

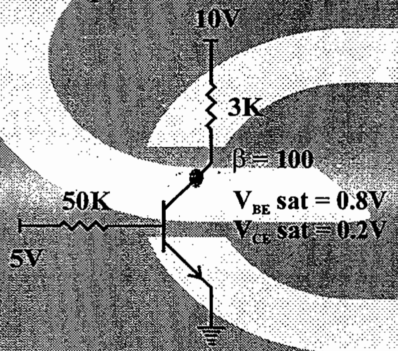
(a) 19.9 MV (b) 0.02V (c) 1.99MV (d) 1.99V

29. The circuit of figure represents as


R_s
L₁
C₁
V_s
L₂
R_L
C₂

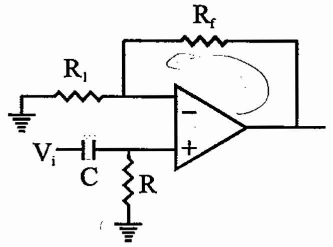
(a) LPF (b) HPF (c) BPF (d) BRF

30. For the given circuit diagram the circuit operates in


10V
3K
β = 100
V_BE sat = 0.8V
V_CE sat = 0.2V
50K
5V

(a) deep saturation (b) cutoff
(c) active (d) saturation

31. For the given OP-AMP network the magnitude of gain of this filter is

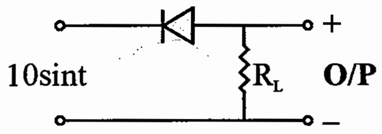


$V\left(\frac{1}{j\omega c}\right)R + V_o R_g$

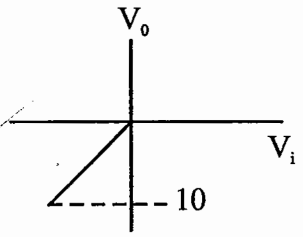
(a) $\left(1+\frac{R_f}{R_1}\right)\frac{1}{\sqrt{1+(\omega RC)^2}}$; *HPF* (b) $\frac{R_f}{R_1\sqrt{1+(\omega RC)^2}}$; *HPF*

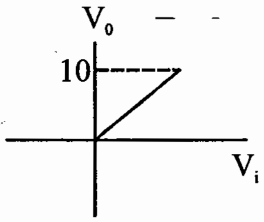
(c) $\left(1+\frac{R_f}{R_1}\right)\frac{[illegible]}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$; *HPF* (d) $\left(1+\frac{R_f}{R_1}\right)\frac{[illegible]}{\sqrt{1+\left(\frac{\omega}{RC}\right)^2}}$; *HPF*

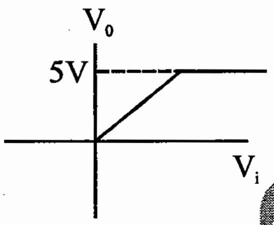
32.

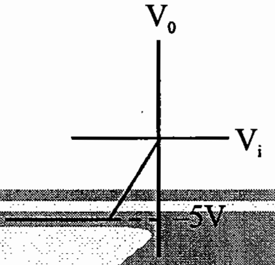


For the given network the transfor characteristic is

(a)


(b)


(c)


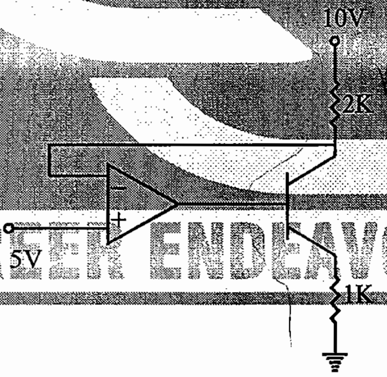
(d)


0 0 0
0 1 – 1
1 1 – 0

$A\bar{B} + \bar{B}\bar{A}$ =

33. The following configuration common collector is having high input resistance due to
(a) Input portion is FB (b) Output portion is RB
(b) Both input and output portion are RB (d) none of the above.

34. For the given network shown below. Determine current I of transistor



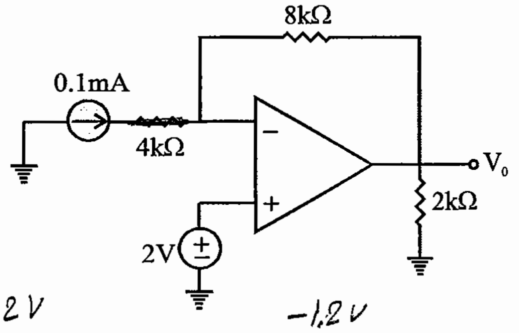
$-10V + 2I_C$
$-10V + 2I_C + I_C 0.7 + 2I_B$
$-10V + 2.7I_C + 2I_B$
$-10V + 4.7I_C + 2I_B$

(a) 2.5 mA (b) 2 mA (c) 5 mA (d) 1 mA

$10V = (4.7 + 2.7)I_C$

$I_C = \frac{10}{7.4}$

35. For the circuit shown below the value of $V_0$ is



$\frac{0.1}{2}$ $V_0$

$V = IR$

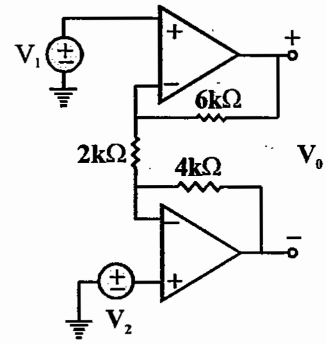
$0.1 \times 2$

$1 + \frac{R_2}{R_1}$

1.5V 1.2V −1.2V 1.5 −1.5V

(a) –30V (b) 18V (c) –18V (d) 28V

36. For the circuit shown below the voltage gain $A_{Vd} = \dfrac{V_0}{(V_1 - V_2)}$ is:



$A = \frac{V_0}{6} =$

$V_0 = -6$

12

$I = \frac{V}{R}$

$12 I = V$

$6I = V$

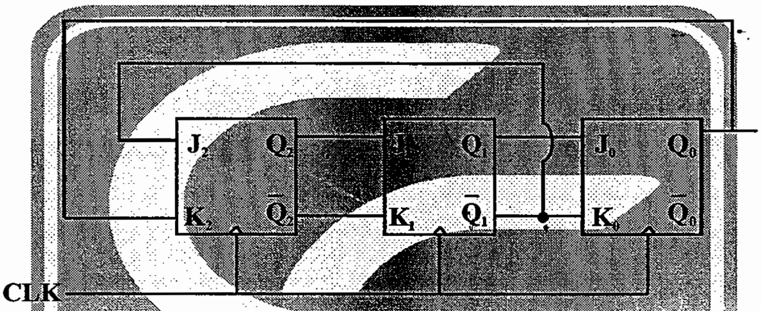
$V = V_1 - V_2$

$V = 12I - 6I$

$V = 6I$

(a) 8 (b) –6 [✓ (c) 6] (d) –8

37. The counter shown in the figure below has initially $Q_2Q_1Q_0 = 000$. The status of $Q_2Q_1Q_0$ after the first pulse is



0 0 0 0

1 0 0

(a) 001 (b) 010 (c) 100 (d) 101

38. Consider the function following function:

$$f(x) = \frac{7}{3}x^3 - 8x^2 + 9x + 71$$

The plot of the function in the 2-D cartesian co-ordinate plane

(a) has three extremum out of which two extremum are in the range $\left[1, \frac{3}{2}\right]$

(b) has three extremum out of which two extremum are in the range $\left[\frac{1}{2}, 1\right]$

(c) has two extremum in the range $\left[1, \frac{3}{2}\right]$

(d) has two extremum in the range $\left[\frac{1}{2}, 1\right]$

$\frac{7}{3} 3x^2 - 8(2) x + 9$

$7(2)x - 16 = 0$

$14x - 16 = 0$

$x = -\frac{16}{14} = \frac{8}{7}$

70 127 128 256

$7x^2 - 16x + 9$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$256 - 277$

$\frac{-21}{14}$ 21

28 - 9 280 160 16 × 16 256

$\frac{(16)^2 \pm \sqrt{(16)^2 - 4(7)(9)}}{2(7)}$

$256 \pm \sqrt{256 - 277}$

14

39. Suppose a random variable ‘X’ can take the following values: 0,1,2,3,4 and $P(x = K) = \beta K^2$ serves as a probability distribution for the random variable ‘X’. The possible value of $\beta$ will be

(a) 1/30 (b) 1/10 (c) 1/3 (d) 1/15

40. Consider the following matrix:

$$A=\begin{bmatrix}1 & i\\ i & -1\end{bmatrix}$$

$-i+i=0$

Which of the given statement is FALSE for the matrix?

(a) $A$ has degenarate eigenvalues (b) $A$ is singular in nature

(c) $A$ has real eigenvalues (d) $A$ is diagonalisable

41. Suppose the laplace transform of two time varying signals $f_1(t)$ and $f_2(t)$ is given to be $g_1(s)$ and $g_2(s)$ and the convolution theorem is defined as:

$$L^{-1}\left[g_1(s)g_2(s)\right]=\int_0^p f_1(t)f_2(p-t)\,dt$$

The Laplace transform of the integral $\int_0^m e^t\sin(m-t)\,dt$ will be

$e^t \frac{1}{m-t}\cos(m-t) + te^t \sin$

$e^t[\ldots$

(a) $\frac{1}{(s-1)(s^2+1)}$ (b) $\frac{1}{(s-1)(s^2-1)}$ (c) $\frac{1}{(s+1)(s^2-1)}$ (d) None of these

42. Consider the following force field:

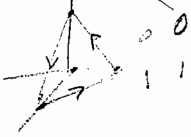
$$\vec{F}=2xyze^{x^2}\hat{i}+ze^{x^2}\hat{j}+ye^{x^2}\hat{k}$$

$2\hat{j}+4\hat{i}+2\hat{j}+\hat{k}$

$4\hat{i}+4\hat{j}+\hat{k}$

Suppose these force is acted upon a particle P and the particle moves along the path OQR:

$$O(0,0,0)\rightarrow Q(1,0,2)\rightarrow R(1,1,2)$$



0 0 0

1 1 0

The work done by the field on the particle will be

(a) 0 (b) $2e$ (c) $e$ (d) $e^2$

43. Suppose if $a\hat{i}+b\hat{j}+c\hat{k}$ is the unit normal vector to the the surface $z=\sqrt{x^2+y^2}$ at $\left(1,1,\sqrt{2}\right)$. Which of the following relations of a, b, c is TRUE?

(a) $a^2+b^2+c^2=0$ (b) $-a^2+b^2+c^2=0$

(c) $a^2-b^2+c^2=0$ (d) $a^2+b^2-c^2=0$

$1+1=2$ ?

44. Consider a $n\times n$ square matrix $P=\{P_{ij}\}$ with $P_{ij}=1$ for $1\le i,j\le n$. The characteristic equation of the matrix P will be

[✓ (a)] $\lambda^n-(n-1)\lambda^{n-1}-\lambda=0$ (b) $\lambda^n-n\lambda^{n-1}-\lambda=0$

(c) $\lambda^n-n\lambda^{n-1}=0$ (d) $\lambda^n-\lambda^{n-1}+n\lambda=0$

45. Consider the following differential equation:

$$(x+y+1)\,dx+(2x+2y+1)\,dy=0$$

Which of following statements is CORRECT for the above differential equation?

(a) The differential equation is linear (b) The differential equation is exact

(c) The integrating factor of the differential equation is $e^{x+y}$.

(d) A suitable substitution transforms the differential equation to the variables separable form.

46. Suppose $y = f(x)$ be a twice continuously differentiable function on $(0,\infty)$ satisfying the following conditions:

$$f(1) = 1 \text{ and } f'(x) = \frac{1}{2} f\left(\frac{1}{x}\right) \quad (x > 0)$$

The differential equation that will be satisfied by $y = f(x)$ will be

[✓ (a) $4x^2 \frac{d^2y}{dx^2} - y = 0$]  (b) $4x^2 \frac{d^2y}{dx^2} + y = 0$

(c) $4x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$  (d) $4x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

$-\frac{y}{2} = \frac{2}{6}$

$y = \frac{4}{6}\ \frac{2}{3}$

$\frac{4}{3}x^3 \frac{dy}{dx} - xy = 0$

$\frac{4}{3(4)}x^4 - \frac{x^2 y}{2} = 0$

$\frac{4}{12}x^4 - \frac{x^2}{2}y = 0$

$\frac{4}{12} - \frac{1}{2}y = 0$

$\frac{4}{12} - \frac{1}{2} - \frac{2}{6} - \frac{1}{2}y$

47. Consider the following complex integral:

$$\int_C \left(z - z^2\right) dz$$

where z is denoted by a point (x,y) in argand plane and C is the upper half of the circle $|z-2| = 3$.
The value of the integral along the lower half of the above given circle will be
(a) 18  (b) -18  (c) 6  (d) -6

48. Consider the following integral:

$$\int_0^{2\pi} \frac{d\theta}{(p + q\cos\theta)} = \frac{2\pi}{\sqrt{p^2 - q^2}}$$

The value of the integral $\int_0^{2\pi} \frac{d\theta}{(p + q\cos\theta)^2}$ will be

(a) $\frac{2\pi p}{\sqrt{p^2 - q^2}}$  (b) $-\frac{2\pi p}{\sqrt{p^2 - q^2}}$  (c) $\frac{2\pi p}{\left(p^2 - q^2\right)^{3/2}}$  [✓ (d) $-\frac{2\pi p}{\left(p^2 - q^2\right)^{3/2}}$]

49. Consider the following complex function:

$$f(z) = \frac{ze^z}{(z-a)^3}$$

$\frac{3e^2}{(3-a)^3} - \frac{1}{(1-a)^3}$

If $R_z(a)$ represents the residue of the function $f(z)$ at its pole of order 3, then $\frac{R_z(3)}{R_z(1)}$ is

(a) $\frac{5}{3}e^2$  (b) $\frac{5}{3}e$  (c) $\frac{3}{5e}$  (d) $\frac{3}{5e^2}$

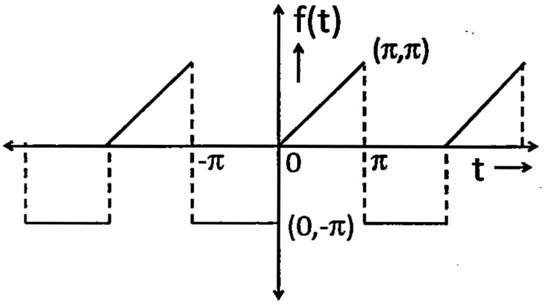
50. A manufacturer knows that the condensers he makes contain on an average 1 percent of defectives. He packs of them in boxes of 100. The probability that a box picked at random will contain 4 or more faulty condensers will be

[✓ (a) $\frac{8}{3e}$]  (b) $\frac{3}{8e}$  (c) $1 - \frac{8}{3e}$  (d) $1 - \frac{3}{8e}$

51. Three elements of a 3 x 3 matrix are chosen at random. The probability that they belong to the same column is

(a) 1/27 (b) 1/28 (c) 1/8 (d) 1/9

52. Suppose the following signal (shown in figure below) is expanded into fourier series as a sum of various frequency terms :



The amplitude of second cosine harmonic term in the fourier series expansion of the above signal, will be

(a) 0 (b) $\frac{2}{9\pi}$ (c) $-\frac{2}{9\pi}$ (d) $\frac{2}{25\pi}$

53. Consider the following function:

$$f(x)=\begin{cases} x^2 & \text{for } |x|\le a \\ 0 & \text{otherwise} \end{cases}$$

In the fourier transform of the above function, the co-efficient of the term $\frac{1}{\omega^3}$ will be proportional to

(a) $\cos(\omega a)$ (b) $\omega a$ (c) 1 (d) $\sin(\omega a)$

54. Consider the following wave equation:

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$$

under the conditions $u(0)=u(\pi)=0$, $\left(\frac{\partial u}{\partial t}\right)_{t=0}=0$ and $u(x,0)=x$.

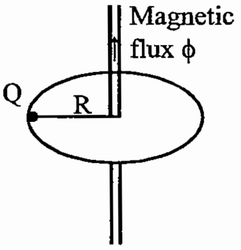
If the solution of above differential equation can be expressed as

$$u(x,t)=\sum_{n=1}^{\infty} b_n \sin nx \cos nat$$

The value of the ratio $\frac{b_2}{b_1}$ will be

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) −2

55. A freely rotating disc is pierced through an infinite superconducting solenoid containing a magnetic flux $\phi$. A charge Q is fixed to disc at a distance R from the centre of the solenoid. If the solenoid is heated and it loses it magnetic flux in time T, then net angular momentum acquired by the disc is given by



$B = 2\phi$
$B\,2\pi R = a\phi$
$B = \frac{a\phi}{2\pi R}$

(a) $\frac{q\phi}{T}$ (b) $\frac{q\phi}{2\pi}$ (c) $q\phi$ (d) $\frac{q\phi}{2\pi R}$

56. Consider an e.m. wave propagating in metallic medium, choose the correct answer from the following statements.

(i) The phase velocity of the electro magnetic wave in good conductor is $\sqrt{\frac{2\omega}{\sigma\mu}}$

(ii) Skin depth of good conductor is $\sqrt{\frac{2}{\mu\sigma\omega}}$

(iii) Energy flows along the direction of propagation without attenuation.

(iv) $\vec{E}$ lags behind the $\vec{B}$ by a phase angle of 45°.

(a) (i), (ii), (iii) (b) (i), (ii), (iv) (c) (i), (ii), (iii) & (iv) (d) (i) and (ii)

57. The electric field components of a plane electromagnetic wave are

$$E_y = E_0 \sin\left(kz - \omega t - \frac{\pi}{6}\right)$$

$$E_x = -E_0 \cos\left(kz - \omega t + \frac{5\pi}{6}\right)$$

The state of polarization of wave is:

(a) Linearly poalrized (b) Elliptically polarized
(c) Left circularly polarized (d) Right circularly polarized.

58. The electric field of an electromagnetic wave propagating through vacuum is given by

$$\vec{E}(\vec{r},t) = E_0\hat{z}\cos\left(100\sqrt{3}\pi x - 100\pi y - \omega t\right)$$

Wave vector $\vec{k}$ and frequency $\omega$ are given by, respectively?

(a) $200\pi\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right), \frac{150}{\pi}\times 10^4$ rad/sec. (b) $200\pi\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right), \frac{150}{\pi}\times 10^4$ rad/sec

(c) $100\pi\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right), \frac{300}{\pi}\times 10^4$ rad/sec (d) $100\pi\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right), \frac{300}{\pi}\times 10^4$ rad/sec

59. A scientist designed an experiment to study the electromagnetic responce of silver at microwave frequency $(\sim 10^{10}\text{ Hz})$. Given that the resistivity of silver is $4\times 10^{-8}\,\Omega - \text{m}$. How thick a silver coating would be needed? (Given permeability of silver is $\mu_0 = 4\pi\times 10^{-7}$ H / m)

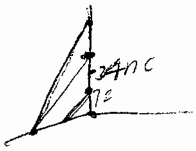
(a) 3.2 Å (b) 50 Å (c) 8Å (d) 6.4Å (e) 1μm

60. Assume that a lamp radiates power P uniformly in all the directions. What is the amplitude of electric field strength at a distance r from the lamp.

(a) $\frac{P}{\pi c\varepsilon_0 r^2}$ (b) $\frac{P}{2\pi c\varepsilon_0 r^2}$ [✓ (c) $\sqrt{\frac{P}{2\pi c\varepsilon_0 r^2}}$] (d) $\sqrt{\frac{P}{\pi c\varepsilon_0 r^2}}$

61. Two point charges $q_1$ and $q_2$ are respectively located at (5, 0, 3) and (2, 0, 2). If $q_2 = 4$nc. Find the charge $q_1$ such that z-component of electric field at (5, 0, 6) becomes zero?

[✓ (a) $q_1 = -\frac{144}{125}$nc] (b) $q_1 = -\frac{144}{25}$nc (c) $q_1 = -5$nc (d) $q_1 = -\frac{12}{5}$nc



**Linked Answer Type Q.62 and Q.63.**

Let $\vec{E} = \hat{x}E_0 \exp\left(i\vec{k}.\vec{r} - \omega t\right)$, where $\vec{k} = \hat{z}(k\cos\phi + ik\sin\phi)$ and $\hat{x}, \hat{y}$ and $\hat{z}$ are cartesian unit vectors, represent an electric field of a plane electromagnetic wave of frequency $(\omega)$.

62. Which one of the following statement is true?
[✓ (a) The magnitude of the electric field is attenuated as wave propagates.]
(b) The energy of the e.m. wave flows along the x-direction.
(c) The magnitude of the electric field of the wave is constant.
(d) The speed of the wave is same as c (speed of light in in free space)

63. The magnetic field $\tilde{B}$ of the wave is:

(a) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp\left[i(zk\cos\phi - \omega t)\right]$

[✓ (b) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp\left[i(zk\cos\phi - \omega t + \phi)\right]$]

[✓ (c) $\hat{y}\frac{k}{\omega}E_0 \exp\left[i(zk\cos\phi - \omega t + \phi)\right]$] (d) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\cos\phi)\exp\left[i(zk\sin\phi - \omega t)\right]$.

**Linked Answer Type Q.64 and Q.65.**

An air filled square wave–guide is operated with a wave of frequency of 3 GHz. The cut off frequency of $TE_{11}$ mode is 2.82 GHz.

64. Dimensions of the wave guide are
(a) $a = 7.5$cm, $b = 15$cm [✓ (b) $a = 7.5$cm, $b = 7.5$cm]
(c) $a = 15$cm, $b = 15$cm (d) $a = 30$cm, $b = 30$cm

65. Which one of the following expression for the magnetic field $(\vec{B})$ inside the wave–guide is incorrect? (Where x and y are in cm.)

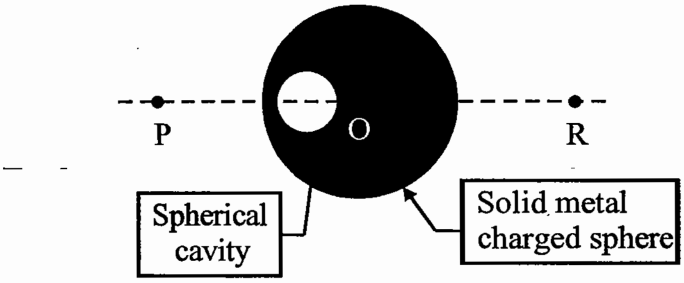
(a) $H_z = H_0 \cos\left(\frac{2\pi x}{15}\right)\cos\left(\frac{2\pi y}{15}\right)e^{-\gamma z}$ [✓ (b) $H_z = H_0 \cos\left(\frac{4\pi x}{15}\right)e^{-\gamma z}$]

(c) $H_z = H_0 \cos\left(\frac{2\pi x}{15}\right)e^{-\gamma z}$ (d) $H_z = H_0 \cos\left(\frac{2\pi y}{15}\right)e^{-\gamma z}$

66. Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is maximum. Through what angle must either be turned so that the transmitted light intensity drops by 75%?

[✓ (a) 30°] (b) 45° [✓ (c) 60°] (d) 90°

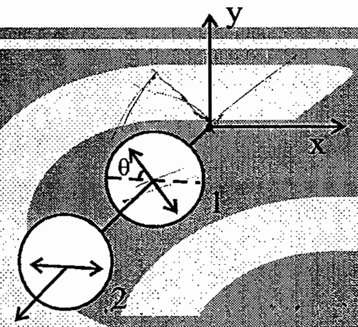
67. A solid metal sphere with a spherical cavity as shown below has a total charge +Q



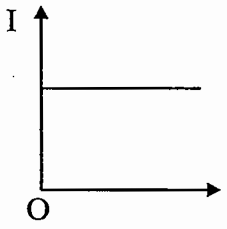
O is the center of the sphere, and P and R are two points equidistant from it. If $E_P$ and $E_R$ represent the magnitude of the electric field at P and R respectively, which of the following statements is correct?

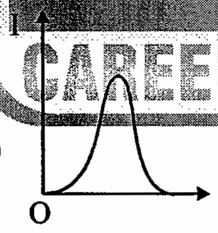
[✓ (a) $E_P = E_R$] [✓ (b) $E_P = 0$ and $E_R = 0$] (c) $E_P > E_R$ (d) $E_P < E_R$

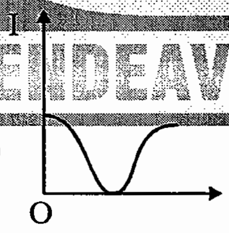
68. A plane polarized electromagnetic wave with polarization along y-axis, is incident on a system of two polarizing sheets as shown below.

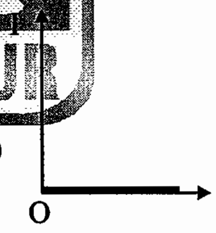


The polarization axis of polarizer 2 is fixed along the x-axis, while polarizer 1 can be rotated in the x-y plane. If θ is the angle between the polarization axes of polarizers 1 and 2, which of the following figures give the correct description of the intensity at the output of polarizer 2 as a function θ between zero and $\pi/2$?

(i)


(ii)


(iii)


(iv)


(a) (i) (b) (ii) (c) (iii) [✓ (d) (iv)]

69. A charge q is placed in front of an infinite metal plate at a distance d. If instead a charge 2q were to experience the same force, at what distance should it be placed from the plate?

(a) d [✓ (b) 2d] (c) d/2 (d) 4d

70. Can the following scalar and vector potential describe an electromagnetic field

$$\phi(\vec{r}, t) = 3xyz - 4t$$

$$\vec{A}(\vec{r}, t) = (2x - \omega t)\hat{i} + (y - 2z)\hat{j} + (z - 2\,e^{\omega t})\hat{k}$$

where '$\omega$' is a constant

(a) yes, in the coulomb gauge [✓ (b) yes, in the lorentz gauge]

(c) yes, provided $\omega = 0$ [✓ (d) no, they donot describe electromagnetic field.]

# TEST SERIES FOR GATE

## BOOKLET SERIES B

**Paper Code: PH**

**Test Type: TEST SERIES**

**Duration: 3:00 Hours**

# PHYSICS

**Date: 13-01-2015**

**Maximum Marks: 100**

**Read the following instructions carefully:**

1. Attempt all the questions.
2. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory. The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.
   The subject specific GATE paper section consists of **55** questions, out of which question numbers 11 to 35 are of 1-mark each, while question numbers 36 to 65 are of 2-mark each.
3. The question paper may consist of questions of **multiple choice type** (MCQ) and **numerical answer type**.
4. Multiple choice type questions will have four choices against (a), (b), (c), (d), out of which only **ONE** is the correct answer.
5. For numerical answer type questions, each question will have a numerical answer and there will not be any choices.
6. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of ⅓ marks for a **1-mark** question and ⅔ marks for a **2-mark** question.
7. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
8. Non-programmable type Calculator is allowed

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T : 011-65462244, 27241845, (0) 9899628311, 9540292991
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## Some Useful Constants

| | |
|---|---|
| Speed of light in free space | $c = 3 \times 10^{8}\ m/s$ |
| Boltzmann constant | $k_B = 1.380 \times 10^{-23}\ J/K$ |
| Planck's constant | $h = 6.626 \times 10^{-34}\ J.s$ |
| Electron charge | $e = 1.602 \times 10^{-19}\ C$ |
| Permittivity of free space | $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/N.m^2$ |
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7}\ H/m$ |
| Mass of electron | $m_e = 9.31 \times 10^{-31}\ kg$ |
| Mass of proton | $m_p = 1.67 \times 10^{-27}\ kg$ |
| Mass of neutron | $m_n = 1.675 \times 10^{-27}\ kg$ |

# PHYSICS-PH

Date: 13-01-2015

## GATE TEST SERIES-II

### ANSWER SHEET

1. (d) 2. (c) 3. (b) 4. (c) 5. (c) 6. (c) 7. (d)
8. (d) 9. (c) 10. (d)

11. (a) 12. (c) 13. (c) 14. (1.05 to 1.15) 15. (d) 16. (c) 17. (a)
18. (b) 19. (a) 20. (4) 21. (a) 22. (b) 23. (3.7) 24. (b)
25. (270) 26. (13126) 27. (c) 28. (b) 29. (d) 30. (d) 31. (c)
32. (c) 33. (b) 34. (d) 35. (c) 36. (a) 37. (d)
38. (0.74 to 0.78)
39. (c) 40. (a) 41. (d) 42. (d) 43. (c) 44. (0.80 to 0.83)
45. (b)
46. (80) 47. (b) 48. (b) 49. (d) 50. (b) 51. (0.05) 52. (a)
53. (c) 54. (c) 55. (d) 56. (d) 57. (b) 58. (b) 59. (b)
60. (d) 61. (b) 62. (0.11 to 0.13) 63. (c) 64. (c) 65. (d)

**Q.1-Q. 5 carry ONE mark each.**

1. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

**Venison:Deer**

(a) Herd:Sheep (b) Kitten:Cat (c) Rumination: Law [✓ (d) Beef:Ox]

2. Choose the word from the options given below that is most nearly similar in meaning to the given word **Vindictive.**

(a) Void (b) Vindictive (c) Rerengful **(d)** Valid

3. Several identical cubes are arranged in a close packed single layer. If the area of the layer is $A$ and volume of the layer is $V$, then the number of cubes in the collection is :

(a) $\frac{V}{A}$ (b) $\frac{A^3}{V^2}$ (c) $\frac{V^2}{A^3}$ **(d)** $\frac{A}{V}$

4. Let $A = \frac{1^{13}+2^{13}+3^{13}+.......+100^{13}}{100}$, $B = \frac{1^{13}+3^{13}+5^{13}+.......+99^{13}}{50}$ and

$C = \frac{2^{13}+4^{13}+6^{13}+.......+100^{13}}{50}$

Which of the following is true ?

(a) $B < C < A$ (b) $A < B < C$ (c) $B < A < C$ (d) $C < A < B$

5. Consider the following equation $x^2+4y^2+9z^2 = 14x+28y+42z-147$ where $x, y$ and $z$ are real numbers. Then the value of $x+2y+3z$ is

(a) 7 (b) 14 (c) 21 (d) Not unique

**Q.6-Q. 10 carry TWO marks each.**

6. Time and again it has been shown that students who attend colleges with low faculty/student ratio get the most well rounded education. As a result, when my children are ready to attend college. I will sure they attend a school with a very small student population.

Which of the following if true the greatest flaw in the reasoning above?

[✓ (a) A low faculty/student ratio is the effect of a well rounded education, not its source.]

(b) Intelligence should be considered the result of childhood envirnment and not advanced education.

(c) A very small student population does not ensure a low faculty/student ratio.

(d) parental desires and preferences rarely determine a childs choice of a college or university.

7. Philistine : Uncultured : : Precocious : ?

(a) Clever [✓ (b) Mature] (c) Polish **(d)** Intelligent

8. By the begining of the 20$^{th}$ century, several hypothesses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of the star which was directly behind our sun.

Which of the following inference(s) may be drawn from the above passage ?

I. Our understanding of the universe changes based on the positions of stars.

[✓ II.] Paradigm shifts usually occur at the begining of centuaries.

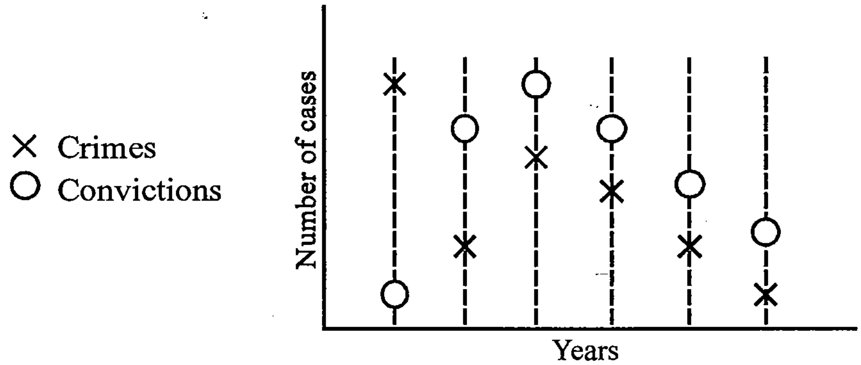
III. Stars are important objects in the universe.

IV. Experimental evidence was important in confirming this paradigm shift.

(a) I, II and IV [✓ (b) III only] (c) I and IV **(d)** IV only

9. A person purchases one kg of tomatoes from each of the 4 places at the rate of 1 kg, 2 kg, 3 kg and 4 kg per rupee respectively. On the average he has purchased $x$ kg of tomatoes per rupee, then the value of $x$ is
(a) 2 (b) 2.5 (c) 1.92 (d) None of these

10. The graph below shows crime rates and convictions for 5 years in a certain society



Which of the following is correct ?
(a) The cause of convictions being on the rise is better law enforcement.
(b) Falling crime rates have slowly reduced the convictions rate also.
(c) There are fewer convictions because crime rate has fallen.
(d) None of these.

**Q.11-Q.35 carry one mark each.**

11. Consider the following surface $x^2+2y^2-z^2=9$. A possible unit tangent vector to the surface at the point (1,0,-2) will be

(a) $\frac{2}{\sqrt{5}}\hat{i}-\frac{1}{\sqrt{5}}\hat{k}$ (b) $\frac{2}{5}\hat{i}-\frac{1}{5}\hat{k}$ (c) $\frac{2}{\sqrt{5}}\hat{i}+\frac{1}{\sqrt{5}}\hat{j}$ (d) $\frac{2}{5}\hat{i}+\frac{1}{5}\hat{j}$

12. The contour integral $\oint_C \frac{dz}{z^2-4z+1}$ is to be evaluated on the circle $C:|z-3|=1$. It will have the contributions only from the point
(a) $2-\sqrt{3}$ (b) $-2+\sqrt{3}$ (c) $2+\sqrt{3}$ (d) $-2-\sqrt{3}$

13. The fourier transform of the time domain function $f(t)=\cos at$ in the frequecy domain will consists of

(a) $\delta(\omega)$ and $\delta(\omega+a)$ (b) $\delta(\omega)$ and $\delta(\omega-a)$

(c) $\delta(\omega+a)$ and $\delta(\omega-a)$ (d) $\delta(\omega),\delta(\omega+a)$ and $\delta(\omega-a)$

14. Consider a system of distinguishable particles with energy levels $0,\varepsilon,2\varepsilon,3\varepsilon,4\varepsilon,\ldots\ldots$ . For a system with 2 particles and energy $2\varepsilon$, the entropy of the system is ________(in the units of $k$).

15. Among the given four quantities :
1. Electric polarization 3. Refractive index
2. Magnetization 4. Molar entropy
Which is /are not extensive quantity?
(a) 1 and 2 (b) 1, 2 and 3 (c) 1, 2 and 4 (d) 1, 2, 3 and 4

16. Consider a system of two particles A and B. Each particle can occupy one of three possible quantum states $|1\rangle$, $|2\rangle$ and $|3\rangle$ .The ratio of the probability that the two particles are in the same state to the probability that the two particles are in different states is calculated for bosons and classical (Maxwell-Boltzmann) particles. They are respectively
(a) 1 and 0 (b) ½ and 1 (c) 1 and ½ (d) 0 and ½

17. Let $E_T$ be the energy obtained variationally by using trial function $Ae^{-\alpha r^2}$ (with $\alpha$ as a variational parameter) for harmonic oscillator. If the exact ground state energy is $E_H$, then

(a) $E_T = E_H$ (b) $E_T > E_H$ (c) $E_T \geq E_H$ (d) $E_T < E_H$

18. For a spin −1/2 particle, the expectation value of the $iS_xS_yS_z$ in the state $\frac{1}{\sqrt{2}}\left(\left|+\frac{1}{2}\right\rangle + \left|-\frac{1}{2}\right\rangle\right)$ is

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{\hbar^3}{8}$ (c) $\frac{i\hbar^3}{4}$ (d) 0

19. An electron and proton each having an energy of 5 eV high, are incdent on a barrier of 10 eV high and 1 A$^0$ wide. Which of the following is a correct statement?

(a) The electron will have greater transmission compared to proton

(b) The proton will have greater transmission compared to electron.

(c) Both electron and proton will have equal tranmission probabilities

(d) Neither electron nor protons can cross the barrier.

$h\nu > E$

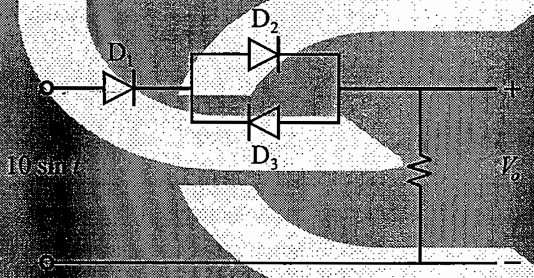
$h\nu < E$

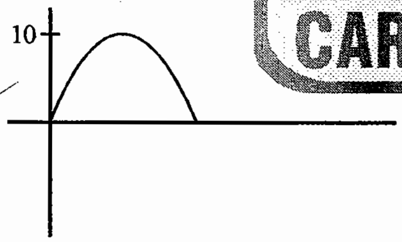
20. A silicon diode under forward bias condition with cut in voltage of (0.7 V) having total voltage 1.1 V if the forward circuit is 100 mA. The value of forward resistance of diode is (in the units of Ω)______

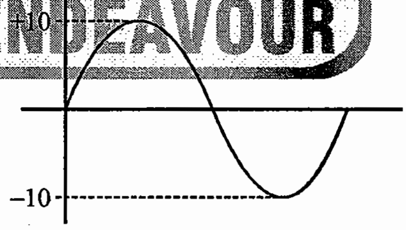
$R = \rho \frac{l}{A}$

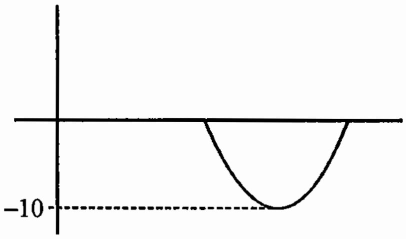
$J = neAV_d$

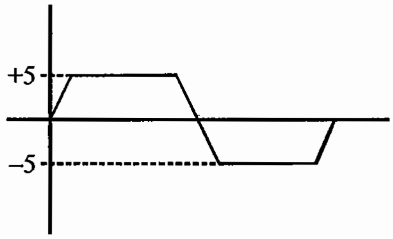
21. For the given circuit diagram the output wave form is (assume ideal diode)



(a)


(b)


(c)


(d)


22. For an ideal operational amplifier of differential mode gain is 10,000 and common mode gain is 100 then CMRR is

(a) 10 (b) 100 (c) 1000 (d) 10000

Toc

23. The half life of $^{226}$Ra is 1620 years, the radioactivity of 1 gram of $^{226}$Ra is______ × $10^{10}$ disintegration/second

24. Which of the following process is allowed ?

(a) $\Omega^- \to \Omega^+ + \pi^- + \pi^+$ (b) $p + \bar{p} \to \pi^+ + \pi^-$

(c) $\pi^+ + K^\circ \to \bar{K}^\circ + e^+ + \nu_e$ (d) $p + \bar{n} \to p + \pi^\circ$

25. The degeneracy of the excited state of nitrogen having configuration, $1s^2 2s^2 2p3d^2$ __________

26. A positronium atom is a system consists of a positron and an electron. The wavelength of the first Balmer line for positronium is (in the units of $A^0$) __3331____
(Given: Wavelength of the first Balmer line for hydrogen is 6563 A⁰) . 6.8

27. The Lagrangian of a free particle in spherical polar coordinates is given by

$$L = \frac{1}{2}m\left[\dot{r}^2 + r\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta\right]$$

The quantity that conserved is

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$ ~~(c)~~ $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}} + \dot{r}\dot{\theta}$

28. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination $\theta$. The number of generalised coordinate required to describe the motion of the system is

(a) 1 ~~(b)~~ 2 (c) 4 (d) 6

29. A bead of mass m slides along a straight frictionless rigid wire rotating in a horizontal plane with a constant angular speed $\omega$. The axis of rotation is perpendicular to the wire and passes through one end of the wire. If $r$ is the distance of the mass from the axis of rotation and $v$ is the its speed, then the magnitude of the Coriolis force is

(a) $\frac{mv^2}{r}$ (b) $\frac{2mv^2}{r}$ (c) $mv\omega$ ~~(d)~~ $2mv\omega$

30. Two events are separated by a distance of $6\times10^5$ km and the first event occurs 1s before the second event. The interval between two events is
(a) time like ~~(b)~~ light like (c) space like
(d) cannot be determined from the information given.

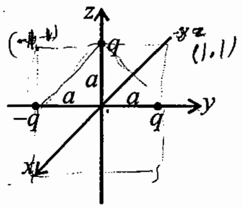
31. Electric field at a distance of 1cm from a long wire near its middle is 9 × 10⁶ V/m charge density on the wire is $9\times10^6$ V/m $\rho = \frac{4}{3}\pi r^3$

(a) 2.5μC (b) 2μC (c) 5μC (d) 10μC

32. Electric dipole moment of charge configuration shown in the figure is

(a) $qa\hat{z}$ (b) $-2qa\hat{y} + qa\hat{z}$

(c) $2qa\hat{y} + qa\hat{z}$ (d) $2qa\hat{x} + qa\hat{z}$



$qa\hat{z} \to qa\hat{y} + qa\hat{y}$

33. Dimension of E/H, where E is electric field and H is magnetic field intensity is same as $V/\omega$
(a) frequency (b) resistance (c) capacitance ~~(d)~~ inductance

34. Consider X-ray diffraction from a crystal with a face centred cubic (fcc) lattice. The lattice plane for which there is no diffraction peak is

(a) (2, 1, 2) ~~(b)~~ (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

35. The Hall coefficient $R_H$ of sodium depends on

(a) The effective charge carrier mass and carrier density

(b) The charge carrier density and relaxation time

[✓ (c) The charge carrier density only]

(d) The effective charge carrier mass.

**Q.36-Q.65 carry TWO marks each.**

36. Consider a vector $\vec{P} = \hat{i} + 2\hat{j} + \hat{k}$ in the co-ordinate system $\left(\hat{i}, \hat{j}, \hat{k}\right)$. The axes are rotated in the clockwise direction about the z axis by an angle $45^0$. The vector $\vec{P}$ in the rotated co-ordinate system will be

(a) $-\frac{1}{\sqrt{2}}\hat{i} + \frac{3}{\sqrt{2}}\hat{j} + \hat{k}$ (b) $\frac{1}{\sqrt{2}}\hat{i} - \frac{3}{\sqrt{2}}\hat{j} + \hat{k}$

(c) $\frac{3}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$ (d) $\frac{3}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$

37. Consider the following differential equation:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

$(D^2 + 4D) + \cancel{4} = 0$

$D^2 + 4D = -4$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$

$\frac{-4 \pm \sqrt{16 - 16}}{2}$

$-\frac{4}{2}$ $-2$

under the condition $y\left(x = \frac{1}{5}\right) = 0$. What can you say with **certainty** about the solution $y$?

(a) The solution first increases and then decreases to zero.

[✓ (b) The solution first decreases and then increases to zero.]

(c) The solution oscillates initially and then decreases to zero.

(d) The solution has a extremum in the range $0.5 < x < 1.5$

38. In a particular metal at 300 K, the electrons each have chemical potential -2.03 eV. A certain quantum state has energy -2.00 eV, and it can contain no more than one electron. The probabilities that it is empty is ________

39. An assembly of N fixed particles with spin ½ and magnetic moment $\mu_0$ is in a static uniform applied magnetic field of intensity H. The spins interact with the applied field but are otherwise essentially free and the system is in thermal equilibrium at temperature T. The heat capacity of the system is

(a) $\frac{N\mu_0 H\left(\frac{\mu_0 H}{kT}\right)^2}{\cosh^2\left(\frac{\mu_0 H}{kT}\right)}$ (b) $\frac{N\mu_0 H\left(\frac{\mu_0 H}{kT}\right)^2}{\sinh^2\left(\frac{\mu_0 H}{kT}\right)}$ (c) $\frac{Nk\left(\frac{\mu_0 H}{kT}\right)^2}{\cosh^2\left(\frac{\mu_0 H}{kT}\right)}$ (d) $\frac{Nk\left(\frac{\mu_0 H}{kT}\right)}{\cosh^2\left(\frac{\mu_0 H}{kT}\right)}$

40. The equation of state for one mole of a gas is given by $\left(P + \frac{a}{TV^2}\right)(V - b) = RT$, where a, b are positive constants of appropriate dimensions and R is universal gas constant, the value of critical temperature of the gas is

(a) $\sqrt{\left(\frac{8a}{27bR}\right)}$ (b) $\frac{8a}{27bR}$ (c) $\frac{3a}{27bR}$ (d) $\sqrt{\left(\frac{a}{27bR}\right)}$

41. Consider that the density of states $g(E)$ for superrelativestic gas is defined as

$$g(E)=\begin{cases}1, & 0<E<E_D\\ 0, & E>E_D\end{cases}$$

The specific heat at constant volume $C_V$ for gas of bosons and fermions in very low temperature limit ($T\to 0$) varies as

(a) $C_V \alpha T^3$ for bosons and $C_V \alpha T$ for fermions

(b) $C_V \alpha T$ for bosons and $C_V \alpha T^3$ for fermions

(c) $C_V \alpha T^2$ for bosons and $C_V \alpha T$ for fermions

(d) $C_V \alpha T$ for bosons and $C_V \alpha T$ for fermions

42. Consider the system of three non-interacting electrons in ground state of one-dimensional infinite square well of width L

$$V(x)=\begin{cases}0 & \text{if } x<L\\ \infty & \text{otherwise}\end{cases}$$

$\mu_B\sigma$

$\frac{n\pi^2\hbar^2}{2mL^2}$

The system is subjected to a uniform magnetic field $\vec{B}$. The ground state energy of the sytsem is $\left(\mu_B=\frac{e\hbar}{2m}=\text{Bohr magnetron}\right)$

(a) $\frac{3h^2}{8mL^2}\pm\mu_B B$ (b) $\frac{3h^2}{4mL^2}\pm\mu_B B$ (c) $\frac{3h^2}{mL^2}+\mu_B B$ (d) $\frac{3h^2}{4mL^2}-\mu_B B$

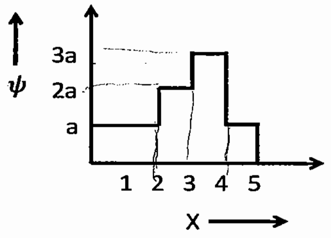
43. A particle of mass $m$ moves (non-relativistically) in three-dimensional potential

$$V=\frac{1}{2}k\left(x^2+y^2+z^2+\lambda xy\right)$$

where, $\lambda$ is a small parameter.
The first excited state energy corresponding to unperturbed Hamiltonian, is

(a) $\frac{1}{2}\hbar\omega-\lambda$ (b) $\frac{3}{2}\hbar\omega-\lambda\hbar\omega$ (c) $\frac{5}{2}\hbar\omega$ (d) $\frac{3}{2}\hbar\omega$

44. The wave function of a particle constrained to move in one dimension is shown in the graph below:



$14^{-16}$

12

The probability that the particle will be found between x = 2 and x = 4, will be 3a

45. Suppose $|\varphi_m\rangle$ and $|\varphi_n\rangle$ stationary state wave functions of a particle in a deep square well potential. Which of the following statements about $\langle\varphi_m\,|\,p_x^r\,|\,\varphi_n\rangle=0$, is true?

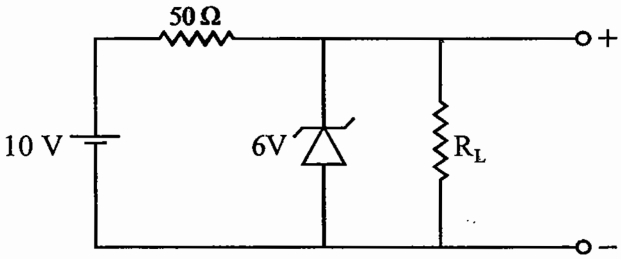
(a) r is odd integer and (m + n) is odd

(b) r is odd integer and (m + n) is even

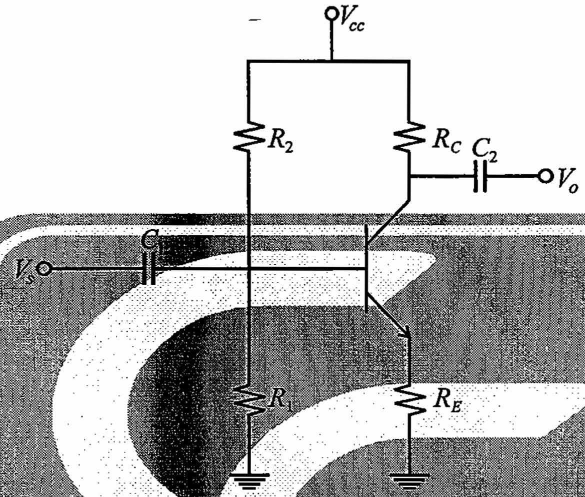
(c) r is even integer and (m + n) is even

(d) none of these

**46.** For the given circuit diagram the value of $R_L$ if $V_Z = 6V$, $I_Z = 5$ mA is(in the unit of $\Omega$) ______

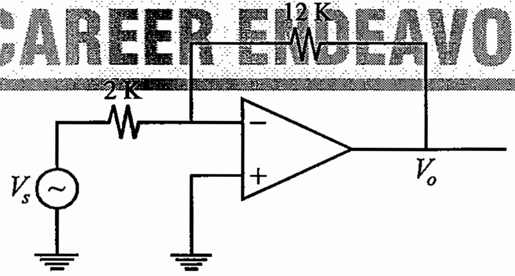


**47.** For the given circuit diagram shown determine voltage gain (approximate)



(a) $\frac{+R_C}{R_E}$ (b) $\frac{-R_C}{R_E}$ (c) $\frac{-(R_C+R_E)}{2}$ (d) None

**48.** For the given opamp network shown determine voltage gain if source impedance is 1 K



(a) – 6 (b) – 4 (c) 7 (d) 5

**49.** When non-relativistic neutrons of kinetic energy $E_0$ collide head-on with stationary nuclei of mass number $A$, the smallest energy that elastically scattered neutrons can have is (approximately)

(a) $E_{\min} = E_0\left(\frac{A-1}{A+1}\right)$ (b) $E_{\min} = E_0\left(\frac{A+1}{A-1}\right)$

(c) $E_{\min} = E_0\left(\frac{A+1}{A-1}\right)^2$ (d) $E_{\min} = E_0\left(\frac{A-1}{A+1}\right)^2$

50. A $\pi^\circ$ **meson** with momentum 135 GeV decays **into two photons.** If the proper meanlife of $\pi^\circ$ is 8.5 × $10^{-17}$ sec, **and the** rest mass is 135 $\frac{\text{MeV}}{c^2}$. The distance **travelled** by $\pi^\circ$ before decaying is

(a) $2.55\times10^{-6}m$ (b) $2.55\times10^{-5}m$ (c) $2.55\times10^{-4}m$ (d) $2.55\times10^{-8}m$

51. Consider the $\alpha$-decay of $^{240}_{94}\text{Pu}$

$$^{240}_{94}\text{Pu} \longrightarrow {}^{236}_{92}\text{U} + {}^{4}_{2}\text{He}$$

The **emitted** $\alpha$-particles are observed to have **energies** 5.17 MeV and 5.12 MeV. The energy of the emitted photon **is**________MeV.

52. A **certain** spectral line of wavelength 4266.73Å **produces a** normal Zeeman pattern when it is placed under a **external magnetic** field 4 $Wb/m^2$. The wavelengths **of the** $\sigma$-components of the Zeeman pattern will be

(a) 4266.4Å, 4267.06Å (b) 4266.1Å, 4267.36Å
(c) 4265.8Å, 4267.66Å (d) 4265.1Å, 4267.66Å

53. The **rotational** spectra of rigid diatomic molecule contains spectral lines separated by a distance 0.6Å. The moment of inertia of the molecule will be

(a) $9.3\times10^{-34}$ kg-m$^2$ (b) $6.3\times10^{-34}$ kg-m$^2$ (c) $9.3\times10^{-46}$ kg-m$^2$ (d) $6.3\times10^{-46}$ kg-m$^2$

54. The **far infrared** rotational absorption spectrum of a diatomic molecule shows equidistant lines with spacing 20 cm$^{-1}$. The position of the first stokes line in the rotational Raman spectrum of this molecule with respect to Rayleigh line is

(a) 20 cm$^{-1}$ (b) 40 cm$^{-1}$ (c) 60 cm$^{-1}$ (d) 120 cm$^{-1}$

55. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation
$Q = q^\alpha \cos\beta p, P = q^\alpha \sin\beta p$ is canonical for

(a) $\alpha = 2, \beta = \frac{1}{2}$ (b) $\alpha = 2, \beta = 2$ (c) $\alpha = 1, \beta = 1$ (d) $\alpha = \frac{1}{2}, \beta = 2$

56. The **Lagrangian** of a diatomic molecule is given by $L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}x_1x_2$ where, m is the mass each atom and $x_1, x_2$ are displacements from equilibrium position **and** k > 0. The normal frequencies are

(a) $\pm\left(\frac{k}{m}\right)^{1/2}$ (b) $\pm\left(\frac{k}{m}\right)^{1/4}$ (c) $\pm\left(\frac{k}{2m}\right)^{1/4}$ (d) $\pm\left(\frac{k}{2m}\right)^{1/2}$

57. The **moment** of inertia tensor of a rigid body is **given by** $I = \begin{bmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{bmatrix}$

$\hat{n} I \hat{n}$

$4 + 4\sqrt{3}$

Moment of inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is

$4 \quad 2$
$2\sqrt{3} \quad \sqrt{3} \quad 2+\sqrt{3}+2$
$-2 \quad 2$

(a) 6 (b) 5 (c) 2 (d) 8/3

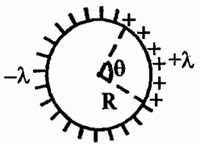
58. The Hamiltonian of a particle is $H = \dfrac{p^2}{2m} + pq$, where $q$ is generalised coordinate and $p$ is the corresponding canonical momentum. The Lagrangian is

H= pq - L
L = pq - H

(a) $\dfrac{m}{2}\left(\dfrac{dq}{dt}+q\right)^2$ (b) $\dfrac{m}{2}\left(\dfrac{dq}{dt}-q\right)^2$

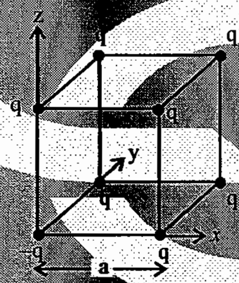
(c) $\dfrac{m}{2}\left[\left(\dfrac{dq}{dt}\right)^2+q\dfrac{dq}{dt}-q^2\right]$ (d) $\dfrac{m}{2}\left[\left(\dfrac{dq}{dt}\right)^2-q\dfrac{dq}{dt}+q^2\right]$

59. In the figure shown some portion of the ring has $+\lambda$ charge density while remaining portion has $-\lambda$ charge density radius of the ring is R. Electric field at the centre of the ring is



(a) $\dfrac{\lambda \sin\frac{\theta}{2}}{\pi \epsilon_0 R}$ (b) $\dfrac{2\lambda \sin\frac{\theta}{2}}{\pi \epsilon_0 R}$ (c) $\dfrac{\lambda \sin\frac{\theta}{2}}{2\pi \epsilon_0 R}$ (d) $\dfrac{\lambda \sin\frac{\theta}{2}}{4\pi \epsilon_0 R}$

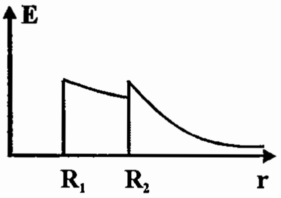
60. The cube shown in figure has charge +q at seven corners and a charge −q at one corner. Electric field vector at the centre of the cube is

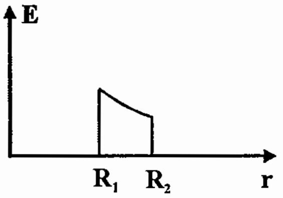


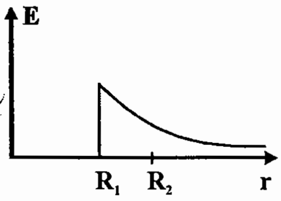
(a) $-\dfrac{q}{3\pi\varepsilon_0}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (b) $\dfrac{q}{3\sqrt{3}\pi\varepsilon_0}\left(\hat{i}+\hat{j}+\hat{k}\right)$

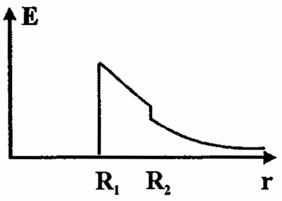
(c) $\dfrac{2q}{3\pi\varepsilon_0}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (d) $\dfrac{-2q}{3\sqrt{3}\pi\varepsilon_0}\left(\hat{i}+\hat{j}+\hat{k}\right)$

61. A dielectric shell of inner and outer radii $R_1$ and $R_2$ hsa polarization $\vec{P} = \dfrac{P_0\hat{r}}{r^2}$. The radial variation of electric field is given by which of the following graph.

(a)


(b)


(c)


(d)


62. Three straight long wires each carrying $15\sqrt{3}$ Amp current are placed at corners of an equilateral triangle of side 15cm. The wires are perpendicular to the plane of triangle and extend upto infinity both above and below it. If current in two wires are same direction and one in opposite direction then magnetic field at the centroid of triangle is ____________ $\times 10^{-3}$ T.

63. An intrinsic semiconductor with mass of a hole $m_h$, and mass of an electron $m_e$ is at a finite temperature T. If the top of the valence band energy is $E_V$ and the bottom of the conduction band energy is $E_C$, the Fermi energy of the semiconductor is

(a) $E_F = \left(\frac{E_V + E_C}{2}\right) - \frac{3}{4} k_B T \ln\left(\frac{m_h}{m_e}\right)$

(b) $E_F = \left(\frac{k_B T}{2}\right) + \frac{3}{4}(E_V + E_C) \ln\left(\frac{m_h}{m_e}\right)$

(c) $E_F = \left(\frac{E_V + E_C}{2}\right) + \frac{3}{4} k_B T \ln\left(\frac{m_h}{m_e}\right)$

(d) $E_F = \left(\frac{k_B T}{2}\right) - \frac{3}{4}(E_V + E_C) \ln\left(\frac{m_h}{m_e}\right)$

64. The energy $E(\vec{k})$ of electrons of wave vector $\vec{k}$ in a solid is given by $E(\vec{k}) = Ak^2 + Bk^4$, where A and B are constants. The effective mass of the electron at $|\vec{k}| = k_0$ is

(a) $Ak_0^2$

(b) $\frac{\hbar^2}{2A}$

(c) $\frac{\hbar^2}{2A + 12Bk_0^2}$

(d) $\frac{\hbar^2}{Bk_0^2}$

65. For a conventional superconductor, which of the following statements is not TRUE?

(a) Specific heat is discontinuous at transition temperature $T_C$.

(b) The resistivity falls sharply at $T_C$.

(c) It is diamagnetic below $T_C$.

(d) It is paramagnetic below $T_C$.

# TEST SERIES FOR GATE

## BOOKLET SERIES A

**Paper Code: PH**

**Test Type: TEST SERIES**

**Duration: 3:00 Hours**

# PHYSICS

**Date: 07-01-2015**

**Maximum Marks: 100**

### Read the following instructions carefully:

1. Attempt all the questions.
2. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory. The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.
   The subject specific GATE paper section consists of **55 questions**, out of which question numbers 11 to 35 are of 1-mark each, while question numbers 36 to 65 are of 2-mark each.
3. The question paper may consist of questions of **multiple choice type** (MCQ) and **numerical answer type**.
4. Multiple choice type questions will have four choices against (a), (b), (c), (d), out of which only **ONE** is the correct answer.
5. For numerical answer type questions, each question will have a numerical answer and there will not be any choices.
6. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of ⅓ marks for a **1-mark** question and ⅔ marks for a **2-mark** question.
7. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
8. Non-programmable type Calculator is allowed

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**South Delhi Centre:**
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**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
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## Some Useful Constants

| | |
|---|---|
| Speed of light in free space | $c = 3 \times 10^{8}\, m/s$ |
| Boltzmann constant | $k_B = 1.380 \times 10^{-23}\, J/K$ |
| Planck's constant | $h = 6.626 \times 10^{-34}\, J.s$ |
| Electron charge | $e = 1.602 \times 10^{-19}\, C$ |
| Permittivity of free space | $\varepsilon_0 = 8.854 \times 10^{-12}\, C^2/N.m^2$ |
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7}\, H/m$ |
| Mass of electron | $m_e = 9.31 \times 10^{-31}\, kg$ |
| Mass of proton | $m_p = 1.67 \times 10^{-27}\, kg$ |
| Mass of neutron | $m_n = 1.675 \times 10^{-27}\, kg$ |

**Q.1-Q. 5 carry ONE mark each.**

1. Choose the most appropriate word from the options given below to complete the following sentence.
**The great scientists like Newton and Galileo thought of science as an account of the world as it is, and the only responsibility that they _____________ was to tell the truth.**
[✓ (a) acknowledged] (b) negated (c) behaved (d) mistimed

2. Which one of the following options is the closest in meaning to the word given below?
**Cantankerous**
(a) Freedom (b) meticulous (c) bad tempered (d) coercion

3. There are 2188 bag of rice look alike same but one is slightly heavier. To determine the heavier one, maximum how many times you use balance sheet.
(a) 2188 (b) 2187 (c) 8 (d) can not say

4. If $a$ and $b$ are real numbers and $a > b$, then which of the following is true always
(a) $|a| > |b|$ (b) $a^2 > b^2$ [✓ (c) $a(a+1) > b(b+1)$] (d) $2b - 1 < 2a - 1$

5. Which of the following term is wrong.
52, 63, 94, 46, 18, 100
(a) 52 (b) 46 (c) 18 (d) 100

**Q.6-Q. 10 carry TWO marks each.**

6. One of the parts (A, B, C D) in the sentence given below contains an error. Which one of the following is incorrect?
**An advertisement can be defined as a massage intended to inform, persuade and remind both the present and prospective customers about a product or an event.**
(a) be defined as a massage
(b) to inform, persuade and remind
(c) both the present and prospective customers
(d) about a product or an event

7. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
**Exercise: Strong**
(a) Perform: Timid (b) Guard: Alert (c) Decide: Shrewd (d) Read: Knowledgeable

8. Consider the following statement
(I) where there is smoge, there is fire
(II) where there is fire, there is always.
Which of the following is true
(a) only I [✓ (b) only II] (c) either I or II (d) Neither I nor II

9. What is the remainder when $2^{11}(2^{10} + 1)$ is divided by 15
(a) 2 (b) 3 (c) 5 [✓ (d) 10]

10. A country, playing cricket, wanted its success record for the season to be 80%. If out of 40 grams already played, it has won 34 games, how many games does it have to win from the remaining 20 games to achieve this target. What is the required success rate in the remaining games?
(a) 16, 80% (b) 14, 70% (c) 12, 60% [✓ (d) 10, 50%]

$2^{11}$
2×2×2×2×2×2×2
1024
2048
1024
512

80 40
40 20
34

$-7$ $\frac{\pi}{2}$ $-13$

**Q.11-Q.35 carry one mark each.**

$-25 \pm \frac{3}{2}$ $\frac{-b^2 \pm \sqrt{b^2-4ac}}{2a}$

11. The inverse Laplace's transform of 1 is:

(a) 1 [✓ (b) δ(t)] (c) δ(t–1) (d) t

$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 4-\lambda \end{bmatrix}$ $(1-\lambda)(\ldots-\lambda) =$

$4-\lambda-4\lambda+\lambda^2$

$\lambda^2-45\lambda+4$

12. The matrix P is given by $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$. The largest eigenvalue of the matrix $4A^{-1}+3A+2I$ is equal to ________

13. To change $\left(x^2y^2+xy\right)dx = dy$ to linear form of first order, the necessary substitution needed is equal to

$y^2 = v$ $2x\,dx = dv$

$2y\,dy = dv$ $x\,dx = \frac{dv}{2}$

$y\,dy = \frac{dv}{2}$ $+\frac{1}{x} = \frac{dx}{dv}$

[✓ (a) $x^2 = v$] (b) $y^2 = v$ (c) $-\frac{1}{x} = v$ (d) $-\frac{1}{y} = v$

14. The vector potential $\vec{A}$ corresponding to a constant magnetic field $\vec{B}$ in z-direction can be represented by

(a) $\frac{B}{2}\left(\hat{i}x-\hat{j}y\right)$ (b) $\frac{B}{2}\left(\hat{j}x-\hat{i}y\right)$ (c) $-Bz\hat{k}$ [✓ (d) $B\left(\hat{j}x-\hat{i}y\right)$]

15. The resultant of these two components of electric field:

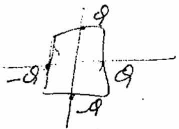
$$E_x = E_0 \sin\left(kz-\omega t+\frac{\pi}{3}\right);\ E_y = E_0 \sin\left(kz-\omega t-\frac{\pi}{6}\right)$$

is ________________________ polarised

[✓ (a) linearly] (b) circularly (c) elliptically (d) unpolarised

16. Four point charges are placed in a plane at following positions

$+Q(1,0), -Q(-1,0), +Q(0,1)$ and $-Q(0,-1)$.



At large distances, the electrostatic potential due to this charge distribution will be dominated by

(a) Monopole moment (b) Dipole moment

[✓ (c) Quadrapole moment] (d) Octapolemoment.

$\int_{-\infty}^{\infty} \delta(x)\,dx = \phi$

17. The S.I. Unit of probability current density in 3 dimensional position space:

(a) $sec^{-1}$ (b) $m^{-2}sec^{-1}$ (c) $m^{-1}sec^{-1}$ (d) dimensionless

18. State of a particle in a infinite potential well is given to be

$\psi(x) = A\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)$ where $0 \le x \le L$

Which of the following values will the energy measurement give?

[✓ (a) $\frac{\pi^2\hbar^2}{2mL^2}, \frac{9\pi^2\hbar^2}{2mL^2}$] (b) $\frac{2\pi^2\hbar^2}{mL^2}, \frac{9\pi^2\hbar^2}{2mL^2}$ (c) $\frac{\pi^2\hbar^2}{mL^2}, \frac{2\pi^2\hbar^2}{mL^2}$ (d) $\frac{8\pi^2\hbar^2}{mL^2}, \frac{2\pi^2\hbar^2}{mL^2}$

19. Which of the following wavefunctions is the eigenfunction of the parity operator?

(a) $\psi(x) = x^2 e^{-\left(\frac{x-a}{b^2}\right)^2}$ (b) $\psi(x) = Axe^{-\frac{x^2}{a^2}}$ (c) $\psi(x) = A(x+a)^2 e^{-\frac{x^2}{a^2}}$ (d) All of the above

20. A particle of mass m is in a simple harmonic oscillator potential $V = x^2$. If the ground state wave function is $Ae^{-ax^2}$, what is the expression for constant a equal to?

$\alpha^2 = \frac{2m}{\hbar^2}$ $\frac{2m}{\frac{h}{2\pi}} = \frac{4\pi m}{h}$ $\frac{2\pi^2 m^2}{h^2}$

$H = \frac{p^2}{2m} + x^2$ $\frac{\hbar^2 \pi^2 n^2}{2mx^2}$

(a) $\frac{2\pi\left(\frac{m}{2}\right)^{1/2}}{h}$ [✓ (b) $\frac{2\pi\left(\frac{1}{m}\right)^{1/2}}{h}$] (c) $\frac{h\left(\frac{m}{2}\right)^{1/2}}{2\pi}$ (d) $\frac{h(m)^{1/2}}{2\pi}$

21. Four particles of equal mass (m) and negligible radius are connected at ends of a massless square. Moment of inertia of the system about any one of the sides is (in the units of $ml^2$) $ml^2/4$

22. If a particle of mass 1 kg is dropped from a height 100 m above the surface of the earth at a latitude 40° N. What is the displacement of the point of impact due to coriolis force?
(a) 0.037 m eastwards (b) 0.027 m eastwards (c) 0.017 m eastwards, (d) 0.007 m eastwards.

1kg
100m

23. The Lagrangian for a particle is given by $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - mgz$, then

[✓ (a) z - compoennt of linear momentum is conserved]
(b) x - component of linear momentum is conserved
(c) y - component of linear momentum is conserved
(d) x & y component of linear momentum are conserved

24. When a system is held at constant temperature and pressure in a state of equilibrium, then it attains a minimum value of:

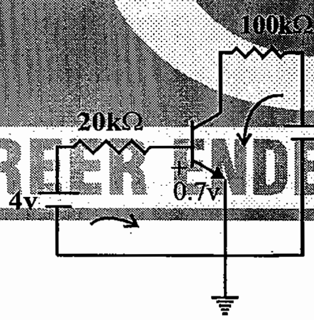
(a) Internal energy (b) Enthalpy [✓ (c) Helmholtz energy] (d) Gibb's free energy

25. The specific heat of an ideal Fermi gas in 3-D at very low temperature T varies as

(a) T [✓ (b) $T^{3/2}$] (c) $T^2$ (d) $T^{1/2}$

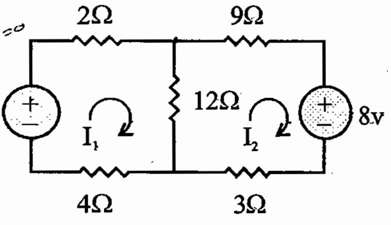
26. Heat Q is added to a monatomic ideal gas under conditions of constant volume, resulting in a temperature change $\Delta T$. The heat that will be required to produce the same temperature change, if it is added under conditions of constant pressure is (in the units of Q) ________

27. For the given transistor shown in figure which is operating in active mode. The value of $I_C$ and $V_{CE}$ is:



(a) 8.25 mA, 0.2 v (b) 1.25 mA, 5.175 v (c) 8.25 mA, 5.175 v (d) 1.25 mA, 0.2v

28. For the given circuit shown the value of $I_1$ is (in the units of Amp) 0.5



$-12 - I_1 2 + (I_1 - I_2) 12 + I_1 2 = 0$
$-12 - I_1 2 + I_1 12 + I_1 4 = 0$
$-12 - 2I_1 + 26I_1 = 0$
$-12 + 24I_1 = 0$
$24I_1 = 12$

29. About FET correct option is:
(a) The minimum current occurs at $V_{GS}$ = pinch of voltage
(b) The maximum current for JFET occurs at $V_{GS}$ = 0V
(c) JFET is non-linear voltage controlled device
[✓ (d) All above are correct]

30. A free neutron decays to a proton but a free proton does not decay to a neutron. This is because:

(a) neutron is a composite particle made of a proton and an electron wheras proton is a fundamental particle.
(b) neutron is an uncharged particle whereas proton is a charged particle
(c) neutron has large rest mass than the proton
(d) weak forces can operate in a neutron but not in a proton

31. Which nuclei among (i) ${}^{7}_{3}\text{Li}$ & ${}^{8}_{3}\text{Li}$, (ii) ${}^{13}_{6}\text{C}$ & ${}^{15}_{6}\text{C}$ will be more stable?

(a) ${}^{7}_{3}\text{Li},\ {}^{13}_{6}\text{C}$ (b) ${}^{7}_{3}\text{Li},\ {}^{15}_{6}\text{C}$ (c) ${}^{8}_{3}\text{Li},\ {}^{13}_{6}\text{C}$ (d) ${}^{8}_{3}\text{Li},\ {}^{15}_{6}\text{C}$

32. The translational vectors of a space lattice are given:

$$\vec{P}=\frac{\hat{i}}{2}+\frac{\sqrt{3}}{2}\hat{j},\quad \vec{Q}=-\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}, \vec{R}=\hat{k}$$

$P(Q \times R) =$

3. 1

The volume of the cell is equal to

(a) $\frac{2}{\sqrt{3}}$ [✓ (b) $\frac{\sqrt{3}}{2}$] (c) $2\sqrt{3}$ (d) $3\sqrt{2}$

33. The ratio between the kinetic energies of an electron in a two-dimensional square lattice which has $k_x = k_y = \frac{\pi}{a}$ and an electron has $k_x = \frac{\pi}{a}$ and $k_y = 0$, is ________________

34. The hyperfine structure arises due to:
(a) interaction between nuclear spin angular momentum & internal magnetic field produced by the surrounding electrons in the atom only.
(b) is due to the presence of different of isotopic species in the element under consideration only.
(c) the interaction between nuclear spin angular momentum & internal magnetic field produced by the surrounding electrons in the atom & due to isotope effect both
[✓ (d) spin-orbit interaction of electron & relativistic correction both]

35. The wave numbers of stokes and anti-stokes lines in a Raman spectrum are 18116 cm$^{-1}$ and 18514 cm$^{-1}$ respectively. The wave number of exciting line is ________________

**Q.36-Q.65 carry TWO marks each.**

$k = \frac{1}{\lambda} + \frac{1}{\lambda'}$

36. The value of the complex integral $\oint_C \tan z\, dz$, where C is the closed curve $|z-\pi/2| = \frac{\pi}{2}$ is (in the units of $\pi i$)

________________

37. The value of integral $\int_0^\infty x^{n-1} e^{-m^2x^2} dx$ is equal to

[✓ (a) $\frac{1}{2m^n}\Gamma\left(\frac{n}{2}\right)$] (b) $\frac{1}{2m^n}\Gamma\left(\frac{n}{2}-1\right)$ (c) $\frac{1}{2m^{n/2}}\Gamma\left(\frac{n}{2}\right)$ (d) $\frac{1}{2m^{n/2}}\Gamma\left(\frac{n}{2}-1\right)$

38. The value of the integral $\int_{-1}^{+1} P_n(x)\left(1-2xt+t^2\right)^{-1/2} dx$ can be simplified to $\left[P_n(x)\right.$ is the legendre polynomial of order 'n']

[✓ (a) $\frac{2}{2n+1}$] (b) $\frac{2t^n}{2n+1}$ (c) $\frac{t^n}{2n+1}$ (d) None of these

39. The wavefunction of a particle of mass in a deep square well potential, extended from x = 0 to L, at t = 0 is, $|\psi(0)\rangle = \frac{1}{\sqrt{5}}\left[|\phi_1\rangle + 2|\phi_2\rangle\right]$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are the ground and first excited state wave functions. The wavefunction at time $t = \frac{2mL^2}{\pi\hbar^2}$ is:

(a) $\frac{-1}{\sqrt{5}}\left[|\phi_1\rangle + 2|\phi_2\rangle\right]$ (b) $\frac{+1}{\sqrt{5}}\left[2|\phi_1\rangle + |\phi_2\rangle\right]$ (c) $\frac{1}{\sqrt{5}}\left[|\phi_1\rangle - 2|\phi_2\rangle\right]$ (d) $\frac{-1}{\sqrt{5}}\left[|\phi_1\rangle - 2|\phi_2\rangle\right]$

1 + 8

40. A spin-1/2 particle is in a state described by the $A\begin{pmatrix}1+i\\2\end{pmatrix}$. Where A is normalization constant. The probability of finding the particle with spin projection $S_z = -\frac{\hbar}{2}$ is ________________

41. A particle of mass m is moving in a potential weil of the type:

$V(x) = \begin{matrix} 0 & \text{it } 0 < x < 2L \\ \infty & \text{otherwise} \end{matrix}$

If we perturb the system simply by raising the floor of the well by a constant amount $V_0$ only half way across the well. The first order correction in energy is (in the units of $V_0$) $\frac{\pi^2\hbar^2}{2mL^2} + V$

42. The one-dimensional quantum mechanical potential energy of a particle of mass m is given by

$V(x) = V_0\delta(x), \quad -a < x < \infty$

$= \infty, \quad x < -a,$

At t = 0, the wave function of the particle is completely confined to region −a < x < 0. The normalized wave function of the particle at time t = 0 is

(a) $\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \quad -a < x < \infty,$
$0, \quad x < -a$

(b) $0, \quad -a < x < \infty$
$\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \quad x < -a$

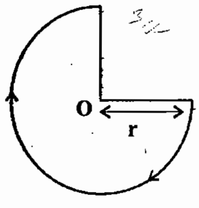
(c) $\frac{\sqrt{\frac{2}{a}}\sin n\pi}{a}(x+a), \quad -a < x < \infty$
$0, \quad x < -a$

(d) 0, everywhere

43. An infinite cylindrical region of radius R and axis along z-direction has a uniform increasing magnetic induction $\vec{B} = B(t)\hat{k}$. Direction of induced electric field $\vec{E}(t)$ and poynting vector $\vec{S}$ at the point on the surface of the cylinder will be, respectively? $S = \frac{E \times B}{\mu}$

(a) $-\hat{\phi}, \hat{r}$ (b) $\hat{\phi}, \hat{r}$ (c) $-\hat{\phi}, -\hat{r}$ (d) $\hat{r}, -\hat{\phi}$

44. A current I flows in the loop having circular arc of radius 'r' (shown in figure). The magnetic field at point 'O' is equal to



(a) $\frac{\mu_0 I}{2r}$ (b) $\frac{3\mu_0 I}{8r}$ (c) $\frac{3\mu_0 I}{4r}$ (d) $\frac{\mu_0 I}{4r}$

45. A carnot engine has a efficiency of 1/6 on reducing the sink temperature by 65°C, the efficiency becomes $\frac{1}{3}$. The source temperature (in the units of K) is ____________

46. A zipper has N links. Each link has two states : state 1 means it is closed and has energy 0 and state 2 means it is open with energy $\in$ . The zipper can only unzip from the left end and the sth link cannot open unless all the links to its left (1,2,3.............. ,s– 1) are already open. The partition function for the zipper is

(a) $1-\exp\left(-\frac{(N+1)\in}{kT}\right)$ (b) $\frac{1-\exp\left(-\frac{N\in}{kT}\right)}{1-\exp\left(-\frac{\in}{kT}\right)}$ (c) $\frac{1-\exp\left(-\frac{(N+1)\in}{kT}\right)}{1-\exp\left(-\frac{\in}{kT}\right)}$ (d) $1+\exp\left(-\frac{N\in}{kT}\right)$

47. Imagine earth to be symmetric top $(I_1 = I_2 < I_3)$ undergoing a torque free motion. If $\left(\frac{I_3 - I_1}{I_1}\right) = 0.00333$, the period of precession of the axis of rotation about the symmetry axis is ____________ days.

48. For the force $\underline{F}(r) = -kr^{-4}\hat{r}$, the relation between the average kinetic energy & potential energy is:

(a) $2\overline{T}+\overline{V}=0$ (b) $2\overline{T}+3\overline{V}=0$ (c) $\overline{T}+\overline{V}=0$ (d) $3\overline{T}+2V=0$

$T = \frac{(\alpha+1)}{2} V$

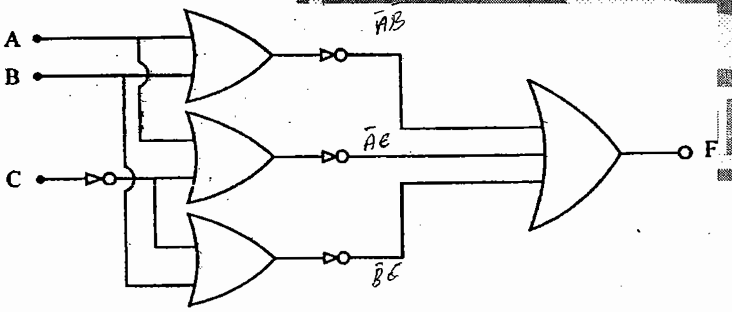
49. A particle of mass 'm' is moving in a potential of the form $V(x,y,z) = \frac{1}{2}m\omega^2(3x^2+3y^2+2z^2+2xy)$. The oscillation frequencies of three normal modes of the particle are given by,

(a) $\omega$, $\sqrt{3}\omega$ and $\sqrt{3}\omega$ (b) $\sqrt{2}\omega$, $\sqrt{3}\omega$ and $\sqrt{3}\omega$

(c) $\sqrt{2}\omega$, $\sqrt{2}\omega$ and $2\omega$ (d) $\sqrt{2}\omega$, $2\omega$ and $2\omega$

$\omega = 2\pi\delta$

50.

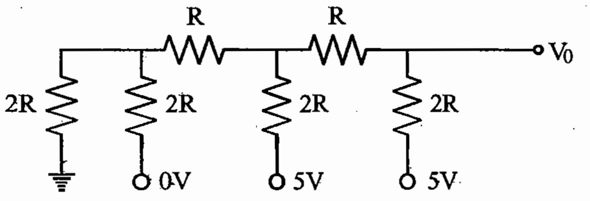


Which one of the following gives the output F for the logic diagram show above?

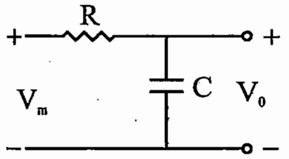
(a) $\bar{A}\,\bar{B}+\bar{A}\,C+\bar{B}\,C$ (b) $\overline{AB}+\overline{A\bar{C}}+\overline{B\bar{C}}$

(c) $\left(\overline{A+B}\right)+\left(\overline{B+C}\right)+\left(\overline{C+A}\right)$ (d) $A+B+\bar{C}$

51. In the R-2R ladder type DAC is shown in figure. For $S_0 = 5V$, $S_1 = 5V$, $S_2 = 5V$, the output of DAC is (in the units of volts) ____________



52. For the given circuit diagram determine 3-dB cut off frequency shown in the figure.



(a) $\frac{1}{2\pi RC}$ rad / sec (b) $\frac{1}{RC}$ Hertz (c) $\frac{1}{2\pi RC}$ Hertz (d) $2\pi RC$ rad/ sec

53. Consider the successive radioactive disintegration

$$X \rightarrow Y \rightarrow Z$$

If $\lambda_1$ and $\lambda_2$ are the decay constants of X and Y and $N_1$ and $N_2$ be the number of atoms of X and Y at an instant 't' then we can write. The time at which $N_2$ will be maximum

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}\{\exp(-\lambda_2 t) - \exp(-\lambda_1 t)\}$$

[where $N_{10} \equiv$ number of atoms of X at time t = 0]

(a) $\frac{1}{\lambda_1}\ell n\frac{\lambda_1}{\lambda_2}$ (b) $\frac{1}{\lambda_2}\ell n\frac{\lambda_1}{\lambda_2}$ (c) $\frac{1}{\lambda_1 + \lambda_2}\ell n\frac{\lambda_1}{\lambda_2}$ (d) $\frac{1}{\lambda_1 - \lambda_2}\ell n\left(\frac{\lambda_1}{\lambda_2}\right)$

54. Supply the missing particles in the reaction

$$K^- + p \longrightarrow \Omega^- + \ldots + \ldots$$

(a) $\pi^+$, $\pi^0$ (b) $\Lambda^0$, n (c) $\Xi^0$, $\Delta^+$ (d) $K^+$, $K^0$

55. The number of fission per second ina 100-MW reactor is

(a) $10^6$ (b) $10^{12}$ (c) $10^{18}$ (d) $10^{24}$

56. The $J = 0 \rightarrow J = 1$ rotational absorption line occurs at $1.153 \times 10^{11}$ cycles/sec in ${}^{n}C^{16}O$ and at $1.102 \times 10^{11}$ cycles / sec in ${}^{12}C^{16}O$. The mass number of unknown carbon isotope is ____________

57. The density of copper is $8.94 \times 10^3$ kg/m³, and its atomic mass is 63.5 amu. If resistivity of the copper at 20°C is: $\rho = 1.72 \times 10^{-8}\ \Omega - m$. What is relaxation time $(\tau)$ of electron? Each copper atom contributes one free electron to the metal.

(a) $2.5 \times 10^{-16}$ sec. (b) $2.5 \times 10^{-15}$ sec. (c) $2.5 \times 10^{-14}$ sec. (d) $2.5 \times 10^{-13}$ sec.

58. We consider a gas of N diatomic non-interacting diatomic molecule in thermal equilibrium at temprature T. The rotational partition function of gas is:

(a) $\frac{V^N}{h^{3N}}(2\pi mkT)^{3N/2}$ (b) $\frac{8\pi^2 IkT}{h^2}$ (c) $\frac{V}{h^3}(2\pi mkT)^{3/2}$ (d) $\left(\frac{8\pi^2 IkT}{h^2}\right)^N$

59. An electron is moving with a velocity of 0.85*c* in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is (in the units of c) ____________

60. The tight binding dispersion (E-k) diagram relation for electrons in one-dimensional array of atoms having lattice constant 'a' and length 'L' is:

$$\boxed{E = \alpha - \beta - \gamma \cos ka}$$

where $\alpha, \beta$ and $\gamma$ are constants and 'k' is wave-vector.
The density of states (including spin degeneracy) in the band for (ka<<1) is given by:

(a) $\frac{L}{2\pi\gamma ka^2}$ [✓ (b)] $\frac{L}{2\pi\gamma a^2}$ (c) $\frac{L}{\pi\gamma ka^2}$ (d) $\frac{2L}{\pi\gamma ka^2}$

61. The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where $a$ and $E_0$ are positive constants. The temperature of the system is

(a) negative for some energies (b) increases monotonically with energy
(c) decreases monotonically with energy [✓ (d)] zero

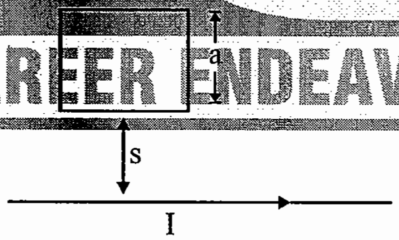
62. The energy levels of hydrogen atom are given by $E_n = -\frac{z^2}{n^2}(\text{Rydberg})$. The excitation energy of n = 2 level to n = 1 level for $He^+$ is (in the units of eV)

63. A charge distribution has the charge density given by $\rho = Q\{\delta(x - x_0) - \delta(x + x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

[✓ (a)] $\frac{2Q\hat{x}}{9\pi\epsilon_0 x_0^2}$ (b) $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^3}$ (c) $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^2}$ (d) $\frac{Q\hat{x}}{16\pi\epsilon_0 x_0^2}$

64. The spacing between vibrational energy levels in CO molecule is found to be $8.441 \times 10^{-2}$ eV. Given that the reduced mass of CO is $1.14 \times 10^{-26}$ kg, Planck's constant is $6.626 \times 10^{-34}$ Js and 1eV = $1.6 \times 10^{-19}$J. The force constant of the bond is CO molecule is (in the units of N/m)

65. A square loop of wire (side a) lies on a table at a distance s from a very long straight whcih carries a current I as shown in fig.



If someone now pull the loop directly away from the wire, at speed v.

The resistance of the loop is R then the magnitude & direction of the induced current is:

(a) $\frac{\mu_0 I}{2\pi sR}\frac{a^2 v}{(s+a)}$ (counter clockwise) (b) $\frac{\mu_0 Ia}{2\pi R}\ell n\left(\frac{s+a}{s}\right)$ (clockwise)

[✓ (c)] $\frac{\mu_0 Iav}{2\pi R}\ell n\left(\frac{s+a}{s}\right)$ (counter clockwise) (d) $\frac{\mu_0 Ia^2}{2\pi(s+a)}\frac{v}{R}$ (clock wise)

# TEST SERIES UGC-CSIR-NET/JRF Dec. 2014

BOOKLET SERIES E

Paper Code 05

Test Type: TEST SERIES

## PHYSICAL SCIENCES

**Duration: 3:00 Hours**

**Date: 09-12-2014**

**Maximum Marks: 200**

**Read the following instructions carefully:**

* Single Paper Test is divided into three Parts.

**Part - A:** This part shall carry 20 questions. The candidate shall be required to answer any 15 questions. Each question shall be of **2 marks.**

**Part - B:** This part shall contain 25 questions covering the topics given in the Part 'B' of syllabus. The candidates are required to answer any 20 questions. Each question shall be of **3.5 Marks**..

**Part - C:** This part shall contain **30** questions from Part - C of the syllabus. The candidates are required to answer any 20 questions. Each question shall be of **5 Marks**.

* Darken the appropriate bubbles with HB pencil/Ball Pen to write your answer.

* There will be negative marking @25% for each wrong answer.

* The candidates shall be allowed to carry the Question Paper Booklet after completion of the exam.

* For rough work, blank sheet is attached at the end of test booklet.

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48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
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# USEFUL FUNDAMENTAL CONSTANTS

| | | |
|---|---|---|
| m | Mass of electron | $9.11 \times 10^{-31}$ Kg |
| h | Planck's constant | $6.63 \times 10^{-34}$ J sec |
| e | Charge of electron | $1.6 \times 10^{-19}$ C |
| k | Boltzmann constant | $1.38 \times 10^{-23}$ J/K |
| c | Velocity of Light | $3.0 \times 10^{8}$ m/Sec |
| $1_eV$ | $1.6 \times 10^{-19}$ J | |
| amu | $1.67 \times 10^{-27}$ kg | |
| G | $6.67 \times 10^{-11}$ Nm$^2$kg$^{-2}$ | |
| $R_y$ | Rydberg constant | $1.097 \times 10^{7}$ m$^{-1}$ |
| $N_A$ | Avogadro number | $6.023 \times 10^{23}$ mole$^{-1}$ |
| $\varepsilon_0$ | $8.854 \times 10^{-12}$ Fm$^{-1}$ | |
| $\mu_0$ | $4\pi \times 10^{-7}$ Hm$^{-1}$ | |
| R | Molar Gas constants | $8.314$ J K$^{-1}$ mole$^{-1}$ |

## Part-A

$A \to >$
$B \to \geq$
$C \; =$
$D \; \leq$
$E \; \leq$
$F$

A
B

1. If $A > B$, $B \geq C$, $C = D$, $D \leq E$ and $E \leq F$, then which of the following is true
(a) $D = F$  (b) $B > D$  [✓ (c) $C \leq E$]  (d) $E > B$

2. To determine "if a, b, c are integers, is a + b + c even" which of the following statement is sufficient.
(I) a – b + c is even.  (ii) a + b – c is even.
(a) Only I  (b) Only II  (c) Both I and II  [✓ (d) Neither I nor II]

1 + 1 + 1 = 3
1 + 1 – 1 = 2
3 + 3 + 3 – 1
2 + 2 + 2 = 3

3. Fill in the blanks : 0010, 0011, 0101, ..., 1011
(a) 0001  (b) 0111  (c) 1010  (d) 1111

4. The area of the circle whose centre is at (0, 0) is $25\pi$. The circle passes through all the points excepts.
[✓ (a) (–5, 0)]  (b) (5, 0)  (c) (5, 5)  [✓ (d) (0, 5)]

5. A classroom has *r* rows of desks with *d* desks in each row. On a particular day when all pupils are present 3 seats are left vacant (one student per desk). The number of pupils in the class is
[✓ (a) dr–3]  (b) d + r +3  (c) dr + 3  (d) $\frac{r}{d}+3$

6. If $2 < r < 8$ and $1 < s < \frac{5}{2}$, which of the following expresses all possible values of rs
(a) $1 < rs < 5$  (b) $2 < rs < 20$  (c) $\frac{5}{2} < rs < B$  (d) $\frac{5}{2} < rs < 20$

7. The sum of the digits in the sum of the two digit prime numbers ending with 1 is
(a) 6  [✓ (b) 7]  (c) 8  [✓ (d) 9]

8. To see a circus the ticket rate is Rs. 25 for gents, Rs. 20 for ladies and Rs. 10 for children between 3 and 12 years. On a particular day, the total number of tickets sold is 2140 and the total collection is Rs. 35800. Which one of the following figure is infeasible to be the number of ladies who saw the circus on that day.
(a) 660  (b) 720  (c) 805  (d) 825

9. From a square board the maximum possible circle is cutout and the remaining area is called 'A'. Another identical square board is first devided into 64 equal squares as in chess board and the maximum possible circle is cut out from each one of the 64 squares. The remaining area is called 'B'. Which one of the following is true.
(a) A = 4B  [✓ (b) B = 4A]  (c) A = 2B  (d) A = B

**Common Data for Q.10 and Q.11:**
A farmer on being asked how many animals he had in his farm replied, they all are buffaloes except 48, all cows except 47, all goats except 46 and all horses except 45.

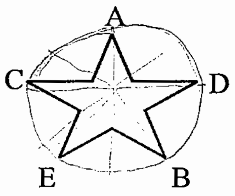
10. How many animals did the farmer have
(a) 186  (b) 124  (c) 93  (d) 62

11. How many cows are there in the farm
(a) 36  (b) 16  (c) 15  (d) 14

48
47
46
48

29
189

12. What is the sum of the 5 angles A, B, C, D and E in the following type of 5 cornered star.



(a) 180°  [✓ (b) 240°]  (c) 300°  (d) 360°

13. A landlord wanted to give some area of land for his sincere most worker. He called that worker and gave a fence of 120 meters long and told him to cover as much area as possible by fencing the land in a rectangular form. If the worker is clever enough, how much area of land he will get
(a) 900 $m^2$  (b) 3600 $m^2$  (c) 2000 $m^2$  (d) 800 $m^2$

14. There are 1248 students in college to choose a president. If 4 condidate contest for this post, what should be the minimum number of votes to be polled in favour of successful candidate with absolute majority
(a) 313  (b) 417  (c) 624  (d) 625

15. How many numbers are between 100 to 1000 which do not contain any '3' as a digit.
(a) 612  (b) 532  (c) 647  (d) 648

16. If $f(x)$ denotes the greatest integer less than or equal to the cube root of $3x$, then the sum of the series $\Sigma f(n)$ where n varies from 1 to 10 is
(a) 13  (b) 19  (c) 20  (d) 39

17. If the cost of a cone ice-cream is Rs. 12, what is the cost of the lower half portion of the ice-cream in the cone having the vertex
(a) 1.50 rs  (b) 3.00 rs  (c) 4.50 rs  (d) 6.00 rs

18. Find the next term
1, 4, 10, 20, 35, 56, 84, 120, ........ is
(a) 165  (b) 175  (c) 180  (d) 192

19. In the following list, one of the word is my password. If I were to tell you any letter of my password, then you would be able to tell me the number of vowels in my password. Which is my password
(a) CASH  (b) SNOW  (c) TIME  (d) IDEA

20. Find the odd man out
(a) TESRIS  (b) HATFER  (c) HOTREM  [✓ (d) LOVINI]

## Part-B

21. The vector $\vec{F} = z\hat{i} - 2x\hat{j} + y\hat{k}$ can be expressed in cyllindrical co-ordinates $(\rho, \varphi, z)$ as

(a) $(z\cos\varphi - 2\rho\sin\varphi\cos\varphi)\hat{e}_\rho - (z\sin\varphi + 2\rho\cos^2\varphi)\hat{e}_\varphi + \rho\sin\varphi\hat{e}_z$

(b) $(z\sin\varphi - 2\rho\sin\varphi\cos\varphi)\hat{e}_\rho - (z\sin\varphi + 2\rho\cos^2\varphi)\hat{e}_\varphi + \rho\cos\varphi\hat{e}_z$

(c) $(z\sin\varphi - 2\rho\sin\varphi\cos\varphi)\hat{e}_\rho + (z\sin\varphi + 2\rho\cos^2\varphi)\hat{e}_\varphi - \rho\cos\varphi\hat{e}_z$

(d) $(z\cos\varphi - 2\rho\sin\varphi\cos\varphi)\hat{e}_\rho + (z\sin\varphi + 2\rho\cos^2\varphi)\hat{e}_\varphi - \rho\sin\varphi\hat{e}_z$

22. The eigenvalues of the matrix $P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ are

(a) $\frac{1}{2}(\sqrt{3} \pm i)$ When $\theta = 45°$  (b) $\frac{1}{2}(1 \pm \sqrt{3}i)$ When $\theta = 60°$

(c) $\frac{1}{2}(\sqrt{3} \pm i)$ When $\theta = 60°$  (d) $\frac{1}{2}(1 \pm \sqrt{3}i)$ When $\theta = 30°$

23. If $A$ and $B$ are $3 \times 3$ real matrices such that rank$(AB) = 1$, then the rank$(BA)$ cannot be
[✓ (a) 0]  (b) 1  (c) 2  (d) 3

24. Consider the following complex integral:

$$\int_C z^2 (z-2)^{n-1}\, dz \;(\text{where } n \text{ is integer})$$

where $C : |z| = 3$, traversing in the anticlockwise direction. If $z = 2$ is a singular point of the given complex function and the value of above integral vanishes, which of the following statement is correct?

(a) $n < -2$ (b) $n > -2$ (c) $n = -2$ (d) none of these

25. The differential equation representing all circles centered at (1,0) is

(a) $x - y\dfrac{dy}{dx} = 1$ (b) $x + y\dfrac{dy}{dx} = 1$ (c) $y + x\dfrac{dy}{dx} = 1$ (d) $y - x\dfrac{dy}{dx} = 1$

26. A particle has the wave function

$$\psi(r) = N\, e^{-ar}$$

Where N is the normalization constant and $\alpha$ is known real parameter.
The expectation value of x-component of position is

(a) $\dfrac{3}{2\alpha}$ (b) $\dfrac{3}{\alpha}$ [✓ (c) $\dfrac{1}{2\alpha}$] (d) 0

27. A free particle is initially (at t=0) in a state corresponding to the wave function

$$\psi(r) = \left(\frac{\gamma}{\pi}\right)^{3/4} e^{-\gamma r^2/2}$$

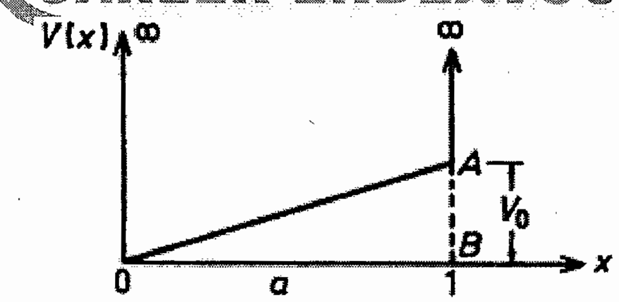
(a) The wave function is an eigen function of Hamiltonian only.

(b) The wave function is an eigen function of $L^2$ and $L_z$ only.

(c) The wave function is an eigen function of H, $L^2$ and $L_z$ at time $t = 0$ but not at all time.

(d) The wave function is an eigen function of H, $L^2$ and $L_z$ at time $t = 0$ and also at all time t.

28. The energy corrected to first order of second excited state for a particle confined to move in an infinite potential well of width a whose portion AB has been sliced off (AB is a straight line) is



(a) 0 (b) $\dfrac{9\hbar^2\pi^2}{2ma^2}$ [✓ (c) $\dfrac{9\hbar^2\pi^2}{2ma^2} + \dfrac{V_0}{2}$] (d) $\dfrac{9\hbar^2\pi^2}{2ma^2} + V_0$

29. Using the WKB approximation the allowed energies of nth state( for large n) of particle of mass m moving in the potential $V(x) = kx^8$, where $k$ is any constant, is given by

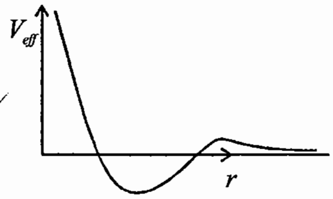
(a) $E_n \sim n^{4/5}$ [✓ (b) $E_n \sim n^{4/3}$] (c) $E_n \sim n^{5/4}$ (d) $E_n \sim n^{8/5}$

30. Let $E_T$ be the energy obtained variationally by using trial function $A\,e^{-\alpha r^2}$ (with $\alpha$ as a variational parameter) for harmonic oscillator. If the exact ground state energy is $E_H$, then

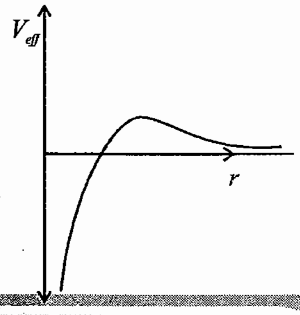
(a) $E_T = E_H$ (b) $E_T > E_H$ (c) $E_T \geq E_H$ (d) $E_T < E_H$

31. A particle is moving under a central potential $V(r) = -\frac{k}{r}\exp\left(\frac{-r}{a}\right)$, where $k > 0$, $a > 0$. The radial variation of effective potential is

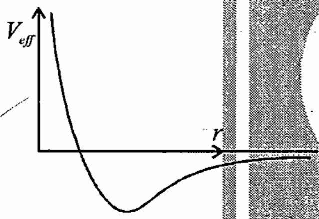
(a)



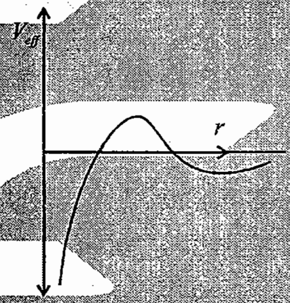
(b)



(c)



(d)



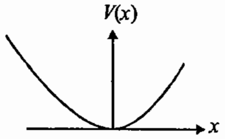
32. Generalised potential for a system is $u(q,\dot{q}) = \frac{(q+\dot{q})^2}{2}$, corresponding generalised force is

(a) $\dot{q}+\ddot{q}$ (b) $\ddot{q}-\dot{q}$ (c) $\ddot{q}+\dot{q}$ (d) $\ddot{q}-q$

33. A double star consits of two stars of masses $m_1$ and $m_2$ separated by a distance r and revolving about their common centre of mass due to their mutual gravitational attraction. Kinetic energy of the system is

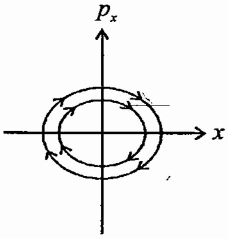
(a) $\frac{Gm_1m_2}{r}$ (b) $\frac{Gm_1m_2}{2r}$ (c) $\frac{Gm_1m_2}{4r}$ (d) $\frac{2Gm_1m_2}{r}$

34. A particle is moving in one dimensional potential $V(x)$ which is shown below.

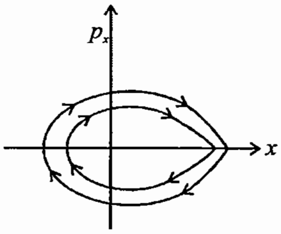


Which of the following represents phase space trajectory of the particle?

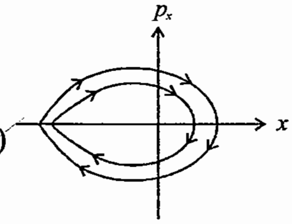
(a)



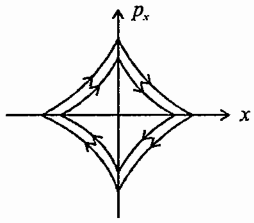
(b)



(c)



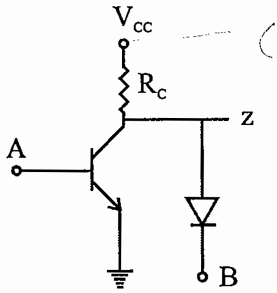
(d)



35. A 5 bit digital to analog converter having 10V applied as $V_{ref}$ the resolution is

(a) $\frac{20}{31}$ (b) $\frac{10}{31}$ (c) $\frac{1}{31}$ (d) 31

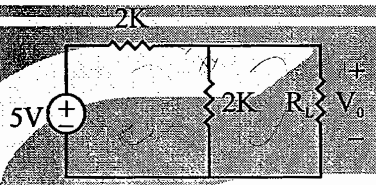
36. For the given network shown below the logic function 'z' behave as



4

(a) AB (b) $\bar{A}B$ (c) $A\bar{B}$ (d) $\bar{A}\bar{B}$

37. For the given circuit diagram shown. Determine $V_0$:



(a) 5V (b) 2.5V (c) 1.25V (d) 1 V

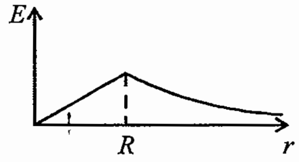
38. A sphere has uniform volume charge density. Ratio of electrostatic energy stored inside and outside the sphere is

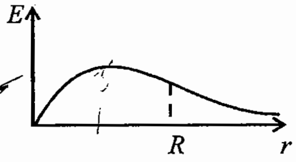
(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 5

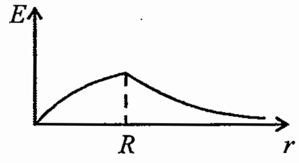
39. A large sheet having surface charge density $\sigma$ is lying in $x$-$y$ plane. It is moved with speed v in $y$-direction. Poynting vector above the sheet is

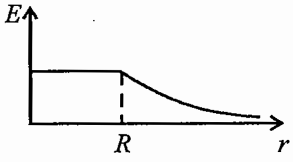
(a) $\frac{\sigma^2 v}{4\varepsilon_0}\hat{x}$ (b) $\frac{\sigma^2 v}{4\varepsilon_0}\hat{y}$ (c) $\frac{\sigma^2 v}{4\varepsilon_0}\hat{z}$ (d) $\frac{\sigma^2 v}{4\sqrt{2}\varepsilon_0}(\hat{x}+\hat{y})$

40. Charge density inside a sphere of radius R is $\rho = k(R-r)$. Radial variation of electric field is

(a)


(b)


(c)


(d)


41. A thin conducting rod of length l is rotated with uniform angular velocity $\omega$ about a perpendicular axis through one end. There exists a uniform magnetic field $\vec{B}$ in space parallel to $\vec{\omega}$. Induced magnitude of electric field inside the rod at a disance $r$ from fixed end is

(a) $\omega B(\ell - r)$ (b) $\frac{\omega B}{\ell}(\ell^2 - r^2)$ (c) $\omega Br$ (d) $\frac{\omega Br^2}{\ell}$

42. A long solenoide having $n$ number of turns per unit length carries a current I. If axis of the solenoide lies along $z$-axis. Magnetic vector potential inside the solenoide can be expressed as

(a) $\frac{\mu_0 nI\left(-x\hat{j}+y\hat{i}\right)}{2\left(x^2+y^2\right)}$ [✓ (b)] $\frac{\mu_0 nI}{2}\left(-x\hat{j}+y\hat{i}\right)$ (c) $\mu_0 nI\left(-x\hat{j}+y\hat{i}\right)$ (d) $\mu_0 nI\left(\frac{-x\hat{j}+y\hat{i}}{x^2+y^2}\right)$

43. The equation of state of a gas is given by P(V-b) = NkT, where 'b' is a constant. The isothermal compressibility $\chi_T$ is:

(a) $\frac{1}{P}$ (b) $\frac{1}{P}\left(1+\frac{b}{V}\right)$ [✓ (c)] $\frac{1}{P}\left(1-\frac{b}{V}\right)$ (d) $\frac{1}{P}\left(1+\frac{b^2}{V^2}\right)$

44. A particle has three energy levels with energies $-\in, 0, +\in$. The first excited state is doubly degenerate while others are non-degenerate. The average energy when the temperature $T \to \infty$ is:

(a) $\in$ (b) $\frac{2\in}{3}$ (c) $\frac{\in}{2}$ (d) 0

45. Consider the free electron gas comprising of N molecules in one dimension, confined in a region of length L at zero temperature. The Fermi-Energy is

(a) $\frac{h^2}{2m}\left(\frac{3N}{8\pi L}\right)^{2/3}$ [✓ (b)] $\frac{h^2}{2m}\left(\frac{N}{2L}\right)^{2/3}$ (c) $\frac{h^2}{2m}\left(\frac{N}{2L}\right)$ (d) $\frac{h^2}{2m}\left(\frac{N}{2L}\right)^2$

## Part-C

46. Consider the following statements about the roots of the equation:

$$f(x)=2x^3-x^2+2x-5=0$$

P: At least one root is positive; Q: At least one root is negative; R: There is a root between x = 1 and x = 2.

Which one of the following is TRUE?

(a) P, Q, R are all valid statements

[✓ (b)] P and Q are valid statements but R is an invalid statement

(c) P and R are valid statements but Q is an invalid statement

(d) Q and R are valid statements but P is an invalid statement

47. Consider the following recurrence relation:

$$2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$$

where $H_n(x)$ is the Hermite polynomial of order '$n$'. Then the value of the integral

$$\int_{-\infty}^{\infty} xe^{-x^2}H_n(x)H_m(x)\,dx$$

(a) $\sqrt{\pi}\left[2^{n-1}n!\delta_{n-1,m}+2^n(n+1)!\delta_{n+1,m}\right]$ (b) $\sqrt{\pi}\left[2^{n-1}n!\delta_{n-1,m}+2^{n+1}(n+1)!\delta_{n+1,m}\right]$

[✓ (c)] $\sqrt{\pi}\left[2^n n!\delta_{n-1,m}+2^{n+1}(n+1)!\delta_{n+1,m}\right]$ (d) $\sqrt{\pi}\left[2^{n-1}(n-1)!\delta_{n-1,m}+2^n(n)!\delta_{n+1,m}\right]$

48. Vehicles pass through a junction on a busy road at an average rate of 300 per hour. The probability that no vehicle passes in a given minute and the expected number of vehicles passing in two minutes, are respectively

(a) 0.007, 5 (b) 0.007, 10 (c) 0.012, 5 (d) 0.012, 10

49. Consider the following function:

$$f(x)=\begin{cases}-1 & for\ -\pi<x<0\\ 0 & for\ x=0\\ 1 & for\ 0<x<\pi\end{cases}$$

The Fourier series expansion the above function is given by,

$$f(x)=\frac{4}{\pi}\left(\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+........\right)$$

The sum of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}.......$ will be

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{16}$ (d) $\pi$

50. A three level system of atoms has $N_1$ atoms in level $E_1$, $N_2$ atoms in level $E_2$ and $N_3$ atoms in level $E_3$ respectively $(N_2>N_1>N_3$ and $E_1<E_2<E_3)$. Laser emission is possible between the levels

(a) $E_2\to E_1$ (b) $E_3\to E_1$ (c) $E_3\to E_2$ (d) $E_2\to E_3$

51. Consider a transition from an electronic state p to an s state and a spectral line is emitted. Under the presence of a strong magnetic field, the spectral line will split into

(a) 3 Components (b) 4 Components (c) 5 Components (d) 6 Components

52. At a given temparature, for a rigid rotator, the probabilty that the system is in rotational state J = 0 is 0.5, in state J = 1 is 0.3 and in J = 2 is 0.2. The average energy of the rigid rotator at the given temparature is
(a) 1.2 B (b) 1.8 B (c) 2.4 B (d) 3.6 B

53. Consider a system which is initially in the state

$$\psi=\frac{1}{\sqrt{5}}|1,-1\rangle+\sqrt{\frac{3}{5}}|1,0\rangle+\frac{1}{\sqrt{5}}|1,1\rangle$$

If we measure the z-component of the orbital angular momentum of the system, we got the result $-\hbar$. The uncertainity in the measurement of x-component of the orbital angular momentum will be

(a) $\frac{\hbar}{2}$ (b) $\frac{\hbar}{\sqrt{2}}$ (c) $\frac{\hbar}{4}$ (d) $\frac{\hbar}{\sqrt{3}}$

54. An electron in hydrogen atom is in its ground state i.e. 1s state and its wavefunction is given as

$$\psi_{100}=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}\left(a_0 \text{ is the first Bohr radius }=\frac{4\pi\varepsilon_0\hbar^2}{me^2}\right)$$

The expectation value of the potential energy of the electron will be

(a) $-\frac{me^4}{16\pi^2\varepsilon_0^2\hbar^2}$ (b) $-\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}$ (c) $-\frac{me^4}{8\pi^2\varepsilon_0^2\hbar^2}$ (d) none of these

55. Suppose N non-interacting fermions (where N is even) are placed in an infinite potential well described as follows:

$$V(x) = 0 \quad \text{for } 0 < x < a$$
$$= \infty \quad \text{otherwise}$$

The ground state energy of the systen will be

(a) $\frac{\pi^2\hbar^2}{24ma^2}N(N+1)(N+2)$ (b) $\frac{\pi^2\hbar^2}{6ma^2}N(N+1)(N+2)$

[✓ (c)] $\frac{\pi^2\hbar^2}{6ma^2}N(2N+1)(N+2)$ (d) $\frac{\pi^2\hbar^2}{24ma^2}N(2N+1)(N+2)$

56. A simple harmonic oscillator of mass $m_0$ and angular frequency $\omega$ is perturbed by an additional potential $bx^3$. The second order correction to the ground state energy of the oscillator will be

[✓ (a)] $-\frac{9b^2\hbar^2}{8m_0^3\omega^4}$ (b) $-\frac{2b^2\hbar^2}{8m_0^3\omega^4}$ (c) $-\frac{11b^2\hbar^2}{8m_0^3\omega^4}$ (d) $-\frac{7b^2\hbar^2}{8m_0^3\omega^4}$

57. The Higgs boson has a decay mode into a photon and $Z$-boson. If the decaying Higgs boson is at rest and rest mass of emitted $Z$-boson is 90 GeV/c$^2$ and energy of emitted photon is 30 GeV. The rest mass of the Higgs boson is approximately.

(a) 250 GeV/c$^2$ (b) 500 GeV/c$^2$ (c) 100 GeV/c$^2$ (d) 125 GeV/c$^2$

58. The electric octupole moment of the nucleus $^A_zX$

(a) Increases with $Z$ but independent of $A$ (b) Decreases with $Z$ but independent of $A$

(c) Is independent of both $Z$ and $A$ (d) Is always zero

59. The ratio of rotational energies of first and second excited states of even-even nucleus is

(a) $\frac{3}{10}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

60. Lagrange's equation of motion $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i$ can also be written as

(a) $\frac{\partial \dot{T}}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i$ (b) $\frac{\partial \dot{T}}{\partial \dot{q}_i} - 2\frac{\partial T}{\partial q_i} = Q_i$ (c) $\frac{\partial \dot{T}}{\partial \dot{q}_i} + \frac{\partial T}{\partial q_i} = Q_i$ (d) $2\frac{\partial \dot{T}}{\partial q_i} - \frac{\partial T}{\partial q_i} = Q_i$

61. A particle of rest mass $m_0$ initially at rest is acted upon by a constant force F. What is kinetic energy of the particle if force acts for a time $t$.

(a) $\sqrt{F^2t^2c^2 + m_0^2c^4}$ (b) $m_0c^2\left[\sqrt{1+\frac{F^2t^2}{m_0^2c^2}} - \frac{Ft}{m_0c}\right]$

[✓ (c)] $m_0c^2\left[\sqrt{1+\frac{F^2t^2}{m_0^2c^2}} - 1\right]$ (d) Ftc

62. When a solid sphere of mass $m$ rolls on an inclined plane of inclination $\theta$, a friction force of magnitude $f_0$ acts on it. What will be friction force on a circular disc of same mass rolling the inclined plane.

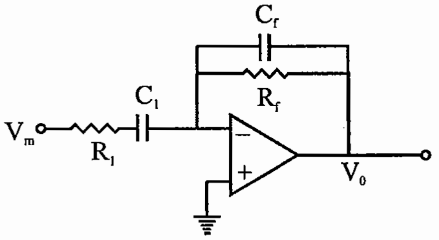
(a) $\frac{2f_0}{5}$ (b) $\frac{7f_0}{3}$ (c) $\frac{5f_0}{3}$ (d) $\frac{7f_0}{6}$

63. Lagrangian of a system is $L=\frac{1}{2}mv^2+m\vec{v}.(\vec{\omega}\times\vec{r})+\frac{1}{2}m(\vec{\omega}\times\vec{r})^2-V$, equation of motion of the system is ($\vec{\omega}$ is a constant vector)

(a) $m\frac{d\vec{v}}{dt}-2m\vec{v}\times\vec{\omega}-m\vec{\omega}\times(\vec{\omega}\times\vec{r})+\vec{\nabla}V=0$ (b) $m\frac{d\vec{v}}{dt}+2m\vec{v}\times\vec{\omega}+m\vec{\omega}\times(\vec{\omega}\times\vec{r})+\vec{\nabla}V=0$

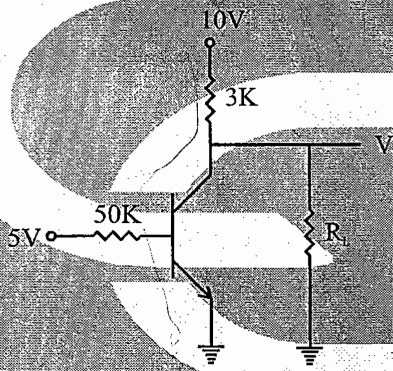
[✓ (c) $m\frac{d\vec{v}}{dt}-2m\vec{v}\times\vec{\omega}+m\vec{\omega}\times(\vec{\omega}\times\vec{r})+\vec{\nabla}V=0$] (d) $m\frac{d\vec{v}}{dt}+2m\vec{v}\times\vec{\omega}-m\vec{\omega}\times(\vec{\omega}\times\vec{r})+\vec{\nabla}V=0$

64. For the given op-amp network shown below identify the type of filter



[✓ (a) BPF] (b) BRF (c) HPF (d) LPF

65. For the given circuit shown below. Determine $V_0$.



Asssume $\beta=100$, $V_{BE}=0.7V$

(a) $V_0=V_{CE}$ (b) $V_0=V_{BE}$ [✓ (c) $V_0=\frac{V_{CE}+V_{BE}}{2}$] (d) $V_0=\frac{V_{BE}}{2}$

$-10V \times I_c 3 + 1004\ 0.7$
$+ I_B\ 5V$

$-10\ \cancel{BE}\ 3 + 100.7 + I_c$

$90.7 = 3I_3 + I_{c5}$
$8I_3$
$\frac{90.7}{8} =$

66. A type-I super conductor is characterized with a band gap $E_g = 3\text{meV}$ and critical magnetic field $H_C$=2000 A/m at 0K. The value of critical magnetic field $(H_C)$ at 5 K is:

(a) 1500 A/m (b) 500 A/m (c) 1000 A/m (d) 2000 A/m

67. In an abrupt p-n junction the doping concentration on p side and n-side are $N_A = 9\times10^{16}/\text{cm}^3$ and $N_D = 1\times10^{16}/\text{cm}^3$ respectively. The p-n junction is reverse biased and total depletion width is 3μm. The depletion width on p side of the junction is:

(a) 2.7 μm (b) 0.3 μm (c) 2.25 μm (d) 0.75 μm

68. X-rays are diffracted from a set of planes with miller indices (110) in NaCl crystal at Bragg angle of 30°. If the lattice parameter (a) of the crystal is 4.2Å, the wavelength λ of the X-rays is:

(a) 2.96Å [✓ (b) 5.94Å] (c) 1.48Å (d) 4.2Å

69. Two electromagnetic waves superpose to give resultant electric field $\vec{E} = E_0 \cos kz \sin \omega t \hat{i}$ . which of the following statements is NOT correct

(a) corresponding magnetic field is $\vec{B} = \dfrac{E_0 k}{\omega} \sin kz \cos \omega t\ \hat{j}$

(b) the waves are propagating in opposite direction along $z$-axis.

(c) average value of energy density is $\dfrac{1}{2}\varepsilon_0 E_0^2$

(d) average value of energy density is $\dfrac{1}{4}\varepsilon_0 E_0^2$

70. Consider a system of $N$ and distinguishable particles (in which particles can move in one dimensional segment $q = 0$ and $q = L$) in thermal equilibrium at temperature $T$ . The single particle Hamiltonian is given by

$$H = \frac{p^2}{2m} - \alpha \ell n\left(\frac{q}{L_0}\right),$$

where $\alpha$ is positive constant and $L_0$ is characteristic length. The equation of state of the system is

(a) $\dfrac{NKT}{L}$ (b) $\dfrac{\alpha NKT}{L}\left(1-\dfrac{\alpha}{kT}\right)$ [✓ (c)] $\dfrac{NKT}{L}\left(1+\dfrac{\alpha}{kT}\right)$ (d) $\dfrac{NKT}{\alpha L}$

71. A one-dimensional quantum harmonic oscillator (whose ground state energy is $\hbar\omega/2$) is in thermal equilibrium with a heat bath at temperature T, the value of root-mean –square fluctuations in energy about $\langle E \rangle$?

(a) $\dfrac{2}{\sinh\left(\dfrac{\hbar\omega}{2kT}\right)}$ (b) $\dfrac{\hbar\omega}{2}$ (c) $\dfrac{\hbar\omega}{2\sinh\left(\dfrac{\hbar\omega}{2kT}\right)}$ (d) $\dfrac{\hbar\omega}{2}\coth\left(\dfrac{\hbar\omega}{2kT}\right)$

72. The partition function for an ideal gas is in 2-D for indistinguishable particles is:

(a) $\dfrac{1}{N!}\left(\dfrac{A}{\lambda_T^2}\right)^N$ , [A = area, $\lambda_T$ = thermal de-Broglie wavelength]

(b) $\left(\dfrac{A}{\lambda_T^2}\right)^N$ [✓ (c)] $\dfrac{1}{N!}\left(\dfrac{\lambda_T^2}{A}\right)^N$ (d) $\dfrac{1}{N}\left(\dfrac{\lambda_T^2}{A}\right)^{-N}$

73. An electric dipole of dipole moment $p_0$ is rotated with angular velocity $\omega$ . Average power radiated by the dipole is

[✓ (a)] $\dfrac{p_0^2\omega^4}{6\pi \in_0 c^3}$ (b) $\dfrac{p_0^2\omega^4}{12\pi \in_0 c^3}$ [✓ (c)] $\dfrac{p_0^2\omega^2}{6\pi \in_0 c^3}$ (d) $\dfrac{p_0^2\omega^2}{12\pi \in_0 c^3}$

74. In a rectangular waveguide for which $a = 1.5$cm, $b = 0.8$ cm,

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(\pi\times10^{11}t-\beta z\right)A/m$$

Which of the following is CORRECT.

(a) Mode is $TE_{13}$ and cutoff frequency is 28.57 MHz

(b) Mode is $TM_{13}$ and cutoff frequency is 28.57 GHz

(c) Mode is $TE_{13}$ and cutoff frequency is 14.57 GHz

(d) Mode is $TM_{13}$ and cutoff frequency is 14.57 GHz

75. An alternating sinusoidal current of frquency $\omega = 1000\,s^{-1}$ flows in the winding of a straight solenoid whose corss-sectional radius is equal to R = 6.0 cm. the ratio of peak values of electric and magnetic energies within the solenoid is

(a) $5\times10^{-14}$ (b) $5\times10^{-15}$ (c) $5\times10^{-16}$ (d) $5\times10^{-13}$

# TEST SERIES UGC-CSIR-NET/JRF Dec. 2014

**BOOKLET SERIES** **F**

**Paper Code** **05**

**Test Type: TEST SERIES**

## PHYSICAL SCIENCES

**Duration: 3:00 Hours**

**Date: 15-12-2014**

**Maximum Marks: 200**

**Read the following instructions carefully:**

* Single Paper Test is divided into three Parts.

**Part - A:** This part shall carry 20 questions. The candidate shall be required to answer any 15 questions. Each question shall be of **2 marks.**

**Part - B:** This part shall contain 25 questions covering the topics given in the Part 'B' of syllabus. The candidates are required to answer any 20 questions. Each question shall be of **3.5 Marks..**

**Part - C:** This part shall contain **30** questions from Part - C of the syllabus. The candidates are required to answer any 20 questions. Each question shall be of **5 Marks.**

* Darken the appropriate bubbles with HB pencil/Ball Pen to write your answer.

* There will be negative marking @25% for each wrong answer.

* The candidates shall be allowed to carry the Question Paper Booklet after completion of the exam.

* For rough work, blank sheet is attached at the end of test booklet.

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**South Delhi Centre:**
28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## USEFUL FUNDAMENTAL CONSTANTS

| | | |
|---|---|---|
| m | Mass of electron | $9.11 \times 10^{-31}$ Kg |
| h | Planck's constant | $6.63 \times 10^{34}$ J sec |
| e | Charge of electron | $1.6 \times 10^{-19}$ C |
| k | Boltzmann constant | $1.38 \times 10^{-23}$ J/K |
| c | Velocity of Light | $3.0 \times 10^{8}$ m/Sec |
| $1_eV$ | $1.6 \times 10^{-19}$ J | |
| amu | $1.67 \times 10^{-27}$ kg | |
| G | $6.67 \times 10^{-11}$ Nm$^2$kg$^{-2}$ | |
| $R_y$ | Rydberg constant | $1.097 \times 10^{7}$ m$^{-1}$ |
| $N_A$ | Avogadro number | $6.023 \times 10^{23}$ mole$^{-1}$ |
| $\varepsilon_o$ | $8.854 \times 10^{-12}$ Fm$^{-1}$ | |
| $\mu_o$ | $4\pi \times 10^{-7}$ Hm$^{-1}$ | |
| R | Molar Gas constants | $8.314$ J K$^{-1}$ mole$^{-1}$ |

## Part-A

1. If 10 machines can produce 10000 identical items in 10 days by operating for **10** hrs/day. How many days does it take for 5 machines to produce 5000 such items by operating for 5 hrs/day.
(a) 5 (b) 10 (c) 20 (d) 40

2. If $x$ and $y$ are real numbers and $x > y$, then which of the following is always true.
(a) $|x|>|y|$ (b) $x^2 > y^2$ (c) $x(x+1)>y(y+1)$ (d) $2y-1<2x-1$

3. In a group of 60 students, 40 students have studied operation research and 27 students have studied graph theory. Almost how many of them have not studied either of these two subjects.
(a) 7 (b) 13 (c) 20 (d) 27

4. If second october falls on monday in a year, then on what day 25th december of that year falls.
(a) Sunday (b) Monday (c) Tuesday (d) Friday

5. How many pairs of letters are there in the word STRENGTH which have as many letters between them as in the alphabet
(a) one (b) two (c) more than two (d) no such pair

**Directions (Q.6 and Q.7):** For a motorist there are 3 ways are going from city A to city C. By way of a bridge the distance is 20 miles and the toll is Rs. 75. A tunnel between the two cities is a distance of 10 miles and the toll is Rs. 100 for the vehicle and driver plus Rs. 10 for each passenger. A two lane highway without tolls goes east for 30 miles to city B and then 20 miles in a north west direction to city C.

6. Martin usually drive alone from City C to city A every working day. His firm deducts a percentage of employees pay for the lateness. Which factor would most probably influence his choice of the bridge or the tunnel
(a) Whether his wife goes with him.
(b) Scenic interest of each route
(c) Traffic condition on the road, bridge and the tunnel.
(d) Saving of Rs. 25 in tolls.

7. In choosing between the use of the bridge and the tunnel, the chief factors would be
(I) Traffic and road conditions.
(II) Number of passengers in the car
(III) Location of one's home in the centre or outskirts of one of the cities.
(IV) Desire to save Rs. 25
(a) I only (b) II only (c) II and III (d) I and II only.

8. One manufacturing plant built 150 tanks in the last six months in 1975. This was increase at 150% over the number built in the preceding six months. Find the number of tanks built in the preceding six months.
(a) 100 (b) 50 (c) 0 (d) 60

9. Find the missing number
7, 10, 63, ............., 215
(a) 26 (b) 28 (c) 80 (d) 126

10. If $6x^{2k-4}+7x^{k-2}+x^{k-3} = (6x+1)(x+1)$ then the value of k must be
(a) 1 (b) 2 (c) 3 (d) 4

11. Pointing to a picture in photograph, a woman said, " this person is the father of the brother of my mother's one of the two son-in-low". How is the person related to the woman
(a) Father (b) Father-in-law (c) Brother (d) Husband

12. The coordinates of the centre of the conic $x^2-3xy+y^2+1=0$ is given by
(a) (1, 2) (b) (0, 0) (c) (1, 1) (d) (0, 2)

13. There are two candles of equal length and of different thickness. The thicker one lasts for 6 hours. The thinner one lasts two hour less than the thicker one. Radha lights the two candles at the same time when she went to bed, she saw the thicker one is twice the length of the thinner one. How long ago did Radha light the two candles?

(a) 1 hour (b) 2 hours (c) 3 hours (d) 4 hours

14. Rishabh and Richa solved a quadratic equation. In solving it, Rishabh made a mistake in the constant term and got the roots as 6 and 2. While Richa made a mistake in the coefficient of $x$ only and obtained the roots as –7 and –1. The correct roots are

(a) –6, –7 (b) 6, –1 (c) 7, 1 (d) 2, –7

15. Manufacturer of a tennis ball anounced that their new ball is best in the market. If one were drop it from any height, it would only rise to the extent of 10% of the height from which it was dropped. If the ball was dropped 45 feet, how much would the ball travel before coming to rest.

(a) 45 ft (b) 50 ft (c) 55 ft (d) 49.5 ft

16. From a leaking tab 'a' drop come out in 'b' minutes. If there are 'c' drops in a liter, then in how many hours one liter of water will be wasted

(a) 60 a/bc (b) a/bc (c) abc/60 (d) bc/60a

17. Fresh grapes contain 68% water and dry grapes contain 20% water. How much fresh grapes are required to obtained 80kg of dry grapes

(a) 100 kg (b) 200 kg (c) 300 kg (d) 400 kg

18. The value of $16^{\log_4^5}$

(a) 16 (b) 5 (c) 25 (d) 4

19. Given that $f(y)=|y|/y$ and $q$ is an non-zero real number. The value of $|f(q)-f(-q)|$ is

(a) 0 (b) –1 (c) 1 (d) 2

20. A king ordered that a golden crown be made from him from 8kg of gold and 2kg of silver. The goldsmith took away some amount of gold and replaced it by an equal amount of silver and the crown when made, weighed 10 kg. The king knew that under water gold lost 1/20th of its weight while silver lost 1/10th. When crown was weighed under water, it was 9.25 kg. How much gold was stolen by the goldsmith

(a) 0.5kg (b) 1kg (c) 2kg (d) 3kg

## Part-B

21. A thin rod of proper length $l_0$ is moving with speed 3c/5. In +x direction with respect to a frame S. What would be length of rod in another frame $S'$ which is moving with speed 4c/5 with respect to S frame in X-direction.

(a) $\frac{12\ell_0}{37}$ (b) $\frac{7\ell_0}{37}$ (c) $\frac{4\ell_0}{5}$ (d) $\frac{3\ell_0}{5}$

22. A thin hollow sphere of mass $M$ and radius R is rolling without sliding on a horizontal surface. If the translational speed of the sphere is $v$ its angular momentum about a point on its path is

(a) MVR (b) $\frac{2}{3}MVR$ (c) $\frac{2}{5}MVR$ (d) $\frac{5}{3}MVR$

23. A wire whose equation is $Z = a\left(x^2 + y^2\right)$ is rotated about z axis with constant angular velocity $\omega$. Z being vertical. A bead of mass m slides on the wire. Lagrangian of the bead is

(a) $\frac{1}{2}m\left(\dot{r}^2+r^2\omega^2\right)-mgar^2$ (b) $\frac{1}{2}m\left[\dot{r}^2\left(1+4r^2a^2\right)+r^2\omega^2\right]-mgar^2$

(c) $\frac{1}{2}m\left(\dot{r}^2+r^2\sin^2\theta\omega^2\right)-mgar^2$ (d) $\frac{1}{2}m\left[\dot{r}^2\left(1+4r^2a^2\right)+r^2\omega^2\sin^2\theta\right]-mgar^2$

24. Value of poisson bracket $\left[r^2, p^2\right]$ is equal to

(a) $2rp$ (b) $2\hat{r}.\hat{p}$ (c) zero (d) $2\vec{r}.\vec{p}$

25. Let, $T(x,y,z)=xy^2+2z-x^2z^2$ be the temparature at the point $(x,y,z)$. The unit vector in the direction in which the temparature decreases most rapidly at $(1,0,-1)$ is

(a) $-\frac{1}{\sqrt{5}}\hat{i}+\frac{2}{\sqrt{5}}\hat{k}$ (b) $\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{k}$

(c) $\frac{2}{\sqrt{14}}\hat{i}+\frac{3}{\sqrt{14}}\hat{j}+\frac{1}{\sqrt{14}}\hat{k}$ (d) $-\frac{2}{\sqrt{14}}\hat{i}-\frac{3}{\sqrt{14}}\hat{j}-\frac{1}{\sqrt{14}}\hat{k}$

26. Consider a $3\times3$ non-singular matrix G such that,

$$Tr(G)=8, Tr\left(G^2\right)=30, Tr\left(G^3\right)=134$$

Which of the following sets will a possible eigenvalues of matrix G?

(a) 2,3,3 (b) 0,3,5 (c) 1,2,5 (d) 0,4,4

27. Consider the following system of linear equations:

$$x+y+z=3$$
$$x+az=b$$
$$y+2z=3$$

This system has infinite number of solutions if

(a) $a=-1, b=0$ (b) $a=-1, b=1$ (c) $a=1, b=1$ (d) $a=0, b=1$

28. Consider the following differential equation:

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\ln x$$

The linearly independent solutions of the equation are

(a) $x^3, \frac{1}{x}, -\frac{1}{3}x^2\left(\ln x+\frac{2}{3}\right)$ (b) $e^{3x}, e^{-x}, -\frac{1}{3}x^2\left(\ln x+\frac{2}{3}\right)$

(c) $e^{3x}, e^{-x}, -\frac{1}{3}e^{2x}\left(x+\frac{2}{3}\right)$ (d) $x^3, \frac{1}{x}, -\frac{1}{3}e^{2x}\left(x+\frac{2}{3}\right)$

29. Consider the following integral:

$$\int_0^\infty \frac{\cos 3x}{\left(x^2+1\right)\left(x^2+4\right)}dx$$

The value of the integral will be

(a) $\frac{\pi}{2}\left[\frac{e^3}{3}-\frac{e^6}{6}\right]$ (b) $\frac{\pi}{2}\left[\frac{e^{-3}}{3}-\frac{e^{-6}}{6}\right]$ (c) $\pi\left[\frac{e^{-3}}{3}-\frac{e^{-6}}{6}\right]$ (d) $\pi\left[\frac{e^3}{3}-\frac{e^6}{6}\right]$

30. A one dimensional wave function corresponding a particle is given by $\psi(x)=\sqrt{a}e^{-a|x|}$. The probability of finding the particle between x=1/a to x=2/a is equal to

(a) $e^{-2}-e^{-4}$ (b) $\frac{e^{-2}-e^{-4}}{2}$ (c) $\frac{e^{-2}-e^{-4}}{4}$ (d) $\frac{e^{-2}+e^{-4}}{4}$

31. The wave function of a beam of particle in a certain region is described by

$$\psi(x)=3Ae^{ikx}+2Ae^{-ikx}$$

The ratio of the number of particle going in the positive x-direction and the number of particles going in the negative x-direction is

(a) 3:2 (b) 2:3 (c) 9:4 (d) 4:9

32. Let $|0\rangle$ and $|1\rangle$ are the normalized eigen states corresponding to the ground and first excited state of a 1-D harmonic osciilator. The uncertainty in position in the state $\frac{1}{\sqrt{2}}[|0\rangle+|1\rangle]$ will be

(a) 0 (b) $\sqrt{\frac{\hbar}{2m\omega}}$ (c) $\sqrt{\frac{\hbar}{m\omega}}$ (d) $\sqrt{\frac{2\hbar}{m\omega}}$

33. Consider a particle of mass 'm' is confined in a finite potential well given as

$$V(x)=\begin{cases}0 & for\,|x|<L\\ V_0 & for\,|x|\geq L\end{cases}$$

If the particle is found to be in a bound state with energy $\frac{\pi^2\hbar^2}{32mL^2}$, then the depth of the well will be

(a) $\frac{\pi^2\hbar^2}{2mL^2}$ (b) $\frac{\pi^2\hbar^2}{4mL^2}$ (c) $\frac{\pi^2\hbar^2}{8mL^2}$ (d) $\frac{\pi^2\hbar^2}{16mL^2}$

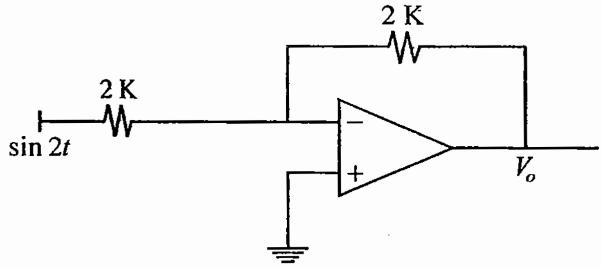
34. Suppose the kets $|j,m_j\rangle$ are the simultaneous eigenkets of the operators $J^2$ and $J_z$ with eigenvalues $j(j+1)\hbar^2$ and $m_j\hbar$ respectively. Then $|j,m_j\rangle$ will be eigenkets of the operators $[J_x,J_+]$ and $[J_y,J_+]$ with respective eigenvalues

(a) $m\hbar^2, im\hbar^2$ (b) $m\hbar^2,-im\hbar^2$ (c) $-m\hbar^2, im\hbar^2$ (d) $-m\hbar^2,-im\hbar^2$

35. For the given 4-bit DAC flash type if $V_i$ changes from 10V to 20V the conversion time (max) **[3 marks]**

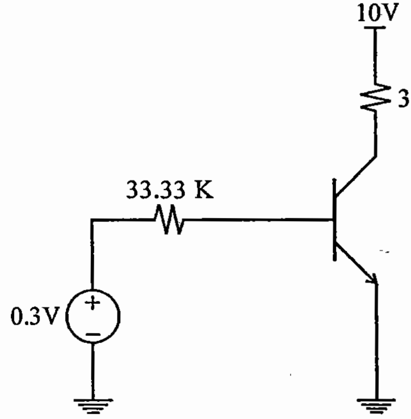
(a) Remain same (b) Double (c) Half (d) None

36. For the given opamp network determine laplace transform of $V_o$



(a) $\frac{2}{s^2+4}$ (b) $\frac{-2}{s^2+4}$ (c) $\frac{2}{s^2+2}$ (d) $\frac{-2}{s^2+2}$

37. For the given transistor circuit shown below operating in



(a) Saturation region (b) Active region (c) Cut off region (d) Inverse active

38. An electric dipole of dipole moment p is placed at d distance from a grounded infinite conducting sheet, with $\vec{p}$ being perpendicular to the sheet. Force on the dipole is

(a) $\frac{3p^2}{16\pi \epsilon_0 d^4}$ (b) $\frac{3p^2}{32\pi \epsilon_0 d^4}$ (c) $\frac{3p^2}{64\pi \epsilon_0 d^4}$ (d) $\frac{p^2}{64\pi \epsilon_0 d^4}$

39. A sphere of radius R has volume charge density $\rho = \beta r$ where $\beta$ is a constant and $r$ is distance from centre. If radius of sphere is increased to 2R keeping the form of charge density unchanged, its electrostatic energy will become

(a) 8 times (b) 32 times (c) 128 times (d) 256 times

40. A sphere made of a dielectric has polarization $\vec{P} = \beta\vec{r}$. Electric field at a point inside the sphere is

(a) $-\frac{\beta\vec{r}}{\epsilon_0}$ (b) $-\frac{\beta\hat{r}}{\epsilon_0}$ (c) $-\frac{\beta\vec{r}}{3\epsilon_0}$ (d) $-\frac{\beta\vec{r}}{4\epsilon_0}\hat{r}$

41. A sphere of radius R has surface charge density $+\sigma$ on one half and $-\sigma$ on the other half. Which one of the following statements is NOT correct?

(a) Electric potential at centre of sphere is zero but electric field is not zero.

(b) Electric potential and field vary as $\frac{1}{r^2}$ and $\frac{1}{r^3}$ respectively at $r >> R$

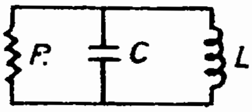
(c) Both electric potential and field are continuous function of $r$ (distrance from centre)

(d) Electrostatic energy of sphere is negative.

42. Magnetic field in a region through which electromagnetic waves are propagating is given to be $\vec{B} = B_0 \cos kz \sin \omega t\, \hat{j}$. Average value of poynting vector is

(a) $\frac{B_0^2\omega}{2\mu_0 k}\hat{z}$ (b) $\frac{B_0^2\omega}{4\mu_0 k}\hat{z}$ (c) $\frac{B_0^2\omega}{8\mu_0 k}\hat{z}$ (d) zero

43. The circuit shown in the figure is in thermal equilibrium with its surroundings at a temperature T. What is the classical expression for the root mean square current through the inductor?



(a) $\sqrt{\frac{2kT}{L}}$ (b) $\sqrt{\frac{3kT}{L}}$ (c) $\sqrt{\frac{kT}{L}}$ (d) $\sqrt{\frac{L}{2kT}}$

44. Consider an ideal gas whose entropy is given by $S=\frac{n}{2}\left[\sigma+5R\ell n\frac{U}{n}+2R\ell n\frac{V}{n}\right]$, where n = number of particles, R = universal gas constant, U = internal energy, V =volume and $\sigma$ =constant. Then equation of state is represented by

(a) $p=\frac{2}{3}\frac{U}{V}$ (b) $p=\frac{1}{3}\frac{U}{V}$ (c) $p=\frac{2}{5}\frac{U}{V}$ (d) $p=\frac{U}{V}$

45. If $\mu_{photon}$, $\mu_{BE}$ and $\mu_{FD}$ represent, the chemical potentials of photon gas, BE gas and FD gas respectively, then which of the following is true.

(a) $\mu_{photon}>0, \mu_{BE}=\mu_{FD}$ (b) $\mu_{photon}<0, \mu_{BE}>\mu_{FD}$

(c) $\mu_{FD}\geq 0, \mu_{BE}<0, \mu_{photon}=0$ (d) $\mu_{FD}<0, \mu_{BE}>0$

## Part-C

46. A particle of mass m and energy E moves in one dimensional potential $V(x)=\frac{1}{4}kx^4$. Motion of the particle is

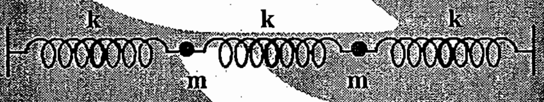
(a) simple harmonic

(b) periodic with time period proportional to $E^{1/2}$

(c) periodic with time period proportional to $E^{-1/4}$

(d) periodic with time period independent of E.

47. Two particles of masses m are attached to three springs as shown in figure. Frequencies of normal modes for longitudinal small oscillation of particles, are



(a) $\sqrt{\frac{k}{m}},\sqrt{\frac{k}{m}}$ (b) $\sqrt{\frac{2k}{m}},\sqrt{\frac{k}{m}}$ (c) $\sqrt{\frac{3k}{m}},\sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{3k}{m}},\sqrt{\frac{2k}{m}}$

48. The invariance of the Lagrangian $L=\frac{1}{2}m\left(\frac{dz}{dt}\right)\left(\frac{dz^*}{dt}\right)-V\left(zz^*\right)$ (where $z=re^{i\theta}=x+iy$ is a complex coordinate under the transformation $Z\rightarrow e^{i\alpha}z, z^*\rightarrow e^{-i\alpha}z^*$ (for a real number $\alpha$), leads to the conservation of

(a) Angular momentum (b) linear momentum along x.

(c) linear momentum along y (d) energy

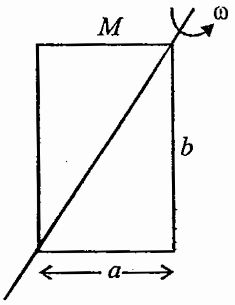
49. A rectangular plate of sides *a* and *b* and mass *M* is rotated about its diagonal as shown in the figure. Which one of the following statements is correct?

(a) angular momentum of the plate is parallel to $\vec{\omega}$

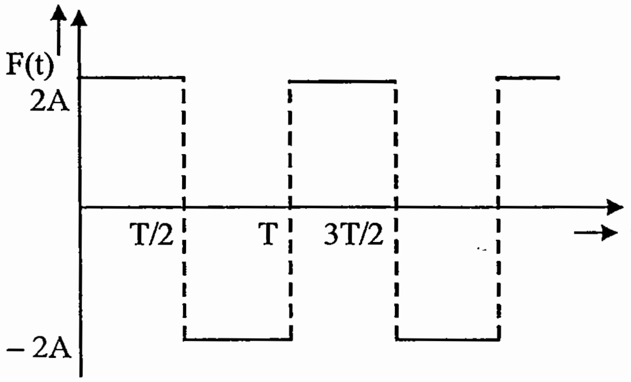
(b) Exerternal torque on the plate is zero.

(c) Diagonal is a principal axis of the plate.

(d) Kinetic energy of the plate is $\frac{Ma^2b^2\omega^2}{12\left(a^2+b^2\right)}$



50. The Laplace transform of the following square wave



is equal to

(a) $\frac{A}{S}\tanh\frac{ST}{4}$ (b) $\frac{A}{S}\coth\frac{ST}{4}$ (c) $\frac{2A}{S}\tanh\frac{ST}{4}$ (d) $\frac{2A}{S}\coth\frac{ST}{4}$

51. 'A' speaks truth in 75% cases and 'B' speaks truth in 80% cases. The percentage of cases in which, they are likely to contradict each other in explaining the same story?

(a) 5% (b) 35% (c) 60% (d) 75%

52. The Laurrent series expansion of the complex function $f(z)=\frac{1}{1+z^2}$ about $z=i$ will be

(a) $\frac{1}{i}(z-i)^{-1}+\sum_{n=0}^{\infty}2^{-n-2}i^{-n}(z-i)^n$ (b) $\frac{1}{i}(z-i)^{-1}+\sum_{n=0}^{\infty}2^{-n+2}i^{-n}(z-i)^n$

(c) $\frac{1}{2i}(z-i)^{-1}+\sum_{n=0}^{\infty}2^{-n+2}i^{-n}(z-i)^n$ (d) $\frac{1}{2i}(z-i)^{-1}+\sum_{n=0}^{\infty}2^{-n-2}i^{-n}(z-i)^n$

53. The following data is obtained in an expriment that measures the moment of inertia $I$ of diatomic meolecule as a fuction of internuclear distance $r$ is given as following:

| $r\left(in\ A^0\right)$ | $I\left(in\ kg-m^2\right)$ |
|---|---|
| 2.1 | $5.2\times10^{-47}$ |
| 9.8 | $605.5\times10^{-47}$ |
| 19.2 | $4713.7\times10^{-47}$ |

The relation that describes the dependency of $I$ on $r$ will be approximately

(a) $I\approx r^{3/4}$ (b) $I\approx r^{3/2}$ (c) $I\approx r^2$ (d) $I\approx r^3$

54. The ground and first excited vibrational energy levels of a diatomic molecule are separated by 2143 cm$^{-1}$. If the anharmonicity constant of the molecule is 14 cm$^{-1}$, then the first overtone will be observed at

(a) 4286 cm$^{-1}$ (b) 4298 cm$^{-1}$ (c) 4314 cm$^{-1}$ (d) 4258 cm$^{-1}$

55. The rotational Raman spectra of certian molecule shoes a series of stokes lines at 19230.769 cm$^{-1}$, 19227.238 cm$^{-1}$, 19223.707 cm$^{-1}$. The rotational constant of the molecule in the units of GHz, is

(a) 26.48 (b) 52.97 (c) 105.94 (d) 3.53

56. A hydrogen atom in its ground state is collided with an electron of kinetic energy 13.377 eV. The maximum factor by which the radius of the atom would increase is

(a) 7 (b) 8 (c) 49 (d) 64

57. In a classical model, a scalar (spin 0) meson consists of a quark and an antiquark bound by the potential

$$V(r) = ar + \frac{b}{r}$$

where, $a = 100$ MeV fm$^{-1}$. If the masses of quark and antiquark are negligible and the mass of the meson is 135 MeV/c$^2$ then the value of $b$ is

(a) 91 MeV fm (b) 45.5 MeV fm (c) 200 MeV fm (d) 100 MeV fm

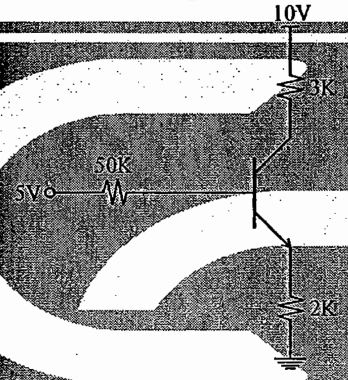
58. The radius of $^{49}_{25}Mn$ nucleus is 4.8 fm and $e^2$ is approximately 1.0 MeV. fm. The difference in the coulomb energy between the mirror nuclei $^{49}_{24}Cr$ and $^{49}_{28}Mn$

(a) 24 MeV (b) 18 MeV (c) 12 MeV (d) 6 MeV

59. A proton of energy 820 GeV collides with an electron the rest mass of the heaviest particle that can be produced in the collision is 300 GeV/c$^2$. The energy of the electron is approximately.

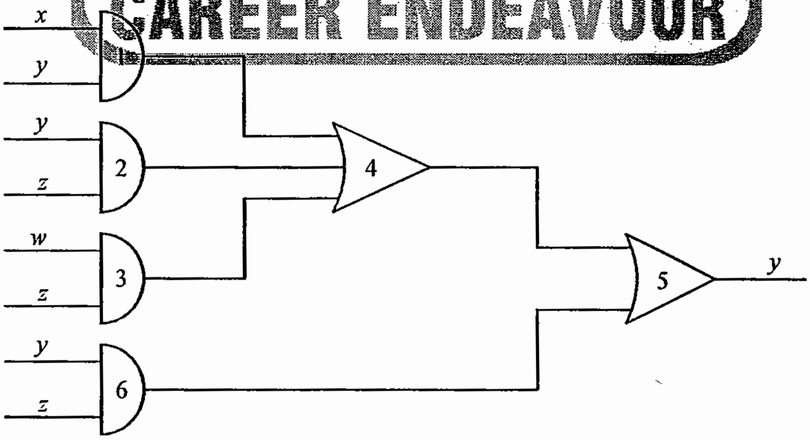
(a) 520 GeV (b) 27 MeV (c) 27 GeV (d) 0.51 MeV

60. For the given transistor circuit shown below the slope of load line is



(a) $\frac{+1}{5000}$ (b) $\frac{+1}{4000}$ (c) $\frac{-1}{5000}$ (d) $\frac{-1}{4000}$

61. For the given digital circuit shown below the gate which is redundant



(a) 1 (b) 2 (c) 3 (d) 5

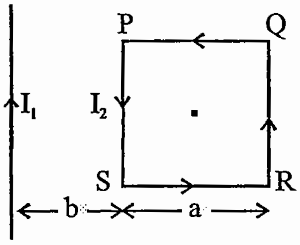
62. A particle of mass $m$ and charge $q$ is attached to light spring of spring constant $k$. When spring mass system is made to oscillate, energy of system decreases due to radiation loss. Ratio of energy loss in one period and initial value of energy is

(a) $\frac{q^2 k}{6\pi \epsilon_0 c^3 m}$ (b) $\frac{q^2 k}{6\pi \epsilon_0 c^3 m^2}$ (c) $\frac{q^2 k}{12\pi \epsilon_0 c^3 m^2}$ (d) $\frac{q^2 k^2}{6\pi \epsilon_0 c^3 m^2}$

63. Dielectric strength of air is $3\times10^6$ V/m. The minimum intensity of electromagnetic wave which can ionize air on being passed through it is

(a) $1.2\times10^{10}\ W/m^2$ (b) $1.2\times10^{8}\ W/m^2$ (c) $1.2\times10^{6}\ W/m^2$ (d) $1.2\times10^{12}\ W/m^2$

64. A current carrying square loop PQRS is placed near a current carrying long wire as shown in the figure. Which one of the following statements is NOT correct.



(a) Magnetic force on the loop is $\dfrac{\mu_0 I_1 I_2 a^2}{2\pi b(b+a)}$

(b) Magnetic force on side PQ is $\dfrac{\mu_0 I_1 I_2}{2\pi}\ln\left(\dfrac{a}{b}\right)$

(c) Magnetic flux through the loop due to $I_1$ is $\dfrac{\mu_0 I_1 a}{2\pi}\ln\left(1+\dfrac{a}{b}\right)$

(d) If $a=b$ and $\dfrac{I_1}{I_2}=6\sqrt{2}$ then net magnetic field at centre of loop is zero.

65. When two plane electromagnetic waves whose electric fields are $\vec{E}_1 = E_0\hat{i}\sin\left(kz-\omega t+\dfrac{\pi}{3}\right)$ and $\vec{E}_2 = E_0\hat{j}\sin\left(kz-\omega t-\dfrac{\pi}{6}\right)$ are superposed the resulting wave is

(a) A standing wave
(b) Left elliptically polarized progressive wave
(c) Left circularly polarized progressive wave
(d) Right circularly polarized progressive wave

66. A fermion gas has energy spectrum given by $\in = ap^s$, where 'a' and 's' are constants, $\in$ and p are energy and momentum of the fermion particle. There are N fermions contained in d-dimension. The average energy per particle at zero temperature is given by (in terms of fermi energy $E_F$) by

(a) $\left(\dfrac{d}{d+2}\right)E_F$ (b) $\left(\dfrac{d}{d-s}\right)E_F$ (c) $\left(\dfrac{d}{d+s}\right)E_F$ (d) $\left(\dfrac{d^2}{d^2+s^2}\right)E_F$

67. The excitations of a three-dimensional solid are bosonic in nature with their frequency $\omega$ and wave-number are related by $\omega \propto k$. If the chemical potential is zero, the behavior of the specific heat of the solid at low temperature is proportional to

(a) $T^{1/2}$ (b) $T^{3/2}$ (c) $T^3$ (d) T

68. Consider a one-dimensional Ising model as a linear chain consisting of N spins with neighbour-next neighbour coupling constant J. Its canonical partition function is given by

(a) $(2\cos h\beta J)^N$ (b) $(\cosh\beta J)^N$ (c) $N(\cosh\beta J)^N$ (d) $\left(\dfrac{1}{2}\cosh\beta J\right)^N$

69. Consider a particle in a central potential. Given that $|\ell, m\rangle$ is an eigenstate of $L^2$ and $L_z$. If $\Delta L_x$ and $\Delta L_y$ represent root mean square deviation in $x$ and $y$ component of orbital angular momentum respectively. Then the value of $(\Delta L_y)^2+(\Delta L_y)^2$ is:

(a) 0 (b) $\hbar^2[\ell(\ell+1)-m^2]$

(c) $2\hbar^2[\ell(\ell+1)-m^2]$ (d) $4\hbar^2[\ell(\ell+1)-m^2]$

70. If Hamiltonian of a system is given by $H = c\,\alpha.p + \beta mc^2$

Where $\alpha$ and $\beta$ are the Dirac's matrices, p is the momentum operator. If $\sigma'$ is the Pauli's spin operator in four dimension defined as $\sigma' = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$, then the value of $[\sigma'.p, H]$ is

(a) H (b) $\sigma'.p$ (c) $\alpha.p$ (d) 0

71. Consider the one-dimensional harmonic oscillator whose unperturbed Hamiltonian is $H_0 = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$

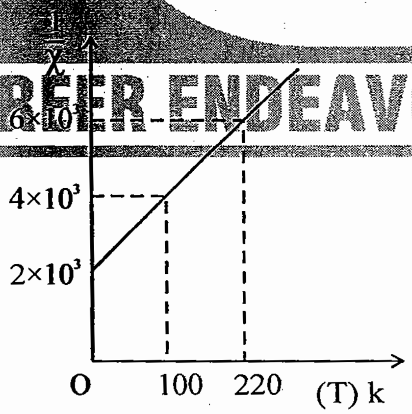
The system is now subjected to a perturbation $H' = gq^3$, the second order correction to ground state energy is

(a) 0 (b) $-\frac{11}{8}\frac{g^2}{\hbar\omega}\left(\frac{\hbar}{2m\omega}\right)^3$ (c) $-\frac{9}{8}\frac{g^2}{\hbar\omega}\left(\frac{\hbar}{2m\omega}\right)^3$ (d) $+\frac{9}{8}\frac{g^2}{\hbar\omega}\left(\frac{\hbar}{2m\omega}\right)^3$

72. Consider the system of three noninteracting electrons in ground state of one-dimensional infinite square well of width L $\left(V(x) = \begin{cases} 0, & \text{if } x < L \\ \infty, & \text{otherwise} \end{cases}\right)$. The system is subjected to a uniform magnetic field B. The ground state energy of the system is $\left(\mu_B = \frac{e\hbar}{2m} = \text{Bohr magnetron}\right)$

(a) $\frac{3\hbar^2}{8mL^2} \pm \mu_B B$ (b) $\frac{3\hbar^2}{4mL^2} \pm \mu_B B$ (c) $\frac{3\hbar^2}{mL^2} + \mu_B B$ (d) $\frac{3\hbar^2}{4mL^2} - \mu_B B$

73. According to mean field theoy, plot of inverse of magnetic susceptibility $\left(\frac{1}{\chi}\right)$ and temperature (T) for MnO crystal (antiferromagnetic) shown in the graph. The Neel temperature ($T_N$), curie constant (C) and mean field constants ($\mu$) are respectively.



(a) $240\text{ K}, 3\times10^{-2}, 8\times10^3$ (b) $120\text{ K}, 3\times10^{-2}, 4\times10^3$

(c) $120\text{ K}, 6\times10^{-2}, 2\times10^3$ (d) $240\text{ K}, 6\times10^{-2}, 4\times10^3$

74. X-rays of 10 keV energy are used to deermine the crystal lattice structure of fcc solid. The lattice constant of solid is a = 1.24Å. The angle of diffraction for (111) plane is :

(a) 30° (b) 45° (c) 60° (d) 120°

75. An intrinsic sample of silicon is doped with P and Al with doping densities of $1.5\times10^{16}/\text{cm}^3$ and $2.5\times10^{16}/\text{cm}^2$ respectively. If intrinsic carrier concentration of Si is $1.5\times10^{10}/\text{cm}^3$. The electron and hole dinsities per cm³ are, respectively.

(a) $2.25\times10^4, 1\times10^{16}$ (b) $1.5\times10^4, 2.5\times10^{16}$

(c) $1\times10^{16}, 2.25\times10^4$ (d) $1.5\times10^{16}, 1\times10^4$

# TEST SERIES FOR GATE

## BOOKLET SERIES C

**Paper Code: PH**

**Test Type: TEST SERIES**

**Duration: 3:00 Hours**

# PHYSICS

**Date: 19-01-2015**

**Maximum Marks: 100**

**Read the following instructions carefully:**

1. Attempt all the questions.
2. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory. The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.
   The subject specific GATE paper section consists of 55 questions, out of which question numbers 11 to 35 are of 1-mark each, while question numbers 36 to 65 are of 2-mark each.
3. The question paper may consist of questions of **multiple choice type** (MCQ) and **numerical answer type**.
4. Multiple choice type questions will have four choices against (a), (b), (c), (d), out of which only **ONE** is the correct answer.
5. For numerical answer type questions, each question will have a numerical answer and there will not be any choices.
6. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of ⅓ marks for a **1-mark** question and ⅔ marks for a **2-mark** question.
7. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
8. Non-programmable type Calculator is allowed

**CAREER ENDEAVOUR**
ACADEMY PRIVATE LIMITED

**South Delhi Centre:**
28-A/11, Jia Sarai, Near-IIT, Hauz Khas, New Delhi-16
T : 011-26851008, 26861009, (0) 9873744231, 9899628311

**North Delhi Centre:**
48, First Floor, Mall Road, G.T.B. Nagar (Metro Gate No.3), Delhi-09
T : 011-65462244, 27241845, (0) 9899628311, 9540292991
E: info@careerendeavour.com W: careerendeavour.com

## Some Useful Constants

| | |
|---|---|
| Speed of light in free space | $c = 3 \times 10^{8}\ m/s$ |
| Boltzmann constant | $k_B = 1.380 \times 10^{-23}\ J/K$ |
| Planck's constant | $h = 6.626 \times 10^{-34}\ J.s$ |
| Electron charge | $e = 1.602 \times 10^{-19}\ C$ |
| Permittivity of free space | $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/N.m^2$ |
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7}\ H/m$ |
| Mass of electron | $m_e = 9.31 \times 10^{-31}\ kg$ |
| Mass of proton | $m_p = 1.67 \times 10^{-27}\ kg$ |
| Mass of neutron | $m_n = 1.675 \times 10^{-27}\ kg$ |

**Q.1-Q. 5 carry ONE mark each.**

1. Choose the most appropriate alternative from the options given below to complete the following sentence:
It is hard to __________ why people kept their weapons in the kitchen as revealed from the latest excavations.
(a) Decipher (b) Divulge (c) Deluge (d) Amplify

2. Which of the following options is closest in meaning to the word given below?
FAD:
(a) Apathetic (b) Expensive (c) Vogue (d) Benevolent

3. Oct 02, 2001 is a palindrome when written in the formaat of MMDDYYYY (a string that reads the same forwards as it does backwards, example 10/02/2001 .............. 10022001. When was the latest century before oct.02, 2001 that is also a palindrome.
(a) 13th century (b) 14th century (c) 17th century (d) 20th century

10 02 2001
02 2001

4. The difference between the squares of two consecutive odd integers is always divisible by which of the following numbers.
(a) 6 (b) 8 (c) 12 (d) 16

x+4
2 6 8
4×4
$2^2+2^2$

5. Consider the equation : $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.
(a) 1634 (b) 1737 (c) 3142 (d) 3162

75126 − 3142
3142
3484
7526
3162
4364

**Q.6-Q. 10 carry TWO marks each.**

6. Choose the word from the options that is most nearly same in meaning of **Ostensible.**
(a) Apparent (b) Acclaimed (c) Profound (d) Genuine.

7. Given below is a pair of words. Choose the most appropriate and related alternative from the options given below:
MENDACIOUS: TRUTHFUL
(a) Gelid: Icy (b) Scorching: Hot
(c) Cognisance: Recognition (d) Capricious: Constant

8. The grass domestic product (GDP) in rupees grew at 7% during 2012-2013. For international comparision, the GDP is compared in US dollars (USD) after conversion based on the market exchange rate. During the period 2012-13 the exchange rate for the USD increased from Rs. 50/USD to Rs. 60/USD. India's GDP in USD during the period 2012-13:
(a) increased by 5% (b) decreased by 13%
(c) decreased by 20% (d) decreased by 11%

48 + 48 = 95
45 + 54 + 48 + 40

9. What will come in place of ? mark?

| | | |
|---|---|---|
| 5 | | 8 |
| | 126 | |
| 9 | | 6 |

| | | |
|---|---|---|
| 4 | | 9 |
| | 78 | |
| 6 | | 7 |

| | | |
|---|---|---|
| 4 | | 12 |
| | ? | |
| 11 | | 5 |

24
48 × 48
44+60
40 56
54 30
66 24 + 63
42 87
44 + 60
104

(a) 240 (b) 336 (c) 180 (d) none of these

10. The table below has question wise data on the performance of students, in an examination. The mark for each questions are also listed. There is no negative or partial marking in the exam.

| Q.No. | Marks | Answered correctly | Answered wrongly | Not attempted |
|---|---|---|---|---|
| 1. | 2 | 21 | 17 | 6 |
| 2. | 3 | 15 | 27 | 2 |
| 3. | 2 | 23 | 18 | 3 |

What is the average of the marks obtained by the class in the examination ________________

**Q.11-Q.35 carry one mark each.**

11. Consider a particle is moving in the $X$-$Y$ plane and the co-ordinate of the position of the particle follows the system of differential equations:

$$\frac{dx}{dt}=-y \quad \text{and} \quad \frac{dy}{dt}=x$$

The trajectory of the particle will be:

(a) circle [✓ (b) straight line] (c) hyperbola (d) parabola

$\frac{dy}{dt}=\frac{-y^2}{2}t$ $dy=\frac{-y^2}{2}+dt$ $y=\frac{(y^2)^2}{4}t^2$

$dx=-y\,dt$ $x=\frac{-y^2}{2}t$ $x+y=0$

$dy=x\,dt$ $\frac{2dy}{y^2}=-tdt$ $y=\frac{x^2}{2}t$ $y=12x^2$

$\frac{x^2}{2}+\frac{y^2}{2}=0$ $y=\frac{y^4}{4}$

12. The imaginary part of the value of the integral $\int_0^{1+i}\left(x^2-iy\right)dz$ along the path $y=x^2$ (in the complex plane) is reciprocal to ________________

13. The shortest wavelength produced in an X-ray tube operated at 1 million volts will be of the order of

[✓ (a) $10^{-1}A^0$] (b) $10^{-2}A^0$ (c) $10^{-4}A^0$ (d) $10^{-6}A^0$

14. In a HCl molecule, the energy gap between the two vibrational levels is 0.36 eV. Its zero point energy will be ___________ eV.

$E=(v+1/2)\hbar\omega_0$ $E_g=0.36\,eV$ $\lambda=\frac{12.28}{\sqrt{V}}=\frac{12.28}{.6}=$

15. The intensity of a particular spectral line will be maximumn if

[✓ (a) $\Delta L=-1,\Delta J=0$] (b) $\Delta L=+1,\Delta J=0$ (c) $\Delta L=+1,\Delta J=-1$ (d) $\Delta L=-1,\Delta J=-1$

16. $^{228}$Th emits an alpha particlwe to reduce to $^{224}$Ra. The kinetic energy of alpha particle emitted in the decay

$$^{228}Th\longrightarrow{}^{224}R_a^*+\alpha$$

$$^{224}Th^*\longrightarrow{}^{224}Ra+\gamma\,(217\;keV)$$

is ________________ MeV. Atomic mass of $^{228}$Th is 228.028726u, that of $^{224}$Ra is 224.020196u and that of $_2^4He$ is 4.00260u.

17. The count rate of nuclear radiation coming from a radioactive sample containing $^{128}$I varies with time as follows

| Time t(minute) : | 0 | 25 | 50 | 75 | 100 |
|---|---|---|---|---|---|
| Count rate R($\times 10^9$ s$^{-1}$) : | 30 | 16 | 8.0 | 3.8 | 2.0 |

$\frac{250}{5}=50$

The mean life is

(a) 25 min (b) 36 min [✓ (c) 50 min] (d) 30 min

18. A uranium reactor develops thermal energy at a rate of 300 MW. Average energy released per fission is 200 MeV, the amount of $^{235}$U being consumed every second is

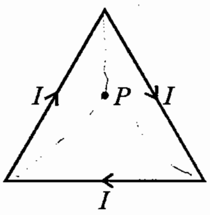
(a) 74 g (b) 37g (c) 7.4 g (d) 3.7 g

19. A uniform circular disc of mass 1 kg and radius 1 meter is rotating with angular speed 2 rad/sec about axis through its center and perpendicular to its plane. Angular momentum of disc about a parallel axis through a point on its periphery is______ kgm²/sec.

20. Polar equation of orbit of a particle moving under a central force is given to be $r\theta$ = constant. If force on the particle varies with $r$ as $F\,\alpha\,r^n$ then value of $n$ is

(a) 3 [✓ (b) – 4] (c) – 3 (d) – 5

21. A current I flows in a triangular loop of each side $a$. Magnetic field at centroide of the triangle is



$\frac{\sqrt{3}}{2}a=\frac{a}{2}$

$\frac{\sqrt{3}}{2}\theta=\frac{a}{2}$

$\frac{B2\mu_0 I}{2\pi r}$

$\frac{3\sqrt{3}\mu_0 I}{2\pi a}$

$\frac{2\sqrt{3}}{a}$

(a) $\frac{3\mu_0 I}{2\pi a}$ (b) $\frac{3\mu_0 I}{\pi a}$ [✓ (c) $\frac{3\sqrt{3}\mu_0 I}{2\pi a}$] (d) $\frac{9\mu_0 I}{2\pi a}$

22. Two capacitors of $2\mu F$ and $4\mu F$ are connected in parallel across a 300 volts potential difference. Total energy stored in the system is_______Joule.

$\frac{c_1 c_2}{c_1+c_2} = \frac{8}{6}\left(\frac{4}{3}\right)^2 \times 2$ $\frac{1}{2}c'$

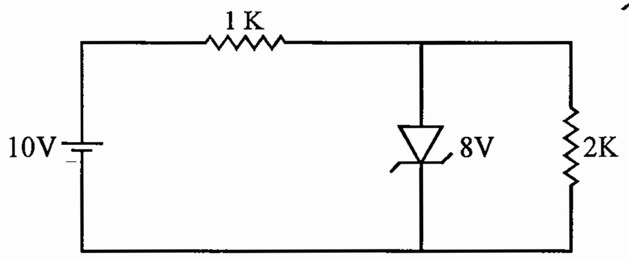
$\frac{16 \times 300}{9}$

$c = \frac{1}{c_1} +$

23. If electrostatic potential in a charge free region be $V(x,y,z) = 3x^2 + \beta y^2 + 4z^2$ then value of $\beta$ is

(a) – 1 (b) – 5 [✓ (c) – 7] (d) 2

24. For the given zener diode circuit



Assume forward drop across zener diode is 0.7 V. The current across 1 kΩ resistors:

[✓ (a) 9.3 mA] (b) 9.7 mA (c) 2 mA (d) 2.6 mA

$-10V + I_1 1k + 0.7V = 0$

$-10V + (I_1 + I_2)2 + 8.7 = 0$

$I_1 = 1.3$

$2I_1 + 2I_2 + 1.3 = 0$

$-10V + I 1k + 8.7 = 0$

$I\,1k = 10 - 8.7$

$I = 1.3$

$-10V + I1k + 0.7 = 9.3$

$I = 9.3\ mA$

$6x + 2\beta y + 8$

$2\beta y = 6x -$

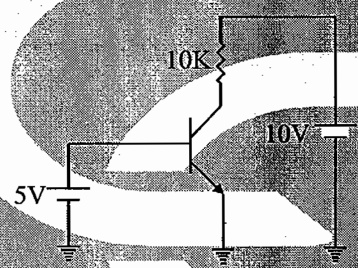
$\beta = \frac{6x}{2y} -$

$6 + 2\beta + 8 = 0$

$2\beta + 14 = 0$

$\beta = \frac{-14}{2}$

25. For the given transistor amplifier $h_{fe} = 100$, $h_{ie} = 10k$



The current gain is______________

26. The angle between [001] and [111] directions in cubic crystal is___$1/\sqrt{3}$___(in units of degree).

$\frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2}\sqrt{h_2^2 + k_2^2 + l_2^2}}$

$\Rightarrow \frac{1}{\sqrt{1}\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = \cos^{-1}(1/\sqrt{3})$

$15^\circ$

27. Consider the free electron gas comprising of N molecules in one dimension, confined in a region of length L at zero temperature. The Fermi-Energy is

(a) $\frac{h^2}{2m}\left(\frac{3N}{8\pi L}\right)^{2/3}$ [✓ (b) $\frac{h^2}{2m}\left(\frac{N}{2L}\right)^{2/3}$] (c) $\frac{h^2}{2m}\left(\frac{N}{2L}\right)$ (d) $\frac{h^2}{2m}\left(\frac{N}{2L}\right)^2$

28. For pure metals the electrical conductivity and thermal conductivity vary with temperature respectively as:

(a) $\sigma \propto T^{-1}$ and $\sigma_T$ = Constant for $T < \theta_D$ (b) $\sigma \propto T^{-5}$ and $\sigma_T \propto T^{-2}$ for $T > \theta_D$

(c) $\sigma \propto T^{-1}$ and $\sigma_T \propto T^{-2}$ for $T > \theta_D$ (d) $\sigma \propto T^{-5}$ and $\sigma_T \propto T^{-2}$ for $T < \theta_D$

29. A second order phase transition is one in which

(a) the plot of entropy as a function of temperature shows a discontinuity

(b) the plot of specific heat as a function of temperature shows a discontinuity

(c) the plot of volume as a function of pressure shows a discontinuity

[✓ (d) the plot of comprehensibility as a function of temperature is continuous.]

30. Unit mass of a liquid of volume $V_1$ completely turns into a gas of volume $V_2$ at constant atmospheric pressure $P_0$ and temperature T. The latent heat of vaporization is L. Then the change in internal energy of the gas is

(a) $L$ (b) $L + P(V_2 - V_1)$ [✓ (c) $L - P(V_2 - V_1)$] (d) zero

31. An ideal monoatomic gas at 300K expands adiabatically to twice its volume. The final temperature is __________________K.

32. Electrons emitted from a hot filament are accelarated through a voltage drop of 100 V. The wavelength of the electron is (in the units of $A^0$) __________

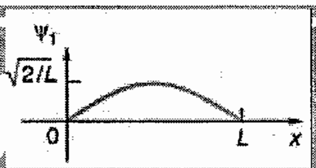
33. Consider a system whose state is given in terms of an orthonormal set of three vectors $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ as

$$|\psi\rangle = \frac{\sqrt{3}}{3}|\phi_1\rangle + \frac{2}{3}|\phi_2\rangle + \frac{\sqrt{2}}{3}|\phi_3\rangle$$

Now, consider an ensemble of 900 identical systems, each one of them in the state $|\psi\rangle$. If measurement is done on all of them, the number of systems which will be found in the state $|\phi_3\rangle$.

(a) 100 [✓ (b) 200] (c) 300 (d) 400

34. The lowest energy state for a particle in a box looks like this. How many electrons can have a wave-function that looks like this?



(a) 0 [✓ (b) 1] (c) 2 (d) Any number

35. A spin - 1/2 particle is in a state described by the $A\begin{pmatrix}1+i\\2\end{pmatrix}$. Where A is normalization constant. The probability of finding the particle with spin projection $S_z = -\frac{\hbar}{2}$ is:

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

**Q.36-Q.65 carry TWO marks each.**

36. Which of the Fourier co-efficients corresponding to the following function, is zero?

$$f(x) = \begin{cases} 2 \text{ for } -2 \le x \le 0 \\ x \text{ for } 0 \le x \le 2 \end{cases}$$

(a) $a_n$ for odd $n$ (b) $a_n$ for even $n$ (c) $b_n$ for even $n$ (d) $b_n$ for odd $n$

37. The matrix $A = \begin{bmatrix} 1 & p \\ p & 1 \end{bmatrix}$ is transformed to the diagonal form $D = T^{-1}AT$ using similarity transformation, where $T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. The value of $\theta$ for which $D$ will be a diagonal matrix, is (in the units of degree)

__________________

38. Consider the follwoing integral $\oint_c (x+y)\,dx + x^2 dy$

Where '$c$' is the triangle with vertices (0, 0), (2, 0), (2, 4) in the clockwise direction. The value of the integral will be __________

39. The inverse Laplace transform of $f(s)=\frac{3s+7}{s^2-2s-3}$ is

(a) $2e^{3x}-e^{-x}$ (b) $4e^{3x}+e^{-x}$ [✓ (c)] $2e^{3x}+e^{-x}$ (d) $4e^{3x}-e^{-x}$

40. In the ground state electronic configuration of nitrogen $\left({}^{14}_{7}N\right)$ the $L$, $S$ and $J$ quantum numbers are

(a) $L=1, S=\frac{1}{2}, J=\frac{1}{2}$ [✓ (b)] $L=1, S=\frac{1}{2}, J=\frac{3}{2}$

(c) $L=0, S=\frac{1}{2}, J=\frac{1}{2}$ (d) $L=0, S=\frac{3}{2}, J=\frac{3}{2}$

N = 7 ~~P = 8~~

$1s \; 2p^2 \; 2p^3$

N = 14 − 7 = 7

P = 7

41. The total numbers of components observed in the electronic transition ${}^2P \rightarrow {}^2S$ under a strong magnetic field will be

[✓ (a)] 3 (b) 5 (c) 6 (d) 8

42. The separation between the lines in the microwave spectrum of a certain molecule is 20.68 cm$^{-1}$. The $J=14 \rightarrow J=15$ transition occurs at ________ cm$^{-1}$.
(Given the reduced mass of the molecule $1.62\times10^{-27}\,kg$ )

43. A permanently deformed even-even nucleus with $J^P=2^+$ has rotational energy 120 keV. The energy of third excited state is ________ MeV.

44. A nucleus undergoes the transition $\left(\frac{3}{2}\right)^+$ to $\left(\frac{5}{2}\right)^-$. The radiation emitted would be

(a) $M_1, E_2, M_3, E_4$ (b) $M_2, E_3, M_4, E_5$ (c) $E_1, M_2, E_3, M_4$ (d) $E_2, M_3, E_4, M_5$

45. The quark content of $\Xi^0$ and $\Delta^{++}$ are
(a) $\bar{u}\,\bar{s}\,\bar{s}$ and $ddd$ respectively (b) $uus$ and $\bar{u}\,\bar{u}\,\bar{u}$ respectively
(c) $dss$ and $uuu$ respectively [✓ (d)] $uss$ and $uuu$ respectively.

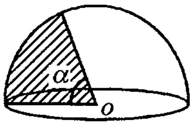
46. A uniform plate of side 1 cm lies in $x$-$y$ plane. Diagonals of the plate are along the straight lines $y=x+1$ and $y=-x+3$. If moment of inertia of the plate about $z$-axis be 62 gm cm$^2$ then mass of square plate is ______ gm.

47. Mass density of a circular disc of radius $R$ and mass $M$ is proportional to $n^{\text{th}}$ power of distance from center of disc. If moment of inertia of disc about its axis passing through center and perpendicular to its plane lie $\frac{5}{7}MR^2$ then value of $n$ is

(a) 1 (b) 2 (c) 3 `(d) 4

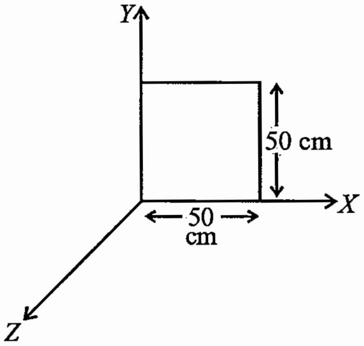
48. A elementary particle of rest mass 1180 $MeV/c^2$ decays into two identical particles each of mass 245 $MeV/c^2$. The momentum of each identical particle is
(a) 1154.29 $MeV/c$ (b) 981.45 $MeV/c$ (c) 765.23 $MeV/c$ (d) 652.17 $MeV/c$

49. Electric field at the center $o$ of a uniformly charged hemisphere is $E_0$. Electric field at $o$ due to shaded portion of sphere shown in the figure is



(a) $E_0\cos\frac{\alpha}{2}$ [✓ (b)] $E_0\sin\frac{\alpha}{2}$ (c) $\frac{E_0}{2}(1-\cos\alpha)$ (d) $\frac{E_0(1+\cos\alpha)}{2}$

50. A plane electromagnetic wave $\vec{E} = E_0\hat{j}\cos(wt - kx)$ propagating in space. If $E_0 = 5\times10^{-4}\,V/m$ and $w = 10\pi\times10^6$ rad/sec amplitude of e.m.f. induced in the loop shown in figure is_______ volt.



51. A sphere of radius 'R' has uniform volume charge density. Electric field at the surface of sphere is $E_s$. Electrical potential at the centre of sphere will be

(a) $\frac{E_sR}{2}$ (b) $-\frac{E_sR}{2}$ (c) $\frac{3}{2}E_sR$ (d) $-\frac{3}{2}E_sR$

$V = \frac{4}{3}\pi R^3$

52. A particle of mass m and charge $q$ is given an initial velocity $v_0\hat{i}$ at (0, 0, 0). There exists a uniform electric field in space along $y$-direction. If slope of the equation of path of particle at $(a, a, 0)$ be 2. Magnitude of electric field must be

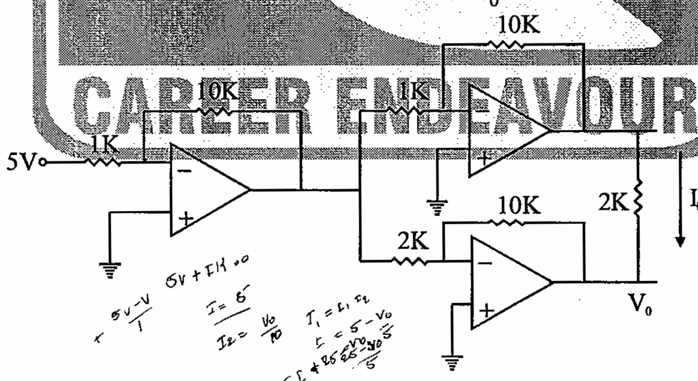
(a) $\frac{mv_0^2}{qa}$ (b) $\frac{2mv_0^2}{qa}$ (c) $\frac{mv_0^2}{2qa}$ (d) $\frac{4mv_0^2}{qa}$

53. For a 5 bit DAC (weighted resistor) maximum value of resistance used if the resistance of MSB is 1kΩ

(a) 8kΩ (b) 4kΩ (c) 2kΩ (d) 16 kΩ

$2^5$ 32

54. For the given OP-AMP circuit shown. Calculate the current $I_0$.



$5V + IK = 0$

$\frac{5V - V}{1}$

$I = \frac{5}{}$

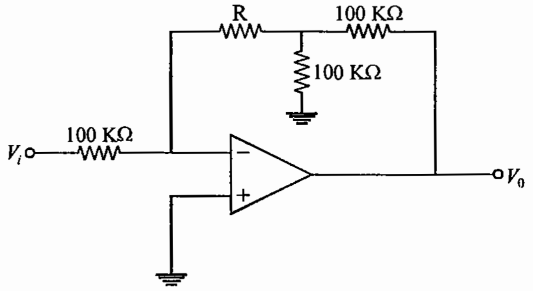
$I_2 = \frac{V_0}{10}$

$I_1 = I_1 + I_2$

$I = 5 - \frac{V_0}{5}$

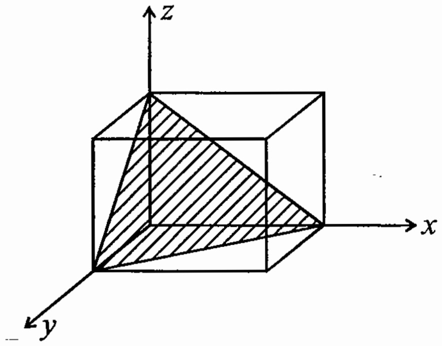
(a) 125 mA (b) 12.5 mA (c) 1.25 mA (d) None of these

55. For the circuit shown below gain is $A_V = \frac{V_0}{V_i} = -10$. The value of $R$ is (in KΩ)______



56. A powder pattern is obtained for an crystal with lattice parameter 3.52 A° by using x-rays of wave length 1.79 A°. The highest reflection possible is ________

57. Planar density in SC on (111) plane is



(a) $\frac{0.707}{a^2}$ (b) $\frac{1}{a^2}$ (c) $\frac{0.577}{a^2}$ (d) $\frac{0.288}{a^2}$

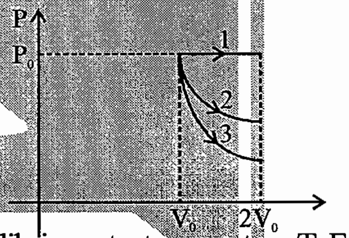
58. The band gap in a BCS superconductor is measured to be 3 meV at 0K.
The critical temperatuer ($T_C$) of the superconductor is:

(a) 5K (b) 10K (c) 2K (d) 15K

59. A gas is expanded from volume $V_0$ to $2V_0$ under three different processes.

Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic.

Let $\Delta U_1, \Delta U_2$ and $\Delta U_3$ be the change in internal energy of the gas in these three processes. Then



(a) $\Delta U_1 > \Delta U_2 > \Delta U_3$ (b) $\Delta U_2 < \Delta U_1 < \Delta U_3$

(c) $\Delta U_1 < \Delta U_2 < \Delta U_3$ (d) $\Delta U_2 < \Delta U_3 < \Delta U_1$

60. Consider a collection of N two level systems in thermal equilibrium at a temperature T. Each system has only two states 0 and an excited state of energy $\in$. Probability that a system will be found in excited state and entropy respectively at very high temperature, vary as

(a) $P \to 0, S \to 0$ (b) $P \to \infty, S \to \infty$

(c) $P \to 1/2, S \to N k\ell n\, 3$ (d) $P \to 1/2, S \to N k\ell n\, 2$

61. Water would boil at 120°C under pressure ____________ atm. (One gm of steam occupies a volume of 1677 $cm^3$. Latent heat of steam = 540 cal/g, J = $4.2 \times 10^7$ erg/cal, 1 atmosphere pressure = $1.0 \times 10^6$ dyne/$cm^2$ ).

62. A system consists of N very weakly interacting particles at a temperature sufficiently high such that classical statistics are applicable. Each particle has mass m and oscillates in one direction about its equilibrium position. The heat capacity at temperature T, if restoring force is proportional to $x^3$.

(a) $\frac{3}{2}Nk$ (b) $\frac{5}{2}Nk$ (c) Nk (d) $\frac{3}{4}Nk$

63. The de-Broglie wavelength of a particle having a mass 900 $MeV/c^2$ and kinetic energy 2 $MeV$, is equal to ____________ $\times 10^{-14} m$.

64. A particle of mass m moves (non-relativistically) in three-dimensional potential

$$V=\frac{1}{2}k\left(x^2+y^2+z^2+\lambda xy\right)$$

Where $\lambda$ is a small parameter.

The energy of first excited energy level corrected to first order is/are

(a) $\left(\frac{3}{2}-\frac{\lambda}{4}\right)\hbar\omega, \frac{3}{2}\hbar\omega, \left(\frac{3}{2}+\frac{\lambda}{4}\right)\hbar\omega$ (b) $\frac{3}{2}\hbar\omega$

(c) $\frac{5}{2}\hbar\omega$ (d) $\left(\frac{5}{2}-\frac{\lambda}{4}\right)\hbar\omega, \frac{5}{2}\hbar\omega, \left(\frac{5}{2}+\frac{\lambda}{4}\right)\hbar\omega$

65. A quantum-mechanical system is known to possess only two energy eigenstates denoted $|1\rangle$ and $|2\rangle$. The system has an observable P. The states $|1\rangle$ and $|2\rangle$ are normalized but they are not necessarily eigenstates of P. Given that $\langle 1|P|1\rangle=\frac{1}{2}, \langle 1|P^2|1\rangle=\frac{1}{4}$. The value of matrix element $\langle 1|P|2\rangle$ is:

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{2}}$